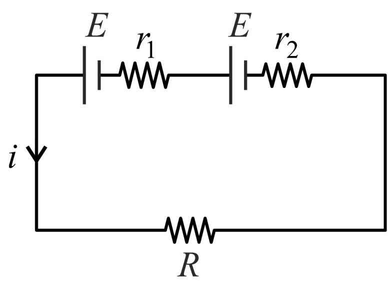
JEE Main Exam 2022 - Session 1

25 June 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. Two cells of the same EMF E but different internal resistances r_1 and r_2 are connected in series with an external resistance R as shown in the figure. The terminal potential difference across, the second cell is found to be zero. The external resistance R must then be:



A) $r_1 r_2$

B) $r_1 - r_2$

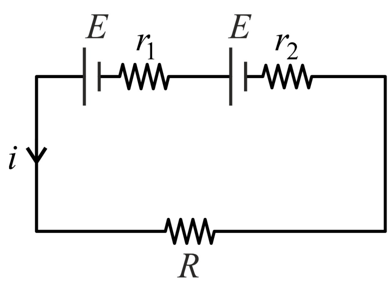
C) $r_2 - r_1$

D) $r_1 + r_2$

E) None of these

Answer: $r_2 - r_1$

Solution:



In this question we have to find the Terminal potential difference of second cell, we are given the internal resistances of both the cells and an external resistance. Applying the following expression:-

$V = E - i r_2$

$\Rightarrow 0 = E - i r_2$

Applying the expression of current

$0 = E - \left(\frac{2E}{r_1 + r_2 + R}\right) \cdot r_2$

$\Rightarrow R = r_2 - r_1$

Q.2. A long solenoid carrying a current produces a magnetic field B along its axis. If the current is doubled and the number of turns per cm is halved, the new value of the magnetic field is

A) $2B$

B) $4B$

C) $\frac{B}{2}$

D) B

Answer: B

Solution: Magnetic field due to long solenoid along its axis is given as,

$B=\mu_0 nI$, where n is the number of turns in the solenoid per unit length, I is current in the solenoid and μ_0 is a constant called the permeability of free space.

$\Rightarrow \frac{B_1}{B_2}=\frac{n_1 I_1}{n_2 I_2}$

According to the question, $n_2=\frac{n_1}{2}$ and $I_2=2I_1$

$\Rightarrow \frac{B}{B_2}=\frac{n_1}{\left(\frac{n_1}{2}\right)}\times\frac{I_1}{2I_1}\Rightarrow B_2=B$; means, the magnetic field remains the same.

Q.3. The correct relation between the degrees of freedom f and the ratio of specific heat γ is:

A) $f=\frac{2}{\gamma-1}$

B) $f=\frac{2}{\gamma+1}$

C) $f=\frac{\gamma+1}{2}$

D) $f=\frac{1}{\gamma+1}$

Answer: $f=\frac{2}{\gamma-1}$

Solution: From Mayer's relation $C_P=C_V+R$ and $C_V=\frac{f}{2}R$

Ratio of specific heat,

$\gamma=\frac{C_P}{C_V}=1+\frac{R}{\frac{f}{2}R}=1+\frac{2}{f}$

$\Rightarrow f=\frac{2}{\gamma-1}$

Q.4. A parallel plate capacitor have capacitance C and distance between plates as d. If distance between plates gets halved, then the ratio of old to new capacitances will be

A) $2:1$

B) $1:2$

C) $1:1$

D) $4:1$

Answer: $2:1$

Solution: Capacitance of a parallel plate capacitor is given by $C = \frac{A\varepsilon_0}{d}$, where, A is area of plate, ε_0 is permittivity of space and d is the distance between plates.

Now, if the distance gets halved, then $C' = \frac{A\varepsilon_0}{\frac{d}{2}} = 2C$.

Thus, the ratio is $\frac{C'}{C} = \frac{2}{1} = 2:1$

Q.5. The ratio of intensities of two concurrent sources of light are in ratio $4:1$. If $\frac{(I_{\max}+I_{\min})}{(I_{\max}-I_{\min})}$ is $\frac{5}{x}$, find x.

A) 4

B) 1

C) 2

D) 3

Answer: 4

Solution: The ratio of the intensities is $\frac{I_1}{I_2} = \frac{4}{1}$

After taking square root of above equation on both sides, we get, $\frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{2}{1}$

Apply componendo and dividendo,

$\Rightarrow \frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}} = \frac{2+1}{2-1} = \frac{3}{1}$

$\Rightarrow \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \frac{I_{\max}}{I_{\min}} = \frac{9}{1}$

$\Rightarrow \frac{I_{max}+I_{min}}{I_{max}-I_{min}} = \frac{10}{8} = \frac{5}{4}$

Therefore, the answer is 4.

Q.6. Brewster's law is valid

A) When reflected ray and refracted ray are perpendicular to each other

B) When reflected ray and refracted ray are parallel to each other

C) When reflected ray and refracted ray are in same direction

D) None

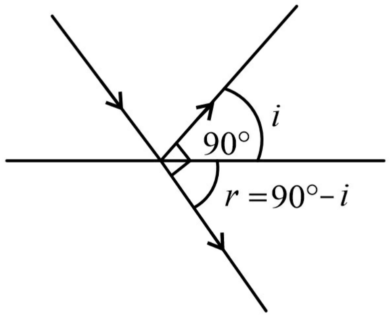
Answer: When reflected ray and refracted ray are perpendicular to each other

Solution: As per the Brewster's Law, the polarisation takes place when the angle is 90° in between the reflected ray and refracted ray. The polarizations angle is called Brewster's angle.

It states that the refractive index of the medium is numerically equal to the tangent of angle of polarization,

$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin 90^\circ - i}$

$\mu = \tan\, i_p$



Q.7. Two masses are in a circular motion about a planet in the orbits of radii are $3200\ \mathrm{km}$ and $800\ \mathrm{km}$. Their orbital velocities are in the ratio of:

A) $1:2$

B) $2:1$

C) $1:1$

D) $1:3$

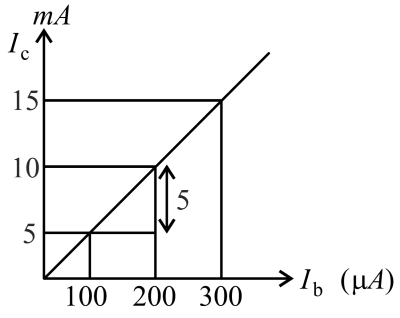
Answer: $1:2$

Solution: Orbital velocity of a satellite in circular motion around a planet,

$v_o = \sqrt{\frac{GM}{r}} \propto \frac{1}{\sqrt{r}}$

Now, $\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{800}{3200}} = \frac{1}{2}$

Q.8. Characteristic of n-p-n transistor is shown below, find the voltage gain. (Input resistance $= 200\ \Omega$ and output resistance $= 60\ \Omega$)



A) 15

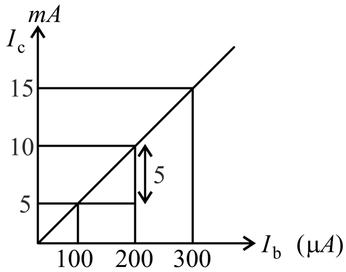
B) 20

C) 25

D) 30

Answer: 15

Solution:



Voltage gain, $A_V = ?$
Input Resistance $= 200\ \Omega$
Output Resistance $= 60\ \Omega$

$$\beta = \frac{\Delta I_c}{\Delta I_b} = \frac{(5-0)\times 10^{-3}}{(100-0)\times 10^{-6}} = 50$$

$$A_V = \beta \frac{R_c}{R_b} = 50 \times \frac{60}{200} = 15$$

Q.9. A cube has total surface area equal to $24\ \text{cm}^2$. If the temperature is changed by $10\ ^{\circ}\text{C}$, then change in its volume is equal to $\left(\alpha = 1.5 \times 10^{-3}{}^{\circ}\text{C}^{-1}\right)$

A) $0.12\ \text{cm}^3$

B) $0.84\ \text{cm}^3$

C) $0.54\ \text{cm}^3$

D) $1.12\ \text{cm}^3$

Answer: $0.12\ \text{cm}^3$

Solution: Total surface area of a cube is $6a^2 = 24\ \text{cm}^2 \Rightarrow a^2 = 4 \Rightarrow a = 2\ \text{cm}$

Volume of a cube $V = a^3 = 8\ \text{cm}^3$

Change in volume is given by,

$\Delta V = V_0 \gamma \Delta T$, where $\gamma = 3\alpha$

$$\Delta V = [8] \times \left[1.5 \times 10^{-3}\right] \times [10]\ \text{cm}^3 \Rightarrow \Delta V = 0.12\ \text{cm}^3$$

Q.10. Displacement function of two particles are given as, $x_a = \alpha t + \beta t^2$ & $x_b = \alpha t^2 - \beta t$. Find the time when $v_a = v_b$.

A) $\frac{(\alpha+\beta)}{(\alpha-\beta)}$

B) $\frac{(\alpha-\beta)}{(\alpha+\beta)}$

C) $\frac{(\alpha-\beta)}{2(\alpha+\beta)}$

D) $\frac{(\alpha+\beta)}{2(\alpha-\beta)}$

Answer: $\frac{(\alpha+\beta)}{2(\alpha-\beta)}$

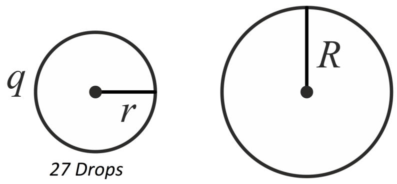
Solution: As we know, $v=\frac{dx}{dt}$.

$v_a=\frac{dx_a}{dt}=\alpha+2\beta t$ and $v_b=2\alpha t-\beta$

When $v_a=v_b$

$\Rightarrow \alpha+2\beta t=2\alpha t-\beta \Rightarrow \alpha+\beta=2(\alpha-\beta)t \ \Rightarrow t=\frac{\alpha+\beta}{2(\alpha-\beta)}$

Q.11. 27 spherical drops of radius r each at potential $22\ \mathrm{V}$ is combined to form big spherical drop of radius R. Potential of the bigger drop is



A) $198\ \mathrm{V}$

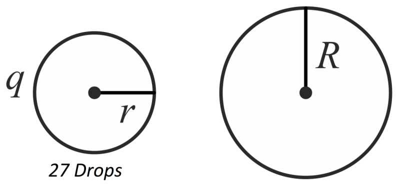
B) $200\ \mathrm{V}$

C) $202\ \mathrm{V}$

D) $204\ \mathrm{V}$

Answer: $198\ \mathrm{V}$

Solution:



Total volume of the drop will remain unchanged, $27\times\left(\frac{4}{3}\pi r^3\right)=\frac{4}{3}\pi R^3$

$\Rightarrow R=3r$

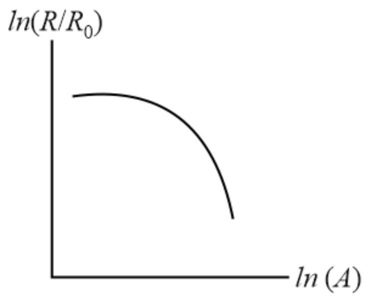
Potential of the small drops, $22=\frac{kq}{r}$

For bigger drop, potential will be

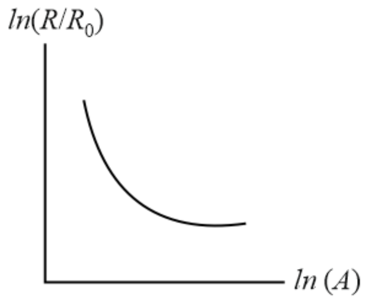
$V=\frac{k(27q)}{(3r)}=9\left(\frac{kq}{r}\right)=9\times 22=198\ \mathrm{V}$

Q.12. Which of the following is the correct graph between $\ln\left(\frac{R}{R_0}\right)$ and $\ln A$ where R is radius and A is atomic mass number?

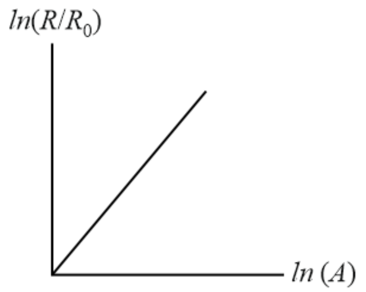
A)



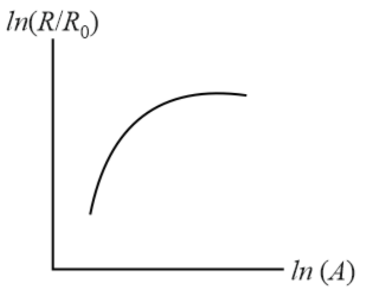
B)



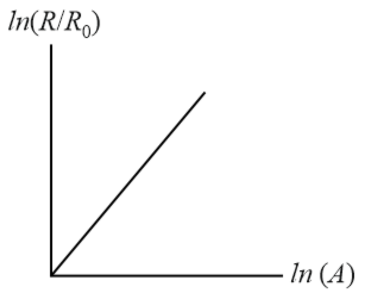
C)



D)



Answer:



Solution: We know the relation between radius and atomic mass number is $R=R_0A^{\frac{1}{3}} \Rightarrow \ln R=\ln R_0+\frac{1}{3}\ln A$

$\Rightarrow \ln\left(\frac{R}{R_0}\right)=\frac{1}{3}\ln A$

It is similar to $y=mx$ which is equation of line passing through origin.

Q.13. If electron, neutron, proton and α - particle have equal kinetic energy, their de-Broglie wavelengths would be in the order -

A) $\lambda_e>\lambda_p>\lambda_n>\lambda_\alpha$

B) $\lambda_e=\lambda_p>\lambda_n>\lambda_\alpha$

C) $\lambda_e>\lambda_p=\lambda_n>\lambda_\alpha$

D) $\lambda_e>\lambda_p=\lambda_\alpha>\lambda_n$

Answer: $\lambda_e>\lambda_p>\lambda_n>\lambda_\alpha$

Solution: The de-Broglie wavelength $\lambda=\frac{\mathrm{h}}{\sqrt{2\mathrm{m}(\mathrm{K.E.})}}$

where, h and $\mathrm{K.E.}$ are constant

$\lambda\propto\frac{1}{\sqrt{\mathrm{m}}}$

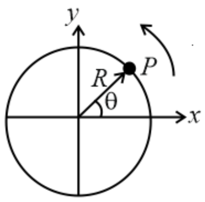
$\mathrm{m}=$ Mass of the particle

Thus, with increasing mass of particle de-Broglie wavelength decreases.

Order of mass $m_e<m_p<m_n<m_\alpha$

Order of de-Broglie wavelength is $\lambda_e>\lambda_p>\lambda_n>\lambda_\alpha$

Q.14. What is the acceleration of the particle moving in a circle of radius R with constant speed v?



A) $\frac{v^2}{R}\left(-\sin\theta\hat{\mathrm{i}}+\cos\theta\hat{\mathrm{j}}\right)$

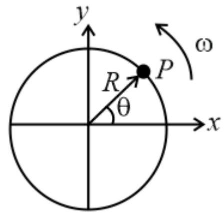
B) $\frac{v^2}{R}\left(\sin\theta\hat{\mathrm{i}}+\cos\theta\hat{\mathrm{j}}\right)$

C) $\frac{v^2}{R}\left(-\sin\theta\hat{\mathrm{i}}-\cos\theta\hat{\mathrm{j}}\right)$

D) $\frac{v^2}{R}\left(-\cos\theta\hat{\mathrm{i}}-\sin\theta\hat{\mathrm{j}}\right)$

Answer: $\frac{v^2}{R}\left(-\cos\theta\hat{\mathrm{i}}-\sin\theta\hat{\mathrm{j}}\right)$

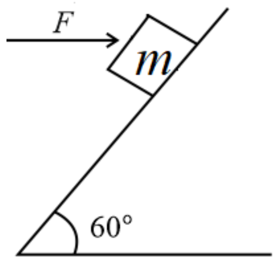
Solution:



Acceleration of a particle in uniform circular motion is directed towards the centre of the circle and its magnitude is $\frac{v^2}{R}$. The radius vector is $\overrightarrow{r} = R\cos\theta\hat{\mathrm{i}} + R\sin\theta\hat{\mathrm{j}}$

Therefore, $\overrightarrow{a} = -\frac{v^2}{R}\hat{\mathrm{r}} = \frac{v^2}{R}\left(-\cos\theta\hat{\mathrm{i}} - \sin\theta\hat{\mathrm{j}}\right)$

Q.15. Block of mass $m = 0.2\ \mathrm{kg}$ is kept on a smooth inclined plane with inclination 60°. Find horizontal F to keep the block stationary. $\left(g = 10\ \mathrm{m\ s^{-2}}\right)$



A) $2\sqrt{3}\ \mathrm{N}$

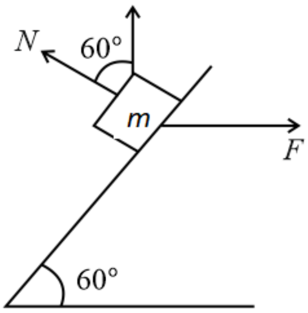
B) $2\ \mathrm{N}$

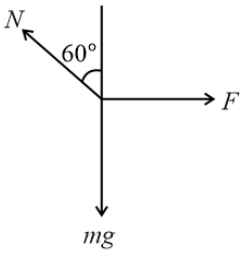
C) $3\ \mathrm{N}$

D) $\frac{2}{\sqrt{3}}\ \mathrm{N}$

Answer: $2\sqrt{3}\ \mathrm{N}$

Solution:





Balancing the forces in horizontal direction $N \sin \theta = F$

And balancing the forces in vertical direction $N \cos \theta = mg$

$\Rightarrow \tan \theta = \frac{F}{mg} \Rightarrow F = mg \tan \theta$

$F = 0.2 \times 10 \times \sqrt{3} = 2\sqrt{3}\ \mathrm{N}$

Q.16. A LCR circuit connected across AC source of $V = 220\ \mathrm{V}$ with $R = 10\ \Omega,\ L = 100\ \mathrm{mH},\ C = 100\ \mu\mathrm{F}$. If frequency of the AC source is $50\ \mathrm{Hz}$, find the current?

(Take, $\pi^2 = 10$)

A) $22\ \mathrm{A}$

B) $24\ \mathrm{A}$

C) $26\ \mathrm{A}$

D) $28\ \mathrm{A}$

Answer: $22\ \mathrm{A}$

Solution: $\omega = 2\pi f = 100\pi\ \mathrm{rad\ s^{-1}}$

$X_L = \omega L = (100\pi)\left(100 \times 10^{-3}\right) = 10\pi\ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{(100\pi)\times\left(100\times10^{-6}\right)} = \frac{100}{\pi}\ \Omega = 10\pi\ \Omega$

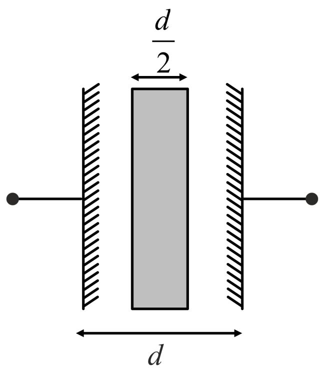
Now, $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = R = 10\ \Omega$

Therefore,

$I_{\mathrm{RMS}} = \frac{V_{\mathrm{RMS}}}{Z}$,

$= \frac{220}{10} = 22\ \mathrm{A}$

Q.17. A metal of thickness $\frac{d}{2}$ is inserted between plates of capacitor having capacitance C and distance between plate is d. If new capacitance is C' the value of $\frac{C}{C'}$ is:



A) $1:2$

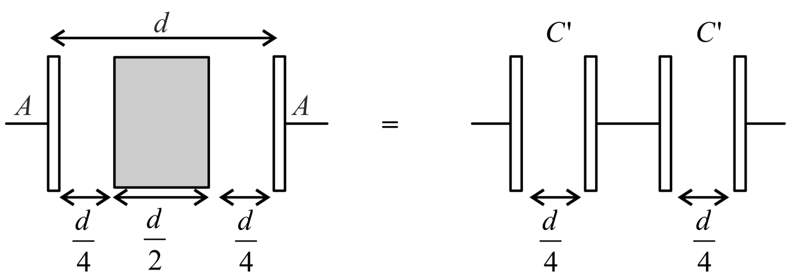
B) $2:1$

C) $1:1$

D) $2:3$

Answer: $1:2$

Solution:



Before the metal sheet is inserted, capacitance is given by $C=\frac{\varepsilon_0 A}{d}$.

After the sheet is inserted, the system is equivalent to two capacitors in series, each of capacitance $C'=\frac{\varepsilon_0 A}{\left(\frac{d}{4}\right)}=4C$.

The equivalent capacity is now $\frac{1}{C'}=\frac{1}{4C}+\frac{1}{4C}\Rightarrow C'=2C$.

The ratio is $\frac{C}{C'}=\frac{1}{2}=1:2$

Q.18. The velocity of an electromagnetic wave in a medium is $2\times 10^{8}\ \mathrm{m\ s^{-1}}$. If the relative permeability is 1, then find the relative permittivity of the medium.

A) $\frac{9}{4}$

B) $\frac{13}{4}$

C) $\frac{7}{4}$

D) $\frac{15}{4}$

Answer: $\frac{9}{4}$

Solution: Here, $v = 2 \times 10^{8}\,\mathrm{m\,s^{-1}}$ and $\mu_{\mathrm{r}} = 1$

The speed of electromagnetic waves in a medium is given by $v = \frac{1}{\sqrt{\mu\varepsilon}} \ \ldots(i)$

where μ and ε are absolute permeability and absolute permittivity of the medium.
Now, $\mu = \mu_0\mu_{\mathrm{r}}$ and $\varepsilon = \varepsilon_0\varepsilon_{\mathrm{r}}$
Then, we have,

$$v = \frac{1}{\sqrt{\mu_0\mu_{\mathrm{r}} \times \varepsilon_0\varepsilon_{\mathrm{r}}}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \times \frac{1}{\sqrt{\mu_{\mathrm{r}}\varepsilon_{\mathrm{r}}}} = \frac{c}{\sqrt{\mu_{\mathrm{r}}\varepsilon_{\mathrm{r}}}}$$

$$or\ \ \varepsilon_{\mathrm{r}} = \frac{c^2}{v^2\mu_r} = \frac{\left(3 \times 10^{8}\right)^2}{\left(2 \times 10^{8}\right)^2} = \frac{9}{4}$$

Q.19. Consider four symmetrical objects of mass M and radius R, a solid sphere, a solid cylinder, a thin disk & a thin ring. Moment of inertia of sphere about diameter is I_1, that of cylinder about the axis of cylinder is I_2, that of disk about diameter of disc is I_3 and that of ring about diameter of ring is I_4 are related as $2\left(I_2 + I_3\right) + I_4 = xI_1$, the value of x is

A) 3

B) 4

C) 5

D) 6

Answer: 5

Solution: $\because\ 2\left(I_2 + I_3\right) + I_4 = xI_1$

Now $I_1 = \frac{2}{5}MR^2$, $I_2 = \frac{1}{2}MR^2$, $I_3 = \frac{1}{4}MR^2$ and $I_4 = \frac{1}{2}MR^2$

$$\Rightarrow 2\left(\frac{MR^2}{2} + \frac{MR^2}{4}\right) + \frac{MR^2}{2} = x \times \frac{2}{5}MR^2$$

$$\Rightarrow 2\left(\frac{3}{4}\right) + \frac{1}{2} = x \times \frac{2}{5}$$

$$\Rightarrow 2 = x \times \frac{2}{5}$$

$$\Rightarrow x = 5$$

Q.20. When temperature is increased the susceptibility of paramagnetic and ferromagnetic materials will

A) Increase.

B) Decrease.

C) remains constant.

D) None of these

Answer: Decrease.

Solution: When temperature is increased the susceptibility of paramagnetic and ferromagnetic materials will decrease.

Q.21. If the metal has threshold frequency of $5 \times 10^{14}\ \mathrm{Hz}$, then find out the work function of the metal.

A) $3.06\ \mathrm{eV}$

B) $4.13\ \mathrm{eV}$

C) $2.06\ \mathrm{eV}$

D) $3.14\ \text{eV}$

Answer: $2.06\ \text{eV}$

Solution: Work function $\phi = h\nu_0$

$\phi = \left(4.13 \times 10^{-15}\right)\ \text{eV}\ \ \text{Hz}^{-1} \times \left(5 \times 10^{14}\right)\ \text{Hz} = 2.06\ \text{eV}$

Section B: Chemistry

Q.1. When an ideal gas filled in a closed vessel is heated through $1^\circ C$, its pressure increases by 0.4%. The initial temperature of the gas was:

A) $250\ K$

B) $2500\ K$

C) $250^\circ C$

D) $25^\circ C$

Answer: $250\ K$

Solution: Let the initial temperature $= T\ K$

Final temperature $= (T + 1)\ K$.

Let, initial pressure $= P$

Final pressure $= P + \frac{P \times 0.4}{100}$

Applying $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ on the gaseous sample.

$(V_1 = V_2 \text{ as the container is closed})$

Or, $\frac{P}{T} = \frac{\left(P + \frac{P \times 0.4}{100}\right)}{(T+1)}$

Or, $T = 250\ K$

Q.2. What will be solubility of A_2X_3 if its solubility product (K_{sp}) is equal to 1.08×10^{-23}?

A) 10^{-5} mol L^{-1}

B) 3.7×10^{-4} mol L^{-1}

C) 1.2×10^{-3} mol L^{-1}

D) 7.5×10^{-4} mol L^{-1}

Answer: 10^{-5} mol L^{-1}

Solution: A_2X_3 will get dissociated as:

$$A_2X_3 \rightleftharpoons \underset{2s}{2A^{+3}} + \underset{3s}{3X^{-2}}$$

Here,

$K_{sp} = (2s)^2(3s)^3 = 108\ s^5$

$s = \left(\frac{K_{sp}}{108}\right)^{1/5}$

$s = \left(\frac{1.08 \times 10^{-23}}{108}\right)^{1/5}$

$s = 10^{-5}\ \text{mol}\ L^{-1}$

Q.3. How many of the following are non-polar molecules.

NH_3 ,HCl, H_2O , BeF_2, BH_3, CCl_4

A) 1

B) 3

C) 2

D) 5

Answer: 3

Solution: Among the given molecules BeF_2, CCl_4 and BH_3 has zero dipole moment. BeF_2 has linear structure and BH_3 has symmetrical trigonal planar structure and CCl_4 also has symmetrical tetrahedral structure. Others have non-symmetrical structure.

Q.4. Which antiseptic contain 6π electron

A) Bithional

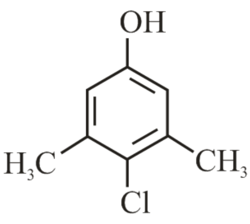
B) Chloroxyenol

C) Chlorophenol

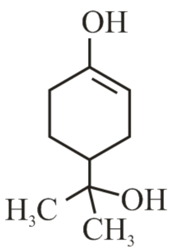
D) Terpineol

Answer: Chloroxyenol

Solution:



Chloroxylenol



Terpineol

dettol is a mixture of chloroxylenol and terpineol. chloroxylenol has 6π electrons and terpineol has 2π electrons

Q.5. Assertion: Water with BOD value 17 is considered polluted

Reason: The amount of oxygen required by the bacteria to break down the inorganic and organic matter in certain volume of water is called BOD value.

A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

B) Both Assertion and Reason are true but Reason is NOT the correct explanation of Assertion.

C) Assertion is true but Reason is false.

D) Assertion is false but Reason is true.

Answer: Assertion is true but Reason is false.

Solution: The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water, is called Biochemical Oxygen Demand (BOD). The amount of BOD in the water is a measure of the amount of organic material in the water, in terms of how much oxygen will be required to break it down biologically. Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

Q.6. Novalac has molar mass 393. Find number of its monomers in it.

A) 1

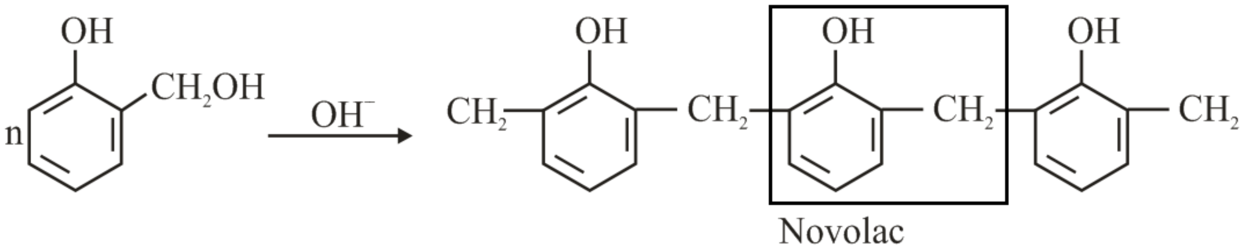
B) 2

C) 3

D) 4

Answer: 4

Solution:



The molar mass of the monomer unit $= 124$

Assume there are n number of monomer units, Hence, $n-1$ number of water molecules removed during the formation of novalac.

$n \times 124 = 393 + (n-1)18$

$n \simeq 4$

Now the difference in the mass is due to terminal OH group.

Q.7. $t_{1/2}$ of a reaction and the initial pressure of the reactant is given. Find the order of the reaction.

$P^\circ{}_R$	$t_{1/2}$
10 atm	5 min
20 atm	10 min

A) 1 st order

B) 2nd order

C) Zero order

D) 1/2 order

Answer: Zero order

Solution:

$$t_{\frac{1}{2}} \propto \left(\frac{P_0}{2k}\right)^{1-n}$$

$$\frac{\left(t_{\frac{1}{2}}\right)_1}{\left(t_{\frac{1}{2}}\right)_2} = \left(\frac{(P_0)_1}{(P_0)_2}\right)^{1-n}$$

$$\frac{5}{10} = \left(\frac{10}{20}\right)^{1-n}$$

$n = 0$

So, this is a zero order reaction.

Q.8. What is the correct oder of electron gain enthalpy of Cl, F, Te, Po

A) $\mathrm{F} > \mathrm{Cl} > \mathrm{Te} > \mathrm{Po}$

B) $\mathrm{Cl} > \mathrm{F} > \mathrm{Te} > \mathrm{Po}$

C) $\mathrm{Te} > \mathrm{Po} > \mathrm{Cl} > \mathrm{F}$

D) $\mathrm{Po} > \mathrm{Te} > \mathrm{F} > \mathrm{Cl}$

Answer: $\mathrm{Cl} > \mathrm{F} > \mathrm{Te} > \mathrm{Po}$

Solution: Electron gain enthalpy values of the elements are given below in $\mathrm{kJ} / \mathrm{mol}$,

$$\begin{array}{cccc} \mathrm{F} & \mathrm{Cl} & \mathrm{Te} & \mathrm{Po} \\ -333 & -349 & -190 & -174. \end{array}$$

Down the group electron affinity decreases. The size of fluorine atom is small. The addition of electron to small fluorine atom results in electron-electron repulsions. Hence, fluorine has less electron affinity than chlorine.

Hence, the correct order is

$\mathrm{Cl} > \mathrm{F} > \mathrm{Te} > \mathrm{PO}$

Q.9. Assertion: The amphoteric behaviour of H_2O is explained by Lewis acid base theory

Reason: H_2O acts as acid with NH_3 and base with H_2S

A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

B) Both Assertion and Reason are true but Reason is NOT the correct explanation of Assertion.

C) Assertion is true but Reason is false.

D) Assertion is false but Reason is true.

Answer: Assertion is false but Reason is true.

Solution: $\underbrace{H_2O}_{\text{Acid}} + \underbrace{\ddot{N}H_3}_{\text{Base}} \rightarrow {}^{-}OH + NH_4^+$

$\underbrace{H_2O}_{\text{Base}} + \underset{\text{Acid}}{H_2S} \longrightarrow H_3O^+ + HS^{\ominus}$

The amphoteric behaviour of H_2O is explained by Brønsted theory not the Lewis acid base theory.

So, we can say Assertion is false, but reason is true.

Q.10. Arrange the following in decreasing order of reduction potential

1) $Cl_2 \mid Cl^-$ 2) $I_2 | I^-$

3) $Li^+ \mid Li$ 4) $Na \mid Na^+$

(5) $Ag^+ | Ag$

A) $1 > 5 > 2 > 4 > 3$

B) $5 > 3 > 2 > 1 > 4$

C) $1 > 4 > 3 > 2 > 1$

D) $3 > 5 > 4 > 2 > 1$

Answer: $1 > 5 > 2 > 4 > 3$

Solution:

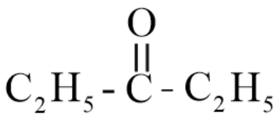
$$E^0_{Cl_2|Cl^-} = 1.36$$
$$E^\circ_{I_2|I^-} = 0.54$$
$$E_{Li^+|Li} = -3.05$$
$$E^\circ_{Ag^+|Ag} = 0.80$$
$$E_{Na^+|Na} = -2.71$$

So, we can clearly see the order of reduction potential.

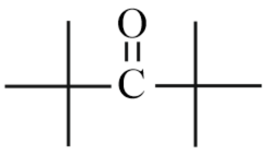
$1 > 5 > 2 > 4 > 3$

Q.11. Which of the following does not give enamine on reaction with 2 degree amine?

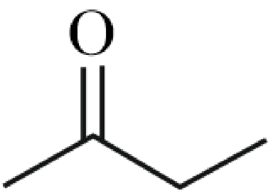
A)



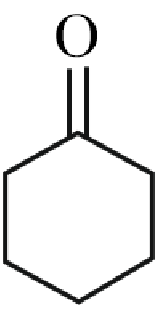
B)



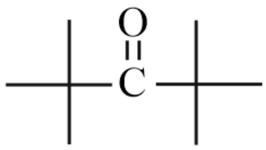
C)



D)



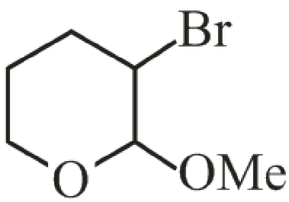
Answer:



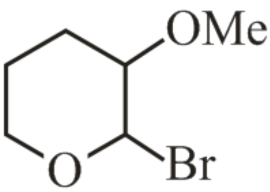
Solution: Due to absence of alpha hydrogen it does not give enamine on reaction with 2 degree amine. All other molecules has alpha hydrogen present.

Q.12. The product of following reaction will be

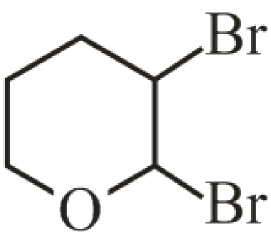
A)



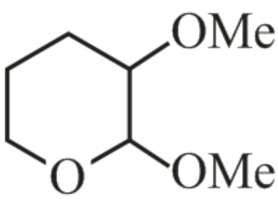
B)



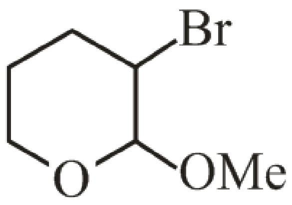
C)



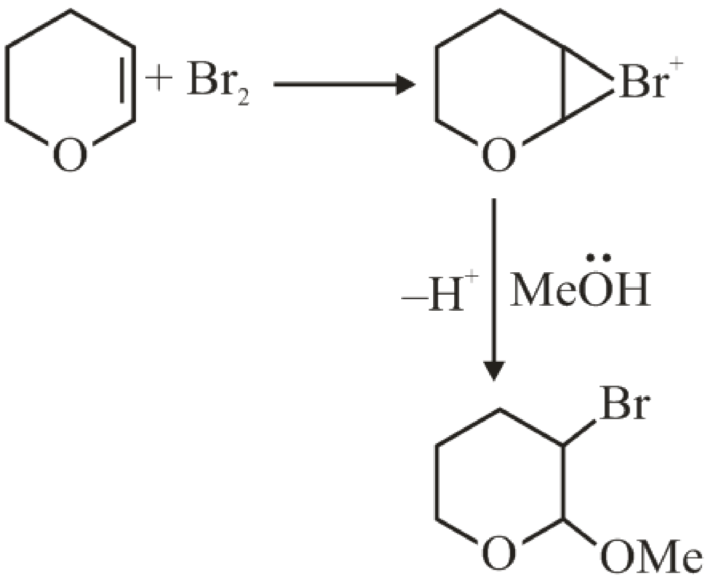
D)



Answer:



Solution:



This is an example of electrophilic addition reaction of alkene.

Q.13. Which will give biuret test

A) Tripeptide

B) Methionine

C) Glycine

D) Cyclalamine

Answer: Tripeptide

Solution: The biuret test, also known as Piotrowski's test, is a chemical test used for detecting the presence of peptide bonds. The Biuret reagent is made of sodium hydroxide ($NaOH$) and hydrated copper(II) sulfate, together with potassium sodium tartrate, the latter of which is added to chelate and thus stabilize the cupric ions. The reaction of the cupric ions with the nitrogen atoms involved in peptide bonds leads to the displacement of the peptide hydrogen atoms under the alkaline conditions.

So, among the given options tripeptide will have peptide bond, so it will show Biuret test.

Q.14. $PCl_5 + H_2O \rightarrow A + HCl$

$A + H_2O \rightarrow B + HCl$

Find number of ionisable hydrogen in B

A) 6

B) 3

C) 2

D) 1

Answer: 3

Solution: PCl_5 is a yellowish white powder and in moist air, it hydrolyses into $POCl_3$ and finally gets converted to phosphoric acid.

$PCl_5 + H_2O \rightarrow POCl_3 + 2HCl$

$POCl_3 + 3H_2O \rightarrow H_3PO_4 + 3HCl$

Phosphoric acid has 3 ionisable hydrogen atoms.

Q.15. Which of the following has the least spin only magnetic moment?

A) Fe^{3+}

B) Fe^{2+}

C) Cu^{2+}

D) Ni^{2+}

Answer: Cu^{2+}

Solution: Magnetic moment is calculated by the formula $\mu = \sqrt{n(n+2)B.M.}$

$Cu^{2+} = [Ar]3d^9$

1 unpaired electrons

$Fe^{3+} = \underbrace{[Ar]3d^5}$

5 unpaired electrons

$Fe^{2+} = [Ar]3d^6$

4 unpaired electrons

$Ni^{2+} = [Ar]3d^8$

2 unpaired electrons

Number of unpaired electrons are directly proportional to magnetic moment.

As Cu^{2+} has only one unpaired electron it has the least magnetic moment

Q.16. If 10 ml of 0.1 M A neutralize 20 ml of 0.05 $M(OH)_2$. Find the basicity of A

A) 4

B) 2

C) 3

D) 6

Answer: 2

Solution: Number of Equivalents of Acid $=$ Number of Equivalents of base

$\mathrm{M} \times \mathrm{V} \times \mathrm{n}_{\text{factor}} = \mathrm{M} \times \mathrm{V} \times \mathrm{n}_{\text{factor}}$

$\frac{1}{10} \times 10 \times \mathrm{n}_{\text{factor}} = \frac{5}{100} \times 20 \times 2$

$\mathrm{n}_{\text{factor}} = 2$

Basicity of an acid $= \mathrm{n}_{\text{factor}} = 2$

Q.17. Electron, proton, neutron and alpha particle have same value of kinetic energy. What is the correct order of de-Broglie wavelength?

A) $\lambda_{\mathrm{P}} = \lambda_{\mathrm{n}} = \lambda_{\mathrm{e}} = \lambda_{\alpha}$

B) $\lambda_{\alpha} < \lambda_{\mathrm{n}} < \lambda_{\mathrm{P}} < \lambda_{\mathrm{e}}$

C) $\lambda_{\mathrm{P}} = \lambda_{\mathrm{e}} = \lambda_{\alpha} = \lambda_{\mathrm{n}}$

D) $\lambda_{\alpha} < \lambda_{\mathrm{P}} < \lambda_{\mathrm{e}} < \lambda_{\mathrm{n}}$

Answer: $\lambda_{\alpha} < \lambda_{\mathrm{n}} < \lambda_{\mathrm{P}} < \lambda_{\mathrm{e}}$

Solution: $\lambda = \frac{\mathrm{h}}{\sqrt{2\mathrm{mKE}}}$

mass : $\alpha > \mathrm{n} > \mathrm{p} > \mathrm{e}$

$\lambda_{\alpha} < \lambda_{\mathrm{n}} < \lambda_{\mathrm{p}} < \lambda_{\mathrm{e}}.$

Q.18. Match the following correctly

(i) Zymase	(a) Stomach
(ii) Urease	(b) Yeast
(iii) Diastase	(c) Malt
(iv) Pepsin	(d)Soyabean

A) (i)-B; (ii) D(iii)-C; (iv)-A

B) (i)-B; (ii)-A; (iii)-C; (iv)-D

C) (i)-A; (ii)-B; (iii)-C; (iv)-D

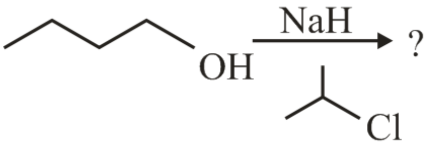
D) (i)-D; (ii)-C; (iii)-B; (iv)-A

Answer: (i)-B; (ii) D(iii)-C; (iv)-A

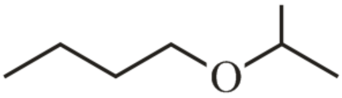
Solution: The enzymes and sources are:

Zymase $\rightarrow$ Yeast
Urease $\rightarrow$ Soyabean
Diastase $\rightarrow$ Malt
Pepsin $\rightarrow$ Stomach

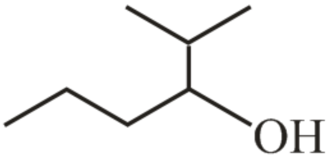
Q.19.



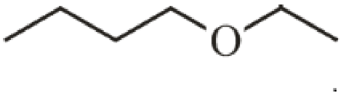
A)



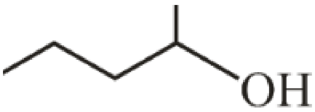
B)



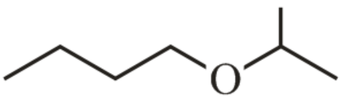
C)



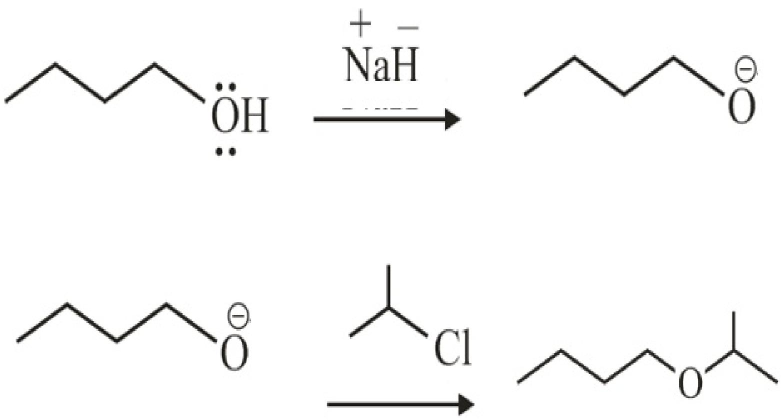
D)



Answer:



Solution: In the first step butoxide ion is generated, which reacts with isopropyl chloride by williamson's synthesis reaction to form the ether.



Q.20. The minimum energy required for the emission of photoelectron from the surface of a metal if threshold frequency is $1.4 \times 10^{15}\ \mathrm{sec}^{-1}$. Given that $\mathrm{h} = 6.6 \times 10^{-34}\ \mathrm{Js}$.

A) $9.24 \times 10^{-19}\mathrm{J}$

B) $9.24 \times 10^{-18}\mathrm{J}$

C) $4.62 \times 10^{-19}\mathrm{J}$

D) $4.62 \times 10^{-18}\mathrm{J}$

Answer: $9.24 \times 10^{-19}\mathrm{J}$

Solution: Radiation energy = threshold energy of metal + kinetic energy of ejected electron

For minimum energy,

Radiation energy = threshold energy of metal

$\mathrm{E} = \mathrm{h}\nu$
$\mathrm{E} = 6.6 \times 10^{-34} \times 1.4 \times 10^{15} = 9.24 \times 10^{-19}\mathrm{J}$

Section C: Mathematics

Q.1. In the expansion of $(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+\ldots+x^{500}$, then the coefficient of x^{101} is

A) $5^{400}\times{}^{501}C_{101}$

B) $5^{399}\times{}^{501}C_{101}$

C) $5^{599}\times{}^{500}C_{101}$

D) $5^{399}\times{}^{501}C_{101}$

Answer: $5^{399}\times{}^{501}C_{101}$

Solution: $\dfrac{(5+x)^{500}\left[\left(\frac{x}{5+x}\right)^{501}-1\right]}{\left(\frac{x}{5+x}\right)-1}=\dfrac{x^{501}-(5+x)^{501}}{-5}$

$=\frac{1}{5}(5+x)^{501}-\frac{x^{501}}{5}$

Now coefficient of x^{101} in $\frac{1}{5}(5+x)^{501}$ will be $\frac{1}{5}\left({}^{501}C_{101}\right)(5)^{501-101}=5^{399}\left({}^{501}C_{101}\right)$

Q.2. If the sum of first n terms of two A.P.'s are in the ratio $3n+8:7n+15$, then find the ratio of their 12th term :

A) $8:7$

B) $7:16$

C) $74:169$

D) $13:47$

Answer: $7:16$

Solution: Given $\frac{S_n}{S_n'}=\frac{3n+8}{7n+15}$

Now $S_n=\dfrac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']}=\dfrac{3n+8}{7n+15}$

$\Rightarrow\frac{S_n}{S_n'}=\frac{2a+(n-1)d}{2a'+(n-1)d'}=\frac{3n+8}{7n+15}\Rightarrow\dfrac{a+\frac{(n-1)d}{2}}{a'+\frac{(n-1)}{2}d'}=\frac{3n+8}{7n+15}$ ......equation (i)

Now we need to find 12^{th} term

$\Rightarrow T_{12}=a+11d$

So taking $\frac{n-1}{2}=11$ in equation (i)

We get $n=23$,

So $\frac{T_{12}}{T_{12}'}=\frac{3\times23+8}{7\times23+15}=\frac{77}{176}=\frac{7}{16}$

Q.3. The value of $\tan^{-1}\left[\dfrac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right]$ is equal to

A) $-\frac{\pi}{8}$

B) $-\frac{4\pi}{9}$

C) $-\frac{5\pi}{12}$

D) $-\frac{\pi}{4}$

Answer: $-\frac{\pi}{8}$

Solution: Given,

$$\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right)$$

$$=\tan^{-1}\left(\frac{\cos\left(4\pi-\frac{\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{1}{\sqrt{2}}-1}{\frac{1}{\sqrt{2}}}\right)$$

$$=\tan^{-1}\left(1-\sqrt{2}\right)\ =-\tan^{-1}\left(\sqrt{2}-1\right)$$

$$=-\frac{\pi}{8}$$

Q.4. The sum of the series $1\times3+2\times3^2+3\times3^3+\cdots+10\times3^9$ is equal to

A) $\frac{3+19\times3^{10}}{4}$

B) $\frac{3-19\times3^{10}}{4}$

C) $\frac{3+20\times3}{4}$

D) $\frac{3-20\times3^{10}}{4}$

Answer: $\frac{3+19\times3^{10}}{4}$

Solution: Let

$$S=1\times3+2\times3^2+3\times3^3+\ \cdots\ +10\times3^9$$
$$3S=\ 0+1\times3^2+2\times3^3+\cdots+9\times3^9+10\times3^{10}$$
$$-\quad-\quad-\quad-\quad-\quad-\quad-$$
$$-----------------------$$
$$-2S=\left(1\times3+3^2+3^3+\cdots+3^9\right)-10\times3^{10}$$

$$\Rightarrow -2S=\frac{3(3^9-1)}{3-1}-10\times3^{10}\Rightarrow -2S=\frac{3(3^9-1)-20\times3^{10}}{2}$$

$$\Rightarrow S=\frac{3^{10}-3-20\times3^{10}}{-4}$$

$$\Rightarrow S=\frac{3+19\times3^{10}}{4}$$

Therefore, the sum of the given series $=\frac{3+19\times3^{10}}{4}$.

Q.5. If there is a biased dice with number on the faces as $2,4,8,16,32,32$ such that the probability of appearing of number n is $\frac{1}{n}$, then the probability of getting the sum as 48 when three dice are thrown is

A) $\frac{1}{2048}$

B) $\frac{1}{4096}$

C) $\frac{13}{4096}$

D) $\frac{5}{4096}$

Answer: $\frac{13}{4096}$

Solution: $P(2)=\frac{1}{2}, P(4)=\frac{1}{4}, P(8)=\frac{1}{8}, P(16)=\frac{1}{16}, P(32)=\frac{1}{32}$

When we get the faces as $8, 8, 32$ then the probability will be $3\left(\frac{1}{8}\times\frac{1}{8}\times\frac{2}{32}\right)=\frac{3}{1024}$

When we get the faces as $16, 16, 16$ then the probability will be $\frac{1}{16}\times\frac{1}{16}\times\frac{1}{16}=\frac{1}{4096}$

Hence, the required probability will be $\frac{3}{1024}+\frac{1}{4096}=\frac{13}{4096}$

Q.6. $2\sin 12^\circ - \sin 72^\circ = ?$

A) $1-\frac{\sqrt{5}}{8}$

B) $\frac{\sqrt{5}\left(1-\sqrt{3}\right)}{4}$

C) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{2}$

D) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

Answer: $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

Solution: To find $\Rightarrow 2\sin 12^\circ - \sin 72$

$\Rightarrow \sin 12^\circ + \sin 12^\circ - \sin 72$

$\Rightarrow \ \sin 12^\circ + 2\cos 42^\circ \sin\left(-30^\circ\right) \ \Rightarrow \sin 12^\circ - \cos 42^\circ$

$\Rightarrow \ \sin 12^\circ - \sin 48^\circ \ \ \Rightarrow 2\cos\left(30^\circ\right)\sin\left(-18^\circ\right)$

$\Rightarrow -\sqrt{3}\sin 18^\circ = -\sqrt{3}\left(\frac{\sqrt{5}-1}{4}\right) \ = \sqrt{3}\left(\frac{1-\sqrt{5}}{4}\right)$

Q.7. If $y=y(x)$ is the solution of the differential equation $2x^2\frac{dy}{dx}-2xy+8y^2=0,\ y(e)=\frac{e}{3}$, then $y(1)$ is equal to

A) $\frac{2}{3}$

B) 3

C) $\frac{3}{2}$

D) -1

Answer: -1

Solution: Given $2x^2\frac{dy}{dx}-2xy+8y^2=0$

Dividing by $2x^2y^2$, we get,

$-\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{xy}=\frac{4}{x^2}$

Let $\frac{1}{y}=p, \frac{-1}{y^2}\frac{dy}{dx}=\frac{dp}{dx}$

$\Rightarrow \quad \frac{dp}{dx}+\frac{1}{x}(p)=\frac{4}{x^2}$

General solution will be $p\cdot e^{\int\frac{1}{x}dx}=\int\frac{4}{x^2}\cdot e^{\int\frac{1}{x}dx}\,dx+c$

$\Rightarrow \quad \frac{x}{y}=\int\frac{4}{x}dx+c \quad \Rightarrow \frac{x}{y}=4\ \ln x+c$

$\because \quad y(e)=\frac{e}{3}\Rightarrow c=-1 \quad \Rightarrow \frac{x}{y}=4\ln x-1$ is the particular solution.

when $x=1;\ \ y(1)=-1$

Q.8. If the angle made by tangent at point (x_0, y_0) on curve $x=12(t+\sin t\cdot\cos t)$, $y=12(1+\sin t)^2$, $0<t<\frac{\pi}{2}$ with the x-axis is $\frac{\pi}{3}$ then y_0 is equal to

A) 36

B) 27

C) 45

D) 48

Answer: 27

Solution: Given,

Curve $x = 12\left(t + \sin t \cos t\right)$

$y = 12(1 + \sin t)^2$

Now $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\tan\left(\frac{\pi}{3}\right) = \frac{\frac{d}{dt}\left(12(1+\sin t)^2\right)}{\frac{d}{dt}(12(t+\sin t\cos t))}$

$\sqrt{3} = \frac{12\times 2(1+\sin t)(\cos t)}{12(1+\cos 2t)}$

$\sqrt{3} = \frac{2(1+\sin t)\cdot\cos t}{2\cos^2 t}$

$\sqrt{3} = \frac{1+\sin t}{\cos t} = \frac{\left(\cos\frac{t}{2}+\sin\frac{t}{2}\right)^2}{\cos^2\frac{t}{2}-\sin^2\frac{t}{2}}$

$\sqrt{3} = \tan\left(\frac{t}{2} + \frac{\pi}{4}\right)$

$\frac{\pi}{3} = \frac{t}{2} + \frac{\pi}{4} \Rightarrow \frac{t}{2} = \frac{\pi}{12} \Rightarrow t = \frac{\pi}{6}$

Now putting $t = \frac{\pi}{6}$ in $y = 12(1 + \sin t)^2$

as (x_0, y_0) lie on curve

So $y_0 = 12\left(1 + \sin\frac{\pi}{6}\right)^2$

$= 12\left(\frac{3}{2}\right)^2 = 27$

Q.9. $A = \{x \in \mathbb{R} : |x+1| < 2\}, B = \{x \in \mathbb{R} : |x-1| \geq 2\}$ then:

A) $A \cup B = \mathbb{R} - [1, 3]$

B) $A \cap B = (-1, 1)$

C) $A \cap B = (-3, -1]$

D) $B - A = \mathbb{R} - (-3, 1]$

Answer: $A \cap B = (-3, -1]$

Solution: Given $|x+1|<2 \quad -2<x+1<2$

$-3<x<1 \quad \ldots \text{(i)}\ x \in(-3,1)$

Also for set B

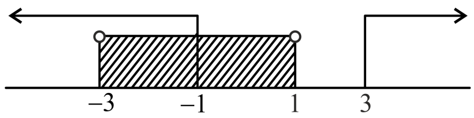
$|x-1| \geqslant 2$

$x-1 \leq -2$ or $x-1 \geqslant 2$

$x \leq -1$ or $x \geqslant 3$

for set B $x \in(-\infty,-1] \cup[3, \infty) \quad \ldots \text{(iii)}$

From equation (i) & (ii)



So By looking at the Graph

we car say $A \cap B=(-3,-1]$

Q.10. The negation of $((\sim q \wedge p) \rightarrow((\sim p) \vee q)$

A) $\sim(p \rightarrow q)$

B) $(p \rightarrow q)$

C) $\sim(p \rightarrow \sim q)$

D) $(p \rightarrow \sim q)$

Answer: $\sim(p \rightarrow q)$

Solution: To find negation of $((\sim q \wedge p) \rightarrow((\sim p) \vee q)$

Let $\sim q \wedge p=A$ & $\sim p \vee q=B$

$\sim(A \rightarrow B) \quad=\sim(\sim A \vee B)=A \wedge \sim B$

Using above formula be get

$(\sim q \wedge p) \wedge \sim(\sim p \vee q) \quad=(\sim q \wedge p) \wedge(p \wedge \sim q)$

$=A \wedge A \quad=A=\sim q \wedge p=p \wedge \sim q$ or $\sim(p \rightarrow q)$

Q.11. The line $y=x+1$ intersect the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$ at the points P and Q. If PQ is the diameter of a circle with radius r, then the value of $(3r)^2$ is

A) 20

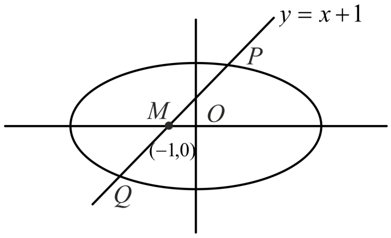
B) $\frac{80}{9}$

C) $\frac{40}{9}$

D) $\frac{20}{3}$

Answer: 20

Solution:



Let P be the point $\left(-1+\frac{p}{\sqrt{2}}, \frac{p}{\sqrt{2}}\right)$

Since it lies on the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$, we get

$\frac{\left(-1+\frac{p}{\sqrt{2}}\right)^2}{4}+\frac{\left(\frac{p}{\sqrt{2}}\right)^2}{2}=1 \Rightarrow \frac{3p^2}{8}-\frac{p}{2\sqrt{2}}-\frac{3}{4}=0$

Here $\left(p_1-p_2\right)^2=\left(\frac{2\sqrt{2}}{3}\right)^2-4(-2)=\frac{8}{9}+8=\frac{80}{9}$

We know, $\left(p_1-p_2\right)^2=PQ^2$

$\Rightarrow PQ^2=\frac{80}{9}$

Now $r=\frac{PQ}{2} \Rightarrow 9r^2=9\frac{(PQ)^2}{4}=\frac{9}{4}\times\frac{80}{9}=20$

Q.12. A circle touching y axis and the line $x+y=0$ find the locus of centre

A) $x^2-y^2-2xy=0$

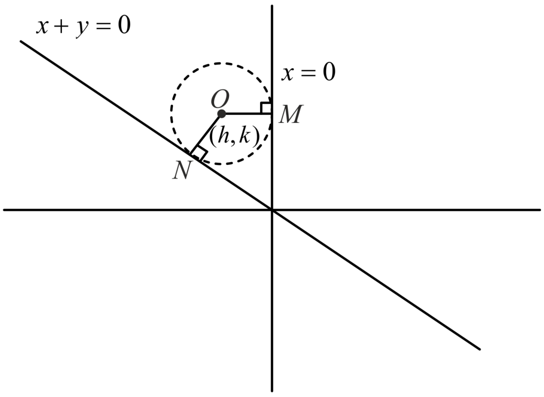
B) $x^2+y^2+2xy=0$..

C) $x^2-y^2+2xy=0$

D) $x^2-y^2+2x=0$

Answer: $x^2-y^2-2xy=0$

Solution:



To find locus, let centre be (h,k). According to question

$|OM|=|ON|$ distance

$\Rightarrow|h|=\frac{|h+k|}{\sqrt{2}}$

squaring both sides we get

$\Rightarrow 2h^2=(h+k)^2$

$\Rightarrow 2h^2=h^2+k^2+2hk$

Replacing h with x & k with y we get

$2x^2=x^2+y^2+2xy$

$\Rightarrow x^2-y^2-2xy=0$

Q.13. Water is increasing in a rigid circular cone with rate $1\ \mathrm{cm}^3/\mathrm{sec}$, then the rate of change of lateral surface of cone is (where height and diameter of cone is 35 and 14 respectively) at $h=10\,\mathrm{cm}$ is

A) $\frac{\sqrt{26}}{10}$

B) $\frac{\sqrt{26}}{5}$

C) $\frac{\sqrt{21}}{5}$

D) 5

Answer: $\frac{\sqrt{26}}{5}$

Solution: Let the volume of the cone be V c.c.

Given $\frac{dv}{dt}=1\,\mathrm{cc}/\mathrm{sec},\ h=35\,\mathrm{cm},\ r=7\,\mathrm{cm}$

i.e. $\frac{h}{r}=5$

We know for a cone $l^2=r^2+h^2$

Lateral surface area, $S=\pi r\sqrt{r^2+h^2}$

$S=\pi\frac{h}{5}\sqrt{\frac{h^2}{25}+h^2}=\pi\frac{\sqrt{26}}{25}h^2$

Also $V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(\frac{h}{5}\right)^2h=\frac{\pi}{75}h^3$

$\Rightarrow\frac{dV}{dt}=\frac{\pi}{25}h^2\frac{dh}{dt}\Rightarrow\frac{\pi}{25}h^2\frac{dh}{dt}=1$

$\Rightarrow\frac{dh}{dt}=\frac{25}{\pi h^2}\qquad\ldots(i)$

Now $\frac{dS}{dt}=\frac{\pi\sqrt{26}}{25}\times 2h\frac{dh}{dt}=\frac{2\sqrt{26}}{h}$ (from (i))

$\left(\frac{dS}{dt}\right)_{h=10}=\frac{\sqrt{26}}{5}$

Q.14. $\lim\limits_{x\to\frac{\pi}{2}}\tan^2x\left(\left(2\sin^2x+4\sin x+3\right)^{\frac{1}{2}}-\left(\sin^2x+6\sin x+2\right)^{\frac{1}{2}}\right)$

A) 0

B) 1

C) $\frac{1}{2}$

D) None ot these

Answer: 0

Solution: Given

$$L=\lim_{x\to\frac{\pi}{2}}\tan^2x\left(\left(2\sin^2x+4\sin x+3\right)^{\frac{1}{2}}-\left(\sin^2x+6\sin x+2\right)^{\frac{1}{2}}\right)$$

Check form by $x=\frac{\pi}{2}$ we get form $(\infty\times 0)$

Now rationalising we get

$$L=\lim_{x\to\frac{\pi}{2}}\tan^2x\left[\frac{2\sin^2x+4\sin x+3-(\sin^2x+6\sin x+2)}{\sqrt{2\sin^2x+4\sin x+3}+\sqrt{\sin^2x+6\sin x+2}}\right]$$

$$l=\lim_{x\to\frac{\pi}{2}}\frac{\sin^2x}{\cos^2x}\left[\frac{\sin^2x-2\sin x+1}{\sqrt{9}+\sqrt{9}}\right]$$

$$L=\lim_{x\to\frac{\pi}{2}}\frac{\sin^2x(\sin x-1)^2}{6(1-\sin^2x)}\ L=\lim_{x\to\frac{\pi}{2}}\frac{-\sin^2x(\sin x-1)}{6(1+\sin x)}$$

Now put $x=\frac{\pi}{2}$

we get

$$L=\frac{-1^2\times(1-1)}{6\times 2}=0$$

Q.15. Let z_1 and z_2 be two complex numbers such that $\bar{z}_1=i\bar{z}_2$ and $\arg\left(\frac{z_1}{\bar{z}_2}\right)=\pi$, then the argument of z_1 is

A) $\frac{\pi}{4}$

B) $\frac{\pi}{2}$

C) $\frac{\pi}{5}$

D) $\frac{\pi}{1}$

Answer: $\frac{\pi}{4}$

Solution: $\bar{z}_1 = i\bar{z}_2 \;\Rightarrow z_1 = -iz_2 \;\Rightarrow iz_1 = z_2 \Rightarrow e^{\frac{i\pi}{2}} z_1 = z_2$

Given, $\arg\left(\frac{z_1}{\overline{z_2}}\right) = \pi$

Let $z_1 = re^{i\theta} \quad \Rightarrow \quad \overline{z_2} = re^{-i\left(\frac{\pi}{2}+\theta\right)}$

So $\arg\left(e^{i\left(2\theta+\frac{\pi}{2}\right)}\right) = \pi \quad \Rightarrow \quad 2\theta + \frac{\pi}{2} = \pi$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

Q.16. The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is.

A) $\frac{3}{4}$

B) $\frac{2}{3}$

C) $\frac{1}{6}$

D) $\frac{1}{3}$

Answer: $\frac{1}{3}$

Solution: We have, $y^2 = 2x - 1 \quad \ldots(1)$ and $y^2 = 4x - 3 \quad \ldots(2)$

We first find the points of intersection of the given parabolas by solving the equations (1) & (2)simultaneously.

This gives, $2x - 1 = 4\,x - 3$

$\Rightarrow \quad 2x = 2 \ \Rightarrow x = 1$

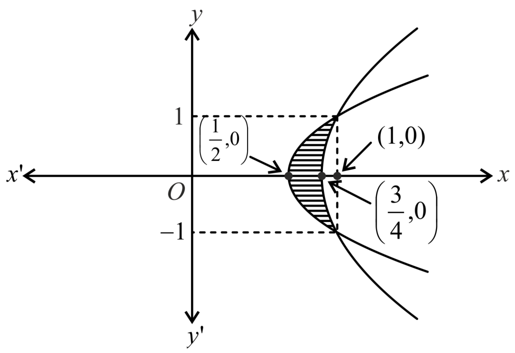
Now, put $x = 1$ in equation (1), we get $y = \pm 1$.

Now, the equations (1) & (2) can be written as

$y^2 = 2\left(x - \frac{1}{2}\right)$

$y^2 = 4\left(x - \frac{3}{4}\right)$ respectively.

The graphs of these two curves are shown in figure.



We have to find the area of the shaded region.

Now, the equations (1) & (2) can be written as

$x = \frac{y^2+1}{2}$ & $x = \frac{y^2+3}{4}$ respectively.

It is clear from the figure, required area $A = 2\int_0^1 (x_2 - x_1)dy$ ($\because$ Area is symmetrical about $x-$ axis)

where $x_1 = \frac{y^2+1}{2}$ and $x_2 = \frac{y^2+3}{4}$

$\Rightarrow A = 2\int_0^1 \left(\frac{y^2+3}{4} - \frac{y^2+1}{2}\right)dy = \frac{2}{4}\int_0^1 (1 - y^2)dy$

$= \frac{1}{2}\left[y - \frac{y^3}{3}\right]_0^1 \ = \frac{1}{2}\left[1 - \frac{1}{3}\right]$

$= \frac{1}{2}\left(\frac{2}{3}\right) = \frac{1}{3}$ sq.units.

Q.17. Find b if $12 \times \int_3^b \frac{1}{(x^2-4)(x^2-1)} = \ln\left(\frac{49}{40}\right)$ & $b > 3$

A) 6

B) 4

C) 9..

D) 12

Answer: 6

Solution: Given $12 \times \int_3^b \frac{1dx}{(x^2-4)(x^2-1)} = \ln\left[\frac{49}{40}\right]$ & $4\, b > 3$

$\Rightarrow$ By Using partial fraction

$\frac{1}{(x^2-4)(x^2-1)} = \frac{1}{3}\left(\frac{1}{x^2-4} - \frac{1}{x^2-1}\right)$

$\Rightarrow \frac{12}{3}\int_3^b \left[\frac{1}{x^2-4} - \frac{1}{x^2-1}\right] = \ln\left(\frac{49}{40}\right)$

$\Rightarrow \quad 4\left[\frac{1}{4}\log\left(\frac{x-2}{x+2}\right) - \frac{1}{2}\log\left(\frac{x-1}{x+1}\right)\right]_3^b = \ln\left(\frac{49}{40}\right)$

$\Rightarrow \quad \log\left(\frac{(b-2)(b+1)^2}{(b+2)(b-1)^2} \times \frac{5}{4}\right) = \log\frac{49}{40}$

$\Rightarrow \quad \frac{(b-2)(b+1)^2}{(b+2)(b-1)^2} \times \frac{5}{4} = \frac{49}{40}$

$\Rightarrow \frac{(b-2)(b+1)^2}{(b+2)(b-1)^2} = \frac{49}{50} \quad \ldots(\mathrm{i})$

Comparing by hit and trial and taking $(b+1)^2 = 49 \Rightarrow b+1 = 7 \Rightarrow b = 6$

Now putting $b = 6$ in equation (i) to corsscheck,

we get $\frac{4\times7^2}{8\times5^2} = \frac{49}{50}$ which is true

So $b = 6$ is correct option,

Hence $b = 6$

Q.18. If $f(x) + 2f(1-x) = x^2 + 1$ then range of $f(x)$ is

A) $\left[\frac{2}{3}, \infty\right)$

B) $\left(\frac{2}{3}, \infty\right)$

C) R

D) $\left[\frac{-2}{3}, \infty\right)$

Answer: $\left[\frac{2}{3}, \infty\right)$

Solution: We have, $f(x) + 2f(1-x) = x^2 + 1 \quad \cdots(1)$

Replace x by $(1-x)$, in equation (1), we get $f(1-x) + 2f(1-(1-x)) = (1-x)^2 + 1$

$\Rightarrow \quad f(1-x) + 2f(x) = (1-x)^2 + 1 \quad \cdots(2)$

On solving equations (1) & (2), we get

$f(x) = \frac{1}{3}\left(x^2 - 2x + 3\right) = \frac{1}{3}\left[(x-1)^2 + 2\right]$

$\because \quad (x-1)^2 \geq 0 \quad \forall x \in R$

$\Rightarrow (x-1)^2 + 2 \geq 2 \quad \forall x \in R$

$\Rightarrow \frac{1}{3}\left[(x-1)^2 + 2\right] \geq \frac{2}{3} \quad \forall x \in R$

$\therefore$ The range of $f(x)$ is $\left[\frac{2}{3}, \infty\right)$

Q.19. Let $a, b \in R$ be such that the equation $ax^2 - 2bx + 15 = 0$ has repeated root α and if α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to:

A) 58

B) 68

C) 37

D) 92

Answer: 58

Solution: Given $ax^2 - 2bx + 15 = 0 \quad \ldots(\text{i})$

Has repeated roots So $D = 0$

$4b^2 - 4 \times 15 \times a = 0$

$\Rightarrow b^2 = 15a \quad \ldots(\text{ii})$

Also given

$$x^2 - 2bx + 21 = 0 \begin{cases} \alpha \\ \beta \end{cases} \text{Root}$$

Now α will satisfy both quadratic

$ax^2 - 2bx + 15 = 0$ & $x^2 - 2bx + 21 = 0$

Putting the value we get

$$\begin{array}{l} a\alpha^2 - 2b\alpha + 15 = 0 \\ \ \alpha^2 - 2b\alpha + 21 = 0 \\ - \quad + \qquad - \\ \hline \end{array}$$

$(a - 1)\alpha^2 = 6$

$\alpha^2 = \frac{6}{a-1}$

Now in equation (1) product of Root $\alpha^2 = \frac{15}{a}$

So $\frac{15}{a} = \frac{6}{a-1} \Rightarrow 2a = 5a - 5 \Rightarrow a = \frac{5}{3}$

Now $b^2 = 15a \Rightarrow b^2 = 15 \times \frac{5}{3} \Rightarrow b^2 = 25$

So $b = \pm 5$

Now in quadratic $x^2 - 2bx + 21 = 0$

Putting the value of b we get

$x^2 - 10x + 21 = 0 \Rightarrow (x - 7)(x - 3) = 0$

So $x = 3$ or 7.

Or

$x^2 + 10x + 21 = 0 \Rightarrow x = -3 \; or \; x = -7$

So $\alpha = \pm 3$ & $\beta = \pm 7$

So $\alpha^2 + \beta^2 = 3^2 + 7^2 = 9 + 49 = 58$

Q.20. The system of equations

$-kx+3y-14z=25$

$-15x+4y-kz=3$

$-4x+y+3z=4$

is consistent, if $k \in$

A) $R-\{13\}$

B) $R-\{-11,13\}$

C) $R-\{-11,11\}$

D) R

Answer: $R-\{-11,11\}$

Solution: For consistent solutions $\Delta \neq 0$

$$\Rightarrow \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} \neq 0$$

$\Rightarrow \quad -k(12+k)-3(-45-4k)-14(-15+16) \neq 0$

$\Rightarrow \quad -12k-k^2+135+12k-14 \neq 0$

$\Rightarrow k^2 \neq 121$

$\Rightarrow k \neq 11,-11$