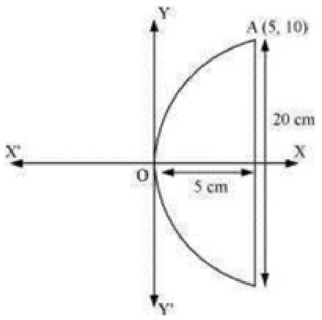
CBSE NCERT Solutions for Class 11 mathematics Chapter 11

Miscellaneous exercise on chapter 11

Q.1. If a parabolic reflector is 20 cm in diameter and 5 cm deep, find the focus.

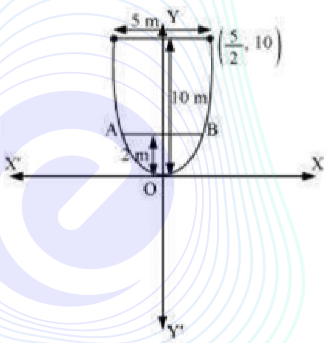
Solution: The origin of the coordinate plane is taken at the vertex of the parabolic reflector in such a way that the axis of the reflector is along the positive x-axis. This can be diagrammatically represented as



The equation of the parabola is of the form y2=4ax (as it is opening to the right). Since the parabola passes through the point A10,5 ⇒ 102=4a5
⇒100=20a
⇒a=10020=5
Therefore, the focus of the parabola is a,0=5,0, which is the mid-point of the diameter.
Hence, the focus of the reflector is at the mid-point of the diameter.

Q.2. An arch is in the form of a parabola with its axis vertical. The arch is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex of the parabola?

Solution: The origin of the coordinate plane is taken at the vertex of the arch in such a way that its vertical axis is along the positive y-axis.
This can be diagrammatically represented as,

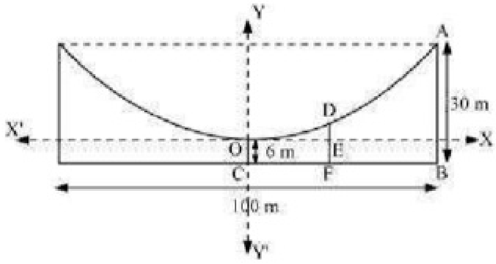


The equation of the parabola is of the form x2=4ay (as it is opening upwards).
Here the parabola is symmetrical about y-axis and it can be clearly seen that the parabola passes through the point 52, 10.
So, 522=4a10
⇒a=254×4×10=532
Therefore, the arch is in the form of a parabola whose equation is x2=58y
When y=2 m,x2=58×2
⇒x2=54
⇒x=54m.∴AB=2×54m≈2.236 m
So, width is approximately 2.23 m.

Q.3. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

Solution: Given that: The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

The vertex is at the lowest point of the cable. The origin of the coordinate plane is taken as the vertex of the parabola, while its vertical axis is taken along the positive y-axis. This can be diagrammatically represented as



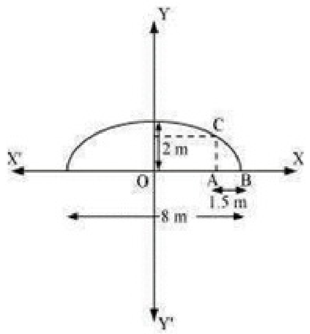
Here, AB and OC are the longest and the shortest wires respectively, attached to the cable.
DF is the supporting wire attached to the roadway, 18 m from the middle.
Here, AB=30 m, OC=6 m, and BC=1002=50 m.
The equation of the parabola is of the form x2=4ay (as it is opening upwards)

The coordinates of point A are 50, 30-6=50, 24
Since A50, 24 ia a point on the parabola,
502=4·a·24⇒a=50×504×24=62524
∴ Equation of the parabola x2=4×62524×y ⇒6x2=625y
The x-coordinate of point D *is* 18
Hence, *at* x=18,
6182=625y ⇒y=6×18×18625⇒y≈3.11
∴DE=3.11 m
DF=DE+EF=3.11 m+6 m=9.11 m
Thus, the length of the supporting wire attached to the roadway 18 m from the middle is approximately 9.11 m

Q.4. An arch is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arch at a point 1.5 m from one end.

Solution: Given that: An arch is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arch at a point 1.5 m from one end.

Since the height and width of the arc from the centre is 2 m and 8 m respectively, it is clear that the length of the major axis is 8 m, while the length of the semi-minor axis is 2 m.
The origin of the coordinate plane is taken as the centre of the ellipse, while the major axis is taken along the x-axis. Hence, the semi-ellipse can be diagrammatically represented as,



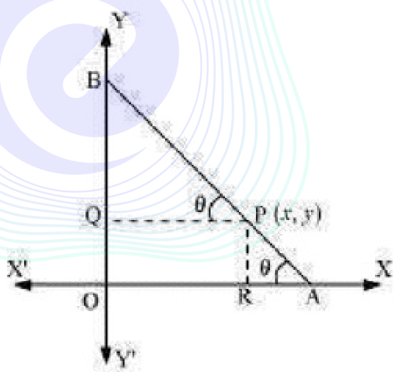
The equation of the semi-ellipse will be of the form x2a2+y2b2=1, y≥0 where a is the semi-major axis.

Accordingly, 2a=8⇒a=4 andb=2.
Therefore, the equation of the semi-ellipse is x216+y24=1, y≥0 …(i)
Let A be a point on the major axis such that AB=1.5 m.
Draw AC perpendicular to OB.
Here, OA=4-1.5 m=2.5 m
The x-coordinate of point C is 2.5.
On substituting the value of x=2.5 in equation (i), we obtain
2.5216+y24=1⇒6.2516+y24=1
⇒y2=41-6.2516⇒y2=49.7516
⇒y2=2.4375 ⇒y≈1.56
∴AC=1.56 m
Thus, the height of the arch at a point 1.5 m from one end is approximately 1.56 m.

Q.5. A rod of length 12 cm moves with its ends always touching the coordinate axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with the x-axis.

Solution: Given that: A rod of length 12 cm moves with its ends always touching the coordinate axes. Determine the equation of the locus of a point P on the rod, which is 3 cm from the end in contact with the x-axis.

Let AB be the rod making an angle θ with OX and Px,y be the point on it such that AP=3 cm.
Then, PB=AB-AP=12-3 cm=9 cm (As AB=12 cm)
From P, draw PQ⊥OY and PR⊥OX.

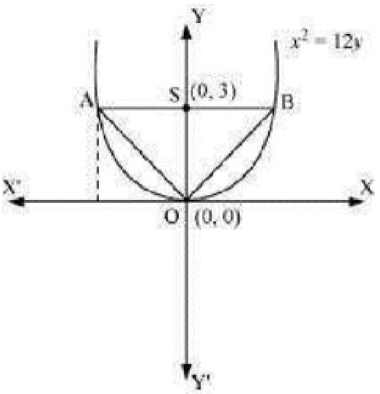


In ΔPBQ, cosθ=PQPB=x9
In ΔPRA, sinθ=PRPA=y3
Since, sin2θ+cos2θ=1
y32+x92=1
or, x281+y29=1
Thus, the equation of the locus of point P on the rod is x281+y29=1

Q.6. Find the area of the triangle formed by the lines joining the vertex of the parabola x2=12y to trends of its latus rectum.

Solution: The given parabola is x2=12y

To find: the area of the triangle formed by joining the vertex and the trends of the latus rectum of the parabola.
On comparing this equation with x2=4ay, we obtain 4a=12⇒a=3
∴The coordinates of foci are S0,a=S0,3
Let AB be the latus rectum of the given parabola.
The given parabola can be roughly drawn as



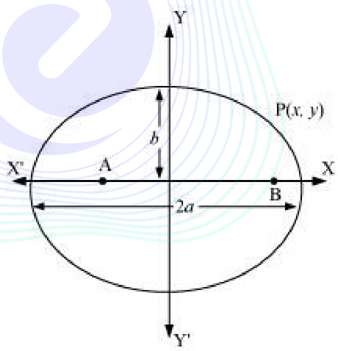
At y=3, x2=123⇒x2=36⇒x=±6
∴ The coordinates of A are -6,3, while the coordinates of B are 6,3
Therefore, the vertices of ΔOAB are O0,0, A-6,3 and B6,3
Area of ΔOAB=1203-3+-63-0+60-3 unit2
=12-63+6-3 unit2
=12-18-18 unit2
=12×36 unit2
=18 unit2
Thus, the required area of the triangle is 18 unit2

Q.7. A man running a racecourse notice that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Find the equation of the path traced by the man.

Solution: Given that: A man running a racecourse notice that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Find the equation of the posts traced by the man.

Let A and B be the positions of the two flag posts and P(x, y) be the position of the man.

Accordingly, PA+PB=10 We know that if distances from two fixed points is constant, then the path is an ellipse and this constant will be the length of the major axis of the ellipses. Therefore, the path described by the man is an ellipse where the length of the major axis is10 m, while points A and B are the foci. Taking the origin of the coordinate plane as the centre of the ellipse, while taking the major axis along the x-axis, the ellipse can be diagrammatically represented as



The equation of the ellipse will be of the form x2a2+y2b2=1, where a is the semi-major axis.

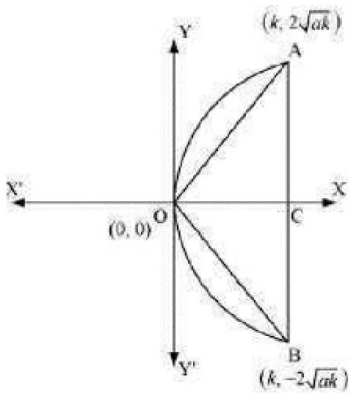
Accordingly, 2a=10⇒a=5

Distance between the foci (2c)=8 ⇒c=4 On using the relation c=a2-b2, we obtain 4=25-b2⇒16=25-b2 ⇒b2=25-16=9⇒b=3 Thus, the equation of the path traced by the man is x225+y29=1

Q.8. An equilateral triangle is inscribed in the parabola y2=4ax, where one vertex is at the vertex of the parabola. Find the length of the side of the triangle.

Solution: Given that: An equilateral triangle is inscribed in the parabola y2=4ax, where one vertex is at the vertex of the parabola. Find the length of the side of the triangle.

Let OAB be the equilateral triangle inscribed in the parabola.
Let AB intersect the x-axis at point C.



Let OC=k, then the coordinates of point C are (k, 0).
So, from the equation of the given parabola, we have y2=4ak⇒y=±2ak
∴ The respective coordinates of points A and B are k, 2ak and k, -2ak
AB=CA+CB=2ak+2ak=4ak

Since OAB is an equilateral triangle, OA=OB⇒OA2=AB2.

Using distance formula we get,

∴(k-0)2+2ak-02=4ak2
⇒k2+2ak2=4ak2
⇒k2+4ak=16ak
⇒k2=12ak
⇒k=12a
∴AB=4ak=4a×12a=412a2=83a
Thus, the side of the equilateral triangle inscribed in parabola y2=4ax is 83a.

Exercise 11.1

Q.1. Find the equation of the circle with centre 0,2 and radius 2.

Solution: We know that, the equation of a circle is given as x-h2+y-k2=r2 where h,k is the centre of the circle and r is the radius of the circle.
Given: centre h,k=0,2 and radius r=2

Therefore, the equation of the given circle can be written as
x-02+y-22=22
⇒x2+y2+4-4y=4
∴ x2+y2-4y=0

Q.2. Find the equation of the circle passing through the points 4,1 and 6,5 and whose centre is on the line 4x+y=16.

Solution: Let the equation of the required circle be x-h2+y-k2=r2.
Since the circle passes through points 4,1 and 6,5, the respective values of x and y will satisfy the equation.
Therefore, 4-h2+1-k2=r2 …(i)
6-h2+5-k2=r2 …(ii)
Since the centre h,k of the circle lies on line 4x+y=16, the values of x and y will satisfy the equation.
4h+k=16 …(iii)
From equations (i) and (ii), we get
4-h2+1-k2=6-h2+5-k2
⇒16-8h+h2+1-2k+k2=36-12h+h2+25-10k+k2
⇒16-8h+1-2k=36-12h+25-10k
⇒4h+8k=44
⇒h+2k=11 …(iv)

On solving equations (iii) and (iv), we get h=3 and k=4.
On substituting the values of h and k in equation (i), we get
4-32+1-42=r2
⇒12+-32=r2
⇒1+9=r2
⇒r2=10
⇒r=10
Thus, the equation of the required circle is
x-32+y-42=102
⇒x2-6x+9+y2-8y+16=10
∴x2+y2-6x-8y+15=0

Q.3. Find the equation of the circle passing through the points 2,3 and -1,1 and whose centre is on the line x-3y-11=0.

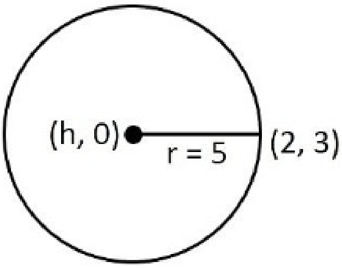
Solution: Let the equation of the required circle be x-h2+y-k2=r2.
Since the circle passes through points 2,3 and -1,1, the points will satisfy the equation of the circle.
Therefore, 2-h2+3-k2=r2 …(i)
-1-h2+1-k2=r2 …(ii)
Since the centre h,k of the circle lies on line x-3y-11=0, the point will satisfy the equation of the line.
Therefore, h-3k=11 …(iii)
From equations (i) and (ii), we get
2-h2+3-k2=-1-h2+1-k2
⇒4-4h+h2+9-6k+k2=1+2h+h2+1-2k+k2
⇒4-4h+9-6k=1+2h+1-2k
⇒6h+4k=11 …(iv)

On solving equations (iii) and (iv), we get h=72 and k=-52.
On substituting the values of h and k in equation (i), we obtain
2-722+3+522=r2
⇒4-722+6+522=r2
⇒-322+1122=r2
⇒94+1214=r2
⇒1304=r2
Thus, the equation of the required circle is
x-722+y+522=1304
⇒2x-722+2y+522=1304
⇒4x2-28x+49+4y2+20y+25=130
⇒4x2+4y2-28x+20y-56=0
⇒4x2+y2-7x+5y-14=0
∴x2+y2-7x+5y-14=0

Q.4. Find the equation of the circle with radius 5, whose centre lies on x-axis and passes through the point (2, 3).

Solution: We know that equation of circle is

(x-h)2+(y-k)2=r2



Centre of circle is denoted by h, k. Since it lies on x-axis, k=0. Hence , Centre of circle =(h, 0) and given radius=5 Now, Distance between centre and point on circle=radius So, Distance between points (h, 0) and (2, 3)=5 We know that the distance between x1, y1 and x2, y2=x2-x12+y2-y12 Therefore, (2-h)2+(3-0)2=5 or (2)2+(h)2-2(2)h+9=5⇒13+h2-4h=5 Squaring both sides we get, (13+h2-4h)2=52⇒13+h2-4h=25 or h2-4h-12=0⇒h2-6h+2h-12=0 or h(h-6)+2(h-6)=0⇒(h+2)(h-6)=0 So, h=-2 or h=6

When h=-2

Equation of circle is:

(x-h)2+(y-k)2=r2⇒x-(-2)2+(y-0)2=52 or (x+2)2+y2=25⇒(x)2+(2)2+2(x)(2)+y2=25 or x2+y2+4x-21=0 When h=6 Equation of circle is: (x-h)2+(y-k)2=r2⇒(x-6)2+(y-0)2=52 or x2+(6)2-2(x)(6)+y2=25⇒x2+y2-12x+36-25=0 or x2+y2-12x+11=0 Hence, the required equation of a circle is x2+y2+4x-21=0 or x2+y2-12x+11=0

Q.5. Find the equation of the circle passing through 0,0 and making intercepts a and b on the coordinate axes.

Solution: Let the equation of the required circle be x-h2+y-k2=r2.
Since the centre of the circle passes through (0, 0), the point will satisfy the equation.
0-h2+0-k2=r2
⇒h2+k2=r2
The equation of the circle now becomes x-h2+y-k2=h2+k2.
It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points a,0 and 0,b.
Therefore,a-h2+0-k2=h2+k2 …(i)
0-h2+b-k2=h2+k2 …(ii)
From equation (i), we obtain a2-2ah+h2+k2=h2+k2
⇒a2-2ah=0
⇒aa-2h=0
⇒a=0 or a-2h=0
However, a≠0; hence, a-2h=0⇒h=a2

From equation (ii), we obtain h2+b2-2bk+k2=h2+k2
⇒b2-2bk=0
⇒bb-2k=0
⇒b=0 or b-2k=0
However, b≠0; hence, b-2k=0⇒k=b2.
Thus, the equation of the required circle is
x-a22+y-b22=a22+b22
⇒2x-a22+2y-b22=a2+b24
⇒4x2-4ax+a2+4y2-4by+b2=a2+b2
⇒4x2+4y2-4ax-4by=0
⇒x2+y2-ax-by=0

Q.6. Find the equation of a circle with centre 2,2 and passes through the point 4,5.

Solution: The centre of the circle is given as h,k=2,2.
Since the circle passes through point 4,5, the radius r of the circle is the distance between the points 2,2 and 4,5.
$\therefore$ r=2-42+2-52=-22+-32=4+9=13

$\because$Distance between (a, b) and (x, y) is (x-a)2+(y-b)2

Thus, the equation of the circle is
x-h2+y-k2=r2
$\Rightarrow$x-22+y-22=132
$\Rightarrow$x2-4x+4+y2-4y+4=13
$\therefore$ x2+y2-4x-4y-5=0

Q.7. Does the point -2.5, 3.5 lie inside, outside or on the circle x2+y2=25?

Solution: The equation of the given circle is x2+y2=25.
x2+y2=25
$\Rightarrow$x-02+y-02=55,
which is of the form x-h2+y-k2=r2, where h=0,k=0 and r=5.
$\therefore$ Centre =0,0 and radius =5
Now, the distance between point -2.5, 3.5 and centre 0, 0.
=-2.5-02+3.5-02
$\because$Distance between (a, b) and (x, y) is (x-a)2+(y-b)2
=6.25+12.25
=18.5
$\approx$4.3<5
Since the distance between point -2.5, 3.5 and centre 0, 0 of the circle is less than the radius of the circle, point -2.5, 3.5 is lies inside the circle.

Q.8. Find the equation of the circle with centre -2, 3 and radius 4.

Solution: The equation of a circle is given as
x-h2+y-k2=r2

Where (h, k) is the centre of the circle and r is the radius of the circle.

Given: centre h,k=-2,3 and radius r=4. Therefore, the equation of the circle is x+22+y-32=42
$\Rightarrow$x2+4x+4+y2-6y+9=16
$\therefore$ x2+y2+4x-6y-3=0

Q.9. Find the equation of the circle with centre 12,14 and radius 112.

Solution: The equation of a circle with centre h,k and radius r is given as x-h2+y-k2=r2

Where, (h, k) is the centre of the circle and r is the radius of the circle.
Given that: Centre h,k=12,14 and radius r=112
Now, the equation of the circle can be written as
x-122+y-142=1122
x2-x+14+y2-y2+116=1144
x2-x+14+y2-y2+116-1144=0
144x2-144x+36+144y2-72y+9-1=0

144x2-144x+144y2-72y+44=0 36x2-36x+36y2-18y+11=0 36x2+36y2-36x-18y+11=0

Q.10. Find the equation of the circle with centre 1,1 and radius 2.

Solution: The equation of a circle with centre h,k and radius r is given as
x-h2+y-k2=r2

Where, (h, k) is the centre of the circle and r is the radius of the circle.
Given that: Centre h,k=1,1 and radius r=2.
Now, the equation of the circle can be written as
x-12+y-12=22
x2-2x+1+y2-2y+1=2
x2+y2-2x-2y=0

Q.11. Find the equation of the circle with centre -a,-b and radius a2-b2.

Solution: We know that, the equation of a circle is given as x-h2+y-k2=r2 where h,k is the centre of the circle and r is the radius of the circle.

Given that: Centre h,k=-a,-b and radius r=a2-b2.
Therefore, the equation of the circle is
x+a2+y+b2=a2-b22
x2+2ax+a2+y2+2by+b2=a2-b2
x2+y2+2ax+2by+2b2=0

Q.12. Find the centre and radius of the circle x+52+y-32=36.

Solution: We know that, the equation of a circle is given as x-h2+y-k2=r2 where h,k is the centre of the circle and r is the radius of the circle.

The equation of the given circle is x+52+y-32=36.
We can write x+52+y-32=36 as,
⇒x--52+y-32=62,
which is of the form x-h2+y-k2=r2,

Here h=-5,k=3 and r=6.
Thus, the centre of the given circle is -5, 3, while its radius is 6.

Q.13. Find the centre and radius of the circle x2+y2-4x-8y-45=0

Solution: We know that, the equation of a circle is given as x-h2+y-k2=r2 where h,k is the centre of the circle and r is the radius of the circle.

The equation of the given circle is x2+y2-4x-8y-45=0.
It can also be written as x2-4x+y2-8y=45

Using completing the square method we get,
x2-2x2+22+y2-2y4+42-4-16=45
⇒x-22+y-42=65
⇒x-22+y-42=652,
which is of the form x-h2+y-k2=r2, where h=2,k=4 and r=65,
Thus, the centre of the given circle is 2, 4, while its radius is 65.

Q.14. Find the centre and radius of the circle x2+y2-8x+10y-12=0

Solution: The equation of the given circle is x2+y2-8x+10y-12=0

x2+y2-8x+10y-12=0

⇒x2-8x+y2+10y=12 ⇒x2-2(x)(4)+42+y2+2(y)(5)+52-16-25=12 ⇒(x-4)2+(y+5)2=53
⇒(x-4)2+{y-(-5)}2=(53)2, which is of the form (x-h)2+(y-k)2=r2, where h=4, k=-5, and r=53 Thus, the centre of the given circle is (4,-5), while its radius is 53

Q.15. Find the centre and radius of the circle 2x2+2y2-x=0

Solution: The equation of the given circle is 2x2+2y2-x=0.

General for of the equation of a circle of radius r and center (h,k) is given by,

(x-h)2+(y-k)2=r2 So to know the center and radius of the circle, we need to convert the given equation to the standard form. Therefore, 2x2+2y2-x=0
⇒2x2-x+2y2=0
⇒2x2-x2+y2=0
⇒x2-2x14+142+y2-142=0
⇒x-142+(y-0)2=142
Comparing the above equation with standard form we get, h=14, k=0 and r=14. Thus, the centre of the given circle is 14,0, while its radius is 14.

Exercise 11.2

Q.1. Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for y2=12x

Solution: The given equation is y2=12x

Here, the coefficient of x is positive. Hence, the parabola opens towards the right.

On comparing this equation with y2=4ax, we obtain
4a=12⇒a=3 ∴ Coordinates of the focus =(a, 0)=(3, 0) Since the given equation involves y2, the axis of the parabola is the x-axis. Equation of directrix, x=-a i.e. x=-3 or x+3=0 Length of latus rectum=4a=4×3=12

Q.2. Find the equation of the parabola that satisfies the given conditions: Vertex 0, 0 focus -2, 0

Solution: Vertex 0, 0 focus -2, 0

Since the vertex of the parabola is 0, 0 and the focus lies on the negative x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form y2=-4ax.

Since the focus is -2, 0=(-a, 0)⇒a=2 Thus, the equation of the parabola is y2=-4×2x i.e. y2=-8x

Q.3. Find the equation of the parabola that satisfies the given conditions: Vertex 0, 0 passing through 2, 3 and axis is along x-axis.

Solution: Since the vertex is 0, 0 and the axis of the parabola is the x-axis, the equation of the parabola is either of the form y2=4ax or y2=-4ax.

The parabola passes through point 2, 3, which lies in the first quadrant.

Therefore, the equation of the parabola is of the form y2=4ax, while point 2, 3 must satisfy the equation y2=4ax. ∴32=4a2⇒a=98 Thus, the equation of the parabola is y2=498x
y2=92x
2y2=9x Therefore, the equation of the parabola is 2y2=9x.

Q.4. Find the equation of the parabola that satisfies the given conditions: Vertex 0, 0, passing through 5, 2 and symmetric with respect to y-axis

Solution: Since the vertex is 0, 0 and the parabola is symmetric about the y-axis, the equation of the parabola is either of the form x2=4ay or x2=-4ay.

The parabola passes through point 5, 2 which lies in the first quadrant.

Therefore, the equation of the parabola is of the form x2=4ay, while point 5,2 must satisfy the equation x2=4ay. ∴52=4×a×2⇒25=8a⇒a=258 Thus, the equation of the parabola is
x2=4258y
2x2=25y Therefore, the equation of the parabola is 2x2=25y.

Q.5. Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for x2=6y

Solution: The given equation is x2=6y

Here, the coefficient of y is positive.

Hence, the parabola opens upwards. On comparing this equation with x2=4ay, we obtain 4a=6⇒a=32 ∴ Coordinates of the focus =(0,a)=0,32 Since the given equation involves x2, the axis of the parabola is the y-axis. Equation of directrix, y=-a i.e. y=-32 Length of latus rectum=4a=6

Q.6. Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for y2=-8x

Solution: The given equation is $y2=-8x$.

Here, the coefficient of x is negative. Hence, the parabola open towards the left.

On comparing this equation with $y2=-4ax$ we obtain
$-4a=-8\Rightarrow a=2$ $\therefore$ Coordinates of the focus $=-a,\ 0=-2,\ 0$ Since the given equation involves y2, the axis of the parabola is the x-axis. Equation of directrix, $x=a$ i.e., $x=2$ Length of latus rectum $=4a=8$

Q.7. Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x2=-16y$

Solution: The given equation is $x2=-16y$.

Here, the coefficient of y is negative. Hence, the parabola opens downwards.

On comparing this equation with $x2=-4ay$, we obtain,
$-4a=-16\Rightarrow a=4$ $\therefore$Coordinates of the focus $=0,\ -a=0,\ -4$ Since the given equation involves x2, the axis of the parabola is the y-axis. Equation of directrix, $y=a$ i.e. $y=4$ Length of latus rectum $=4a=16$

Q.8. Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y2=10x$

Solution: The given equation is $y2=10x$.

Here, the coefficient of x is positive.

Hence, the parabola opens towards the right. On comparing this equation with $y2=4ax$, we obtain,
$4a=10\Rightarrow a=52$ $\therefore$Coordinates of the focus $=a,\ 0=52,\ 0$ Since the given equation involves y2,the axis of the parabola is the x-axis. Equation of directrix, $x=-a$ i.e. $x=-52$ Length of latus rectum $=4a=10$

Q.9. Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x2=-9y$

Solution: The given equation is $x2=-9y$.
Here, the coefficient of y is negative. Hence, the parabola opens downwards.
On comparing this equation with $x2=-4ay$, we obtain
$-4a=-9\Rightarrow a=94$
$\therefore$ Coordinates of the focus $=0,-a=0,-94$
Since the given equation involves x2, the axis of the parabola is the y-axis.
Equation of directrix, $y=a$ i.e. $y=94$
Length of latus rectum $=4a=9$

Q.10. Find the equation of the parabola that satisfies the given conditions: Focus 6, 0; directrix $x=-6$

Solution: Focus 6, 0; directrix, $x=-6$

Since the focus lies on the x-axis, the x-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $y2=4ax$ or $y2=-4ax$. It is also seen that the directrix, $x=-6$ is to the left of the y-axis, while the focus 6, 0 is to the right of the y-axis. Hence, the parabola is of the form $y2=4ax$. Here, $a=6$ Thus, the equation of the parabola is $y2=24x$.

Q.11. Find the equation of the parabola that satisfies the given conditions: Focus 0, -3; directrix $y=3$

Solution: Focus $=0,\ -3$; directrix $y=3$

Since the focus lies on the y-axis, the y-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $x2=4ay$ or $x2=-4ay$. It is also seen that the directrix, $y=3$ is above the x-axis, while the focus 0, -3 is below the x-axis. Hence, the equation of the parabola is of the form $x2=-4ay$. Here, $a=3$ Thus, the equation of the parabola is $x2=-12y$.

Q.12. Find the equation of the parabola that satisfies the given conditions: Vertex 0, 0; focus 3, 0

Solution: Vertex $0, 0$; focus $3, 0$

Since the vertex of the parabola is $0, 0$ and the focus lies on the positive x-axis, x-axis is the

axis of the parabola, while the equation of the parabola is of the form $y2=4ax$. Since the focus is $3, 0=(a, 0) \Rightarrow a=3$ Thus, the equation of the parabola is $y2=4ax$ i.e. $y2=4\times3x=12x$

Exercise 11.3

Q.1. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse x236+y216=1

Solution: The given equation is x236+y216=1.
Here, the denominator of x236 is greater than the denominator of y216.
Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.
On comparing the given equation with x2a2+y2b2=1 we obtain, a=6 and b=4.

∴c=a2-b2=36-16=20=25

Therefore, the coordinates of the foci are 25, 0 and -25, 0. The coordinates of the vertices are 6, 0 and -6, 0. Length of major axis =2a=12 Length of minor axis =2b=8 Eccentricity, e=ca=256=53 Length of latus rectum =2b2a=2×166=163

Q.2. Find the equation for the ellipse that satisfy the given conditions: Vertices ±5, 0, foci ±4, 0.

Solution: Vertices ±5, 0, foci ±4, 0
Here, the vertices are on the x-axis.
Therefore, the equation of the ellipse will be of the form x2a2+y2b2=1, where a is the semi-major axis and b is the semi-minor axis.
Accordingly, a=5 and c=4.
It is known that a2=b2+c2
∴52=b2+42
⇒25=b2+16
⇒b2=25-16
⇒b=9=3
Thus, the equation of the ellipse is x252+y232=1 or x225+y29=1.

Q.3. Find the equation for the ellipse that satisfy the given conditions: Vertices 0, ±13, foci 0, ±5.

Solution: Vertices 0, ±13, foci 0, ±5
Here, the vertices are on the y-axis.
Therefore, the equation of the ellipse will be of the form x2b2+y2a2=1, where a is the semi-major axis.
Accordingly, a=13 and c=5.
It is known that a2=b2+c2
∴132=b2+52
⇒169=b2+25
⇒b2=169-25
⇒b=144=12
Thus, the equation of the ellipse is x2122+y2132=1 or x2144+y2169=1.

Q.4. Find the equation for the ellipse that satisfy the given conditions: Vertices ±6, 0, foci ±4, 0.

Solution: Vertices ±6, 0, foci ±4, 0
Here, the vertices are on the x-axis.
Therefore, the equation of the ellipse will be of the form x2a2+y2b2=1, where a is the semi-major axis.
Accordingly, a=6,c=4.
It is known that a2=b2+c2
∴62=b2+42
⇒36=b2+16
⇒b2=36-16
⇒b=20
Thus, the equation of the ellipse is x262+y2202=1 or x236+y220=1

Q.5. Find the equation for the ellipse that satisfy the given conditions: Ends of major axis ±3, 0, ends of minor axis 0, ±2.

Solution: Ends of major axis ±3, 0, ends of minor axis 0, ±2 Here, the major axis is along the x-axis.
Therefore, the equation of the ellipse will be of the form x2a2+y2b2=1, where a is the semi-major axis.
Accordingly, a=3 and b=2.
Thus, the equation of the ellipse is x232+y222=1 i.e., x29+y24=1.

Q.6. Find the equation for the ellipse that satisfy the given conditions: Ends of major axis 0, ±5, ends of minor axis ±1, 0.

Solution: Ends of major axis 0, ±5, ends of minor axis ±1, 0.
Here, the major axis is along the y-axis. Therefore, the equation of the ellipse will be of the form x2b2+y2a2=1, where a is the semi-major axis.
Accordingly, a=5 and b=1.
Thus, the equation of the ellipse is x212+y252=1 or x21+y25=1.

Q.7. Find the equation for the ellipse that satisfy the given conditions: Length of major axis 26, foci ±5, 0.

Solution: Length of major axis =26; foci =±5, 0.
Therefore, the equation of the ellipse will be of the form x2a2+y2b2=1, where a is the semi-major axis.
Accordingly, 2a=26⇒a=13 and c=5.
It is known that a2=b2+c2
∴132=b2+52
⇒169=b2+25
⇒b2=169-25
⇒b=144=12
Thus, the equation of the ellipse is x2132+y2122=1 or x2169+y2144=1

Q.8. Find the equation for the ellipse that satisfy the given conditions: Length of minor axis 16, foci 0, ±6.

Solution: Length of minor axis =16; foci =0, ±6.
Since the foci are on the y-axis, the major axis is along the y-axis.
Therefore, the equation of the ellipse will be of the form x2b2+y2a2=1, where a is the semi-major axis.
Accordingly, 2b=16⇒b=8 and c=6.
It is known that a2=b2+c2.
∴a2=82+62=64+36=100
⇒a=100=10
Thus, the equation of the ellipse is x282+y2102=1 or x264+y2100=1.

Q.9. Find the equation for the ellipse that satisfy the given conditions: Foci ±3, 0, a=4.

Solution: Foci ±3, 0, a=4
Since the foci are on the x-axis, the major axis is along the x-axis.
Therefore, the equation of the ellipse will be of the form x2a2+y2b2=1, where a is the semi-major axis.
Accordingly, c=3 and a=4.
It is known that a2=b2+c2
∴42=b2+32
⇒16=b2+9
⇒b2=16-9=7
Thus, the equation of the ellipse is x216+y27=1

Q.10. Find the equation for the ellipse that satisfy the given conditions: b=3, c=4, centre at the origin; foci on the x axis.

Solution: It is given that b=3, c=4, centre at the origin; foci on the x-axis.
Since the foci are on the x-axis, the major axis is along the x-axis.
Therefore, the equation of the ellipse will be of the form x2a2+y2b2=1, where a is the semi-major axis.
Accordingly, b=3,c=4.
It is known that a2=b2+c2
∴a2=32+42=9+16=25
⇒a=5
Thus, the equation of the ellipse is x252+y232=1 or x225+y29=1.

Q.11. Find the equation for the ellipse that satisfy the given conditions: Centre at 0, 0, major axis on the y-axis and passes through the points 3, 2 and 1, 6.

Solution: Since the centre is at 0, 0 and the major axis is on the y-axis, the equation of the ellipse will be of the form,
x2b2+y2a2=1 …(i)
Where, a is the semi-major axis.
The ellipse passes through points 3, 2 and 1, 6. Hence,
9b2+4a2=1 …(ii)
1b2+36a2=1 …(iii)
On solving equations (ii) and (iii), we obtain b2=10 and a2=40.
Thus, the equation of the ellipse is x210+y240=1 or 4x2+y2=40.

Q.12. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse x24+y225=1

Solution: The given equation is x24+y225=1 or x222+y252=1.
Here, the denominator of y225 is greater than the denominator of x24.
Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis. On comparing the given equation with x2b2+y2a2=1, we obtain b=2 and a=5.
∴c=a2-b2=25-4=21
Therefore,
The coordinates of the foci are
0, 21 and 0, -21.
The coordinates of the vertices are 0, 5 and 0, -5
Length of major axis =2a=10
Length of minor axis =2b=4
Eccentricity, e=ca=215
Length of latus rectum =2b2a=2×45=85

Q.13. Find the equation for the ellipse that satisfy the given conditions: Major axis on the x-axis and passes through the points 4, 3 and 6, 2.

Solution: Since the major axis is on the x-axis, the equation of the ellipse will be of the form
x2a2+y2b2=1 …(i)
Where, a is the semi-major axis
The ellipse passes through points 4, 3 and 6, 2. Hence,
16a2+9b2=1 …(ii)
36a2+4b2=1 …(iii)
On solving equations (ii) and (iii), we obtain a2=52 and b2=13
Thus, the equation of the ellipse is x252+y213=1 or x2+4y2=52

Q.14. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis,the eccentricity and the length of the latus rectum of the ellipse x216+y29=1

Solution: The given equation is x216+y29=1 or x242+y232=1.
Here, the denominator of x216 is greater than the denominator of y29.
Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis. On comparing the given equation with x2a2+y2b2=1, we obtain a=4 and b=3.
∴c=a2-b2=16-9=7
Therefore, the coordinates of the foci are ±7, 0
The coordinates of the vertices are ±4, 0
Length of major axis =2a=8
Length of minor axis =2b=6
Eccentricity, e=ca=74
Length of latus rectum =2b2a=2×94=92

Q.15. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse x225+y2100=1.

Solution: The given equation is x225+y2100=1 or x252+y2102=1.

Here, the denominator of y2100 is greater than the denominator of x225.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis. On comparing the given equation with x2b2+y2a2=1, we obtain b=5 and a=10. ∴c=a2-b2=100-25=75=53 Therefore, the coordinates of the foci are 0,±c=0,±53 The coordinates of the vertices are 0,±a=0,±10 Length of major axis =2a=20 Length of minor axis =2b=10 Eccentricity, e=ca=5310=32 Length of latus rectum =2b2a=2×2510=5

Q.16. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse x249+y236=1.

Solution: The given equation is x249+y236=1 or x272+y262=1.
Here, the denominator of x249 is greater than the denominator of y236.
Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis. On comparing the given equation with x2a2+y2b2=1, we obtain a=7 and b=6.
∴c=a2-b2=49-36=13
Therefore, the coordinates of the foci are ±c, 0=±13, 0.
The coordinates of the vertices are ±a, 0=±7, 0.
Length of major axis =2a=14
Length of minor axis =2b=12
Eccentricity, e=ca=137
Length of latus rectum =2b2a=2×367=727

Q.17. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse x2100+y2400=1.

Solution: The given equation is x2100+y2400=1 or x2102+y2202=1.
Here, the denominator of y2400 is greater than the denominator of x2100.
Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis. On comparing the given equation with x2b2+y2a2=1, we obtain b=10 and a=20.
∴c=a2-b2=400-100=300=103

The coordinates of the foci are 0, ±c=0, ±103.
The coordinates of the vertices are 0, ±a=0, ±20.
Length of major axis =2a=40.
Length of minor axis =2b=20.
Eccentricity, e=ca=10320=32.
Length of latus rectum =2b2a=2×10020=10.

Q.18. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse 36x2+4y2=144

Solution: The given equation is 36x2+4y2=144.
It can be written as
36x2+4y2=144
or, x24+y236=1
or, x222+y262=1 …(i)
Here, the denominator of y262 is greater than the denominator of x222.
Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.
On comparing equation (i) with x2b2+y2a2=1, we obtain b=2 and a=6.
∴c=a2-b2=36-4=32=42
Therefore,
The coordinates of the foci are 0, ±c=0, ±42.
The coordinates of the vertices are 0, ±a=0, ±6.
Length of major axis =2a=12.
Length of minor axis =2b=4.
Eccentricity, e=ca=426=223.
Length of latus rectum =2b2a=2×46=43.

Q.19. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse 16x2+y2=16.

Solution: The given equation is 16x2+y2=16.
It can be written as
16x2+y2=16
or, x21+y216=1
or, x212+y242=1 …(i)
Here, the denominator of y242 is greater than the denominator of x212.
Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.
On comparing equation (i) with x2b2+y2a2=1, we obtain b=1 and a=4.
∴c=a2-b2=16-1=15
Therefore,
The coordinates of the foci are 0, ±c=0, ±15.
The coordinates of the vertices are 0, ±a=0, ±4.
Length of major axis =2a=8.
Length of minor axis =2b=2.
Eccentricity, e=ca=154.
Length of latus rectum =2b2a=2×14=12.

Q.20. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse 4x2+9y2=36.

Solution: The given equation is 4x2+9y2=36.

It can be written as

4x2+9y2=36

or, x29+y24=1

or, x232+y222=1 …(i)

Here, the denominator of x232 is greater than the denominator of y222.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis. On comparing the given equation with x2a2+y2b2=1, we obtain a=3 and b=2.

∴c=a2-b2=9-4=5

Therefore,

The coordinates of the foci are ±c, 0=±5, 0.

The coordinates of the vertices are ±a, 0=±3, 0.

Length of major axis =2a=6.

Length of minor axis =2b=4.

Eccentricity, e=ca=53.

Length of latus rectum =2b2a=2×43=83.

Exercise 11.4

Q.1. Find the coordinates of the foci and the vertices, the eccentricity and the length of the lotus rectum of the hyperbola $x216-y29=1$.

Solution: The given equation is $x216-y29=1$ or $x242-y232=1$
On comparing this equation with the standard equation of hyperbola i.e., $x2a2-y2b2=1$, we obtain $a=4$ and $b=3$
We know that $a2+b2=c2$
$\therefore c2=42+32=25$
$\Rightarrow c=5$
Therefore,
The coordinates of the foci are $\pm5, 0$.
The coordinates of the vertices are $\pm4, 0$.
Eccentricity, $e=ca=54$
Length of latus rectum $=2b2a=2\times94=92$

Q.2. Find the equation of the hyperbola satisfying the given conditions: Foci $\pm5, 0$, the transverse axis is of length 8.

Solution: Foci $\pm5, 0$, the transverse axis is of length 8.
Here, the foci are on the x-axis.
Therefore, the equation of the hyperbola is of the form $x2a2-y2b2=1$.
Since the foci are $\pm5, 0$, $c=5$.
Since the length of the transverse axis is 8, $2a=8\Rightarrow a=4$.
We know that $a2+b2=c2$.
$\therefore 42+b2=52$
$\Rightarrow b2=25-16=9$
Thus, the equation of the hyperbola is $x216-y29=1$.

Q.3. Find the equation of the hyperbola satisfying the given conditions: Foci $0, \pm13$, the conjugate axis is of length 24.

Solution: Foci $0,\pm13$, the conjugate axis is of length 24.
Here, the foci are on the y-axis.
Therefore, the equation of the hyperbola is of the form $y2a2-x2b2=1$
Since the foci are $0, \pm13$, $c=13$.
Since the length of the conjugate axis is 24, $2b=24\Rightarrow b=12$.
We know that $a2+b2=c2$.
$\therefore a2+122=132$
$\Rightarrow a2=169-144=25$
Thus, the equation of the hyperbola is $y225-x2144=1$.

Q.4. Find the equation of the hyperbola satisfying the given conditions: Foci $\pm35, 0$, the latus rectum is of length 8.

Solution: Step1: Foci $\pm35, 0$, the latus rectum is of length 8
Here, the foci are on the x-axis.
Therefore, the equation of the hyperbola is of the form $x2a2-y2b2=1$.

Since the foci are $\pm35, 0$, $c=35$.

Length of latus rectum $=8$.
$\Rightarrow 2b2a=8$
$\Rightarrow b2=4a$

We know that, $a2+b2=c2$
$\therefore a2+4a=45$
$\Rightarrow a2+4a-45=0$
$\Rightarrow a2+9a-5a-45=0$
$\Rightarrow a+9a-5=0$
$\Rightarrow a=-9$ or 5
Since a is non-negative, $a=5$
$\therefore b2=4a=4\times5=20$
Thus, the equation of the hyperbola is $x225-y220=1$

Q.5. Find the equation of the hyperbola satisfying the given conditions: Foci $\pm4, 0$, the latus rectum is of length 12.

Solution: Given that: Foci is ±4, 0 and the latus rectum is of length 12. Here, the foci are on the x-axis.
Therefore, the equation of the hyperbola is of the form x2a2-y2b2=1.
Since the foci are ±4, 0, c=4.

Length of latus rectum =12

⇒2b2a=12 ⇒b2=6a

We know that a2+b2=c2

∴a2+6a=16

⇒a2+6a-16=0 ⇒a2+8a-2a-16=0 ⇒a+8a-2=0 ⇒a=-8,2 Since a is non-negative, a=2 ∴b2=6a=6×2=12
Thus, the equation of the hyperbola is x24-y212=1

Q.6. Find the equation of the hyperbola satisfying the given conditions: Vertices ±7, 0, e=43.

Solution: Given that: Vertices are ±7,0 and e=43.
Here, the vertices are on the x-axis.
Therefore, the equation of the hyperbola is of the form x2a2-y2b2=1
Since the vertices are ±7, 0, a=7.
It is given that e=43.
∴ca=43e=ca
⇒c7=43
⇒c=283

We know that a2+b2=c2
∴72+b2=2832
⇒b2=7849-49
⇒b2=784-4419=3439
Thus, the equation of the hyperbola is x249-9y2343=1.

Q.7. Find the equation of the hyperbola satisfying the given conditions: Foci 0, ±10, passing through 2, 3.

Solution: Given that: Foci are 0, ±10, and the hyperbola is passing through 2,3.
Here, the foci are on the y-axis.
Therefore, the equation of the hyperbola is of the form y2a2-x2b2=1.
Since the foci are 0, ±10, c=10.
We know that a2+b2=c2
∴a2+b2=10
⇒b2=10-a2 …(i)

Since the hyperbola passes through point 2, 3, the point will satisfy the equation of the parabola.
Therefore, 9a2-4b2=1 …(ii)
From equations (i) and (ii), we obtain
9a2-410-a2=1
⇒910-a2-4a2=a210-a2
⇒90-9a2-4a2=10a2-a4
⇒a4-23a2+90=0
⇒a4-18a2-5a2+90=0
⇒a2a2-18-5a2-18=0
⇒a2-18a2-5=0
⇒a2=18 or 5

In hyperbola, c>a, i.e. c2>a2
∴a2=5
⇒b2=10-a2=10-5=5
Thus, the equation of the hyperbola is y25-x25=1

Q.8. Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola y29-x227=1

Solution: The given equation is y29-x227=1 or y232-x2272=1
On comparing this equation with the standard equation of hyperbola y2a2-x2b2=1 i.e., we obtain a=3 and b=27
We know that a2+b2=c2
∴c2=32+272=9+27=36
⇒c=6
Therefore,
The coordinates of the foci are 0, ±6.
The coordinates of the vertices are 0, ±3.
Eccentricity, e=ca=63=2.
Length of latus rectum =2b2a=2×273=18.

Q.9. Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbola 9y2-4x2=36.

Solution: The given equation is 9y2-4x2=36
It can be written as
9y2-4x2=36
Or, y24-x29=1
Or, y222-x232=1 …(i)
On comparing equation (i) with the standard equation of hyperbola y2a2-x2b2=1, we obtain a=2 and b=3
We know that a2+b2=c2
∴c2=4+9=13
⇒c=13
Therefore,
The coordinates of the foci are 0, ±13.
The coordinates of the vertices are 0, ±2.
Eccentricity, e=ca=132.
Length of latus rectum =2b2a=2×92=9.

Q.10. Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbola 16x2-9y2=576.

Solution: The given equation is 16x2-9y2=576
It can be written as
16x2-9y2=576
⇒x236-y264=1
⇒x262-y282=1 …(i)
On comparing equation (i) with the standard equation of hyperbola i.e., x2a2-y2b2=1, we obtain a=6 and b=8
We know that a2+b2=c2
∴c2=36+64=100
⇒c=10
Therefore,
The coordinates of the foci are ±c, 0=±10, 0.
The coordinates of the vertices are ±a, 0=±6, 0.
Eccentricity, e=ca=106=53.
Length of latus rectum =2b2a=2×646=643.

Q.11. Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola 5y2-9x2=36.

Solution: The given equation is 5y2-9x2=36
⇒y2365-x24=1
⇒y2652-x222=1 …(i)
On comparing equation (i) with the standard equation of hyperbola i.e., y2a2-x2b2=1, we obtain a=65 and b=2
We know that a2+b2=c2
∴c2=365+4=565
⇒c=565=2145
Therefore, the coordinates of the foci are 0, ±c=0, ±2145.
The coordinates of the vertices are 0, ±a=0, ±65.
Eccentricity, e=ca=214565=143.
Length of the latus rectum=2b2a=2×465=453.

Q.12. Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the hyperbola 49y2-16x2=784.

Solution: The given equation is 49y2-16x2=784
It can be written as
49y2-16x2=784
Or, y216-x249=1
Or, y242-x272=1 …(i)
On comparing equation (i) with the standard equation of hyperbola i.e., y2a2-x2b2=1, we obtain a=4 and b=7
We know that a2+b2=c2
∴c2=16+49=65
⇒c=65
Therefore,
The coordinates of the foci are 0, ±c=0, ±65.
The coordinates of the vertices are 0, ±a=0, ±4.
Eccentricity, e=ca=654.
Length of the latus rectum =2b2a=2×494=492.

Q.13. Find the equation of the hyperbola satisfying the given conditions: Vertices ±2, 0, foci ±3, 0.

Solution: Vertices ±2,0, foci ±3,0.
Here, the vertices are on the x-axis.
Therefore, the equation of the hyperbola is of the form x2a2-y2b2=1.
Since the vertices are ±2, 0, a=2.
Since the foci are ±3, 0, c=3.
We know that a2+b2=c2.
∴22+b2=32
b2=9-4=5
Thus, the equation of the hyperbola is x24-y25=1.

Q.14. Find the equation of the hyperbola satisfying the given conditions: Vertices 0, ±5, foci 0, ±8.

Solution: Vertices 0,±5, foci 0,±8
Here, the vertices are on the y-axis.
Therefore, the equation of the hyperbola is of the form y2a2-x2b2=1.
Since the vertices are 0, ±5, a=5.
Since the foci are 0, ±8, c=8.
We know that a2+b2=c2.
∴52+b2=82
b2=64-25=39
Thus, the equation of the hyperbola is y225-x239=1.

Q.15. Find the equation of the hyperbola satisfying the given conditions: Vertices 0, ±3, foci 0, ±5.

Solution: Vertices 0, ±3, foci 0, ±5
Here, the vertices are on the y-axis.
Therefore, the equation of the hyperbola is of the form y2a2-x2b2=1
Since the vertices are 0, ±3, a=3.
Since the foci are 0, ±5, c=5.
We know that a2+b2=c2.
∴32+b2=52
⇒b2=25-9=16
Thus, the equation of the hyperbola is y29-x216=1.