JEE Main Exam 2022 - Session 1

24 June 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. The normal reaction N for a vehicle of $800\ \mathrm{kg}$ mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is ___ $\times 10^{3}\ \mathrm{kg\ m\,s^{-2}}$. [Given $\cos 30^{\circ}=0.87,\ \mu_{\mathrm{s}}=0.2$]

A) 10

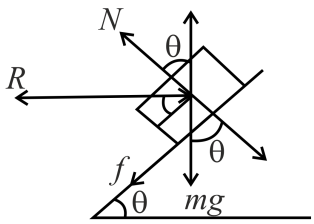
B) 20

C) 30

D) 40

Answer: 10

Solution:



$N\cos\theta = mg + \mu_s N\sin\theta$

$N=\frac{mg}{\cos\theta-\mu_s\sin\theta}=\frac{800\times10}{0.87-0.2\times\frac{1}{2}}$

$\Rightarrow N\simeq 10000\ \mathrm{N}=10\ \mathrm{kN}$

Therefore, answer is 10.

Q.2. Efficiency of carnot engine was 25% at $27\ ^{\circ}\mathrm{C}$. What will be temperature increase required to increase its efficiency by 100%?

A) $200\ \mathrm{K}$

B) $300\ \mathrm{K}$

C) $400\ \mathrm{K}$

D) $500\ \mathrm{K}$

Answer: $200\ \mathrm{K}$

Solution: Efficiency of carnot engine is given by, $\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$.

For initial case: $T_{\text{sink}} = (27 + 273)\ ^{\circ}\mathrm{C} = 300\ \mathrm{K}$

So we can write, $\frac{1}{4} = 1 - \frac{300}{T_1} \Rightarrow T_1 = 400\ \mathrm{K}$

Now for the second case: $T_{\text{sink}} = 300\ \mathrm{K}$

After increasing the previous efficiency by 100%, value of efficiency will get doubled.

Therefore, $\frac{1}{2} = 1 - \frac{300}{T_2} \Rightarrow T_2 = 600\ \mathrm{K}$

Increase in temperature required will be, $600\ \mathrm{K} - 400\ \mathrm{K} = 200\ \mathrm{K}$

Q.3. Stopping potential for electron (e^-) of wavelength $491\ \mathrm{nm}$ is $0.410\ \mathrm{V}$. If incidence wavelength is changed to new value then stopping potential is $1.02\ \mathrm{V}$. Calculate the new wavelength.

A) $396\ \mathrm{nm}$

B) $450\ \mathrm{nm}$

C) $564\ \mathrm{nm}$

D) $296\ \mathrm{nm}$

Answer: $396\ \mathrm{nm}$

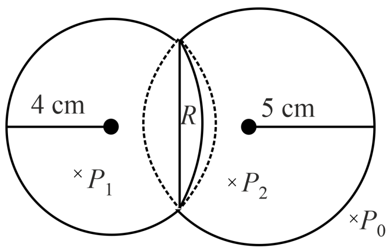
Solution: Stopping potential, $eV = \frac{hc}{\lambda} - \phi$

$e(V_2 - V_1) = hc\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$

$(1.02 - 0.410) = 1240\left[\frac{1}{\lambda_2} - \frac{1}{491}\right]$

$\lambda_2 \approx 396\ \mathrm{nm}$

Q.4. Two air bubbles of radius of curvature $4\ \mathrm{cm}$ & $5\ \mathrm{cm}$ touch each other, then the radius of curvature of common interface to both bubbles will be:



A) $10\ \mathrm{cm}$

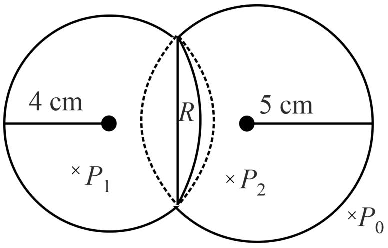
B) $20\ \mathrm{cm}$

C) $30\ \mathrm{cm}$

D) $40\ \mathrm{cm}$

Answer: $20\ \mathrm{cm}$

Solution:



Excess pressure inside air bubble is given as $\Delta P=\frac{4T}{r}$.

$P_1=P_0+\frac{4T}{R_1}$ and $P_2=P_0+\frac{4T}{R_2}$

The interface will have curvature such that,

$P_1-P_2=\frac{4T}{R}$

$\Rightarrow \frac{4T}{R_1}-\frac{4T}{R_2}=\frac{4T}{R}$

$\frac{1}{4}-\frac{1}{5}=\frac{1}{R}\ \Rightarrow R=20\ \text{cm}$

Q.5. If W is weight of a block on earth , find radius of a planet to get $\frac{W}{3}$ weight. Assume density is constant.

A) $\frac{R}{2}$

B) $\frac{R}{3}$

C) $\sqrt{3}R$

D) $3R$

Answer: $\frac{R}{3}$

Solution: Let mass density of the Earth/Planet be ρ.

For Earth: $W=mg$ and $g=\frac{GM}{R^2}=\frac{G\left(\rho\times\frac{4\pi}{3}R^3\right)}{R^2}=\frac{4\pi G\rho R}{3}$

For planet: $W\prime=\frac{W}{3}=\frac{mg}{3}$.

As mass of the object will remain same, so acceleration due to gravity on the surface of planet will be $\frac{g}{3}$.

As we can see, $g\ \propto\ R$ Therefore, $\frac{g}{g'}=\frac{R}{R\prime}$

$R\prime=\frac{g'}{g}R=\frac{R}{3}$

Q.6. A body is projected at angle 30° from an inclined plane of inclination 30° with horizontal, with velocity $10\ \text{m s}^{-1}$. Find range.

A) $\frac{10}{3}$

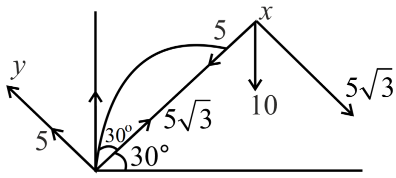
B) $\frac{20}{3}$

C) 10

D) $\frac{40}{3}$

Answer: $\frac{20}{3}$

Solution:



Component of velocity in x direction, $u_x = 10 \cos 30^\circ = 5\sqrt{3}$ m s^{-1} and in y direction $u_y = 10 \sin 30^\circ = 5$ m s^{-1}

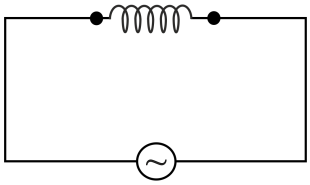
Time of flight, $t = \frac{2u \sin 30^\circ}{g \cos 30^\circ} = \frac{10}{5\sqrt{3}} = \frac{2}{\sqrt{3}}$ s

Range $x = u_x t - \frac{1}{2} a_x t^2$

$= 5\sqrt{3} \times \frac{2}{\sqrt{3}} - \frac{1}{2} \times 5 \times \frac{4}{3}$

$= 10 - \frac{10}{3} = \frac{20}{3}$ m

Q.7. Calculate I_{rms} in the following circuit.



A) 3.5 mA

B) 35 mA

C) 350 mA

D) 3500 mA

Answer: 3.5 mA

Solution: $I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L}$, here, inductive reactance $X_L = \omega L = 2\pi f L$

Thus, $I_{rms} = \frac{220}{2\pi \times 50 \times 200} = 3.5$ mA.

Q.8. A ball is thrown from a tower of height h with initial velocity v. It takes 6 s to fall to the ground when it is thrown upward and it takes 1.5 s when it is thrown downward. How long will it take to fall to the ground if it is dropped?

A) 2 s

B) 5 s

C) 3 s

D) 4.5 s

Answer: 3 s

Solution: When it is thrown upward, $-h = v \times 6 - \frac{1}{2}g(6)^2$

When it is thrown downward, $-h = -v \times 1.5 - \frac{1}{2}g(1.5)^2$

Therefore, $5h = 5[36 + 9]$ $h = 45 \text{ m}$

When the ball is dropped, $h = \frac{1}{2}gt^2 \Rightarrow 45 = \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 3 \text{ s}$

Alternate method, $t = \sqrt{t_1 t_2} = \sqrt{6 \times 1.5} = 3 \text{ s}$

Q.9. If a body of given speed v is sliding on a horizontal surface of frictional coefficient 0.5, in what time will it come to rest?

A) $\frac{v}{5}$

B) $\frac{v}{3}$

C) $3v$

D) $5v$

Answer: $\frac{v}{5}$

Solution: Given initial velocity v.

Acceleration $a = \mu g = 0.5 \times 10 = 5 \text{ m s}^{-2}$

Using, $v = u + at$, here, final velocity $v = 0$.

So, $0 = v - at \Rightarrow t = \frac{v}{5} \text{ s}$

Q.10. Let B_1 be the magnetic field on the centre of a current carrying loop. Let B_2 be the magnetic field of the same coil at a distance $\frac{R}{4}$ along the axis of the coil. Find the ratio of B_1 to B_2.

A) $\frac{(5)^{\frac{3}{2}}}{16}$

B) $\frac{(5)^{\frac{3}{2}}}{32}$

C) $\frac{(17)^{\frac{3}{2}}}{32}$

D) $\frac{(17)^{\frac{3}{2}}}{64}$

Answer: $\frac{(17)^{\frac{3}{2}}}{64}$

Solution: Magnetic field at the centre of the circle is, $B_1 = \frac{\mu_0 i}{2R}$

Magnetic field at the point on the axis is,

$$B_2 = \frac{\mu_0 i R^2}{2(R^2+x^2)^{\frac{3}{2}}} \Rightarrow B_2 = \frac{\mu_0 i R^2}{2\left(R^2+\left(\frac{R}{4}\right)^2\right)^{\frac{3}{2}}} \Rightarrow B_2 = \frac{32\mu_0 i}{(17)^{\frac{3}{2}}R}$$

Therefore, $\frac{B_1}{B_2} = \frac{(17)^{\frac{3}{2}}}{64}$

Q.11. In a metre scale, 2 coins of 10 grams each are placed on the $10\,\text{cm}$ mark. The scale is balanced at $40\,\text{cm}$ using a knife. Find the mass of the metre scale.

A) $60\ \text{g}$

B) $20\ \text{g}$

C) $30\ \text{g}$

D) $50\ \text{g}$

Answer: $60\ \text{g}$

Solution: Let the mass of the scale be m.

The centre of mass of the scale will be at $50\ \text{cm}$.

Applying torque balance,

$$0 = mg \times (50 - 40) - 2 \times 10g \times (40 - 10)$$
$$\Rightarrow m = 60\ \text{g}$$

Q.12. In a Potentiometer the balancing length of a certain cell is $75\,\text{cm}$. The cell is removed and another cell is attached with balancing length x. If the ratio of the EMFs of the two cells is $3:4$, then find the value of x.

A) $56\ \text{cm}$

B) $100\ \text{cm}$

C) $75\ \text{cm}$

D) $66\ \text{cm}$

Answer: $100\ \text{cm}$

Solution: If the potential difference between the potentiometer is V and the potential of the battery is ε.

Then, $\frac{\varepsilon}{V} = \frac{l}{L}$

Therefore, $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2} \Rightarrow \frac{3}{4} = \frac{75}{x} \Rightarrow x = 100\ \text{cm}$

Q.13. Assertion: The speed and Kinetic energy of a charged particle in a uniform magnetic field is constant.

Reason: The force applied by magnetic field is always perpendicular to the velocity of the body.

A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

B) Both Assertion and Reason are true but Reason is NOT the correct explanation of Assertion.

C) Assertion is true but Reason is false.

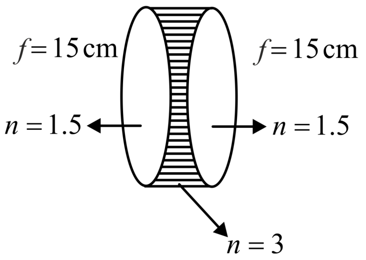
D) Assertion is false but Reason is true.

Answer: Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Solution: The force experienced by the charge particle is perpendicular to the instantaneous velocity at all instants as $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$.

Hence, the magnetic force cannot bring any change in the speed of the charged particle. Since, the speed remains constant, the kinetic energy also remains constant.

Q.14. A liquid of refractive index 3 is placed between two biconvex lenses of focal length $f = 15\ \mathrm{cm}$. The equivalent focal length of the combined system is,



A) $-7.5\ \mathrm{cm}$

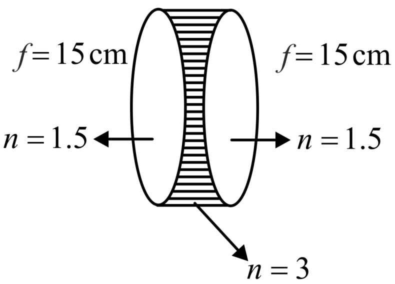
B) $-10\ \mathrm{cm}$

C) $-15\ \mathrm{cm}$

D) $-20\ \mathrm{cm}$

Answer: $-7.5\ \mathrm{cm}$

Solution:



For convex lenses, let radius of curvature is R. Then,

$\frac{1}{15\ \mathrm{cm}} = (1.5 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$

$\Rightarrow \frac{1}{R} = \frac{1}{15} \ \Rightarrow R = 15\ \mathrm{cm}$

Now for concave lens,

$\frac{1}{f_{concave}} = (3 - 1)\left(\frac{1}{-R} - \frac{1}{R}\right)$

$= 2 \times -\frac{2}{R} = \frac{-4}{15}$

$\frac{1}{f_{eff}} = \frac{1}{15} + \frac{1}{15} - \frac{4}{15} = -\frac{2}{15}$

$\Rightarrow f_{eff} = \frac{-15}{2} = -7.5\ \mathrm{cm}$

Q.15. If one end of vertical spring is connected to the ground and other end is connected to horizontal platform at rest. If a ball is dropped on it from height h above platform compresses spring by $\frac{h}{2}$. If $h = 10\ \mathrm{cm}$ then find K.

A) $k = \frac{12mg}{h}$

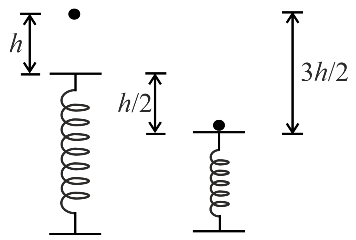
B) $k = \frac{8mg}{h}$

C) $k = \frac{16mg}{h}$

D) $k=\frac{6mg}{h}$

Answer: $k=\frac{12mg}{h}$

Solution:



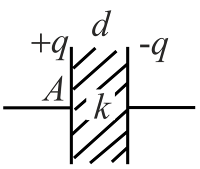
Loss in the gravitational potential energy of the ball= gain in the potential energy by the spring

$-\Delta PE_{\text{gravitational}}=\Delta PE_{\text{spring}}$

$mg\left(h+\frac{h}{2}\right)=\frac{1}{2}k\left(\frac{h}{2}\right)^2\Rightarrow mg\frac{3h}{2}=\frac{kh^2}{8}$

$\Rightarrow k=\frac{12mg}{h}$

Q.16. A dielectric with constant k is inserted between the plates of a parallel plate capacitor of surface area A with charge q. If the electric field strength between the plates is E, find k



A) $\frac{q}{A\varepsilon_0 E}$

B) $\frac{qE}{A\varepsilon_0}$

C) $\frac{qA}{\varepsilon_0 E}$

D) $\frac{q}{2A\varepsilon_0 E}$

Answer: $\frac{q}{A\varepsilon_0 E}$

Solution: Electric field between the plates of the capacitor is given by,

$E=\frac{\sigma}{\varepsilon_0 k}=\frac{q}{A\varepsilon_0 k}$

$\Rightarrow k=\frac{q}{A\varepsilon_0 E}$

Q.17. Equations of two waves are given by

$y=5\sin(\omega t-kx)$

$y=3\sin\left(\omega t-kx+\frac{\pi}{2}\right)$

Find resultant amplitude

A) 8

B) 2

C) 4

D) $\sqrt{34}$

Answer: $\sqrt{34}$

Solution: From the given equations we can conclude that the phase difference between the waves is $\frac{\pi}{2}$. Therefore, the resultant amplitude will be,

$$A=\sqrt{{A_1}^2+{A_2}^2}=\sqrt{5^2+3^2}=\sqrt{34}\text{ unit}$$

Q.18. A liquid drop having mass m is in equilibrium in air. Electric field E is present in vertically upward direction. Find charge in drop.

A) $\frac{mg}{E}$

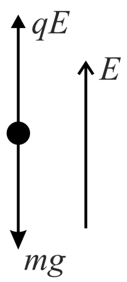
B) $\frac{2mg}{E}$

C) $\frac{2mg}{5E}$

D) Zero

Answer: $\frac{mg}{E}$

Solution:



To balance gravitational force, electric force on the liquid drop should act in upward direction.

For translational equilibrium along vertical direction,

$qE=mg$

$\Rightarrow q=\frac{mg}{E}$

Q.19. If the frequency of light is double that of the threshold frequency (f_0) , the maximum velocity of photoelectron is v_1 and if the frequency become five times the threshold frequency, then the maximum velocity of the photoelectron is v_2. Find the ratio of $\frac{v_2}{v_1}$.

A) 2

B) 3

C) 4

D) 5

Answer: 2

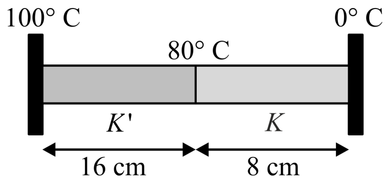
Solution: Maximum kinetic energy is $K_{\max} = hf - \phi$

For first case, $\frac{1}{2}mv_1{}^2 = h[(2f_0) - f_0]$

For second case, $\frac{1}{2}mv_2{}^2 = h[(5f_0) - f_0]$

$\Rightarrow \frac{v_2{}^2}{v_1{}^2} = \frac{4}{1} \Rightarrow \frac{v_2}{v_1} = 2$

Q.20. Find value of K' in terms of K (both rods have same cross-sectional area).



A) $8K$

B) $16K$

C) $4K$

D) $2K$

Answer: $8K$

Solution: Using heat current relation,

$\frac{K'A(100-80)}{16} = \frac{KA(80-0)}{8}$

$\Rightarrow \frac{K'\times 20}{16} = \frac{K\times 80}{8}$

$\Rightarrow K' = 8K$

Q.21. If $\mathrm{B} = 10^9\,\mathrm{Nm}^{-2}$ and fractional change in volume is $\frac{2}{100}$, find the volumetric stress required

A) 2×10^7 Pa

B) 3×10^7 Pa

C) 4×10^7 Pa

D) 5×10^7 Pa

Answer: 2×10^7 Pa

Solution: Bulk modulus $B = \frac{\text{Volumetric stress}}{\text{Volumetric strain}} = \frac{-\Delta P}{\frac{\Delta V}{V}}$

Volumetric stress$= B\times\frac{\Delta V}{V} = 10^9\times\frac{2}{100} = 2\times10^7$ Pa

Section B: Chemistry

Q.1. Which compound is minimum or not found in photochemical smog

A) $CH_2 = O$

B) NO_2

C) O_3

D) N_2

Answer: N_2

Solution: Photochemical smog is a type of smog that is produced when UV light from the sun interacts with the oxides of nitrogen present in the atmosphere. This smog is most commonly seen in highly populated cities that are placed in relatively warm climates. So, the compounds which are responsible for photochemical smog is $HCHO$, O_3 and NO_2

Q.2. Glucose and Galactose are having identical configuration in all the positions except ....position

A) $C - 3$

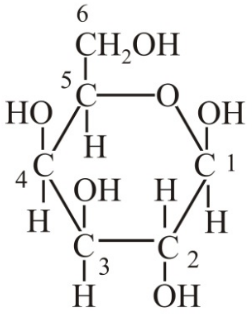
B) $C - 4$

C) $C - 5$

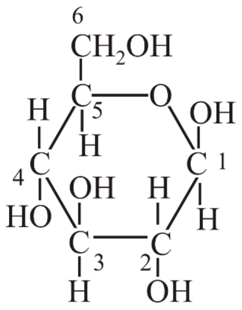
D) $C - 2$

Answer: $C - 4$

Solution: The structures of Glucose and Galactose are:



Glucose



Galactose

Hence, Glucose and Galactose differ only in the arrangement at $C - 4$ carbon

Q.3. Which of the following is not a broad spectrum antibiotic?

A) Amoxycillin

B) Penicillin

C) Chloramphenicol

D) Ampicillin

Answer: Penicillin

Solution: A broad-spectrum antibiotic is an antibiotic that acts on the two major bacterial groups, gram-positive and gram-negative, or any antibiotic that acts against a wide range of disease-causing bacteria.

Penicillin G has a narrow spectrum. Ampicillin and Amoxycillin are synthetic modifications of penicillins. These have broad spectrum.

Chloramphenicol is a broad spectrum antibiotic. It is rapidly absorbed from the gastrointestinal tract and hence can be given orally in case of typhoid, dysentery, acute fever, certain form of urinary infections, meningitis and pneumonia.

Q.4. The main constituent of enamel on the surface of teeth is:

A) $\left[3\,Ca_3\left(PO_4\right)_2\cdot CaF_2\right]$

B) $Ca_3\left(PO_4\right)_2\cdot H_2O$

C) $\left[3\,Ca_3\left(PO_4\right)_2\cdot Ca\left(OH\right)_2\right]$

D) CaF_2

Answer: $\left[3\,Ca_3\left(PO_4\right)_2\cdot Ca\left(OH\right)_2\right]$

Solution: Enamel provides the hard surface to the teeth. It has the carbonate substituted hydroxyapatite crystallites. It is formed from the minerals. It has the calcium phosphate and calcium hydroxide.

Hence, the main constituent of enamel on the surface of teeth is $\left[3\,Ca_3\left(PO_4\right)_2\cdot Ca\left(OH\right)_2\right]$.

Q.5. In an ionic compound of X and Y, X is present at the lattice point of h.c.p. structure and Y is present at $\frac{2}{3}$ of tetrahedral voids. Find the percentage of X in the lattice.

A) 42.86 %

B) 33.33 %

C) 50 %

D) 66.67 %

Answer: 42.86 %

Solution: Number of effective atoms in HCP lattice is =6

Number of atoms of $X=6$

Number of T.V. in h.c.p. =$12\ \left(2\times \text{Number of atoms}\right)$

Number of Y atoms $=\frac{2}{3}\times 12=8$ atoms.

$\%$ of $X=\frac{6}{14}\times 100=42.86\%$

Q.6. Number of π bonds in marshall's acid will be.

A) 4

B) 3

C) 6

D) 2

Answer: 4

Solution: Molecular formula of Marshall's acid is $H_2S_2O_8$ and its structure is

$$\begin{array}{ccccccccccccc} & & & & \mathrm{O} & & & & \mathrm{O} & & & & \\ & & & & \| & & & & \| & & & & \\ \mathrm{H} & - & \mathrm{O} & - & \mathrm{S} & - & \mathrm{O}-\mathrm{O} & - & \mathrm{S} & - & \mathrm{O} & - & \mathrm{H} \\ & & & & \| & & & & \| & & & & \\ & & & & \mathrm{O} & & & & \mathrm{O} & & & & \end{array}$$

As we can see in above structure, there are total of 4π bonds present.

Q.7. Statement 1: Emulsion of water and oil is unstable and separated in two different layers.

Statement 2: It is stabilised by adding excess of electrolytes.

A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

B) Both Assertion and Reason are true but Reason is NOT the correct explanation of Assertion.

C) Assertion is true but Reason is false.

D) Assertion is false but Reason is true.

Answer: Assertion is true but Reason is false.

Solution: Oil and water are not soluble in each other. since oils are non-polar organic compounds and water is a polar compound. So, they will form hydrophobic type of colloid.

Hydrophobic collides are unstable and required some stabilisers like proteins, gums, natural and synthetic soaps, etc..

So, statement 1 is true but statement 2 is false.

Q.8. The stable nitrogen halide among the following

A) NF_3

B) NCl_3

C) NBr_3

D) NI_3

Answer: NF_3

Solution: The size of halogen increases from F to I. The $N-F$ bond is stronger than other $N-X$ bonds due to the high electronegativity of fluorine. Larger the size of the halogen, less is the stability of nitrogen halide:

The stability order is $NF_3 > NCl_3 > NBr_3 > NI_3$.

Q.9. In the structure of $\left[Co_2(CO)_8\right]$, the number bonds of $Co-Co$ bonds is X and the number of $Co-CO$ terminal bonds is Y. Than find the value of $X+Y$

A) 7

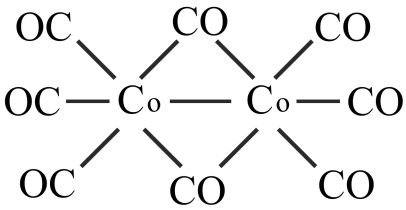
B) 6

C) 3

D) 5

Answer: 7

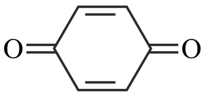
Solution:



The number of $Co - Co$ bonds are 1 and the number of $Co - CO$ terminal bonds are 6. Hence, $x + y = 7$.

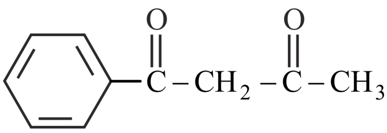
Q.10. Identify the conjugates dione from the following:

A)

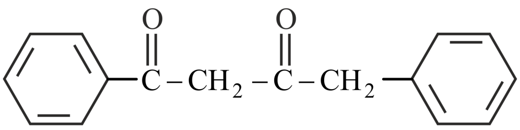


B) $CH_3 - \overset{\overset{O}{\|}}{C} - CH_2 - \overset{\overset{O}{\|}}{C} - CH_3$

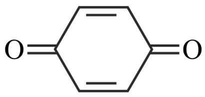
C)



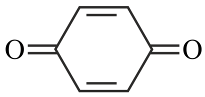
D)



Answer:

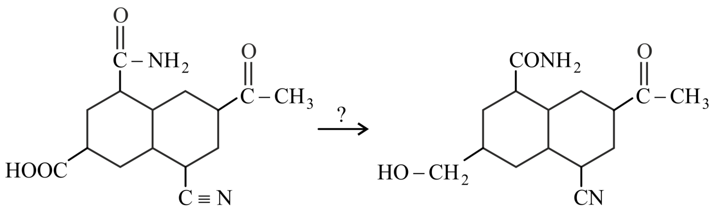


Solution: In the dione, there are two keto groups are present. If π bonds are separated by a single bond, then we can say that there exists a conjugation. In the molecule



all π bonds separated by single bonds and it is a dione.

Q.11. The most suitable reagent for the given conversion will be



A) $NaBH_4$

B) $LiAlH_4$

C) H_2/Pd

D) B_2H_6

Answer: B_2H_6

Solution: B_2H_6 is a reducing Agent which can reduce carboxylic acid into alcohol. It does not affect other groups present in the compound. $LiAlH_4$ is and H_2/Pd can reduce all these functional groups into alcohols or amines. $NaBH_4$ will reduce ketone group into alcohol.

Q.12. $A_{(g)} \longrightarrow B_{(g)} + \frac{1}{2}C_{(g)}$

Find the relationship between K_p, α and equilibrium pressure P.

A) $k_p = \frac{\alpha^{\frac{3}{2}}p^{\frac{1}{2}}}{(1-\alpha)(2+\alpha)^{\frac{1}{2}}}$

B) $k_p = \frac{\alpha^2 p}{(1-\alpha)(2+\alpha)}$

C) $k_p = \frac{\alpha^{\frac{3}{2}}p^{\frac{3}{2}}}{(1-\alpha)(2+\alpha)^{\frac{3}{2}}}$

D) $k_p = \frac{\alpha^{\frac{3}{2}}p^{\frac{3}{2}}}{(1-\alpha)(2+\alpha)^{\frac{1}{2}}}$

Answer:

$$k_p = \frac{\alpha^{\frac{3}{2}}p^{\frac{1}{2}}}{(1-\alpha)(2+\alpha)^{\frac{1}{2}}}$$

Solution:

$$\begin{array}{lccc} & A_{(g)} & \rightleftharpoons B_{(g)} & + \frac{1}{2}C_{(g)} \\ \text{at } t=0 & 1 & 0 & 0 \\ \text{at eq.} & 1-\alpha & \alpha & \frac{\alpha}{2} \end{array}$$

total moles at eq. $= 1-\alpha+\alpha+\frac{\alpha}{2} = 1+\frac{\alpha}{2}$

$P_B = \frac{\alpha}{1+\frac{\alpha}{2}}p \quad P_c = \frac{\frac{\alpha}{2}}{1+\frac{\alpha}{2}}p \quad P_A = \frac{1-\alpha}{1+\frac{\alpha}{2}}p$

$$K_p = \frac{P_B \times P_C^{\frac{1}{2}}}{P_A} = \frac{\left(\frac{\alpha}{1+\frac{\alpha}{2}}p\right)\left(\frac{\frac{\alpha}{2}}{1+\frac{\alpha}{2}}p\right)^{\frac{1}{2}}}{\left(\frac{1-\alpha}{1+\frac{\alpha}{2}}p\right)}$$

$$K_p = \frac{\alpha^{\frac{3}{2}}p^{\frac{1}{2}}}{2^{\frac{1}{2}}\left(1-\alpha\right)\left(1+\frac{\alpha}{2}\right)^{\frac{1}{2}}}$$

$$k_p = \frac{\alpha^{\frac{3}{2}}p^{\frac{1}{2}}}{(1-\alpha)(2+\alpha)^{\frac{1}{2}}}$$

Q.13. Difference in oxidation number between dichromate ion and chromate ions is

A) 6

B) 0

C) 12

D) 3

Answer: 0

Solution: $Cr_2O_7^{2-} + OH^- \longrightarrow 2\,CrO_4^{2-}$

Let the oxidation number of chromium be x

The oxidation state of chromium in $Cr_2O_7^{2-}$
$= 2x + (-2)7 = -2$
$x = +6$

Oxidation state of Cr in CrO_4^{2-}

$= x + (-2)4 = -2$
$x = +6.$

So, the oxidation number of Cr in CrO_4^{2-} is also $+6$.

Hence, the difference in oxidation number between ions is 0

Q.14. The total number of lone pairs of electrons in melamine is

A) 6

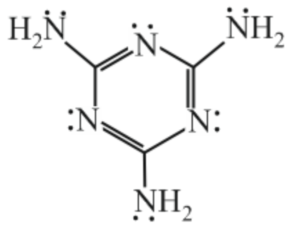
B) 3

C) 4

D) 12

Answer: 6

Solution: Structure of melamine is as follows



Total no. of lone pairs of electrons is '6'.

Q.15.

	Column I		Column II
(1)	calamine	(p)	PbS
(2)	Galena	(q)	$ZnCO_3$
(3)	Sphalerite	(r)	$FeCO_3$
(4)	Siderite	(s)	ZnS

A) $1-q,\ 2-p, 3-s, 4-r.$

B) $1-p, 2-q, 3-r, 4-s$

C) $1-q,\ 2-p, 3-r, 4-s$

D) $1-r, 4-s, 3-q, 4-p$

Answer: $1-q,\ 2-p, 3-s, 4-r.$

Solution: Calamine is the carbonate ore of zinc.It composition is $ZnCO_3$.

Galena is also called lead glance is the natural mineral form of lead (ii) sulfide, PbS

Sphalerite is a sulfide mineral with the formula Zns. It is the most important ore of zinc.

Siderite is a mineral composed of iron(II) carbonate ($FeCO_3$).

Q.16. Which of the following is correct statement?

A) B_2H_6 is Lewis acid

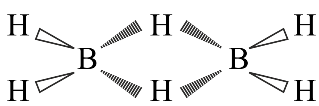
B) All the $B-H$ bonds in B_2H_6 are equal

C) B_2H_6 has planar structure

D) Maximum number of hydrogen in the plane is six.

Answer: B_2H_6 is Lewis acid

Solution: B_2H_6 is deficient in electrons by '2. Hence, it can act as Lewis acid. The structure of the diborane (B_2H_6) is



All the $B-H$ bonds are not equal. Bridge $B-H$ bond length is greater than terminal $B-H$ bonds B_2H_6 is a non-planar molecule. Maximum number of hydrogen in one plane is 4.

Q.17. Which of the following have the maximum melting point?

A) Formic acid

B) Acetic acid

C) Propionic acid

D) Butanoic acid

Answer: Acetic acid

Solution: Melting point depends on molecular mass and intermolecular forces between molecules. Among the given options H-bond is present in all the molecules. But as the number of C increases, extend of H-bonding decreases.

Due to intermolecular H-Bonding Acetic acid exist as Dimer. Due to which it has higher molecular mass, and due to smaller size it has maximum tendency to form H-bond. So, it has the maximum melting point.

formic acid $8^{\circ}\mathrm{C}$,

acetic acid $17^{\circ}\mathrm{C}$,

propionic acid $-22^{\circ}\mathrm{C}$,

butanoic acid $-8^{\circ}\mathrm{C}$

Q.18. Two isomers can be metamers if they have

A) Different functional groups.

B) carbon skeleton is different

C) Number of carbon atoms on Either side of functional groups are different

D) Different molecular formula.

Answer: Number of carbon atoms on Either side of functional groups are different

Solution: Metamers are the structural isomers, which are due to the presence of different alkyl groups on either sides of a functional group.

Example: $\mathrm{C_2H_5-O-C_2H_5}$ and $\mathrm{C_3H_7-O-CH_3}$
are metamers as they have same molecular formula but differ in the alkyl groups on either sides of the oxygen.

Q.19. Which of the following is correct about the solubility of LiF and $\mathrm{MgCl_2}$ in ethanol?

A) LiF is more soluble than $\mathrm{MgCl_2}$

B) $\mathrm{MgCl_2}$ is more soluble than LiF

C) Both are equally soluble in ethanol

D) Both are not soluble in ethanol

Answer: $\mathrm{MgCl_2}$ is more soluble than LiF

Solution: According to Fajan's Rule, the compound formed by smaller cation and larger anion have more covalent character. Among $\mathrm{Li^+}$ and $\mathrm{Mg^{2+}}$, $\mathrm{Mg^{2+}}$ is the smaller cation. Among $\mathrm{F^-}$ and $\mathrm{Cl^-}$, $\mathrm{Cl^-}$ is the larger anion. Hence, $\mathrm{MgCl_2}$ has more covalent character. Hence, $\mathrm{MgCl_2}$ is readily soluble in ethanol.

Q.20. From which of the following, hydrogen is commercially prepared?

A) Carbon

B) Oxygen

C) Chlorine

D) Nitrogen

Answer: Carbon

Solution: $C + \underset{(\text{Steam})}{H_2O} \rightarrow \underbrace{CO + H_2}_{\text{Water gas}}$

The hydrogen concentration can be increased by the following water gas shifting reaction.

$CO + H_2 + H_2O \rightarrow CO_2 + 2H_2$

The above reaction is an example for commercially hydrogen preparation.

Q.21. How many oxides among the following are neutral $Na_2O,\ Cl_2O_7,\ As_2O_3,\ N_2O,\ NO$.

A) 2

B) 5

C) 3

D) 4

Answer: 2

Solution: Metallic oxides are basic in nature, Oxides of non-metals are acidic in nature. Neutral oxides are those which are neither acidic nor basic.

Among the given oxides Na_2O is a metallic oxide, and it is basic in nature.

N_2O and NO are non-metallic oxides but are neutral in nature.

As_2O_3 is an Amphoteric oxide. It reacts with both acids and bases.

Q.22. $2,\ 7-$dimethyl$-2,\ 6-$octadiene $\xrightarrow[\Delta]{H^+}$ A. Find the number of sp^2 hybridized carbon in the product 'A' ?

A) 2

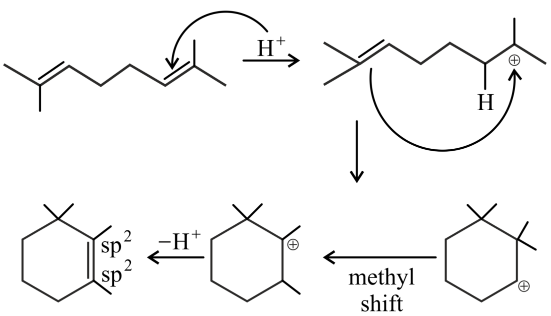
B) 4

C) 6

D) 0

Answer: 2

Solution:



So, there are total of two sp^2 hybrid carbon in the final product.

Q.23. Osmotic pressure of a glucose solution is 7.47 atm at 300 K temperature, calculate the concentration of the solution in $\frac{g}{L}$ (Molecular weight of glucose $= 180$ U)

$\left(R = 0.083 \text{ L atm mol}^{-1} \text{ K}^{-1}\right)$

A) 54

B) 0.3

C) 30

D) 108

Answer: 54

Solution: Osmotic pressure, $\pi = iCRT$

$i =$ van't hoff factor

$C =$ concentration in molarity

$R =$ universal gas constant

$T =$ temperature

$C =$ Molarity

$= \frac{\text{moles}}{\text{Litre}}$

$= \frac{\omega}{GM\omega} / \text{Litre}$

$\pi = iCRT$

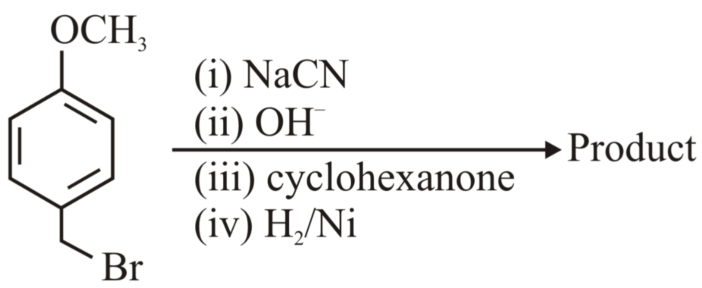
$i = 1$ for glucose as it neither associates nor dissociates in solution.

$7.47 = 1 \times C \times 0.083 \times 300$

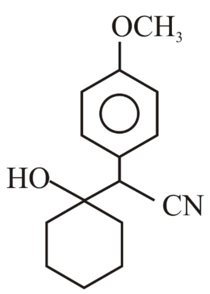
$7.47 = 1 \times \frac{\omega}{180} \times 0.083 \times 300$

$\omega = \frac{7.47 \times 180}{0.083 \times 300} = 54 \text{ g/Lit}$

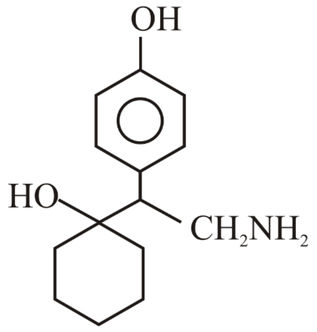
Q.24. The end product in the following reaction sequence is



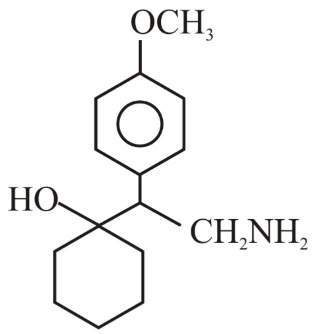
A)



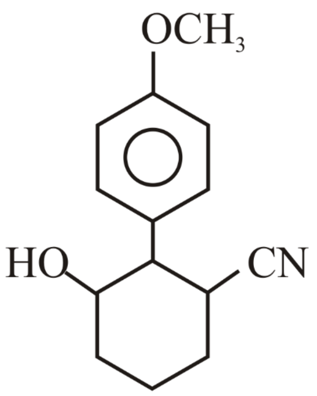
B)



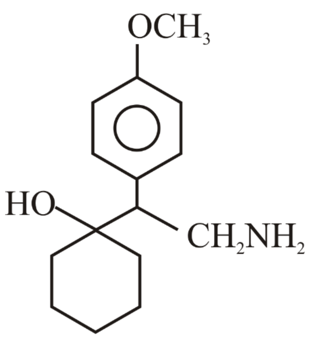
C)



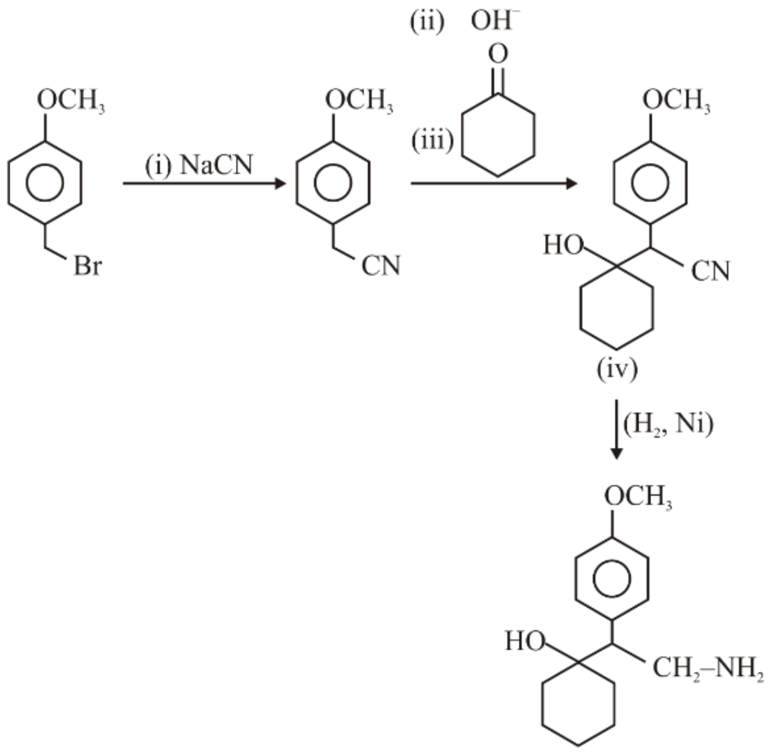
D)



Answer:



Solution:



Q.25. Which of the following vitamin cannot be given to the living organism through food?

A) Vitamin C

B) Vitamin K

C) Vitamin D

D) Vitamin B_5

Answer: Vitamin D

Solution: Vitamin D is a group of fat-soluble secosteroids responsible for increasing intestinal absorption of calcium, magnesium, and phosphate, and many other biological effects. Vitamin D cannot be supplied through diet, but it can be given in the form of supplements.

Section C: Mathematics

Q.1. The equation of a plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$ is

A) $5x-11y+z=17$

B) $\sqrt{2}x+y=3\sqrt{2}-1$

C) $x+y+z=\sqrt{3}$

D) $x-\sqrt{2}y=1-\sqrt{2}$

Answer: $5x-11y+z=17$

Solution: The equation of plane through the intersection of two given planes is

$x+2y+3z-2+k(x-y+z-3)=0$

$\Rightarrow (1+k)x+(2-k)y+(3+k)z-2-3k=0 \quad \ldots(1)$

It is given that distance of plane (1) from point $(3,1,-1)$ is $\frac{2}{\sqrt{3}}$, i.e.,

$\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|=\frac{2}{\sqrt{3}}$

$\Rightarrow \left|\frac{3(1+k)+(2-k)-(3+k)-(2+3k)}{\sqrt{(1+k)^2+(2-k)^2+(3+k)^2}}\right|=\frac{2}{\sqrt{3}}\Rightarrow \left|\frac{-2k}{\sqrt{3k^2+4k+14}}\right|=\frac{2}{\sqrt{3}}$

Squaring on both sides

$\frac{4k^2}{3k^2+4k+14}=\frac{4}{3}\Rightarrow 3k^2+4k+14=3k^2\Rightarrow k=\frac{-7}{2}$

Put the value of k in (1), we get

$\frac{-5}{2}x+\frac{11}{2}y-\frac{z}{2}+\frac{17}{2}=0\Rightarrow 5x-11y+z=17$

Q.2. $(p\rightarrow q)\wedge(q\rightarrow {\sim}p)\equiv$

A) ${\sim}p$

B) p

C) ${\sim}q$

D) q

Answer: ${\sim}p$

Solution: $({\sim}p\vee q)\wedge({\sim}p\vee{\sim}q)$

$\equiv {\sim}p\vee(q\wedge{\sim}q)\equiv{\sim}p\vee f\equiv{\sim}p$

Q.3. If $f(x)=\frac{x^2-1}{x^2+1}$, for every real number x, then the minimum value of $f(x)$

A) is not attained even though f is bounded.

B) is equal to 0

C) is equal to 1.

D) is equal to -1.

Answer: is equal to -1.

Solution: Given $f(x)=\frac{x^2-1}{x^2+1}=1-\frac{2}{x^2+1}$

Now, $f(x)$ will be minimum when $\frac{2}{x^2+1}$ is maximum, i.e., when x^2+1 is minimum, i.e., at $x=0$.

$\therefore$ Minimum value of $f(x)=f(0)=\frac{0-1}{0+1}=-1$.

Q.4. If sum of squares of Reciprocal of root α,β of the equation $3x^2+\lambda x-1=0$ is 15 then the value of $6\left(\alpha^3+\beta^3\right)^2$

A) 46

B) 36

C) 24

D) 18

Answer: 24

Solution: Given $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=15$

For equation $3x^2+\lambda x-1=0$

$\alpha+\beta=\frac{-\lambda}{3}$, $\alpha\beta=\frac{-1}{3}\Rightarrow\alpha^2\beta^2=\frac{1}{9}$

then $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{\lambda^2}{9}-2\left(\frac{-1}{3}\right)=\frac{\lambda^2}{9}+\frac{2}{3}$

$\Rightarrow\alpha^2+\beta^2=\frac{\lambda^2+6}{9}$

Now $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=15\Rightarrow\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}=15$

$\Rightarrow\frac{\lambda^2+6}{9}\times\frac{1}{\alpha^2\beta^2}=15\ \Rightarrow\frac{\lambda^2+6}{9}\times\frac{1}{\frac{1}{9}}=15$

$\Rightarrow\lambda^2+6=15\Rightarrow\lambda=\pm3$

Now $6\left(\alpha^3+\beta^3\right)^2=6\left((\alpha+\beta)\left(\alpha^2-\alpha\beta+\beta^2\right)\right)^2$

$=6\times1\times\left(\frac{15}{9}+\frac{1}{3}\right)^2$

$=6\times1\times(2)^2=6\times4=24$

Q.5. When 3^{2022} is divided by 5, then it leaves remainder

A) 1

B) 2

C) 3

D) 4

Answer: 4

Solution: $\left(3^2\right)^{1011}=(10-1)^{1011}=-(1-10)^{1011}$

$=-\left[1-{}^{1011}C_1 10+{}^{1011}C_2 10^2\ldots\right]=10q-1$

So, here the remainder will be $5-1=4$

Q.6. If $\left(\tan^{-1}x\right)^3+\left(\cot^{-1}x\right)^3=k\pi^3$ has a solution, then the minimum value of k is equal to

A) $\frac{1}{32}$

B) $\frac{1}{64}$

C) $\frac{1}{8}$

D) $\frac{1}{16}$

Answer: $\frac{1}{32}$

Solution: We have, $\left(\tan^{-1}x\right)^3+\left(\cot^{-1}x\right)^3=k\pi^3$

We know that, $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$

$\Rightarrow\left(\frac{\pi}{2}-\cot^{-1}x\right)^3+\left(\cot^{-1}x\right)^3=k\pi^3$

$\Rightarrow\frac{\pi^3}{8}-\left(\cot^{-1}x\right)^3-\frac{3\pi^2}{4}\left(\cot^{-1}x\right)+\frac{3\pi}{2}\left(\cot^{-1}x\right)^2+\left(\cot^{-1}x\right)^3=k\pi^3$

$k=\frac{3}{2\pi^2}\left[\left(\cot^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^2}{48}\right]$

Cleanly, the k is minimum when $\cot^{-1}x=\frac{\pi}{4}$

$\therefore k_{\min}=\left(\frac{3}{2\pi^2}\left(\frac{\pi^2}{48}\right)\right)\Rightarrow k_{\min}=\frac{1}{32}.$

Q.7. The curve representing the differential equation $x\frac{dy}{dx}=2y$ will be

A) $y=cx$

B) $y^2=cx$

C) $y=cx^2$

D) $y^2=cx^2$

Answer: $y=cx^2$

Solution: $x\frac{dy}{dx}=2y\Rightarrow\frac{dy}{y}=2\frac{dx}{x}$

$\Rightarrow\ln|y|=2\ln|x|+C\Rightarrow\ln|y|=\ln x^2+\ln c$

$\Rightarrow y=cx^2$

Q.8. The domain of $y=\frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\log\left(x^2-3x+2\right)}$ is

A) $x \in \left[\frac{-1}{2}, 1\right) \cup (2, \infty) - \{3\}$

B) $x \in \left[\frac{-1}{2}, 1\right] \cup (2, \infty) - \{3\}$

C) $x \in \left(\frac{-1}{2}, 1\right) \cup [2, \infty) - \{3\}$

D) $x \in \left[\frac{-1}{2}, 1\right) \cup [2, \infty) - \{3\}$

Answer: $x \in \left[\frac{-1}{2}, 1\right) \cup (2, \infty) - \{3\}$

Solution: We know that, $\cos^{-1} x$ is defined when $-1 \leqslant x \leqslant 1$ So, $\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)$ is defined when

$-1 \leqslant \frac{x^2-5x+6}{x^2-9} \leqslant 1$ and $x^2 - 9 \neq 0$

$\Rightarrow -1 \leq \frac{(x-3)(x-2)}{(x-3)(x+3)} \leq 1$ and $x \neq \pm 3$

$\Rightarrow -1 \leq \frac{x-2}{x+3} \leq 1$ and $x \neq \pm 3$, $\Rightarrow -1 \leq \frac{x+3-5}{x+3} \leq 1$ and $x \neq \pm 3$

$\Rightarrow -1 \leq 1 - \frac{5}{x+3} \leq 1$ and $x \neq \pm 3$,

$\Rightarrow -2 \leq -\frac{5}{x+3} \leqslant 0$ and $x \neq \pm 3$

$\Rightarrow 0 \leq \frac{5}{x+3} \leqslant 2$ and $x \neq \pm 3$, $\Rightarrow x + 3 \geq \frac{5}{2}$ and $x \neq \pm 3$

$\Rightarrow \ x \geq \frac{-1}{2}$ and $x \neq \pm 3$

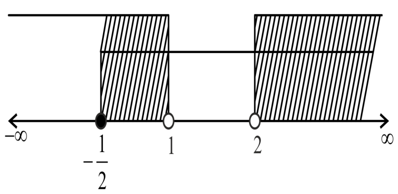
$\therefore \ x \in \left[\frac{-1}{2}, \infty\right) - \{3\} \ \cdots (1)$

Now, we know that $\log(x)$ is defined for $x > 0$.

So, $\log\left(x^2 - 3x + 2\right)$ is defined for $x^2 - 3x + 2 > 0$, $\Rightarrow (x-1)(x-2) > 0$

$\Rightarrow x > 2$ or $x < 1 \ \cdots (2)$

from equations (1) & (2)



$\therefore \ x \in \left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \{3\}$

Q.9. If $f(\theta) = \sin\theta + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin\theta + t\cos\theta) \cdot f(t) d\theta$, then $\left|\int_0^{\frac{\pi}{2}} f(\theta) d\theta\right|$ is

A) $|1 + \pi t f(t)|$

B) $|-1 + \pi t f(t)|$

C) $1 - \pi t + (t)$

D) $1+\pi^2 t f(t)$

Answer: $|1+\pi t f(t)|$

Solution: Here, $f(\theta)=\sin\theta+f(t)[-\cos\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}+t f(t)[\sin\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$\Rightarrow f(\theta)=\sin\theta+2t f(t)$

Now, $\int_0^{\frac{\pi}{2}} f(\theta)d\theta=\int_0^{\frac{\pi}{2}}(\sin\theta+2t f(t))d\theta$

$=\int_0^{\frac{\pi}{2}}\sin\theta d\theta+2t f(t)\int_0^{\frac{\pi}{2}}d\theta=[-\cos\theta]_0^{\frac{\pi}{2}}+2t f(t)[\theta]_0^{\frac{\pi}{2}}$

$=1+\pi t f(t)$

So $\left|\int_0^{\frac{\pi}{2}} f(\theta)d\theta\right|=|1+\pi t f(t)|$

Q.10. If a circle equation $x^2+y^2+ax+by+c=0$ passes through $(0,\ 6)$ and also touches $y=x^2$ at $(2,\ 4)$, then the value of $a+c$ is

A) 16

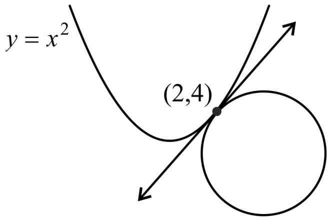
B) 12

C) 10

D) $\frac{17}{5}$

Answer: 16

Solution:



Equation of tangent at $(2,\ 4)$ to the parabola $y=x^2$ is $\frac{1}{2}(y+4)=2x$

$\Rightarrow 4x-y-4=0$

Then family of circle touching this tangent at $(2,\ 4)$ is

$(x-2)^2+(y-4)^2+\lambda(4x-y-4)=0 \quad \ldots\ (\text{i})$

Since it passes through $(0,\ 6)$, we get,

$4+4+\lambda(-10)=0\Rightarrow\lambda=\frac{4}{5}$

So, equation (i) becomes $x^2+y^2-\frac{4}{5}x-\frac{44}{5}y+\frac{84}{5}=0$

Hence, $a+c=\frac{80}{5}=16$

Q.11. The number of solutions of the equation $81^{\sin^2 x}+81^{\cos^2 x}=9$ in $x\in\left[0,\ \frac{\pi}{2}\right]$ is

A) 0

B) 1

C) 2

D) 4

Answer: 0

Solution: We have, $81^{\sin^2 x} + 81^{\cos^2 x} = 9$

$\Rightarrow 81^{\sin^2 x} + 81^{1-\sin^2 x} = 9$

$\Rightarrow 81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}} = 9 \quad \ldots (1)$

Let $81^{\sin^2 x} = y$

We know that, $-1 \leq \sin x \leq 1$

$0 \leq \sin^2 x \leq 1$ and $1 \leq 81^{\sin^2 x} \leq 81$

Hence, $1 \leq y \leq 81$.

So, equation (1) can be written as $y + \frac{81}{y} = 9$

$\Rightarrow y^2 + 81 = 9y$

$\Rightarrow y^2 - 9y + 81 = 0$

Discriminant $= D = 81 - 4(1)(81)$

$= D = -3(81) < 0$

$\therefore$ The above equation does not have real roots.

Hence, the number of solutions of the equation $81^{\sin^2 x} + 81^{\cos^2 x} = 9$ is zero.

Q.12. If a_i is sequence of an $\mathrm{A.P.}$ with common difference 1 and $\sum_{i=1}^{n} a_i = 192$ and $\sum_{i=1}^{\frac{n}{2}} a_{2i} = 120$, then find the value of n, where n is an even integer.

A) 18

B) 36

C) 96

D) 48

Answer: 96

Solution: Given, $\sum_{i=1}^{n} a_i = 192$

$a_1 + a_2 \quad \cdots \ a_n = 192$

$\Rightarrow \ \frac{n}{2}(a_1 + a_n) = 192$

$a_1 + a_n = \frac{384}{n} \qquad \cdots(1)$

Also given $\sum_{i=1}^{\frac{n}{2}} a_{2i} = 120$

$\Rightarrow \ \underbrace{a_2 + a_4 + a_6 \cdots a_n}_{\frac{n}{2}\text{ terms}} = 120$

$\Rightarrow \ \frac{n}{2} \times \frac{1}{2}[a_2 + a_n] = 120$

$a_2 + a_n = \frac{480}{n}$

$a_1 + 1 + a_n = \frac{480}{n} \quad \cdots(2)$

Now equation (2)–equation (1)

$\frac{480}{n} - \frac{384}{n} = (a_1 + a_n + 1) - (a, + a_n)$

$\frac{1}{n}(480 - 384) = 1$

$480 - 384 = n$

$n = 96$

Q.13. Polynomial equation is given as $f(x) = (1+x)(1+2x)\cdots(1+2^{20}x)$. Then find the coefficient of x^{20}

A) $2^{211} - 2^{190}$

B) $2^{211} - 2^{191}$

C) $2^{210} - 2^{190}$

D) $2^{210} - 2^{191}$

Answer: $2^{211} - 2^{190}$

Solution: Given, $f(x) = (1+x)(1+2x) \ \cdots \ (1+2^{20}x)$

Now expanding polynomial we get

$= x^{21}(1 \times 2 \times 2^2 \times \cdots 2^{20}) + x^{20}((1 \times 2 \times 2^2 \cdots 2^{19}) + (1 \times 2 \times 2^2 \ \cdots \ 2^{18} \times 2^{20})$
$+ (1 \times 2 \times 2^2 \ \cdots \ 2^{17} \times 2^{19} \times 2^{20}) \ldots\ldots\ldots\ldots) \ldots\ldots\ldots + 1$

Now coefficient of x^{20} will be

$= 2^{0+1+2+\ldots 20}\left\{\frac{1}{1} + \frac{1}{2} + \frac{1}{2^2} \cdots \frac{1}{2^{20}}\right\}$

$= 2^{210}\left\{\frac{1\left(1-\left(\frac{1}{20}\right)\right)^{24}}{1-\frac{1}{2}}\right]$

$= 2^{211}\left\{1 - \frac{1}{2^{21}}\right\}$

$= 2^{211} - 2^{190}$

Q.14. If matrix $A=\begin{bmatrix}1 & 0 & a\\ 1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$

where $a\in\ N$ (set of natural number)

and $\sum_{a=1}^{50}|Adj\ A|=100\ K$ and then the value of K is:-

A) $\frac{1821}{4}$

B) $\frac{1719}{4}$

C) $\frac{1723}{2}$

D) $\frac{1717}{2}$

Answer: $\frac{1821}{4}$

Solution: We know that

$|Adj\ A|=|A|^{n-1}$

Here n is 3×3 matrix

so $|Adj\ A|=|A|^{3-1}=|A|^{2}$

Now find $|A|=\begin{vmatrix}1 & 0 & a\\ 1 & 1 & 0\\ -1 & 0 & 1\end{vmatrix}$

$=1\times 1-0+a(0+1)$

$=1+a$

so $|A|=1+a$

Now $|Adj\ A|=|A|^{2}=(1+a)^{2}$

Now finding

$\sum_{a=1}^{50}|adj\ A|=\sum_{a=1}^{50}(1+a)^{2}=100\ K$

$\Rightarrow 2^{2}+3^{2}+4^{2}\ldots 51^{2}=100\ K$

$\Rightarrow 1^{2}+2^{2}\ldots 51^{2}-1^{2}=100\ K$

$\Rightarrow \frac{51\times(51+1)(2\times 51+1)}{6}-1=100\ K$

$\Rightarrow \frac{51\times 52\times 103}{6}-1=100\ K$

$\Rightarrow 45526-1=100\ K$

$\frac{45525}{100}=K$

$\frac{1821}{4}=K$

Q.15. If $ax-\mu y=2$ is tangent to hyperbola $\frac{a^{4}x^{2}}{\lambda^{2}}-\frac{b^{2}y^{2}}{1}=4$, then the value of $\left(\frac{\lambda}{a}\right)^{2}-\left(\frac{\mu}{b}\right)^{2}$ is

A) 1

B) 0

C) 2

D) 3

Answer: 1

Solution: Given $ax - \mu y = 2$

$y = \frac{ax}{\mu} - \frac{2}{\mu}$ ... (1) [comparing with $y = mx + c$]

Now we know that condition of tangent to hyperbola is $C = \sqrt{A^2m^2 - B^2}$

Now from equation (1)

$C = \frac{-2}{\mu}$ and $m = \frac{a}{\mu}$ and from hyperbola $\frac{a^4x^2}{4\lambda^2} - \frac{b^2y^2}{4} = 1$

$A^2 = \frac{4\lambda^2}{a^4}$ $B^2 = \frac{4}{b^2}$

Now putting the value in $C = \sqrt{A^2m^2 - B^2}$

We get $\frac{-2}{\mu} = \sqrt{\frac{4\lambda^2}{a^4} \times \left(\frac{a}{\mu}\right)^2 - \frac{4}{b^2}}$

Now squaring both sides

$\Rightarrow \frac{4}{\mu^2} = \frac{4\lambda^2}{a^4} \times \frac{a^2}{\mu^2} - \frac{4}{b^2}$

$\Rightarrow \frac{1}{\mu^2} = \frac{\lambda^2}{a^2\mu^2} - \frac{1}{b^2}$

$\Rightarrow 1 = \frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2}$

So option (A) is correct.

Q.16. If the angle between the vectors $\vec{a}$ and $\vec{c}$ is $\frac{\pi}{12}$ and $\left|\vec{a}\right| = \left|\vec{b}\right| = 1$, and $\vec{b} = \vec{c} + \vec{c} \times \vec{a}$ then the value of $6\left|\vec{c}\right|^2 =$

A) $\frac{8\left(6-\sqrt{3}\right)}{11}$

B) $\frac{8\left(6+\sqrt{3}\right)}{11}$

C) $\frac{8\left(3+\sqrt{3}\right)}{11}$

D) $\frac{24\left(6+\sqrt{3}\right)}{11}$

Answer: $\frac{8\left(6+\sqrt{3}\right)}{11}$

Solution: we have, $\vec{b}=\vec{c}+\vec{c}\times\vec{a}$,

$$\vec{b}\cdot\vec{b}=\left(\vec{c}+\vec{c}\times\vec{a}\right)\cdot\left(\vec{c}+\vec{c}\times\vec{a}\right)$$

$$\Rightarrow\left|\vec{b}\right|^2=\left(\vec{c}\cdot\vec{c}\right)+\vec{c}\cdot\left(\vec{c}\times\vec{a}\right)+\left(\vec{c}\times\vec{a}\right)\cdot\vec{c}+\left(\vec{c}\times\vec{a}\right)\cdot\left(\vec{c}\times\vec{a}\right)$$

$$\Rightarrow 1=\left|\vec{c}\right|^2+\vec{o}+\vec{o}+\left|\vec{c}\times\vec{a}\right|^2\ \left(\because\left|\vec{b}\right|=1\right)\ \Rightarrow 1=\left|\vec{c}\right|^2+\left|\vec{c}\right|^2\left|\vec{a}\right|^2\sin^2\frac{\pi}{12}$$

$$\Rightarrow 1=\left|\vec{c}\right|^2+\left|\vec{c}\right|^2(1)^2\sin^2\frac{\pi}{12}\Rightarrow 1=\left|\vec{c}\right|^2+\left|\vec{c}\right|^2\left(\frac{1-\cos\frac{\pi}{6}}{2}\right)$$

$$\Rightarrow 1=\left|\vec{c}\right|^2\left[1+\frac{1-\frac{\sqrt{3}}{2}}{2}\right]\Rightarrow 1=\left|\vec{c}\right|^2\left[1+\frac{2-\sqrt{3}}{4}\right]\Rightarrow\left|\vec{c}\right|^2=\frac{4}{6-\sqrt{3}}$$

$$\therefore 6\left|\vec{c}\right|^2=\frac{24}{6-\sqrt{3}}\times\frac{6+\sqrt{3}}{6+\sqrt{3}}=\frac{8\left(6+\sqrt{3}\right)}{11}$$

Q.17. If $S=\{z\in C:1\le|z-(1+i)|\le 2\}$ and $A=\{z\in S:|z-(1-i)|=1\}$ then the set A

A) is a null set

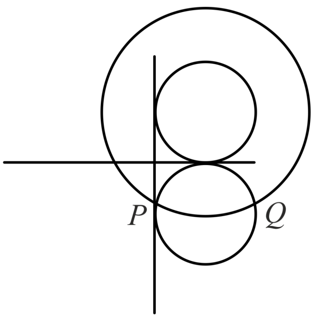
B) is a singleton set

C) has exactly two elements

D) has infinite elements

Answer: has infinite elements

Solution:



S represents the region between the concentric circles with centre $(1,1)$

Set A contains all points on minor arc PQ of the circle $|z-(1-i)|=1$.

Hence, A has infinite elements.

Q.18. If the function $x(y)=x$ and differential equation is given as $y\frac{dx}{dy}=2x+y^3(y+1)e^y$ and $x(1)=0$, then $x(e)$ is equal to

A) (e^e-1)

B) $e^3(e^e-1)$

C) e^e-1

D) e^e-3

Answer: $e^3(e^e-1)$

Solution: Given

$\frac{dx}{dy}-\frac{2x}{y}=y^2(y+1)e^y$

Finding integrating factor $I.F.=e^{\int\frac{-2}{y}dx}=e^{-2\ln y}=\frac{1}{y^2}$

$\Rightarrow \frac{x}{y^2}=\int(y+1)e^y dy$

$\Rightarrow \frac{x}{y^2}=y\cdot e^y+C$

$0=e+C\Rightarrow C=-e$

$\Rightarrow \frac{x}{y^2}=ye^y-e$

Putting $y=e$ in the equation we get

$\Rightarrow \frac{x}{e^2}=e.e^e-e$

$\Rightarrow x=e^3(e^e-1)$

Q.19. A balloon, spherical in shape is inflated and its surface area is increasing with a constant rate, initially the radius is 3 units, after 5 seconds radius is 7 units then the radius after 9 seconds is

A) 9

B) 7

C) 5

D) 3

Answer: 9

Solution: Let surface of the spherical balloon $A=4\pi r^2$

$\frac{dA}{dt}=8\pi r\frac{dr}{dt}=k$ (let) $\quad\ldots(1)$

On integrating on both sides w.r.t t , we get

$4\pi r^2=kt+C$.

Given that, at $t=0,\ r=3$.

$\Rightarrow 36\pi=C$

Also given that, at $t=5,\ r=7$

$\Rightarrow 4\pi\times 49=5k+36\pi$

$\Rightarrow 5k=4\pi(49-9)\quad\Rightarrow 5k=4\pi\times 40$

$\Rightarrow k=32\pi$

On substituting k value in equation (1)we get, $4\pi r^2=32\pi t+36\pi$

$\Rightarrow r^2=8t+9$

Given $t=9$.

$\Rightarrow r^2=81\Rightarrow r=9$.

Q.20. Given $S=\left\{\theta\ :\ \theta\in[-\pi,\ \pi]-\left\{\pm\frac{\pi}{2}\right\}\text{ and }\sin\theta\tan\theta+\tan\theta=\sin 2\theta\right\}$. If $T=\Sigma\cos\ 2\theta$ where $\theta\ \in S$, then $T+n(S)=$

A) 4

B) 5

C) 7

D) 9

Answer: 9

Solution: Given $\sin\ \theta\ \tan\ \theta + \tan\ \theta = \sin\ 2\theta;\ \theta \neq \pm\frac{\pi}{2}$

$\Rightarrow \frac{\sin^2\theta}{\cos\ \theta} + \frac{\sin\ \theta}{\cos\ \theta} = 2\ \sin\ \theta\ \cos\ \theta$

$\Rightarrow \sin\ \theta = 0$ or $\frac{\sin\ \theta}{\cos\ \theta} + \frac{1}{\cos\ \theta} = 2\ \cos\ \theta$

$\Rightarrow \theta = 0,\ \pi,\ -\pi$ or $\sin\ \theta + 1 = 2\left(1 - \sin^2\ \theta\right)$

$\Rightarrow \theta = 0,\ \pi,\ -\pi$ or $2\ \sin^2\ \theta\ + \sin\ \theta - 1 = 0$

$\Rightarrow \theta = 0,\ \pi,\ -\pi$ or $\sin\ \theta = \frac{1}{2},\ \sin\ \theta = -1$

$\Rightarrow \theta = 0,\ \pi,\ -\pi$ or $\theta = \frac{\pi}{6},\ \frac{5\pi}{6}$ or $\theta = \frac{-\pi}{2}$ (rejected)

Hence, $\theta = \left\{0,\ \pi,\ -\pi,\ \frac{\pi}{6},\ \frac{5\pi}{6}\right\}$

$\Rightarrow n\left(S\right) = 5$

Now, $T = \Sigma\cos\ 2\theta$

$= \cos\ 0 + \cos\ 2\pi + \cos\left(-2\pi\right) + \cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{5\pi}{3}\right)$

$= 1 + 1 + 1 + \frac{1}{2} + \frac{1}{2} = 4$

So, $T + n\left(S\right) = 9$