JEE MAIN 2021

Reliable INSTITUTE
Unleashing Potential

PAPER-1 (B.E. / B.TECH)

QUESTIONS & SOLUTIONS

Reproduced from Memory Retention

16 March, 2021 SHIFT-1

09:00 am to 12 Noon

Duration : 3 Hours Max. Marks : 300

SUBJECT - MATHEMATICS


A-10, Road No.1, IPIA, Kota-324005 (Rajasthan), India
Tel. : +91-744-2665544 I Website : www.reliablekota.com I E-mail : info@reliablekota.com

MATHEMATICS

1. Three distinct normal are drawn from the point (a, 0) to the parabola $y^2 = 2x$. The range of 'a' is

(1) $(-\infty, 0)$ (2) $(1, \infty)$ (3) $(-\infty, -1)$ (4) $(0, 1)$

Ans. (2)

Sol. Let the equation of the normal is

$y = mx - 2am - am^3$

here $4a = 2 \Rightarrow a = \frac{1}{2}$

$y = mx - m - \frac{1}{2}m^3$

It passing through A(a, 0) then

$0 = am - m - \frac{1}{2}m^3$

$m = 0, a - 1 - \frac{1}{2}m^2 = 0$

$m^2 = 2(a - 1) > 0$

$\therefore a > 1$

2. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $A^8\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$. Then the equation has

(1) one solution (2) two solution (3) infinite solution (4) no solution

Ans. (4)

Sol. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$$A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A^4 = 4\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = 4\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^8 = 64\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 64\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$$

$$128\begin{bmatrix} x - y \\ -x + y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix} \Rightarrow 128(x - y) = 8$$

$\Rightarrow x - y = \frac{1}{16}$ ....(1)

and $128(-x + y) = 64 \Rightarrow x - y = \frac{-1}{2}$ ....(2)

$\Rightarrow$ no solution

3. If $x^2 + y^2 = 25$ is a circle whose chord is a tangent to a hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$, then locus of mid-point of chord is

(1) $9x^2 + 16y^2 = (x^2 + y^2)^2$ (2) $9x^2 - 25y^2 = (x^2 - y^2)^2$

(3) $9x^2 - 16y^2 = (x^2 + y^2)^2$ (4) $9x^2 + 25y^2 = (x^2 + y^2)^2$

Ans. (3)

Sol. tangent of hyperbola

$y = mx \pm \sqrt{9m^2 - 16}$ .....(i)

which is a chord of circle with mid-point (h, k)

so equation of chord $T = S_1$

$hx + ky = h^2 + k^2$

$y = -\frac{hx}{k} + \frac{h^2 + k^2}{k}$ .....(ii)

by (i) and (ii)

$m = -\frac{h}{k}$ and $\sqrt{9m^2 - 16} = \frac{h^2 + k^2}{k}$

$9\frac{h^2}{k^2} - 16 = \frac{(h^2 + k^2)^2}{k^2}$

locus $9x^2 - 16y^2 = (x^2 + y^2)^2$

4. If a, b, c are three numbers where b = a + c & S.D. of a + 2, b + 2, c + 2 is d then which of the following statement is true.

(1) $a^2 = 3b^2 + 3c^2 - d^2$ (2) $a^2 = b^2 + c^2 - d^2$

(3) $b^2 = 3b^2 + 3c^2 + 9d^2$ (4) $9d^2 = 3a^2 + 3c^2 - b^2$

Ans. (4)

Sol. for a, b, c

mean = $\bar{x} = \frac{a + b + c}{3}$

$\bar{x} = \frac{2b}{3}$

S.D. of a, b, c = d

$d^2 = \frac{a^2 + b^2 + c^2}{3} - \frac{4b^2}{9}$

$b^2 = 3a^2 + 3c^2 - 9d^2$

5. If $\lim\limits_{x\to 0}\dfrac{ae^{x}-b\cos x+ce^{-x}}{x\sin x}=2$ then (a + b + c) equal to

Ans. 4

Sol. $$\lim_{x\to 0}\frac{\left\{a\left(1+x+\frac{x^2}{2!}+....\right)-b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}....\right)+c\left(1-x+\frac{x^2}{2!}....\right)\right\}}{x\left(x-\frac{x^3}{3!}+....\right)}=2$$

$$\therefore \lim_{x\to 0}\frac{(a-b+c)+x(a-c)+x^2\left(\frac{a}{2}+\frac{b}{2}+\frac{c}{2}\right)+....}{x^2\left(1-\frac{x^2}{6}....\right)}=2$$

$\therefore\ a-b+c=0$

$a-c=0$

$\frac{a}{2}+\frac{b}{2}+\frac{c}{2}=2$

$a+b+c=4$

6. which of the following is tautology ?

(1) $(p\wedge q)\to(p\to q)$ (2) $(p\wedge q)\vee(p\to q)$

(3) $(p\wedge q)\wedge(p\to q)$ (4) $(p\vee q)\to(p\to q)$

Ans. (1)

Sol.

p	q	$p\wedge q$	$p\vee q$	$p\to q$	$(p\wedge q)\to(p\to q)$
T	T	T	T	T	T
F	T	F	T	T	T
T	F	F	T	F	T
F	F	F	F	T	T

7. Number of irrational term in $(5^{1/4}+3^{1/8})^{60}$ is n then which of the following is a factor of (n – 1)

(1) 26 (2) 5 (3) 7 (4) 27

Ans. (1)

Sol. $T_{r+1}={}^{60}C_r\ (5^{1/4})^{60-r}\left(3^{1/8}\right)^r$

rational if $\frac{60-r}{4},\frac{r}{8}$ both are whole numbers, $r\in\{0,1,2,....60\}$

$\frac{60-r}{4}\in W\Rightarrow\ r\in\{0,4,8,....60\}$

and $\frac{r}{8}\in W\Rightarrow r\in\{0,8,16,....56\}$

$\therefore$ Common terms $r\in\{0,8,16,....56\}$

So 8 terms are rational

Then Irrational terms $=61-8=53=n$

$\therefore\ n-1=52=13\times 2^2$

factors 1,2,4,13,26,52

8. If $\log_{10}(\sin x) + \log_{10}(\cos x) = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

Find 'n'

(1) 10 (2) 11 (3) 12 (4) 13

Ans. (3)

Sol. $\log_{10}(\sin x) + \log_{10}(\cos x) = -1$

$\sin x \cos x = \frac{1}{10}$ .....(1)

and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \sin x + \cos x = \left(\frac{n}{10}\right)^{\frac{1}{2}}$

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x = \frac{n}{10}$ (squaring)

$\Rightarrow 1 + 2\left(\frac{1}{10}\right) = \frac{n}{10}$ (using Equation (1))

$\Rightarrow \frac{n}{10} = \frac{12}{10} \Rightarrow n = 12$

9. If $f(x) + f(x+1) = 2$, $I_1 = \int_0^8 f(x)\,dx$ and $I_2 = \int_{-1}^3 f(x)\,dx$, then $I_1 + 2I_2$ is

Ans. 16

Sol. $f(x) + f(x+1) = 2$ .....(i)

$x \to (x+1)$

$f(x+1) + f(x+2) = 2$ .....(ii)

by (i) & (ii)

$f(x) - f(x+2) = 0$

$f(x+2) = f(x)$

f(x) is period with T = 2

$I_1 = \int_0^{2\times4} f(x)\,dx = 4\int_0^2 f(x)\,dx$

$I_2 = \int_{-1}^3 f(x)\,dx = \int_0^4 f(x+1)\,dx = \int_0^4 (2 - f(x))\,dx$

$I_2 = 8 - 2\int_0^2 f(x)\,dx$

$I_1 + 2I_2 = 16$

10. If $(|x| - 3)(|x + 4|) = 6$ then find the number of values of x ($x \in R$ given)

(1) 1 (2) 2 (3) 3 (4) 4

Ans. (2)

Sol. Case-1 $x \le -4$

$(-x - 3)(-x - 4) = 6$

$\Rightarrow (x + 3)(x + 4) = 6$

$\Rightarrow x^2 + 7x + 6 = 0$

$\Rightarrow x = -1$ or -6

but $x \le -4$

$x = -6$

Case-2 $x \in (-4, 0)$

$(-x - 3)(x + 4) = 6$

$\Rightarrow -x^2 - 7x - 12 - 6 = 0$

$\Rightarrow x^2 + 7x + 18 = 0$

$D < 0$ No solution

Case-3 $x \ge 0$

$(x - 3)(x + 4) = 6$

$\Rightarrow x^2 + x - 12 - 6 = 0$

$\Rightarrow x^2 + x - 18 = 0$

$x = \frac{-1 \pm \sqrt{1 + 72}}{2}$

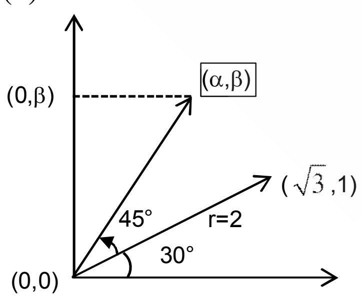
$\therefore x = \frac{\sqrt{73} - 1}{2}$ only

11. If vector $(\sqrt{3}\,\hat{i} + \hat{j})$ is rotated 45° counter clockwise about origin then the new vector is $(\alpha\hat{i} + \beta\hat{j})$ then find the area of triangle whose co-ordinates are (0, 0), (0, β) and (α, β)

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) 2

Ans. (3)

Sol.



$(\alpha, \beta) \equiv (2\cos 75°, 2\sin 75°)$

Area $= \frac{1}{2}(2\cos 75°)(2\sin 75°)$

$= \sin(150°) = \frac{1}{2}$ square unit

12. Number of solution of equation $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in $[0, \pi]$ are

(1) 2 (2) 3 (3) 4 (4) 5

Ans. (3)

Sol. $(81)^{\sin^2 x} + (81)^{1-\sin^2 x} = 30$

$(81)^{\sin^2 x} + \dfrac{81}{(81)^{\sin^2 x}} = 30$

$t + \dfrac{81}{t} = 30 \qquad \Rightarrow t^2 + 81 = 30t$

$t^2 - 30t + 81 = 0$

$t^2 - 27t - 3 + 81 = 0$

$(t - 3)(t - 27) = 0$

$T = 3, 27$

$(81)^{\sin^2 x} = 3, 3^3$

$3^{4\sin^2 x} = 3^1, 3^3$

$4 \sin^2 x = 1, 3$

$\sin^2 x = \dfrac{1}{4}, \dfrac{3}{4}$

in $[0, \pi]$ $\sin x > 0$

$\sin x = \dfrac{1}{2}, \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

Number of solution = 4

13. If one card is missing out of 52 cards deck then out of remaining 51 cards two cards are drawn if both cards are found to be spade then the probability that missing card was NOT spade, is :

(1) $\dfrac{21}{50}$ (2) $\dfrac{13}{50}$ (3) $\dfrac{39}{50}$ (4) $\dfrac{4}{5}$

Ans. (3)

Sol. $P\left(\overline{S}_{\text{missing}} \Big/ \text{both found spade}\right)$

$\dfrac{P\left(\overline{S_m} \cap BFS\right)}{P(BFS)}$

$= \dfrac{\left(1 - \dfrac{13}{52}\right) \times \dfrac{13}{51} \times \dfrac{12}{50}}{\left(1 - \dfrac{13}{52}\right) \times \dfrac{13}{51} \times \dfrac{12}{50} + \dfrac{13}{52} \times \dfrac{12}{51} \times \dfrac{11}{50}}$

$= \dfrac{39}{50}$

14. If $\frac{dy}{dx} + 2y \tan x = \sin x,\ y\left(\frac{\pi}{3}\right) = 0$, then maximum value of function

Ans. $(\frac{1}{8})$

Sol. $\frac{dy}{dx} + 2 \tan x.y = \sin x$

I.F. $= e^{2\ell(\sec x)} = \sec^2 x$

$y.\sec^2 x = \int \sin x \sec^2 x\, dx = \int \tan x \sec x\, dx + c$

$y \sec^2 x = \sec x + c$

$y = \cos x + c \cos^2 x$

$x = \frac{\pi}{3},\ y = 0$

$\Rightarrow \frac{1}{2} + \frac{c}{4} \Rightarrow c = -2$

$\therefore\ y = \cos x - 2 \cos^2 x$

$y = -2\left(\cos^2 x - \frac{1}{2}\cos x\right) = -2\left(\left(\cos x - \frac{1}{4}\right)^2 - \frac{1}{16}\right)$

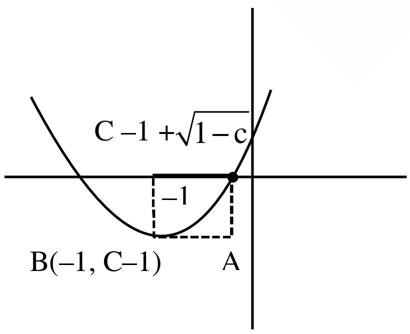
$y = \frac{1}{8} - 2\left(\left(\cos x - \frac{1}{4}\right)^2\right)$

$\therefore\ y_{max} = \frac{1}{8}$

15. If curve $y = f(x)$ satisfy the differential equation $\frac{dy}{dx} = 2(x + 1)$ & area bounded by $y = f(x)$ & x-axis is $\frac{4\sqrt{8}}{3}$, then find f(1)

Ans. (2)

Sol. $y = x^2 + 2x + c$



Area of rectangle (ABCD) = $\left|(c-1)\left(\sqrt{1-c}\right)\right|$

Area of parabola and x-axis = $2\left(\frac{2}{3}((1-c)^{3/2})\right) = \frac{4\sqrt{8}}{3}$

$1-c=2 \Rightarrow c=-1$

Equation of $f(x) = x^2 + 2x - 1$

$f(1) = 1 + 2 - 1 = 2$

16. If $y(x) = \int_0^x (2t^2 - 15t + 10)dt$,

Normal of above curve at P(a,b) is parallel to $3y + x = 5$ then find $|a + 6b|$ (given $a > 1$)

Ans. 406

Sol. $y'(x) = (2x^2 - 15x + 10)$

at point P

$3 = (2a^2 - 15a + 10)$

$\Rightarrow 2a^2 - 15a + 7 = 0$

$\Rightarrow 2a^2 - 14a - a + 7 = 0$

$\Rightarrow 2a(a-7) - 1(a-7) = 0$

$a = \frac{1}{2}$ or 7,

given $a > 1 \therefore a = 7$

also P lies on curve

$\therefore b = \int_0^a (2t^2 - 15t + 10)dt$

$b = \int_0^7 (2t^2 - 15t + 10)dt$

$6b = -413$

$\therefore |a + 6b| = 406$

17. If elements of matrix A are {0,1,2,3} and $AA^T = 9$, trace of (AA^T) is 9 then total number of such matrixes are.

Ans. 766

Sol. $AA^T = \begin{bmatrix} x & y & z \\ a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} x & a & d \\ y & b & e \\ z & c & f \end{bmatrix}$

$= \begin{bmatrix} x^2+y^2+z^2 & ax+by+cz & dx+ey+fz \\ ax+by+cz & a^2+b^2+c^2 & ad+be+cf \\ dx+ey+fz & ad+be+cf & d^2+e^2+f^2 \end{bmatrix}$

$\text{Tr}(AA^T) = x^2 + y^2 + z^2 + a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 9$

all → 1	1
one 3, rest = 0	$\frac{9!}{8!} = 9$
two 2 , one 1 & rest 0	$\frac{9!}{2!6!} = 63\times 4 = 252$
one 2 , five 1, rest 0	$\frac{9!}{5!3!} = 63\times 8 = 504$
	= 766

18. If $f(x) = \log_2\left(1+\tan\frac{\pi x}{4}\right)$ find $\lim\limits_{x\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+.......+f(1)\right)$

Ans. 1

Sol. $E = 2\lim\limits_{x\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$

$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\frac{\pi x}{4}\right)dx$ ….(i)

replacing x → 1 – x

$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\frac{\pi}{4}(1-x)\right)dx$

$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\left(\frac{\pi}{4}-\frac{\pi}{4}x\right)\right)dx$

$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\frac{1-\tan\frac{\pi}{4}x}{1+\tan\frac{\pi}{4}x}\right)dx$

$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(\frac{2}{1+\tan\frac{\pi x}{4}}\right)dx$

$E = \frac{2}{\ell n2}\int_0^1\left(\ell n2 - \ell n\left(1+\tan\frac{\pi x}{4}\right)\right)dx$ …(ii)

equation (i) + (ii)

E = 1

19. $\ell n_{\frac{1}{\sqrt{2}}}\left(\dfrac{|z|+1}{(|z|-1)^2}\right) \le 2$, find the maximum value of $|z|$.

Ans. (7)

Sol. $\dfrac{|z|+1}{(|z|-1)^2} \ge \dfrac{1}{2}$

$2|z| + 22 \ge (|z| - 1)^2$

$2|z| + 22 \ge |z|^2 - 2|z| + 1$

$|z|^2 - 4|z| - 21 \le 0$

$(|z| - 7)\ (|z| + 3) \le 0$

$\Rightarrow\ |z| \le 7$

$\therefore\ |z|_{max} = 7$

20. Find value $\sum_{r=1}^{\infty} \tan^{-1}\left(\dfrac{6^r}{2^{(2r+1)} + 3^{(2r+1)}}\right)$

(1) $\cot^{-1}\dfrac{3}{2}$ (2) $\tan^{-1}\dfrac{3}{2}$ (3) $\dfrac{\pi}{2}$ (4) $\dfrac{\pi}{4}$

Ans. (1)

Sol. $\sum_{r=1}^{\infty} \tan^{-1}\left(\dfrac{6^r(3-2)}{1+\left(\dfrac{3}{2}\right)^{2r+1}}\right)$

$\sum_{r=1}^{\infty} \tan^{-1}\left(\dfrac{2^r\,.\,3^{r+1} - 3^r 2^{r+1}}{2^{2r+1} + 3^{2r+1}}\right)$

$\sum_{r=1}^{\infty} \tan^{-1}\left(\dfrac{\left(\dfrac{3}{2}\right)^{r+1} - \left(\dfrac{3}{2}\right)^{r}}{1+\left(\dfrac{3}{2}\right)^{r+1}\left(\dfrac{3}{2}\right)^{r}}\right) = \sum_{r=1}^{\infty}\left[\tan^{-1}\left(\dfrac{3}{2}\right)^{r+1} - \tan\left(\dfrac{3}{2}\right)^{r}\right] = \dfrac{\pi}{2} - \tan^{-1}\dfrac{3}{2} = \cot^{-1}\dfrac{3}{2}$

21. If $f(x) = \begin{cases} x+2 & x \le 0 \\ x^2 & x > 0 \end{cases}$ and $g(x) = \begin{cases} 3x-2 & x \ge 1 \\ x^3 & x < 1 \end{cases}$ then number of points of non-differentiability of fog(x)

(1) 0 (2) 1 (3) 2 (4) 3

Ans. (2)

Sol. $fog(x) = \begin{cases} x^3+2 & x \le 0 \\ x^6 & 0 \le x \le 1 \\ (3x-2)^2 & x \ge 1 \end{cases}$

$\because$ fog(x) is discontinuous at x = 0 then non-differentiable at x = 0 at x = 1

RHD = $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{(3(1+h)-2)^2-1}{h} = 6$

LHD = $\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0}\frac{(1-h)^6-1}{-h} = 6$

Number of points of non-differentiability = 1

22. Let set A = {4, 8, 16, 32, 21, 26, 11, 15). If four numbers taken from setA they form Ist four terms of AP & again four numbers taken from these they from Ist four terms of GP. If last number of these series is a four digit numbers then find number of common terms in these two series.

Ans. (3)

Sol. AP – 11, 16, 21, 26 …….

GP – 4, 8, 16, 32 …….

So common terms are 16, 256, 4096

23. If f(x) = (4a – 3) (x + ℓn5) + 2(a – 7) $\cot\frac{x}{2}.\sin^2\frac{x}{2}$, then the complete set of values of 'a' for which critical point of f(x) exist is :

(1) $[1,\infty)$ (2) $(-1,0)$ (3) $\left[\frac{-4}{3},2\right]$ (4) $\left[-2,\frac{-4}{3}\right]$

Ans. (3)

Sol. $f(x) = (4a-3)(x+\ell n5) + 2(a-7)\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}.\sin^2\frac{x}{2}\right)$

$f(x) = (4a-3)(x+\ell n5) + (a-7)\sin x$

$f'(x) = (4a-3) + (a-7)\cos x = 0$

$\cos x = \frac{-(4a-3)}{a-7}$

$-1 \le -\frac{4a-3}{a-7} \le 1$

$-1 \le \frac{4a-3}{a-7} \le 1$

$\frac{4a-3}{a-7} - 1 \le 0$ and $\frac{4a-3}{a-7} + 1 \ge 0$

$\Rightarrow \frac{-4}{3} \le a \le 2$