JEE Main Exam 2022 - Session 1

24 June 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. A proton, a deuteron and an alpha particle with the same kinetic energy enter a region of uniform magnetic field B at right angles to the field. The ratio of the radii of their circular paths is

A) 1 : 1 : 1

B) $1 : \sqrt{2} : 1$

C) $\sqrt{2} : 1 : 1$

D) $\sqrt{2} : \sqrt{2} : 1$

Answer: $1 : \sqrt{2} : 1$

Solution:

Radius, $R = \frac{mv}{qB} = \frac{m\left(\sqrt{2(\mathrm{KE})}\right)}{qB\sqrt{m}} \propto \frac{\sqrt{m}}{q}$

$R_P : R_D : R_\alpha = \frac{1}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2} = 1 : \sqrt{2} : 1$

Q.2. Find the magnitude of displacement current when a parallel plate capacitor is charged to $60 \mu\mathrm{C}$. Due to a radioactive source, the plate loses charge at the rate of $1.8 \times 10^{-8}\,\mathrm{Cs}^{-1}$

A) $1.8 \times 10^{-8}\,\mathrm{Cs}^{-1}$

B) $3.6 \times 10^{-8}\,\mathrm{Cs}^{-1}$

C) $4.1 \times 10^{-11}\,\mathrm{Cs}^{-1}$

D) $5.7 \times 10^{-12}\,\mathrm{Cs}^{-1}$

Answer: $1.8 \times 10^{-8}\,\mathrm{Cs}^{-1}$

Solution: The capacitor is charged initially to $60 \mu C$, then it loses charge because of a radioactive source. We need to seek out the displacement current. The displacement current is that current which comes into play within the region during which the electrical field and hence the electric flux is changing with time.

Maxwell found that conduction current (I) and displacement current (I_d) together have the property of continuity, although individually they may not be continuous. Maxwell also predicted that this current produces an equivalent magnetic flux as a conduction current can produce.

Displacement current is given by,

$$I_d = \frac{dq}{dt}$$

As displacement current is the rate of change of electric displacement field. The rate of loss of charge of the capacitor is given as $1.8 \times 10^{-8}\ C\ s^{-1}$.

Therefore, $I_d = \frac{dq}{dt}$

$\therefore I_d = 1.8 \times 10^{-8}\ C\ s^{-1}$

Hence, it is the required answer.

Q.3. A Carnot engine accepted heat of $5000\ \text{kcal}$ at $727^\circ C$ and rejects at $127^\circ C$ what is the work done by the engine?

A) $3000\ \text{kcal}$

B) $2000\ \text{kcal}$

C) $4000\ \text{kcal}$

D) $5000\ \text{kcal}$

Answer: $3000\ \text{kcal}$

Solution: The efficiency of the Carnot cycle is given by,

$$\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}} = \frac{W}{Q}$$

$$\Rightarrow \left(1 - \frac{400}{1000}\right) = \frac{6}{10} = \frac{W}{5000} \Rightarrow W = 3000\ \text{kcal}$$

Q.4. Charge on capacitor is increased by $2\ C$ and energy stored becomes 144%. Find initial charge

A) 10

B) 12

C) 14

D) 16

Answer: 10

Solution: Energy stored in a capacitor is given by $E = \frac{Q^2}{2C}$.

Let initial charge be Q_0.

Then, $1.44\left(\frac{Q_0^2}{2C}\right) = \frac{(Q_0+2)^2}{2C}$

$$\Rightarrow 1.2 Q_0 = Q_0 + 2$$

$$Q_0 = \frac{2}{0.2} = 10$$

Q.5. If the distance between the earth and the sun becomes three times, then the time period in years will be

A) $2\sqrt{3}$ yr

B) $3\sqrt{3}$ yr

C) 3 yr

D) 9 yr

Answer: $3\sqrt{3}$ yr

Solution: According to Kepler's Third Law – The Law of Periods,

$$T^2 \propto R^3$$

where T is time taken by the planet to go once around the sun and R is the semi-major axis (distance) of the elliptical orbit.

$\therefore\ T^2 = k\,R^3 \quad \ldots$(i)

Where k is a constant of proportionality.

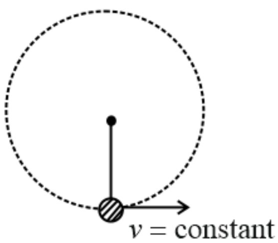
When R becomes 3 times, let the time period be (T').

$\therefore \qquad (T')^2 = k\left(3R\right)^3 \qquad \ldots\left(\text{ii}\right)$

$\therefore \qquad \frac{T^2}{(T')^2} = \frac{1}{27}$

So, time period will be $3\sqrt{3}$ yr.

Q.6. A mass m is tied to a massless string and rotated in a vertical circle with uniform speed. The tension in the string,



A) same throughout.

B) maximum at top.

C) minimum at top.

D) minimum at bottom.

Answer: minimum at top.

Solution: As the speed is constant, centripetal force required towards the centre of the circle will be same. At the top, direction of gravitational force and tension acting on the object will be same. Therefore, we can write,

$mg + T = \frac{mv^2}{r} \ \Rightarrow T = \frac{mv^2}{r} - mg$ Clearly in this case, tension in the string will be minimum.

Q.7. When Q amount of heat is supplied to an ideal monatomic gas, the gas performs $\frac{Q}{4}$ amount of work on its surrounding, then the molar heat capacity for the process is

A) $2R$

B) R

C) $\frac{3\mathrm{R}}{2}$

D) $\frac{5\mathrm{R}}{2}$

Answer: $2R$

Solution: Using first law of thermodynamics,

$Q=\Delta U+W$

$\Rightarrow Q=\Delta U+\frac{Q}{4}$

$\Rightarrow \Delta U=\frac{3Q}{4}$

$\Rightarrow \Delta U=\frac{3}{2}nR\Delta T=\frac{3Q}{4}$

Therefore, $C=\frac{Q}{n\Delta T}$

$\Rightarrow C=2R$

Q.8. The ratio of intensities of two waves is $9:4$. When they superimpose, the ratio of maximum to minimum intensity will become:

A) $9:4$

B) $3:2$

C) $4:1$

D) $25:1$

Answer: $25:1$

Solution: Data provided in the question is, the ratio of the intensities,

$\frac{I_1}{I_2}=\frac{9}{4}$

After taking square root of above equation on both sides, we get,

$\frac{\sqrt{I_1}}{\sqrt{I_2}}=\frac{3}{2}$

Apply componendo and dividendo,

$\Rightarrow \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}=\frac{3+2}{3-2}=\frac{5}{1}$

$\Rightarrow \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2=\frac{I_{\max}}{I_{\min}}=\frac{25}{1}$

Q.9. Two travelling waves moving in opposite directions superimpose with each other. The equation of the resulting wave is $y=10\cos(\pi x)\sin\left(\frac{2\pi t}{T}\right)$ in metres. Find its amplitude at $x=\frac{4}{3}\mathrm{m}$ is

A) $5\ \mathrm{m}$

B) $10\ \mathrm{m}$

C) $12\ \mathrm{m}$

D) $11\ \mathrm{m}$

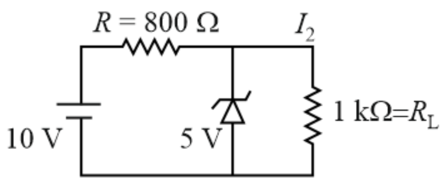
Answer: $5\ \mathrm{m}$

Solution: Amplitude is the maximum displacement from the mean position, for that

$\sin\left(\frac{2\pi t}{T}\right)=1$

Therefore, $A=\left|10\cos\left(\frac{4}{3}\pi\right)\right|=10\times\frac{1}{2}=\ 5\ \text{m}$

Q.10. For the given below circuit, find current through Zener diode.



A) $1.25\ \text{mA}$

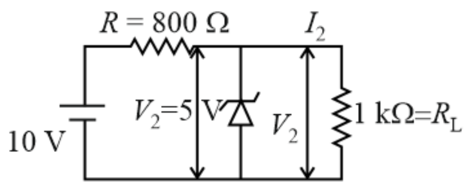
B) $2.25\ \text{mA}$

C) $3.25\ \text{mA}$

D) $4.25\ \text{mA}$

Answer: $1.25\ \text{mA}$

Solution:



Potential drop across the branch containing load resistance will be $5\ \text{V}$ due to zener diode. Therefore, potential drop across $800\ \Omega$ will be, $10\ \text{V}-5\ \text{V}=5\ \text{V}$.

Now, current through battery will be,

$I=\frac{5}{800}\times 1000=6.25\ \text{mA}$

Current through load will be,

$I_L=\frac{5}{1000}\ \text{A}$

$=5\ \text{mA}$

Required current, $I_Z=(6.25-5)\ \text{mA}$

$=1.25\ \text{mA}$

Q.11. A projectile fired at an angle 45° with horizontal. After 2 s its velocity becomes $20\ \text{m s}^{-1}$. Then what is the range of the projectile?

A) $80\ \text{m}$

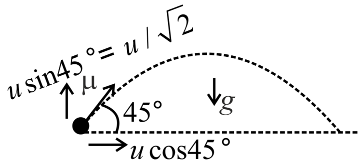
B) $50\ \text{m}$

C) $20\sqrt{3}$ m

D) 60 m

Answer: 80 m

Solution:



After 2 s the velocity is $20 \mathrm{~m~s^{-1}}$

$\Rightarrow v_x^2 + v_y^2 = v^2 = 400$

$\Rightarrow u \cos^2 45^\circ + (u \sin 45^\circ - gt)^2 = \frac{u^2}{2} + \left(\frac{u}{\sqrt{2}} - 20\right)^2 = 400$

$\Rightarrow \frac{u^2}{2} + \frac{u^2}{2} + 400 - \frac{40}{\sqrt{2}}u = 400$

$\Rightarrow u^2 = \frac{40}{\sqrt{2}}u \Rightarrow u = \frac{40}{\sqrt{2}} \mathrm{~m~s^{-1}}$

$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R = \frac{1600}{20} = 80 \mathrm{~m}$

Q.12. A hammer of mass 1.5 kg having speed of $60 \mathrm{~m~s^{-1}}$ hits an iron nail of mass 100 g. If the specific heat of iron is 0.45 cal $\mathrm{g^{-1}\ {}^\circ C^{-1}}$ and one-fourth the energy is converted into heat and went into nail. Then, the rise in the temperature of nail is

A) $3.5^\circ \mathrm{C}$

B) $7.2^\circ \mathrm{C}$

C) $10.5^\circ \mathrm{C}$

D) $12.1^\circ \mathrm{C}$

Answer: $3.5^\circ \mathrm{C}$

Solution: If we heat-up an object, then, the energy is used to either change the temperature of the body or to change the state of the body.
If it changes the temperature, then,

$Q = mS\Delta T$
where, Q is the heat given, m is the mass and ΔT is the change in temperature.

We know that a moving object has an energy in the form of kinetic energy, i.e.,
$KE = \frac{1}{2}mv^2$

When that moving object strikes another object, it loses some energy which changes into the heat energy and this heat energy increases the temperature of the object.
Given a hammer of 1.5 kg is moving with $60 \mathrm{~m~s^{-1}}$.
Kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}(1.5)(60)^2 = 2700$ J

Also, we know that, 1 cal $= 4.2$ J
One-fourth of kinetic energy changes into heat energy.
$\Rightarrow \frac{1}{4}(KE) =$ Heat energy
$\Rightarrow \frac{1}{4}(KE) = m(S)(\Delta T)$
$\Rightarrow \frac{1}{4}(2700) = (100)(0.45 \times 4.2)(\Delta T)$
$\Rightarrow \Delta T \approx 3.5^\circ \mathrm{C}$

Q.13. A particle of charge $-q$ revolves around a long cylinder radius of radius R charge density ρ. Find the kinetic energy of the charge

A) $\frac{q\rho R^2}{4\varepsilon_0}$

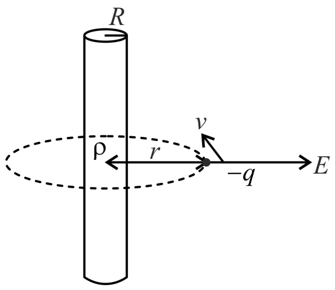
B) $\frac{q\rho R^2}{2\varepsilon_0}$

C) $\frac{q\rho R^2}{16\varepsilon_0}$

D) $\frac{q\rho R^2}{\varepsilon_0}$

Answer: $\frac{q\rho R^2}{4\varepsilon_0}$

Solution:



Electric field due to a long cylinder is,

$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\rho A}{2\pi\varepsilon_0 r}$

If a charge $-q$ is rotating in a circle of radius r

$\frac{mv^2}{r} = qE \Rightarrow \frac{mv^2}{r} = q\frac{\rho A}{2\pi\varepsilon_0 r} \Rightarrow \frac{1}{2}mv^2 = \frac{q\rho\pi R^2}{4\pi\varepsilon_0} = \frac{q\rho R^2}{4\varepsilon_0}$

Q.14. A block of $5\ \mathrm{kg}$ is thrown vertically upwards is experiences constant air resistance $= 10\ \mathrm{N}$. Find the ratio of the time of ascent and descent

A) $\frac{2}{3}$

B) $\frac{1}{1}$

C) $\frac{\sqrt{2}}{\sqrt{3}}$

D) $\frac{\sqrt{3}}{\sqrt{2}}$

Answer: $\frac{\sqrt{2}}{\sqrt{3}}$

Solution: Net force while going upward will be $mg+R=50+10=60\ \mathrm{N}$

$\Rightarrow a_{up}=\frac{60}{5}=12\ \mathrm{m\ s^{-2}}$

Similarly, Net force while going downward will be $mg-R=40\ \mathrm{N}$

$\Rightarrow a_d=\frac{40}{5}=8\ \mathrm{m\ s^{-2}}$

Since

$h=\frac{1}{2}at^2$

$\Rightarrow \frac{a_{up}{t_{up}}^2}{a_d{t_d}^2}=1 \Rightarrow \frac{t_{up}}{t_d}=\sqrt{\frac{8}{12}}=\sqrt{\frac{2}{3}}$

Q.15. Two massless springs, with spring constants $2k$ and $9k$ respectively is attached with 50 g and 100 g mass at free end. Both have same maximum velocity, then find the ratio of amplitudes of vibrations.

A) $\frac{3}{2}$

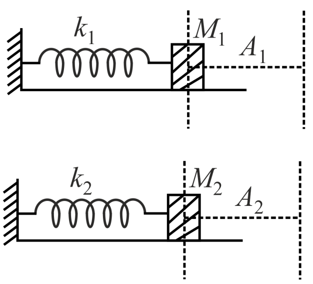
B) $\frac{4}{3}$

C) $\frac{1}{3}$

D) $\frac{1}{2}$

Answer: $\frac{3}{2}$

Solution: Maximum velocity is given by $v_{\max}=\omega A=\sqrt{\frac{k}{m}}A$



Here, $\sqrt{\frac{k_1}{m_1}}A_1=\sqrt{\frac{k_2}{m_2}}A_2$

$\Rightarrow \sqrt{\frac{2k}{50\times10^{-3}}}A_1=\sqrt{\frac{9k}{100\times10^{-3}}}A_2$

$\Rightarrow 2A_1=3A_2$

Thus, ratio of amplitude is $\frac{A_1}{A_2}=\frac{3}{2}$.

Q.16. Two different substances A and B of equal masses 10^{-12} kg are undergoing radioactive decay having half lives 4 s and 8 s respectively. Ratio of molecular weight of A and B is $2:1$. Find the ratio of the active nuclei after 16 s.

A) $\frac{1}{2}$

B) $\frac{1}{4}$

C) $\frac{1}{8}$

D) $\frac{1}{16}$

Answer: $\frac{1}{8}$

Solution:

Ratio of number of moles of element in the beginning will be, $\frac{(N_0)_A}{(N_0)_B}=\frac{\frac{10^{-2}\text{ kg}}{M_A}}{\frac{10^{-2}\text{ kg}}{M_B}}=\frac{M_B}{M_A}=\frac{1}{2}$.

Number of half-life in 16 s for element A will be, $n_A=\frac{16}{4}=4$

Number of half-life in 16 s for element B will be, $n_A=\frac{16}{8}=2$

Hence, ratio of number of nuclei after 16 s will be, $=\frac{(N_0)_A\times\left(\frac{1}{2}\right)^{n_A}}{(N_0)_B\times\left(\frac{1}{2}\right)^{n_B}}=\frac{1}{2}\times\left(\frac{1}{2}\right)^{4-2}=\frac{1}{8}$

Q.17. Photon of 3.8 eV and 1.4 eV are bombarded on metal with work function 0.6 eV. Then the ratio of maximum velocities of ejected electrons are,

A) $2:1$

B) $3:1$

C) $1:2$

D) $1:3$

Answer: $2:1$

Solution: Expression for maximum kinetic energy can be written as, $(KE)_{max}=E-\phi(KE)_{max,\,1}=E_1-\phi=3.8-0.6=3.2\text{ eV}$, $(KE)_{max,\,2}=E_2-\phi=1.4-0.6=0.8\text{ eV}$

$$\frac{(KE)_{max,\,1}}{(KE)_{max,\,2}}=\frac{3.2}{0.8}=4\Rightarrow 4=\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2}$$

$$\Rightarrow\frac{v_1^2}{v_2^2}=4\Rightarrow\frac{v_1}{v_2}=2:1$$

Q.18. A square coil with number of turns 1000 and area 1 m^2 is present in a uniform magnetic field of 0.07 T. It is rotating along its vertical diameter with one revolution per second, find the maximum emf generated.

A) 439.6 V

B) 460 V

C) 489 V

D) 389 V

Answer: 439.6 V

Solution: The maximum emf generated is $\varepsilon_{\max}=NBA\omega,=1000\times0.07\times1\times2\pi(1)=140\pi$

Thus, maximum emf is 439.6 V

Q.19. Two identical charged objects each of mass m and Q are separated by a distance L on a table with coefficient of friction 0.25. If charges are in equilibrium, find the value of L.

A) $2\sqrt{\frac{k}{mg}}Q$

B) $3\sqrt{\frac{k}{mg}}Q$

C) $4\sqrt{\frac{k}{mg}}Q$

D) $5\sqrt{\frac{k}{mg}}Q$

Answer: $2\sqrt{\frac{k}{mg}}Q$

Solution: Normal reaction applied by table on object will be, $N = mg$.

Both charges are same, hence a repulsive force will act between them.

Value of force acting will be, $\frac{kQ^2}{L^2}$.

Now the maximum value of friction force which table can provide is,

$= \mu N = \mu mg$

For equilibrium, $\frac{kQ^2}{L^2} = \mu mg$.

$L = \sqrt{\frac{kQ^2}{\mu mg}} = \sqrt{\frac{k}{0.25mg}}Q = 2\sqrt{\frac{k}{mg}}Q$

Q.20. A glass slab shows the lateral displacement of $4\sqrt{3}\ \text{cm}$, when a light is incident at an angle of 60°. The refractive index of slab is $\sqrt{3}$ and the light emerges parallel to its original path, then the thickness of slab would be __.

A) $12\ \text{cm}$

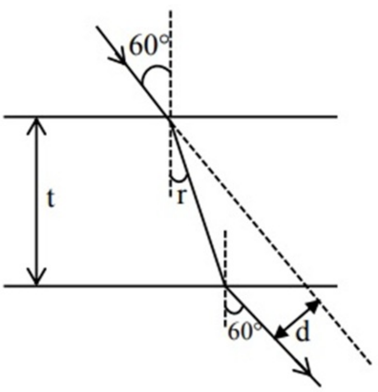
B) $5\ \text{cm}$

C) $6\ \text{cm}$

D) $8\ \text{cm}$

Answer: $12\ \text{cm}$

Solution:



From snell's law, $\frac{\sin i}{\sin r} = \sqrt{3} \Rightarrow \frac{\sin 60^\circ}{\sin r} = \sqrt{3} \Rightarrow r = 30^\circ$

From the diagram shown, $d = \frac{t\sin(i-r)}{\cos r} \Rightarrow \frac{t\times\frac{1}{2}}{\frac{\sqrt{3}}{2}} = 4\sqrt{3} \Rightarrow t = 12\ \text{cm}$

Q.21. If $A = 2\ \Omega,\ B = 4\ \Omega\ \&\ C = 6\ \Omega$. Arrange them such that equivalent resistance is $\mathrm{Req} = \frac{22}{3}\ \Omega$.

A) A and B parallel and C in series

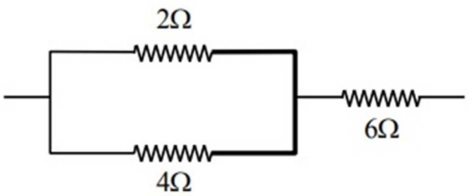
B) A and C series and B parallel

C) B and C series and A parallel

D) B and C parallel and A in series

Answer: A and B parallel and C in series

Solution:



$R_{eq} = \frac{2 \times 4}{2+4} + 6 = \frac{22}{3}\ \Omega$

Q.22. Which of the following are having the same dimensions?

A) Velocity gradient and decay constant

B) Angular velocity and angular momentum

C) Impulse and force

D) Wein's constant and Stephan's constant

Answer: Velocity gradient and decay constant

Solution: Decay constant is measured per unit time, $\lambda = s^{-1}$

So, its dimension is $[\lambda] = [T^{-1}]$

We know velocity gradient is defined as rate of change in velocity per unit of distance. So, dimension of velocity gradient is

$\left[\frac{dv}{dx}\right] = \frac{[L^1T^{-1}]}{[L^1]} = [T^{-1}]$

Section B: Chemistry

Q.1. Which of the following is not a condensation polymer?

A) Nylon $-6,6$

B) Nylon -6

C) Dacron

D) Buna $-S$

Answer: Buna $-S$

Solution: (a) Nylon $-6,6$ is the polymer of hexamethylene diamine and adipic acid. It is made by the condensation of given monomers.

(b) Nylon -6 os also a condensation polymer of caprolactum. It is a polymide.

(c) Dacron is also a condensation polymer of ethylene glycol and terephthalic acid. It is an example of polyester.

(d) Buna $-S$ is an addition polymer of $1,3,$ butadiene and styrene.

Hence, only (4) is not a condensation polymer.

$\therefore$ (4) is the correct option.

Q.2. H_2 gas is produced in the preparation of

A) Na_2CO_3

B) $NaOH$

C) Na metal

D) $NaHCO_3$

Answer: $NaOH$

Solution: $NaOH$ is produced in Nelson cell using Brine solution.

Reaction taking place will be

Cathode: $2H_2O + 2e^- \rightarrow H_2 + 2\,OH^-$

Anode: $2\,Cl^- \rightarrow Cl_2 + 2e^-$

In solution $Na^+ + OH^- \rightarrow NaOH$

So, H_2 gas is released at cathode during preparation of $NaOH$.

Q.3. Which of the following is not a green House gas?

A) H_2O vapour

B) O_3

C) N_2

D) CH_4

Answer: N_2

Solution: About 75% of the solar energy reaching the earth is absorbed by the earth's surface, which increases its temperature. The rest of the heat radiates back to the atmosphere. Some of the heat is trapped by gases such as carbon dioxide, methane, ozone, chlorofluorocarbon compounds (CFCs) and water vapour in the atmosphere. Thus, they add to the heating of the atmosphere. This causes global warming. Hence, Nitrogen gas is not responsible for green house effect.

Q.4. Among the following which one has the highest melting point?

A) Ga

B) Ag

C) Hg

D) Ba

Answer: Ag

Solution: Transition metals have high melting point due to their high enthalpy of atomisation. They form strong metallic bonds with unpaired electrons.

Melting point of $\mathrm{Ag} = 961.8^{\circ}\mathrm{C}$

Melting point of $\mathrm{Ba} = 727^{\circ}\mathrm{C}$

Melting point of $\mathrm{Ga} = 30^{\circ}\mathrm{C}$

Melting point of $\mathrm{Hg} = -38.83^{\circ}\mathrm{C}$

Ga and Hg are liquid at room temperature.

Among the given options Ag has the maximum Melting point, as it belongs to transition metal series.

Q.5. The value of CFSE is maximum for

A) $\left[\mathrm{Mo}\left(\mathrm{H_2O}\right)_6\right]^{3+}$

B) $\left[\mathrm{Cr}\left(\mathrm{H_2O}\right)_6\right]^{3+}$

C) $\left[\mathrm{Mn}\left(\mathrm{H_2O}\right)_6\right]^{3+}$

D) $\left[\mathrm{Fe}\left(\mathrm{H_2O}\right)_6\right]^{3+}$

Answer: $\left[\mathrm{Mo}\left(\mathrm{H_2O}\right)_6\right]^{3+}$

Solution: $\mathrm{H_2O}$ is a weak field ligand. Crystal field stabilisation energy is maximum for d^3 and d^8 cases with weak field ligands among the 3d-series elements.But CFSE increases from 3d to 4d series elements due to the increase in nuclear charge. Hence, $\left[\mathrm{Mo}\left(\mathrm{H_2O}\right)_6\right]^{3+}$ complex case, the CFSE is maximum.

Q.6. Which one is NOT the enamel of teeth?

A) Ca^{+2}

B) P^{+3}

C) F^{-}

D) P^{+5}

Answer: P^{+3}

Solution: For drinking purposes water should be tested for fluoride ion concentration. Its deficiency in drinking water is harmful to man and causes diseases such as tooth decay etc. Soluble fluoride is often added to drinking water to bring its concentration up to $1 \text{ ppm } \left(1 \text{ mg dm}^{-3}\right)$. The F^- ions make the enamel on teeth much harder by converting hydroxyapatite, $\left[3\left(Ca_3(PO_4)_2 \cdot Ca(OH)_2\right]\right.$, the enamel on the surface of the teeth, into much harder fluorapatite, $\left[3\left(Ca_3(PO_4)_2 \cdot CaF_2\right]\right.$

In the above compounds, the ions involved are Ca^{2+}, P^{+5}, F^- etc.,

Q.7. Which metal gives blue flame test?

A) Cesium

B) Lithium

C) Barium

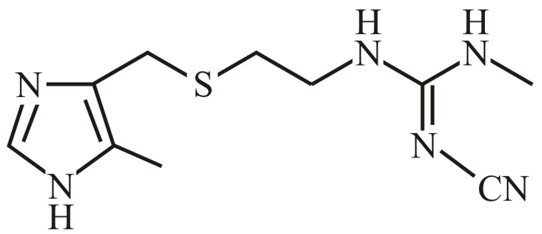
D) Strontium

Answer: Cesium

Solution:

Metal	Flame colour
Li	Crimson red
Na	Yellow
K	Violet
Rb	Violet red
Cs	Blue
Ba	Apple green
Sr	Crimson

Q.8. Compound in the given structure is



A) Codeine

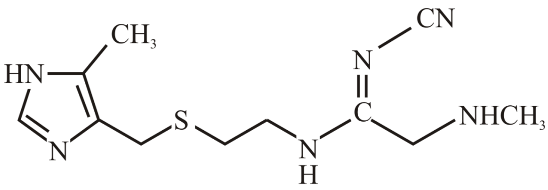
B) Morphine

C) Ranitidine

D) Cimetidine

Answer: Cimetidine

Solution:



The given structure is of Cimetidine. Cimetidine is an antihistamine drug which is used to treat stomach acidity and peptic ulcers. It is a sulphur-containing derivative of imidazole.

Q.9. Which of the following is not a sulphide ore?

A) Baryte

B) Galena

C) Zinc blende

D) Copper pyrites

Answer: Baryte

Solution: Baryte is a mineral consisting of barium sulphate $(BaSO_4)$

Galena, also called lead glance, is the natural mineral of lead (II) Sulphate (PbS)

Zinc blende, is a mineral consisting of ZnS.

Copper pyrites is a copper iron sulphide mineral.

It has the chemical formula $CuFeS_2$

Q.10. Correct order of bond order of C_2^{2-}, N_2^{2-} and O_2^{2-}

A) $C_2^{2-} > N_2^{2-} > O_2^{2-}$

B) $O_2^{2-} > N_2^{2-} > O_2^{2-}$

C) $N_2^{2-} > O_2^{2-} > C_2^{2-}$

D) $C_2^{2-} > O_2^{2-} > N_2^{2-}$

Answer: $C_2^{2-} > N_2^{2-} > O_2^{2-}$

Solution: Bond order $= \frac{N_B - N_{A.B}}{2}$

$N_B =$ Number of electrons in bonding molecular orbitals.

$N_{A.B} =$ Number of electrons in anti bonding molecular orbitals.

C_2^{2-} has 14 electrons, it is isoelectronic with N_2 where Bond order $= 3$,

N_2^{2-} has 16 electrons is isoelectronic with O_2 whose bond order $= 2$

O_2^{2-} has 18 electrons and is isoelectronic with F_2 whose bond order $= 1$

So, the correct order of Bond order is

$C_2^{2-} > N_2^{2-} > O_2^{2-}$.

Q.11. Volume occupied by 3 grams of gas A at $300\ K$ is the same as occupied by 0.2 grams of H_2 gas at $200\ K$, what is the molar mass of A gas.

A) $90 \mathrm{~g}$

B) $22.5 \mathrm{~g}$

C) $45 \mathrm{~g}$

D) $180 \mathrm{~g}$

Answer: $45 \mathrm{~g}$

Solution: $PV = nRT$

$V = \frac{nRT}{P}$

$V_{H_2} = V_A \left(\text{Given}\right)$

$\left(\frac{nRT}{P}\right)_A = \left(\frac{nRT}{P}\right)_{H_2}$

$\frac{3 \times 300}{M_A} = \frac{0.2}{2} \times 200$

Hence, the molar mass of A, $M_A = 45g$

Q.12. Which of the following is used in fire extinguisher?

A) Banking soda

B) Washing soda

C) Caustic sods

D) Soda ash

Answer: Banking soda

Solution: $2\,NaHCO_3(aq) + H_2SO_4(aq) \rightarrow Na_2SO_4(aq) + 2H_2O(l) + 2\,CO_2(g)$

When the fire extinguisher is operated by pressing the knob on it, the sulphuric acid gets mixed with sodium bicarbonate solution producing a lot of carbon dioxide gas. Carbon dioxide gas is neither combustible nor helps combustion.

Q.13. PCl_5 exists but NCl_5 does not. Which of the following statement correctly explains the above?

A) N - does not have vacant d-orbital

B) P - does not have $2d$ orbitals

C) Back bonding in NCl_5 is not possible

D) N atom is more electronegative so does not forms 5 bonds

Answer: N - does not have vacant d-orbital

Solution: Due to the absence of vacant d orbitals nitrogen cannot form penta halides, so NCl_5 is not formed but PCl_5 is formed as phosphorous has vacant d orbitals

Q.14. **Statement I:** π-bond makes the compound unstable.

Statement II: Bond strength of C=C(double bond) is more than C-C(single bond)

A) S1 and S2 both are correct and S2 is correct explanation of S1

B) S1 and S2 both are correct and S2 is not correct explanation

C) S1 is false and S2 is correct

D) S1 is correct and S2 is false

Answer: S1 is false and S2 is correct

Solution: A double bond is composed of a $\pi-$bond and a $\sigma-$bond. In order to break a bond like this, we must provide more energy than the molecule with $\sigma-$bond alone. i.e., $\pi-$bond makes the compound more stable. Hence, the statement-1 is false and statement-2 is true.

Q.15. If, $\mathrm{K}=$ Kinetic energy of H-atom.

$\mathrm{P}=$ Potential energy of H-atom.

$\mathrm{T}=$Total energy of H-atom.

As the principle quantum number increases, then which of the following option is correct?

A) K and P increased but T decreased

B) P and T increased but K decreased

C) All increase

D) All decrease

Answer: P and T increased but K decreased

Solution: According to Bohr's theory for H-atom

$$\mathrm{T.E.}\ =\ -13.6\frac{1}{\mathrm{n}^2}\mathrm{eV}$$

So as value of n increases, value of Total energy also increases.

$$\mathrm{P.E.}\ =\ -27.2\frac{1}{\mathrm{n}^2}\mathrm{eV}$$

So as value of n increases, value of potential energy also increases.

$$\mathrm{K.E.}\ =\ 13.6\frac{1}{\mathrm{n}^2}\mathrm{eV}$$

So as value of n increases, value of kinetic energy decreases.

Q.16. Number of peptide linkages in the given sequence is:

$\mathrm{Ala}-\mathrm{Val}-\mathrm{Gly}-\mathrm{Lys}$

A) 5

B) 4

C) 3

D) 6

Answer: 3

Solution: A peptide bond is a chemical bond formed between two molecules when the carboxyl group of one molecule reacts with the amino group of the other molecule, releasing a molecule of water. This linkage is found along a peptide or protein chain.

There are four amino acids in the given sequence, so they will have three peptide bonds in between them.

Q.17. For the equilibrium , $\mathrm{A(g)} \leftrightharpoons \mathrm{B(g)}, \triangle\mathrm{H}=-40\,\mathrm{kJ}/\mathrm{mol}$. The ratio of the activation energies of the forward $(\mathrm{E_{af}})$ and reverse $(\mathrm{E_{ab}})$ reactions is $2/3$ then, what is $\mathrm{E_{af}}$

A) 80

B) 120

C) 40

D) 20

Answer: 80

Solution: $\Delta H = -40\,\text{kJ mole}$

$\Delta H = E_{a_f} - E_{a_b}$

(E_{a_f} =activation energy of forward reaction)

(E_{a_b} =activation energy of backward reaction)

$E_{a_f} : E_{a_b} = \frac{2}{3} : 1$

$-40 = \frac{2}{3}x - x$

$-40 = \frac{-x}{3}$

$x = 120 \text{ kJ / mol}$

$E_f = \frac{2}{3}x = \frac{2}{3} \times 120$

$= 80 \text{ kJ / mole}$

Q.18. Which of the following sequence of reagent can perform the following conversion?

$CH_3 - CH_2 - CH_2 - OH \longrightarrow CH_3 - CH_2 - CH_2 - CH_2 - NH_2$

A) $SOCl_2, KCN, H_2/Pd$

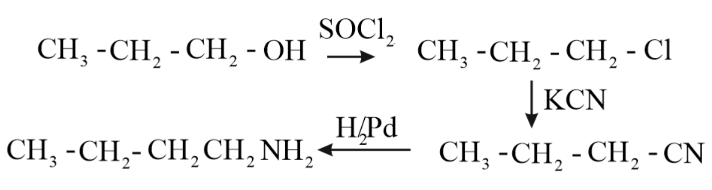
B) $SOCl_2, AgCN, H_2/Pd$

C) $PCl_5, AgCN, H_2/Pd$

D) $\text{Red P}/HI, KCN, H_2/Pd$

Answer: $SOCl_2, KCN, H_2/Pd$

Solution: Propyl alcohol undergo nucleophilic substitution reaction with thionyl chloride to give propyl chloride. Propyl chloride give butane nitrile on reaction with potasium cyanide. Butane nitrile on reduction give butyl amine.



Q.19. How many half life are required to complete 90% of the reaction? [Given the reaction is first order]

A) 3.32

B) 2.07

C) 1.44

D) 4.02

Answer: 3.32

Solution:

$$\left[\frac{\ln 2}{t_{1/2}}\right] t_{90\%} = \ln\left[\frac{100}{10}\right] = \ln 10$$

$$t_{90\%} \times \frac{0.693}{t_{1/2}} = 2.303$$

$$t_{90\%} = \frac{2.303}{0693} \times t_{1/2} = 3.32\, t_{1/2}$$

Q.20. Wavelength of a single photon is $300\ \mathrm{nm}$. Energy of N_A number of such photon is $x \times 10^5 \mathrm{J}$. Find the value of x.

A) 1

B) 2

C) 4

D) 5

Answer: 4

Solution: $\lambda = 300 \times 10^{-9} \mathrm{m}$

Energy of single photon $= \frac{hc}{\lambda}$

Energy of N_A photons $= \frac{6.02 \times 10^{23} \times 6.6 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}}$

$= 4 \times 10^5$

So, $x = 4$

Q.21. A conductivity cell filled with $0.1\mathrm{M}$ electrolyte gives at $25^\circ \mathrm{C}$ a resistance of 20 ohms. If the molar conductivity of solution is $0.154 \times 10^{-3}\ \Omega^{-1} \mathrm{cm}^2 \mathrm{mol}^{-1}$, what is the cell constant?

A) $3.08 \times 10^{-7}\ \mathrm{cm}^{-1}$

B) $30.8 \times 10^{-7}\ \mathrm{cm}^{-1}$

C) $3.08 \times 10^{-9}\ \mathrm{cm}^{-1}$

D) $4.08 \times 10^{-5}\ \mathrm{cm}^{-1}$

Answer: $3.08 \times 10^{-7}\ \mathrm{cm}^{-1}$

Solution: Given,

Resistance, $\mathrm{R} = 20\ \Omega$

Molar conductivity, $\lambda_{\mathrm{m}} = 0.154 \times 10^{-3}\ \Omega^{-1}\mathrm{cm}^{2}\ \mathrm{mol}^{-1}$

Concentration, $\mathrm{C} = 0.1\ \mathrm{M}$

The relation between molar conductivity and conductivity is as follows:

$\lambda_{\mathrm{m}} = \frac{1000 \times \mathrm{k}}{\mathrm{C}}$

$0.154 \times 10^{-3} = \frac{1000 \times \mathrm{k}}{0.1}$

$\mathrm{k} = \frac{0.154 \times 10^{-3} \times 0.1}{1000} = 0.154 \times 10^{-7}\ \Omega^{-1}\mathrm{cm}^{-1}$

The relation between conductivity and cell constant is as follows:

$\mathrm{k} = \frac{1}{\mathrm{R}} \times \frac{\mathrm{l}}{\mathrm{A}}$

$0.154 \times 10^{-7} = \frac{1}{20} \times \frac{\mathrm{l}}{\mathrm{A}}$

Cell constant, $\frac{\mathrm{l}}{\mathrm{A}} = 0.154 \times 20 \times 10^{-7}\ \mathrm{cm}^{-1} = 3.08 \times 10^{-7}\ \mathrm{cm}^{-1}$

Section C: Mathematics

Q.1. If $\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{4}\cos^2(2x)$. Then find Number of solution if $x\in[-3\pi,3\pi]$

A) 8

B) 7

C) 9

D) 0

Answer: 7

Solution: Given,

$\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{4}\cos^2(2x)$

$\Rightarrow 2\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{2}\cos^2(2x)$

$\Rightarrow \cos(2x)+\cos\left(\frac{2\pi}{3}\right)=\frac{1}{2}\cos^2 2x$

$\Rightarrow \cos 2x+\left(-\frac{1}{2}\right)=\frac{1}{2}\cos^2 2x$

$\Rightarrow \cos^2 2x-2\cos 2x+1=0$

$\Rightarrow (\cos 2x-1)^2=0$ or $\cos 2x=1$

$\Rightarrow 2x=-6\pi,-4\pi,-2\pi,0,2\pi,4\pi,6\pi$

So, $x\in\{-3\pi,-2\pi,-\pi,0,\pi,2\pi,3\pi\}$

So total 7 solutions.

Q.2. The number of seven digit numbers which are multiples of 11 using the digits $1,2,3,4,5,6$ and 9 without repetition will be

A) 432

B) 216

C) 144

D) 864

Answer: 432

Solution: For the seven digit number $ABCDEFG$ to be divisible by 11,

$(A+C+E+G)-(B+D+F)=-11$ or 0 or 11

We know, $(A+C+E+G)+(B+D+F)=30$

Let $A+C+E+G=x$ and $B+D+F=y$

So, $x+y=30$.

If $x-y=-11$, no possible values of x

Similarly if $x-y=11$, no possible values of x

Only possible case is $x-y=0$

i.e. $x=y=15 \Rightarrow A+C+E+G=B+D+F=15$

For $B+D+F=15$, the possible cases are $1,5,9$; $2,4,9$; $4,5,6$

Hence, the total number of seven digit numbers are $3\times 3!\times 4!=3\times 6\times 24=432$

Q.3. Find the Remainder when $1+3+3^2...+3^{2021}$ is divided by 50

A) 8

B) 4

C) 12

D) 16

Answer: 4

Solution: Let $S=1+3+3^2+\cdots\cdot 3^{2021}$

$=\frac{3^{2022}-1}{3-1}=\frac{3^{2022}-1}{2}$

Simplifying by using binomial expansion we get $\frac{(3^2)^{1011}-1}{2}=\frac{(10-1)^{1011}-1}{2}$

$\Rightarrow \frac{{}^{1011}C_0 10^{1011}-{}^{1011}C_1 10^{1010}+...+{}^{1011}C_{1010}10^1-{}^{1011}C_{1011}\times 1-1}{2}$

$\frac{100q+1011\times 10-2}{2}$

$\Rightarrow \frac{100q+10110-2}{2}=\frac{100q+10108}{2}=50q+5054$

Now dividing $50q+5054$ by 50

We get 4 as remainder.

Hence option B is correct.

Q.4. The number of real roots of the polynomial $P(x)=x^7-2x+3$ is

A) 0

B) 1

C) 3

D) 7

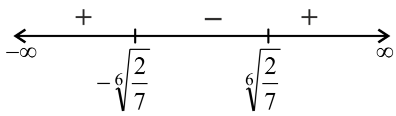
Answer: 1

Solution: We have, $p(x)=x^7-2x+3$

$p'(x)=7x^6-2$

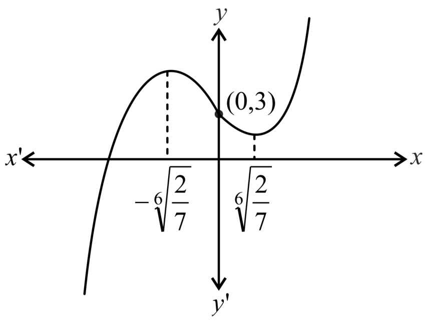
To find maxima (or) minima, put $p'(x)=0$.

$\Rightarrow 7x^6-2=0 \ \Rightarrow x^6=\frac{2}{7} \ \Rightarrow x=\pm\sqrt[6]{\frac{2}{7}}$



It is clear from the sign-scheme method, local maximum occur at $x=-\sqrt[6]{\frac{2}{7}}$ and local minimum occur at $x=\sqrt[6]{\frac{2}{7}}$

Also, $P\left(+\sqrt[6]{\frac{2}{7}}\ \right)=\left(\frac{2}{7}\right)^{\frac{7}{6}}-2\left(\frac{2}{7}\right)^{\frac{1}{6}}+3>0$



It is clear from the diagram, $P(x)$ has only one real root.

Q.5. If $x^3y^2=2^{15}, x, y \in R^+$ then the minimum value of $3x+4y$ is

A) $20\times 2^{2/5}$

B) $40\times 2^{2/5}$

C) $60\times 2^{2/5}$

D) $80\times 2^{2/5}$

Answer: $40\times 2^{2/5}$

Solution: $\frac{x+x+x+2y+2y}{5}\geq\sqrt[5]{x^3\cdot(2y)^2}$

$\Rightarrow \ \frac{3x+4y}{5}\geq\sqrt[5]{4x^3y^2}$

$\Rightarrow \ \frac{3x+4y}{5}\geq\sqrt[5]{4\left(2^{15}\right)} \qquad (\because \ \ x^2y^2=2^{15})$

$\Rightarrow \ 3x+4y\geq 5\cdot(2)^{17/5}$

$\Rightarrow \ 3x+4y\geqslant 5\times 2^3\times 2^{2/5}$

$\Rightarrow \ 3x+4y\geqslant 40\times 2^{2/5}$

$\therefore$ The minimum value of $3x+4y$ is $40\times 2^{2/5}$

Q.6.

X	0	1	2	3	4
$P(x)$	k	$2k$	$3k$	$4k$	$5k$

Then find the value of $P\left(\frac{1<x<4}{x\leq 2}\right)$

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{2}{5}$

Answer: $\frac{1}{2}$

Solution: To find $P\left(\frac{1<x<4}{x\leq 2}\right)$ or $P\left(\frac{A}{B}\right)$

We know that

$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

Given

x	0	1	2	3	4
$P(x)$	k	$2k$	$3k$	$4k$	$5k$

$P(A)=\{2,3\}$

$P(B)=\{0,1,2\}$

$P(A\cap B)=P(x=2)$

$P(B)=P(x=0)+P(x=1)+P(x=2)$

So $P\left(\frac{A}{B}\right)=\frac{P(x=2)}{P(x=0)+P(x=1)+P(x=2)}$

$=\frac{3k}{k+2k+3k}=\frac{3k}{6k}=\frac{1}{2}$

Hence option A is correct.

Q.7. The area between the curves $y^2=2x$ and $x+y=4$ is

A) 10

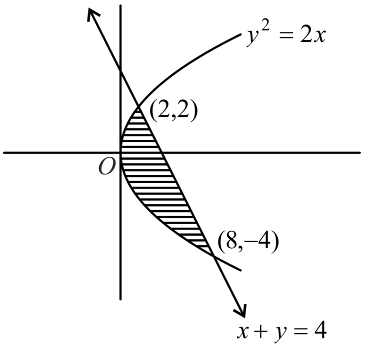
B) 18

C) 21

D) $\frac{13}{3}$

Answer: 18

Solution:



$y^2 = 2x \ \Rightarrow y^2 = 2(4 - y)$

$\Rightarrow y^2 + 2y - 8 = 0 \ \Rightarrow y = -4, 2$

The point of intersection of $y^2 = 2x$ and $x + y = 4$ are $(8, -4)$ and $(2, 2)$

Area $= \int_{-4}^{2} \left((4 - y) - \frac{y^2}{2}\right) dy \ = \left(4y - \frac{y^2}{2} - \frac{y^3}{6}\right)_{-4}^{2}$

$= \left(8 - 2 - \frac{4}{3}\right) - \left(-16 - 8 + \frac{32}{3}\right)$

$= \frac{54}{3} = 18$ Sq.units.

Q.8. If the length of latus rectum of $H \equiv \frac{x^2}{a^2} - y^2 = 1$ and $E \equiv 3x^2 + 4y^2 = 12$ are equal then find the value of $12\left(e_E^2 + e_H^2\right)$

A) 48

B) 36

C) $\frac{41}{2}$

D) 42

Answer: 42

Solution: Given

$E \equiv 3x^2 + 4y^2 = 12 \ \Rightarrow \ \frac{x^2}{4} + \frac{y^2}{3} = 1$

Now, $e_E = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

Length of L.R. $= \frac{2b^2}{a} = \frac{2 \times 3}{2} = 3$

Now $H \equiv \ \frac{x^2}{a^2} - \frac{y^2}{1} = 1$

Length of L.R. $= \frac{2b^2}{a} = \frac{2 \times 1}{a}$

Given length of L.R. of E and H are equal

So $\frac{2}{a} = 3 \ \Rightarrow \ a = \frac{2}{3}$

Now $e_H = \sqrt{\frac{b^2}{a^2} + 1} = \sqrt{\frac{1}{\left(\frac{2}{3}\right)^2} + 1} \ \Rightarrow \ e_H = \sqrt{\frac{9}{4} + 1} = \sqrt{\frac{13}{4}}$

So $12\left(e_H^2 + e_E^2\right) = 12\left(\frac{13}{4} + \frac{1}{4}\right) = 12\left(\frac{14}{4}\right) = 14 \times 3 = 42$

So option D is correct.

Q.9. The solution of the equation $\log_{(x+1)}\left(x^2-x+6\right)^2=4$ is

A) $\frac{5}{3}$

B) $\frac{3}{5}$

C) $\frac{8}{3}$

D) $\frac{2}{5}$

Answer: $\frac{5}{3}$

Solution: We have, $\log_{(x+1)}\left(x^2-x+6\right)^2=4$

Clearly, the above equation is defined when $x>-1,\ x\neq 0$ and $x^2-x+6\neq 0$.

Since, discriminant of $x^2-x+6=0$ is negative. So, $x^2-x+6>0\ \forall x\in R$.

Hence, domain of the given equation is $x\in(-1,\ \infty)-\{0\}$

$\Rightarrow\ \left(x^2-x+6\right)^2=(x+1)^4\ \Rightarrow\left(x^2-x+6\right)^2=\left(x^2+2x+1\right)^2$

$\Rightarrow\left(x^2-x+6\right)^2-\left(x^2+2x+1\right)^2=0$

$\Rightarrow\left[\left(x^2-x+6\right)-\left(x^2+2x+1\right)\right]\left[\left(x^2-x+6\right)+\left(x^2+2x+1\right)\right]=0$

$\Rightarrow\ (-3x+5)\left(2x^2+x+7\right)=0$

$\Rightarrow\ 3x-5=0$ or $2x^2+x+7=0$

$\Rightarrow x=\frac{5}{3}$ or $2x^2+x+7=0$

Now, clearly $2x^2+x+7=0$ does not have real roots because it's discriminant $D=(1)^2-4(2)(7)=-55<0$.

Clearly, $x=\frac{5}{3}$ lies in the domain $x\in(-1,\ \infty)-\{0\}$.

Hence, the solution of the given equation is $\frac{5}{3}$.

Q.10. The sum of roots of equation $\left(e^{2x}-4\right)\left(6e^{2x}-5e^{x}+1\right)=0$ is

A) $\ln 4$

B) $\ln 3$

C) $-\ln 3$

D) $\ln 5$

Answer: $-\ln 3$

Solution: Let $e^x = t$ so, $e^{2x} = t^2$

So, the given equation becomes, $\left(t^2 - 4\right)\left(6t^2 - 5t + 1\right) = 0$

$t^2 - 4 = 0$ (or) $6t^2 - 5t + 1 = 0$

$t^2 = 4$ (or) $6t^2 - 3t - 2t + 1 = 0$

$t = \pm 2$ (or) $3t\,(2\,t - 1) - 1\,(2\,t - 1) = 0$

$t = \pm 2$ (or) $(3t - 1)\,(2t - 1) = 0$

$t = \pm 2$ (or) $t = \frac{1}{3}$ (or) $t = \frac{1}{2}$

But, $e^x = t > 0 \ \ (\because \ \ e^x > 0; \forall x \in R)$

$\therefore$ Only possible values for t are $2, \frac{1}{3}, \frac{1}{2}$

When $t = 2$, $e^x = 2 \ \Rightarrow x = \log_e(2)$ (or) $\ln 2$

When $t = \frac{1}{3}$, $e^x = \frac{1}{3} \ \Rightarrow \ x = \log_e\left(\frac{1}{3}\right)$ or $-\ln 3$

When $t = \frac{1}{2}$, $e^x = \frac{1}{2} \Rightarrow x = \log_e\left(\frac{1}{2}\right)$ or $-\ln 2$

$\therefore$ The sum of roots of given equation $= \ln 2 - \ln 3 - \ln 2 = -\ln 3$.

Q.11. The circle $(x-h)^2 + (y-k)^2 = r^2$ touches x-axis at $(1, 0)$ such that $k > 0$, also $x + y = 0$ intersects the circle at two points P and Q. If the length of chord PQ is 2 units, then the value of $h + k + r$ is

A) 3

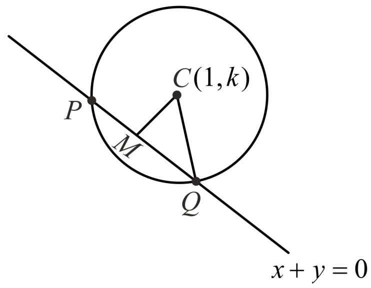
B) 4

C) 6

D) 7

Answer: 7

Solution: Since the circle touches x-axis at $(1, 0)$ so $h = 1$ and $r = k$



In ΔMCQ,

$MQ = 1, CM = \frac{|1+k|}{\sqrt{2}}, CQ = k$

So, $MQ^2 + CM^2 = CQ^2$

$\Rightarrow \ \ 1^2 + \frac{(1+k)^2}{2} = k^2$

$\Rightarrow \ \ k = 3$ or -1 (rejected)

Hence, $h + k + r = 1 + 3 + 3 = 7$

Q.12. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{\left(1+e^{x}\right)\left(\sin^{6} x+\cos^{6} x\right)}$

A) $\frac{\pi}{4}$

B) $\frac{\pi}{2}$

C) π

D) 2π

Answer: π

Solution: $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{\left(1+e^{x}\right)\left(\operatorname{sm}^{6} x+\cos^{6} x\right)} \quad \ldots(\mathrm{i})$

Now we know $\int_{a}^{b} f(x) dx=\int_{a}^{b} f(a+b-x) dx$

$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{x} dx}{\left(1+e^{x}\right)\left(\sin^{6} x+\cos^{6} x\right)} \quad \ldots(\mathrm{ii})$

Adding (i) and (ii)

$2I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\left(1+e^{x}\right) dx}{\left(1+e^{x}\right)\left(\sin^{6} x+\cos^{6} x\right)}$

$I=\frac{1}{2} \times 2 \int_{0}^{\frac{\pi}{2}} \frac{dx}{\left(\sin^{2} x+\cos^{2} x\right)\left(\sin^{4} x+\cos^{4} x-\sin^{2} x \cos^{2} x\right)}$

$=\int_{0}^{\frac{\pi}{2}} \frac{dx}{\left(\sin^{2} x+\cos^{2} x\right)^{2}-3 \sin^{2} x \cos^{2} x}=\int_{0}^{\frac{\pi}{2}} \frac{dx}{1-\frac{3}{4} \sin^{2} 2x}$

$=\int_{0}^{\frac{\pi}{2}} \frac{4 \sec^{2} 2x dx}{4\left(1+\tan^{2} 2x\right)-3 \tan^{2} 2x}=2 \int_{0}^{\frac{\pi}{4}} \frac{4 \sec^{2} 2x}{4+\tan^{2} 2x} dx$

Let $\tan 2x=t$ and $2 \sec^{2} 2x dx=dt$

$I=4 \int_{0}^{\infty} \frac{dt}{2^{2}+t^{2}}=\frac{4}{2}\left[\tan^{-1} \frac{t}{2}\right]_{0}^{\infty}=2\left(\frac{\pi}{2}-0\right)=\pi$

Q.13. If $\Delta_{r}=\begin{vmatrix} 2^{r-1} & \frac{(r+1)!}{\left(1+\frac{1}{r}\right)} & 4r^{3}-2nr \\ a & b & c \\ 2^{n}-1 & (n+1)!-1 & n^{3}(n+1) \end{vmatrix}, n \in N$ then $\sum_{r=1}^{n} \Delta_{r}$ is equal to

A) 0

B) $(n+3)!$

C) abc

D) $a(n!)+b \cdot 2^{n}+c$

Answer: 0

Solution:

$$\sum_{r=1}^{n}\Delta_r=\begin{vmatrix}\sum_{r=1}^{n}2^{r-1} & \sum_{r=1}^{n}\frac{(r+1)!}{\left(1+\frac{1}{r}\right)} & \sum_{r=1}^{n}\left(4r^3-2nr\right)\\ a & b & c\\ 2^n-1 & (n+1)!-1 & n^3(n+1)\end{vmatrix}$$

Now, $\sum_{r=1}^{n}2^{r-1}=2^0+2^1+2^2+\ldots+2^{n-1}=\frac{2^n-1}{2-1}=2^n-1$

$\sum_{r=1}^{n}\frac{(r+1)!}{\left(1+\frac{1}{r}\right)}=\sum_{r=1}^{n}\frac{r(r+1)r!}{(r+1)}=\sum_{r=1}^{n}r(r!)\sum_{r=1}^{n}(r+1-1)r!$

$=\sum_{r=1}^{n}[(r+1)!-r!]$

$=(2!-1!)+(3!-2!)+(4!-3!)+\ldots..+((n+1)!-n!)$

$=(n+1)!-1!=(n+1)!-1$

Now, $\sum_{r=1}^{n}\left(4r^3-2nr\right)$

$=4\sum_{r=1}^{n}r^3-2n\sum_{r=1}^{n}r$

$=4\left[\frac{n(n+1)}{2}\right]^2-2n\left(\frac{n(n+1)}{2}\right)$

$=n^2(n+1)^2-n^2(n+1)$

$=n^2(n+1)[(n+1)-1]=n^3(n+1)$

$$\therefore\quad \sum_{r=1}^{n}\Delta_r=\begin{vmatrix}2^n-1 & (n+1)!-1 & n^3(n+1)\\ a & b & c\\ 2^n-1 & (n+1)!-1 & n^3(n+1)\end{vmatrix}=0$$

$\because\quad R_1=R_3$

Q.14. If S is set of first 100 natural numbers, then find the sum of values of S such that $\{\text{H.C.F. of } 24 \text{ and } S \text{ is } 1\}$

A) 1633

B) 1834

C) 1734

D) 1604

Answer: 1633

Solution: Given $S=\{1,2,3\ldots 100\}$

Now finding the sum of $S=\frac{100\times 101}{2}$

Prime factors of $24=2^3\times 3$

Let $n(A)=$ Multiples of 2

$n(B)=$ Multiples of 3

$n(A\cap B)=$ Multiples of 2 & 3

So $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

To have H.C.F to be 1 we need to subtract the sum of multiples of $2\ \&\ 3$ from sum of set S to get required answer,

So required answer

$=\frac{100\times 101}{2}-$ Sum of $n(A\cup B)$

$=\frac{100\times 101}{2}-\left\{2\times\frac{50\times 51}{2}+\frac{33}{2}(102)-\frac{16}{2}\times 102\right\}=1633$

Q.15. Given $x*y=x^2+y^3$, $(x\ *1)*\ 1$ & $x*(1*1)$ are equal then find the value of $2\sin^{-1}\left[\frac{x^4+x^2-2}{x^4+x^2+2}\right]$

A) $\frac{\pi}{3}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{6}$

D) π

Answer: $\frac{\pi}{3}$

Solution: Given $x*y=x^2+y^3$

Now $(x*1)*\ 1=x*(1*1)$

$\left(x^2+1^3\right)*1=x*\left(1^2+1^3\right)$

$\left(x^2+1\right)^2+1^3=x^2+2^3$

$x^4+1+2x^2+1=x^2+8$

$x^4+x^2-6=0$

Let $x^2=t$

$t^2+t-6=0$

$(t+3)(t-2)=0$

$t\neq-3$ so $t=2$ or $x^2=2$

Now $2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)=2\sin^{-1}\left(\frac{4+2-2}{4+2+2}\right)$

$=2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}\times 2=\frac{\pi}{3}$

Q.16. If the system of equations

$x+y+\alpha z=1$

$x+y+3\,z=2$

$x+2\,z=4$

has unique solution then which of the following is true?

A) $\alpha=3$

B) $\alpha\neq 3$

C) $\alpha\in R$

D) $\alpha\neq -3$

Answer: $\alpha\neq 3$

Solution: If the given system of equation has unique solution then $\Delta\neq 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & \alpha \\ 1 & 1 & 3 \\ 1 & 0 & 2 \end{vmatrix}\neq 0$$

$$\Rightarrow\ 1\,(2-0)-1\,(2-3)+\alpha\,(0-1)\neq 0$$

$$\Rightarrow\ 2+1-\alpha\neq 0\Rightarrow\ \ 3-\alpha\neq 0\Rightarrow \alpha\neq 3.$$

Q.17. $\lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{(n^2+r^2)(n+r)}=$

A) $-\frac{1}{4}\ln 2-\frac{\pi}{8}$

B) $\frac{1}{4}\ln 2-\frac{\pi}{8}$

C) $\frac{1}{4}\ln 2+\frac{\pi}{8}$

D) $\frac{1}{2}\ln 2+\frac{\pi}{4}$

Answer: $\frac{1}{4}\ln 2+\frac{\pi}{8}$

Solution:

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\frac{1}{n}}{\left(1+\frac{r^2}{n^2}\right)\left(1+\frac{r}{n}\right)}=\int_0^1\frac{dx}{(1+x^2)(1+x)}$$

Let $I=\int_0^1\frac{dx}{(1+x^2)(1+x)}$

Now $x=\tan\theta, dx=\sec^2\theta d\theta$

$$I=\int_0^{\frac{\pi}{4}}\frac{d\theta}{1+\tan\theta}=\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{2\cos\theta d\theta}{\sin\theta+\cos\theta}$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{(\sin\theta+\cos\theta)d\theta}{\sin\theta+\cos\theta}+\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{(\cos\theta-\sin\theta)d\theta}{\sin\theta+\cos\theta}$$

$$=\frac{1}{2}\times\frac{\pi}{4}+\frac{1}{2}[\ln|\sin\theta+\cos\theta|]_0^{\frac{\pi}{4}}=\frac{\pi}{8}+\frac{1}{4}\ln 2$$