JEE Main Exam 2022 - Session 1

26 June 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. A capacitor C_1 is charged to a potential difference V. The charging battery is then removed and the capacitor is connected to an uncharged capacitor C_2. The potential difference across the combination is

A) $V\left(1+\frac{C_2}{C_1}\right)$

B) $V\left(1+\frac{C_1}{C_2}\right)$

C) $\frac{VC_2}{(C_1+C_2)}$

D) $\frac{VC_1}{(C_1+C_2)}$

Answer: $\frac{VC_1}{(C_1+C_2)}$

Solution: Charge $Q=C_1V$

The total capacity of the parallel combination is

$C=C_1+C_2$

Potential difference$=\frac{Q}{C}=\frac{C_1V}{C_1+C_2}$

Q.2. Certain amount of an ideal gas is contained in a closed vessel. The vessel is moving with a constant velocity v. The rise in temperature of the gas when the vessel is suddenly stopped is (M is molecular mass,)

A) $\frac{Mv^2(\gamma-1)}{2R}$

B) $\frac{Mv^2(\gamma+1)}{2R}$

C) $\frac{Mv^2}{2R\gamma}$

D) $\frac{Mv^2}{2R(\gamma+1)}$

Answer: $\frac{Mv^2(\gamma-1)}{2R}$

Solution: KE of the vessel $= \frac{1}{2}mv^2$

When the vessel is suddenly stopped, the ordered motion of the gas molecules is converted into disordered motion of the molecules increasing thereby the internal energy of the gas. Thus,

$\Delta U = nC_v\,\Delta T = \frac{1}{2}mv^2 = \frac{1}{2}(nM)v^2$

Where n is number of moles of the gas in the vessel and M is molecular weight of the gas.

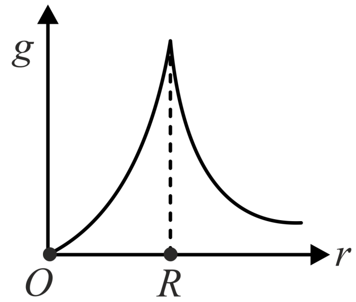
$\therefore \quad \Delta T = \frac{Mv^2}{2C_v}$

As $C_v = \frac{R}{\gamma - 1}$

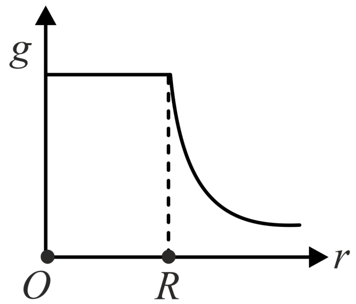
$\therefore \quad \Delta T = \frac{Mv^2(\gamma-1)}{2R}$

Q.3. Starting from the centre of the earth having radius R, the variation of g (acceleration due to gravity) is shown by,

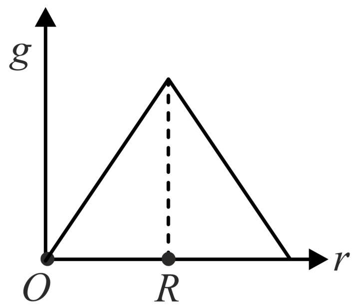
A)



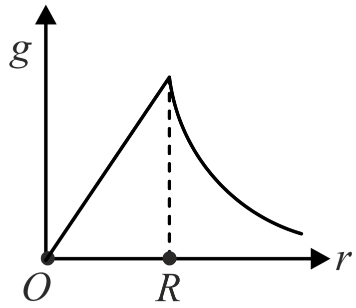
B)



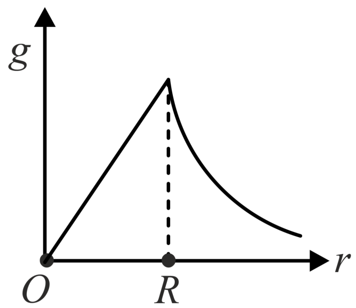
C)



D)



Answer:



Solution: We know that the gravitational field strength varies according to the variation of the distance of an object from the centre of the earth.

So, gravitational field strength inside the earth varies as,

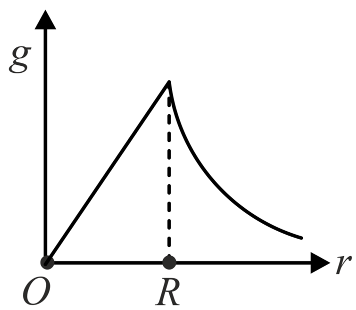
$g=\frac{GMr}{R^3}$ if $0\leq r\leq R$.

So, $g\propto r$.

At the surface, it is maximum, $g=\frac{GM}{R^2}$.

Outside the earth, it varies as, $g\propto\frac{1}{r^2}$ for $(r\geq R)$.

So, this is the possible graph,



Q.4. A ball is thrown vertically upward. At the maximum height. Which of the following is zero?

A) Momentum

B) Potential energy

C) Acceleration

D) Force

Answer: Momentum

Solution: When a ball is thrown vertically upwards, at the maximum height the velocity is zero.

As momentum is $p = mv$, so momentum is also zero at maximum height.

Q.5. If the time period of a simple pendulum is T, then find its time period inside a lift moving upward with an acceleration of g.

A) T

B) $2T$

C) $\frac{T}{2}$

D) $\frac{T}{\sqrt{2}}$

Answer: $\frac{T}{\sqrt{2}}$

Solution: The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$

In an upward accelerating frame the effective $g_{eff} = g + a$

$\Rightarrow T_2 = 2\pi\sqrt{\frac{l}{g+a}} = 2\pi\sqrt{\frac{l}{g+g}}$

$\frac{T}{T_2} = \sqrt{\frac{g+g}{g}} = \sqrt{2}$

$\Rightarrow T_2 = \frac{T}{\sqrt{2}}$

Q.6. Efficiency of a carnot engine between steam point and ice point is

A) $26.67\ \%$

B) $36.71\ \%$

C) $46.71\ \%$

D) $56.61\ \%$

Answer: $26.67\ \%$

Solution: Steam point $= 100\ ^{\circ}\mathrm{C} = (100 + 273)\ \mathrm{K} = 373\ \mathrm{K}$

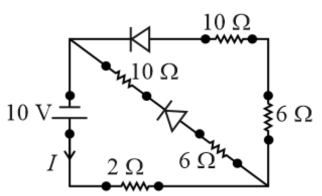
Ice point $= 0\ ^{\circ}\mathrm{C} = (0 + 273)\ \mathrm{K} = 273\ \mathrm{K}$

Now,

$\eta = 1 - \frac{T_{sink}}{T_{source}} = 1 - \frac{273}{373}$

$= \frac{100}{373} \times 100\% \approx 26.67\%$

Q.7. In the diagram as shown, the value of current I is



A) $1 \mathrm{~A}$

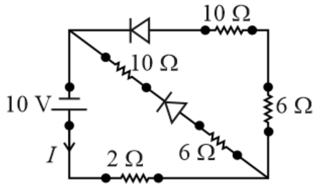
B) $2 \mathrm{~A}$

C) $0.5 \mathrm{~A}$

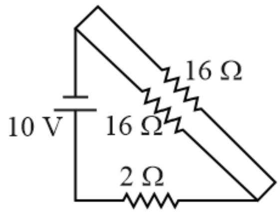
D) $4 \mathrm{~A}$

Answer: $1 \mathrm{~A}$

Solution:



In the given circuit both the diodes are forward biased. Therefore,



$R_{eq}=\frac{16}{2}+2=10 \Omega$

$\Rightarrow I=\frac{10}{10}=1 \mathrm{~A}$

Q.8. An α particle and proton travel with the same velocity, in a magnetic field perpendicular to the direction of their velocities. Find the ratio of the radii of their circular path.

A) $4: 1$

B) $2: 1$

C) $1: 2$

D) $1: 1$

Answer: $2: 1$

Solution: Radius of circular path of charged particle (q) of mass (m) moving with velocity (v) in magnetic field (B) is given by $r=\frac{mv}{qB}$

$\frac{r_\alpha}{r_p}=\frac{\frac{(4m)v}{(2e)B}}{\frac{mv}{eB}}=2: 1$

Q.9. When do you experience weightlessness in a lift?

A) Moving upward with constant velocity

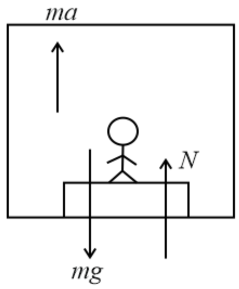
B) Moving upward with constant acceleration

C) Moving downward with constant velocity

D) Moving downward with constant acceleration

Answer: Moving downward with constant acceleration

Solution:



When the pseudo force becomes equal and opposite to the force due to gravity then we will feel weightless in a lift. The pseudo force is equal to the mass multiplied by the acceleration of the frame and its direction is opposite to the acceleration of the frame. Therefore,

$$mg + (-ma) = 0 \quad \Rightarrow \quad a = g$$

Hence, D is the correct answer.

Q.10. If length of wire increases by 0.4% keeping volume constant, find percentage change in resistance?

A) 0.8%

B) 0.4%

C) 0.2%

D) 0.1%

Answer: 0.8%

Solution: Volume of wire having cross-sectional area A and length l is, $V = Al$

Resistance of a wire is given by, $R = \frac{\rho l}{A} = \frac{\rho l}{\frac{V}{l}} = \frac{\rho l^2}{V}$

As change in length is small,

$$\frac{\Delta R}{R} \times 100\% = \frac{2\Delta l}{l} \times 100\% = 2 \times 0.4\% = 0.8\%$$

Q.11. The magnetic flux through a coil perpendicular to the plane is varying according to the relation $\phi = \left(5t^3 + 6t^2 - 6t + 6\right)\ \mathrm{Wb}$. Calculate the induced current through the coil at $t = 2\ \mathrm{s}$, if the resistance of the coil is $\mathrm{R} = 5\ \Omega$.

A) $15.6\ \mathrm{A}$

B) $18.4\ \mathrm{A}$

C) $21.6\ \mathrm{A}$

D) $24.8\ \mathrm{A}$

Answer: $15.6\ \mathrm{A}$

Solution: Given, $\phi = 5t^3 + 6t^2 - 6t + 6$

Magnitude of induced emf $\varepsilon = \left|\frac{d\phi}{dt}\right| = 15t^2 + 12t - 6$

At, $t = 2\ \mathrm{s}$, $\varepsilon = 60 + 24 - 6 = 78\ \mathrm{V}$

Current, $i = \frac{\varepsilon}{R} = \frac{78}{5} = 15.6\ \mathrm{A}$

Q.12. An object of mass $0.5\ \mathrm{kg}$ is dropped from a height $10\ \mathrm{m}$, at what height will the magnitude of velocity be equal to magnitude of gravitational acceleration?(Take $g = 10\ \mathrm{m\ s^{-2}}$)

A) $5\ \mathrm{m}$

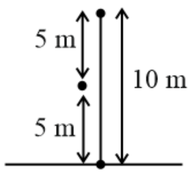
B) $4\ \mathrm{m}$

C) $3\ \mathrm{m}$

D) $1\ \mathrm{m}$

Answer: $5\ \mathrm{m}$

Solution:



Consider downward direction to be positive.

Applying first equation of motion, $v = u + at$

$\Rightarrow v = 0 + gt \ \Rightarrow g = gt \ \Rightarrow t = 1\ \mathrm{s}$

Distance covered in $1\ \mathrm{s}$, $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}g \times 1^2 = \frac{10}{2} = 5\ \mathrm{m}$

Hence, height of the point from the surface $= (10 - 5)\ \mathrm{m} = 5\ \mathrm{m}$

Q.13. Uranium $^{238}_{92}\mathrm{U}$ undergoes successive radioactive decays into $^{206}_{82}\mathrm{Pb}$. Find number of α and β particles decayed.

A) $\alpha = 8,\ \beta = 8$

B) $\alpha = 8,\ \beta = 6$

C) $\alpha = 4,\ \beta = 6$

D) $\alpha = 4,\ \beta = 4$

Answer: $\alpha = 8,\ \beta = 6$

Solution: $^{238}_{92}\mathrm{U} \rightarrow {}^{206}_{82}\mathrm{Pb}$

Change in mass number is $4\alpha = 238 - 206 = 32 \Rightarrow \alpha = 8$

Change in atomic number is $-2\alpha + \beta = -10 \Rightarrow -16 + \beta = -10 \Rightarrow \beta = 6$

Q.14. Change in length of wire on earth is $10^{-4}\ \mathrm{m}$ and the change in length of same wire on a planet P is $6 \times 10^{-5}\ \mathrm{m}$ due to self weight, then find the acceleration due to gravity at the planet in $\mathrm{m\ s^{-2}}$.

A) 6

B) 2

C) 3

D) 4

Answer: 6

Solution: Change in length $\propto$Force$\propto mg$

$mg = kx = \left(\frac{YA}{l}\right)x$

Thus, $x \propto g$

Then, $\frac{10^{-4}}{6\times10^{-5}} = \frac{g}{g'} \Rightarrow g' = 0.6\text{g} = 6$

Q.15. Find direction of magnetic field if electromagnetic wave is travelling along $+z$ axis and electric field is along $-x$ direction.

A) $-y$

B) $+y$

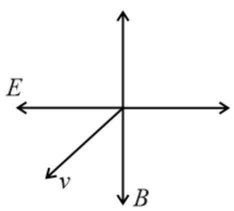
C) $-z$

D) $+x$

Answer: $-y$

Solution: In an electromagnetic wave, the electric field and magnetic field are perpendicular to each other as well perpendicular to the line of propagation of the wave, stated as $\vec{E} \times \vec{B} = \vec{v}$.

When the wave is propagating in the $+z$ direction and the electric field in the$-x$ direction, then the magnetic field will propagate in the $-y$ direction.



Q.16. A wave of frequency $= 3\ \text{GHz}$, strikes a particle of size $\left(\frac{1}{100}\right)^{th}$ of λ, then this phenomenon is called as

A) Diffraction

B) Scattering

C) Reflection

D) Refraction

Answer: Scattering

Solution: Rayleigh scattering applies to particles that are small with respect to wavelengths of light.

Q.17. A ring of mass M and radius R is rotating with angular velocity $\omega = 2\ \text{rad s}^{-1}$. Two point masses of equal mass m are placed gently at diametrically opposite points. The value of new angular velocity (in rad s^{-1}) will be

A) $\frac{M}{2M-m}$

B) $\frac{2M}{M+2m}$

C) $\frac{2m}{M+m}$

D) $\frac{M}{M+2m}$

Answer: $\frac{2M}{M+2m}$

Solution: As the masses are gently placed we can apply conservation of angular momentum.

Initial moment of Inertia $I_1 = MR^2$

Final moment of inertia $I_2 = MR^2 + (2m)R^2$

By conservation of angular momentum

$I_1\omega_1 = I_2\omega_2 \Rightarrow MR^2 \times 2 = (M+2m)R^2 \times \omega_2$

$\Rightarrow \omega_2 = \frac{2M}{M+2m}\ \mathrm{rad\ s^{-1}}$

Q.18. An aeroplane is flying horizontally with the velocity of $200\ \mathrm{m\,s^{-1}}$, when it is just above a cannon. The cannon fires a shell at a speed of $400\ \mathrm{m\,s^{-1}}$ at an angle of θ with horizontal. If the shell hits the aeroplane than value of θ is

A) 45°

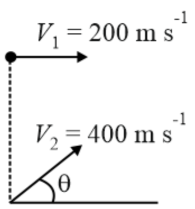
B) 30°

C) 37°

D) 60°

Answer: 60°

Solution:



In order to hit, horizontal velocity of both moving object should be same,

$V_1 = V_2\cos\theta \Rightarrow 200 = 400\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

Q.19. de Broglie wavelength of photon and electron are same, then find the ratio of their energy. Here, c is the speed of light and v is the velocity of electron.

A) $\frac{2c}{v}$

B) $\frac{c}{v}$

C) $\frac{v}{2c}$

D) $\frac{v}{c}$

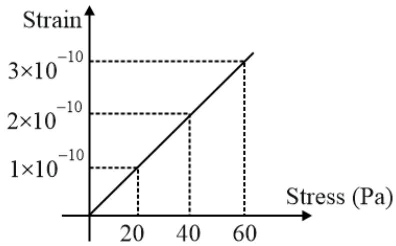
Answer: $\frac{2c}{v}$

Solution: We have, $\lambda_p = \frac{h}{p_p}$ and $\lambda_e = \frac{h}{p_e}$

Given here, $\lambda_e = \lambda_p$, then $p_p = p_e$

Ratio of their energy is $\frac{E_p}{K_e} = \frac{\frac{hc}{\lambda_P}}{\frac{p_e^2}{2\mathrm{m}}} = \frac{2mc}{p_e} = \frac{2mc}{mv} = \frac{2c}{v}$

Q.20. In the graph given for a stretched wire, the value of energy density when strain is 5×10^{-10}, is



A) $4.0 \times 10^{-6}\ \mathrm{J\ m^{-3}}$

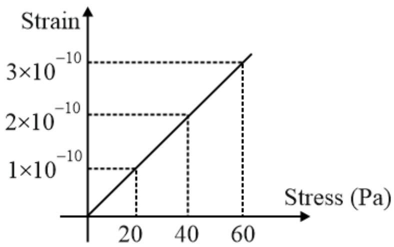
B) $1.5 \times 10^{-8}\ \mathrm{J\ m^{-3}}$

C) $2.5 \times 10^{-8}\ \mathrm{J\ m^{-3}}$

D) $2.5 \times 10^{-10}\ \mathrm{J\ m^{-3}}$

Answer: $2.5 \times 10^{-8}\ \mathrm{J\ m^{-3}}$

Solution:



From the above graph, we can find that when the strain is 5×10^{-10} the stress is $100\ \mathrm{Pa}$. We know that energy density is

$U = \frac{1}{2} \times \text{stress} \times \text{strain} \Rightarrow U = \frac{1}{2} \times 100 \times 5 \times 10^{-10}$

$\Rightarrow U = 2.5 \times 10^{-8}\ \mathrm{J\ m^{-3}}$

Q.21. A dimensionless quantity p is given by $p = \frac{\alpha}{\beta}\ln\left(\frac{Kx}{\beta}\right)$. If K is kinetic energy, x is displacement, then find the dimensions of α .

A) $\mathrm{M^0L^{-1}T}$

B) $\mathrm{ML^3T^{-2}}$

C) $\mathrm{ML^2T^{-1}}$

D) $\mathrm{ML^0T^{-3}}$

Answer: $\mathrm{ML^3T^{-2}}$

Solution: Given $p=\frac{\alpha}{\beta}\ln\left(\frac{Kx}{\beta}\right)$

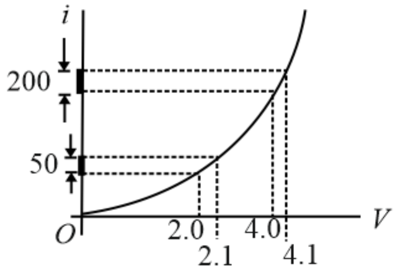
We know that expression inside log is dimensionless.

herefore, $\left[\frac{kx}{\beta}\right]$ is dimensionless

$\Rightarrow[\beta]=[kx]=\mathrm{ML^2T^{-2}}\times\mathrm{L}=\mathrm{ML^3T^{-2}}$

Since p is dimensionless $\therefore$ $[p]=\left[\frac{\alpha}{\beta}\right]=\mathrm{M^0L^0T^0}\Rightarrow[\alpha]=[\beta]\quad\Rightarrow[\alpha]=\mathrm{ML^3T^{-2}}$

Q.22. Find the ratio of dynamic resistance at $2\ \mathrm{V}$ and $4\ \mathrm{V}$ for a semiconductor device having $V-i$ characteristics as shown below?



A) $4:1$

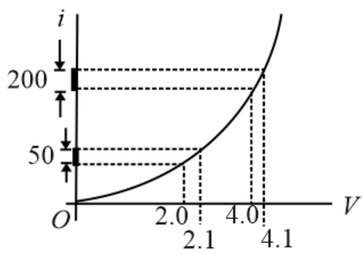
B) $3:1$

C) $2:1$

D) $1:1$

Answer: $4:1$

Solution:



Dynamic resistance is used to quantify the resistance of non-ohmic materials. It is defined as the ratio of differential change in voltage to a differential change in current, $R=\frac{\mathrm{d}V}{\mathrm{d}i}$

Therefore, $R_1=\frac{2.1-2}{50}=\frac{0.1}{50}\ \Omega$ and $R_2=\frac{4.1-4}{200}=\frac{0.1}{200}\ \Omega$

Hence $\frac{R_1}{R_2}=4:1$

Section B: Chemistry

Q.1. The stability of $+1$ oxidation state among Al, Ga, In and Tl increases in the sequence

A) $\mathrm{Al} < \mathrm{Ga} < \mathrm{In} < \mathrm{Tl}$

B) $\mathrm{Tl} < \mathrm{In} < \mathrm{Ga} < \mathrm{Al}$

C) $\mathrm{In} < \mathrm{Tl} < \mathrm{Ga} < \mathrm{Al}$

D) $\mathrm{Ga} < \mathrm{In} < \mathrm{Al} < \mathrm{Tl}$

Answer: $\mathrm{Al} < \mathrm{Ga} < \mathrm{In} < \mathrm{Tl}$

Solution: On moving down the group, the stability of $+3$ oxidation state decreases while $+1$ oxidation state increases due to inert pair effect.

Q.2. Which of the following is the correct order of melting points of group 16 elements?

A) $\mathrm{O} < \mathrm{S} < \mathrm{Se} < \mathrm{Te} < \mathrm{Po}$

B) $0 < \mathrm{S} < \mathrm{Se} < \mathrm{Te} \approx \mathrm{Po}$

C) $\mathrm{O} < \mathrm{S} < \mathrm{Se} < \mathrm{Te} > \mathrm{Po}$

D) $\mathrm{O} < \mathrm{S} < \mathrm{Se} > \mathrm{Te} < \mathrm{Po}$

Answer: $\mathrm{O} < \mathrm{S} < \mathrm{Se} < \mathrm{Te} > \mathrm{Po}$

Solution: Melting point is the temperature where a solid is converted to liquid, both solids and liquids are in equilibrium.

Parameters which affect melting point are:

- Ionic bonds: Greater the charge, greater the electrostatic attraction, stronger the ionic bond, so the melting point is higher.
- Intermolecular force: Stronger the intermolecular forces, more the energy is required, so the melting point is higher.
- Shape of molecules: More symmetry in shape means tighter packing in the solid phases, so the melting point is higher.
- Size of molecules: Greater the size, so the melting point is higher.

Down the group, the size of the molecule increases. Thus, Van der Waal's force increases and the melting point also increases.

But Polonium (Po)shows less melting point than Tellurium (Te) due to decrease in Vanderwaal's radius.

Q.3. The increasing order of bond order of $\mathrm{O_2}$, $\mathrm{O_2}^{+}$, $\mathrm{O_2}^{-}$ and $\mathrm{O_2}^{2-}$ is______.

A) $\mathrm{O_2}^{+} < \mathrm{O_2} < \mathrm{O_2}^{-} < \mathrm{O_2}^{2-}$

B) $\mathrm{O_2}^{2-} < \mathrm{O_2}^{-} < \mathrm{O_2}^{+} < \mathrm{O_2}$

C) $\mathrm{O_2} < \mathrm{O_2}^{+} < \mathrm{O_2}^{-} < \mathrm{O_2}^{2-}$

D) $\mathrm{O_2}^{2-} < \mathrm{O_2}^{-} < \mathrm{O_2} < \mathrm{O_2}^{+}$

Answer: $\mathrm{O_2}^{2-} < \mathrm{O_2}^{-} < \mathrm{O_2} < \mathrm{O_2}^{+}$

Solution: Bond order $= \frac{\text{number of bonding } e^{-} - \text{number of antibonding } e^{-}}{2}$

Electronic configuration of O_2, O_2^{+}, O_2^{-} and O_2^{-2} are:

$$O_2 = \sigma_{1s^2}\sigma^*_{1s^2}\sigma_{2s^2}\sigma^*_{2s^2}\sigma_{2p_x{}^2}\pi_{2p_y{}^2} = \pi_{2p_z{}^2}\pi^*_{2p_y{}^1} = \pi^*_{2p_z{}^1}\left(\text{Bond order} = 2\right)$$

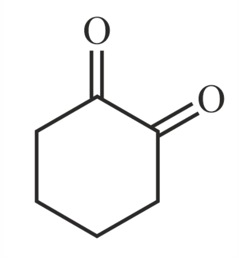
$$O_2^{+} = \sigma_{1s^2}\sigma^*_{1s^2}\sigma_{2s^2}\sigma^*_{2s^2}\sigma_{2p_x{}^2}\pi_{2p_y{}^2} = \pi_{2p_z{}^2}\pi^*_{2p_y{}^1} = \pi^*_{2p_z{}^0}\left(\text{Bond order} = 2.5\right)$$

$$O_2^{-} = \sigma_{1s^2}\sigma^*_{1s^2}\sigma_{2s^2}\sigma^*_{2s^2}\sigma_{2p_x{}^2}\pi_{2p_y{}^2} = \pi_{2p_z{}^2}\pi^*_{2p_y{}^2} = \pi^*_{2p_z{}^1}\left(\text{Bond order} = 1.5\right)$$

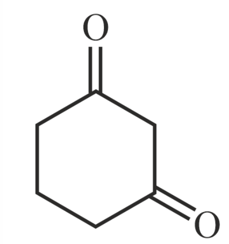
$$O_2^{-2} = \sigma_{1s^2}\sigma^*_{1s^2}\sigma_{2s^2}\sigma^*_{2s^2}\sigma_{2p_x{}^2}\pi_{2p_y{}^2} = \pi_{2p_z{}^2}\pi^*_{2p_y{}^2} = \pi^*_{2p_z{}^2}\left(\text{Bond order} = 1\right)$$

Q.4. Most stable enol will be shown by which of the following?

A)



B)



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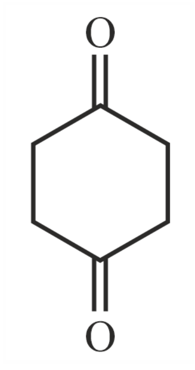
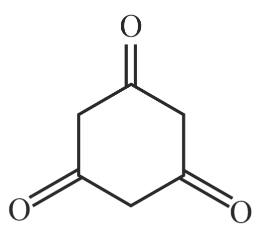
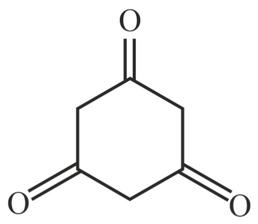


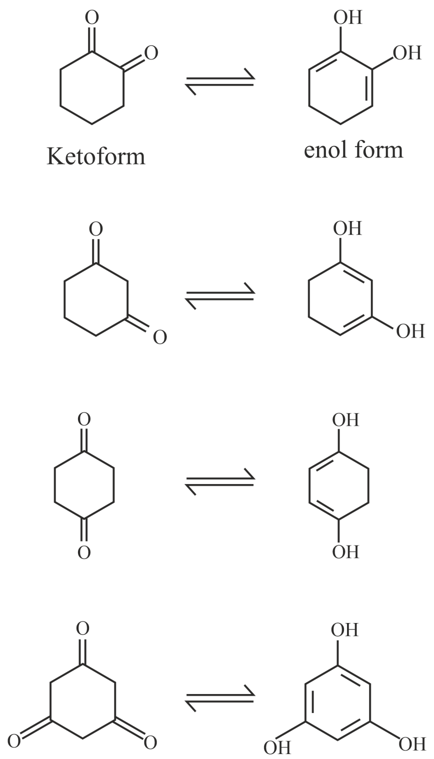
D)



Answer:



Solution: Keto enol tautomerism



As complete conjugation is there in the above enolic form, It is the most stable enol and option D is the correct answer.

Q.5. Which of the following is responsible for the secretion of pepsin

A) Histamine

B) Cimetidine

C) Zantac

D) None of these

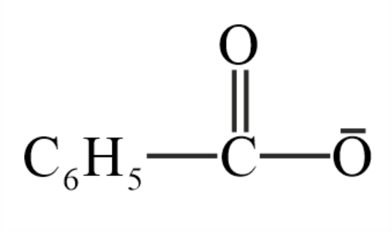
Answer: Histamine

Solution: Histamine stimulates the secretion of pepsin and hydrochloric acid and antihistamines are used to prevent the action of histamines.

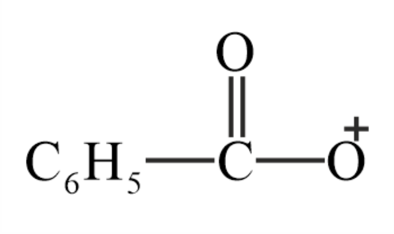
Q.6. The intermediate in the given reaction is:

$$(C_7H_5O_2)_2 \xrightarrow{hv} \left[\text{intermediate}\right] \longrightarrow \dot{C}_6H_5 + CO_2$$

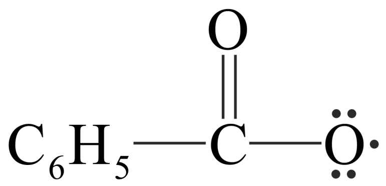
A)



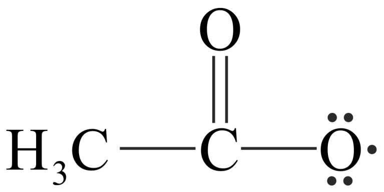
B)



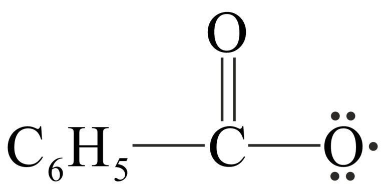
C)



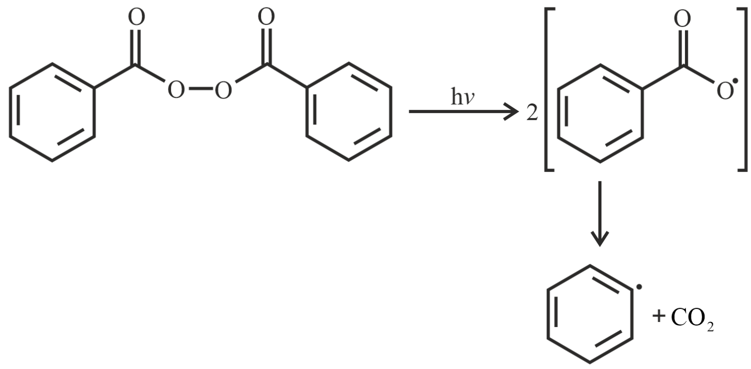
D)



Answer:



Solution:



In presence of sunlight homolytic fission of peroxide linkage will take place.

Q.7. $\underset{\text{white phosphorus}}{P_4}$ + Alkali metal oxide/Hydroxide $\rightarrow$ Product will be

A) White P

B) Red P

C) H_3PO_4

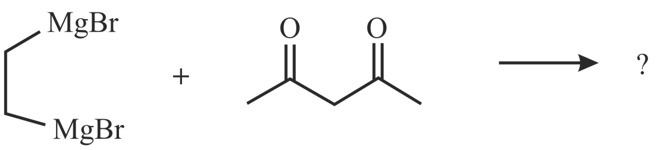
D) $H_2PO_2^-$

Answer: $H_2PO_2^-$

Solution: $P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$

White Phosphorous react with alkali metal hydroxide to form phosphine and a salt of $H_3PO_2^-$. In this reaction P is getting oxidised as well as reduced. So, this is a disproportionation redox reaction.

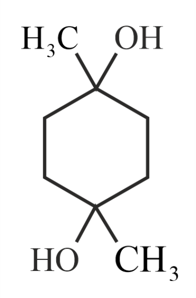
Q.8.



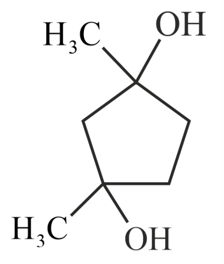
A)

B)

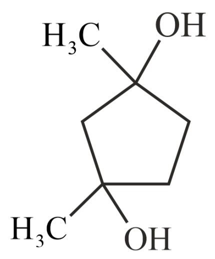
C)



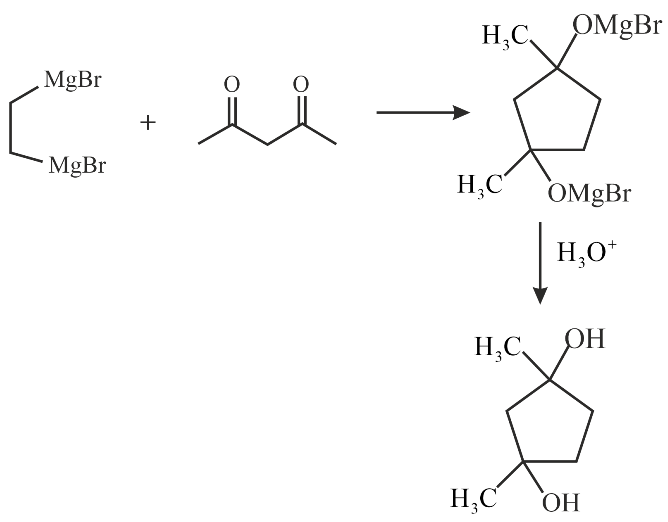
D)



Answer:



Solution:



Q.9. Find the pH of the solution when 50 ml of $0.1\ \mathrm{M}\ \mathrm{CH_3COOH}$ and 25 ml of $0.1\ \mathrm{M}\ \mathrm{NaOH}$ are mixed, $\mathrm{pK_a}$ of $\mathrm{CH_3COOH} = 4.8$

A) 6.8

B) 2.4

C) 4.8

D) 4.0

Answer: 4.8

Solution: $pH = pK_a + \log\frac{[salt]}{[acid]}$

$CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$

$t = 0 \quad 50 \times 0.1 \quad 25 \times 0.1 \quad 0 \quad 0$

$t = t \quad 5 - 2.5 \quad 2.5 - 2.5 \quad 2.5 \quad 2.5$

$= 2.5 \quad = 0$

$pH = 4.8 + \log\frac{[2.5]}{[2.5]}$

$pH = 4.8$

Q.10. The radius of third orbit of H-atom is r_3 pm and radius of fourth orbit of H-atom is r_4 pm. The ratio of r_3 to r_4 is:

A) $9 : 16$

B) $16 : 9$

C) $4 : 3$

D) $3 : 4$

Answer: $9 : 16$

Solution: For H-atom, $Z = 1$

$r_n \propto \frac{n^2}{z}$

$\frac{r_3}{r_4} = \frac{3^2}{4^2} = \frac{9}{16}$

Q.11. $2\,HSO_4^- \xrightarrow[\text{Hydrolysis}]{\text{Electrolysis}} A$

Dihedral angle of A is

A) 111.5°

B) 80°

C) 60°

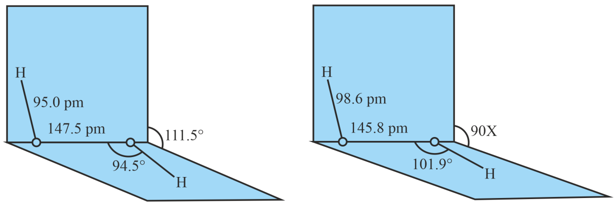
D) 120°

Answer: 111.5°

Solution: Peroxodisulphate, obtained by electrolytic oxidation of acidified sulphate solutions at high current density, on hydrolysis yields hydrogen peroxide.

$$2\,HSO_4^-\left(aq\right) \xrightarrow{\text{Electrolysis}} HO_3SOOSO_3H\left(aq\right) \xrightarrow{\text{Hydrolysis}} 2\,HSO_4^-\left(aq\right) + 2H^+\left(aq\right) + H_2O_2\left(aq\right)$$

Hydrogen peroxide has a non-planar structure. The molecular dimensions in the gas phase and solid phase are shown



(a) Gas phase (b) Solid phase

H_2O_2 structure in gas phase, dihedral angle is 111.5°.

H_2O_2 structure in solid phase at $110\ K$, dihedral angle is 90.2°.

Q.12. Statement I: In Lassaigne's test of organic compound including both N and S, sodium thiocyanate is formed.

Statement II: Excess $Na + NaCNS \rightarrow Na_2S + NaCN$

The correct statement(s) is/are:

A) Both (I) and (II)

B) Only (I)

C) Only (II)

D) None

Answer: Both (I) and (II)

Solution: In case, nitrogen and sulphur both are present in an organic compound, sodium thiocyanate is formed. It gives blood red colour and no Prussian blue since there are no free cyanide ions.

$$Na + C + N + S \rightarrow NaSCN$$

$$Fe^{3+} + SCN^- \rightarrow \underset{\text{Blood red}}{[Fe(SCN)]^{2+}}$$

$$Na + NaCNS \rightarrow Na_2S + NaCN$$

Q.13. Number of oxygen atoms present in structure of base which is replaced in RNA is

A) 1

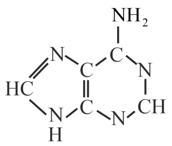
B) 2

C) 3

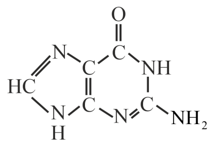
D) 4

Answer: 2

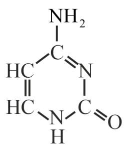
Solution: DNA contains four bases viz. adenine (A), guanine (G), cytosine (C) and thymine (T). RNA also contains four bases, the first three bases are same as in DNA but the fourth one is uracil (U).



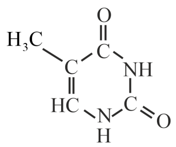
Adenine (A)



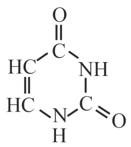
Guanine (G)



Cytosine (C)



Thymine (T)



Uracil (U)

Q.14. Calculate the spin only magnetic moment of the oxide which is the most basic among $\mathrm{V_2O_3}$, $\mathrm{V_2O_4}$, $\mathrm{V_2O_5}$.

A) $4.12\ \mathrm{BM}$

B) $2.83\ \mathrm{BM}$

C) $1.24\ \mathrm{BM}$

D) $3.91\ \mathrm{BM}$

Answer: $2.83\ \mathrm{BM}$

Solution: $\mathrm{V_2O_3}$ is the most basic oxide among the given oxides.

In $\mathrm{V_2O_3}$, Vanadium is in $+3$ oxidation state

$\mathrm{V^{3+}} \Rightarrow [\mathrm{Ar}]3\mathrm{d}^2 4\mathrm{s}^0$

So, 2 unpaired electrons are there,

Its magnetic moment is equal to $\mu = \sqrt{\mathrm{n}(\mathrm{n}+2)}\ \mathrm{BM} = \sqrt{2(2+2)}\ \mathrm{BM}$

$= 2.83\ \mathrm{BM}$

$(\mathrm{n} =$ number of unpaired electrons)

Q.15. 0.3 g of an organic compound on complete combustion gave 0.2 g of carbondioxide an0.1g of water, then the percentage composition of carbon in the compound is

A) 4.5

B) 18

C) 9

D) 16

Answer: 18

Solution: $\%\ \mathrm{of\ C} = \frac{12}{44} \times \frac{\text{Weight of } \mathrm{CO_2} \text{ formed} \times 100}{\text{Weight of organic compound}}$

$= \frac{12 \times 0.2 \times 100}{44 \times 0.3} = 18.18\%$

Q.16. Statement 1 : As the value of ΔG^{o} decreases, the metal oxide becomes more stable.

Statement 2 : As the value of ΔG^{o} increases, the metal having lower value of ΔG^{0} displaces the other metal.

The correct statement(s) is/are:

A) Both (1) and (2)

B) Only (1)

C) Only (2)

D) Neither (1) nor (2)

Answer: Both (1) and (2)

Solution: Any metal will reduce the oxide of other metals which lie above it in the Ellingham diagram because the free energy will become more negative by an amount equal to the difference between the two graphs at that particular temperature.

Q.17. 35% by mass of HCl solution has density $1.46\ g/mL$. Find the Molarity.

A) $12\ M$

B) $14\ M$

C) $9\ M$

D) $16\ M$

Answer: $14\ M$

Solution: Molarity $M = \frac{x \times d \times 10}{m_B}$

$= \frac{35 \times 1.46 \times 10}{36.5} = 14\ M$

$m_B =$ molecular mass of solute

$d =$ density of the solution in g/L

$x = \%$ solute by mass.

Q.18. $0.5\%\ KCl$ solution has depression in freezing point of $0.24\ K$. Calculate the degree of dissociation of KCl.

Given: $K_f(\text{water}) = 1.86\ K\ kg\ mol^{-1}$

A) 1.84

B) 0.92

C) 0.68

D) 1.22

Answer: 0.92

Solution: $KCl \rightarrow K^{+} + Cl^{-}$

$1 \quad - \quad -$

$1-\alpha \quad \alpha \quad \alpha$

$i = 1 - \alpha + 2\alpha = 1 + \alpha$

$\Delta T_f = i\,K_f \cdot m$

$0.24 = (1+\alpha)1.86 \times \frac{0.5}{74.5} \times \frac{1000}{99.5}$

$\alpha = 0.92$

Q.19. Which of the following is incorrect about brown ring test?

1. Brown ring forms at the junction of two solutions

2. The brown ring complex is $\left[Fe\,(H_2O)_5\,NO\right]SO_4$

3. It consists of ferrous nitro sulphate complete.

4. $NO_3^- + H_2SO_4\,(conc.) \rightarrow$ Brown fumes are evolved

A) 3

B) 2

C) 1

D) 4

Answer: 3

Solution: Brown ring test is usually carried out by adding dilute ferrous sulphate solution to an aqueous solution containing nitrate ion, and then carefully adding concentrated sulphuric acid along the sides of the test tube. A brown ring at the interface between the solution and sulphuric acid layers indicates the presence of nitrate ion in solution.

$NO_3^- + H_2SO_4 \rightarrow HNO_3 + HSO_4^-$

$6\,FeSO_4 + 3H_2SO_4 + 2\,HNO_3 \rightarrow 3\,Fe_2\,(SO_4)_3 + 2NO + 4H_2O$

$NO + FeSO_4 + 5H_2O \rightarrow \left[Fe\,(H_2O)_5\,NO\right]SO_4$ (Brown ring)

Brown ring complex formed is $\left[Fe\,(H_2O)_5\,NO\right]SO_4$

IUPAC name of the above complex is Pentaaquanitrosyllron(I) Sulphate.

Nitrate ions reacts with sulphuric acid to form nitrogen dioxide which is responsible for brown fumes.

Hence, statement 3 is incorrect.

Q.20. The increasing order of melting point of alkaline earth metals is

A) $Mg < Ca < Sr < Be$

B) $Ca < Sr < Be < Mg$

C) $Mg < Sr < Ca < Be$

D) $Ca < Mg < Be < Sr$

Answer: $Mg < Sr < Ca < Be$

Solution:

Alkali metal	Melting point
Mg	924 K
Sr	1062 K
Ca	1124 K
Be	1560 K

Q.21. 2-ethylanthraquinol is oxidized to form 2-ethylanthraquinone and compound X. The dihedral angle of X in solid state is:

A) 90.2

B) 111.5

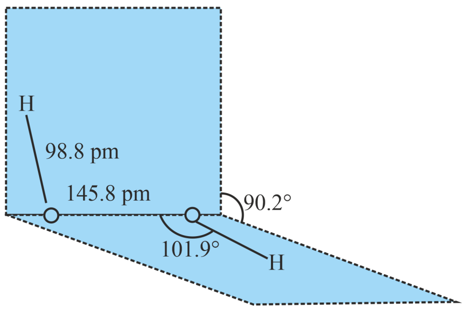
C) 94.8

D) 101.9

Answer: 90.2

Solution:

$$2-\text{ethylanthraquinol} \underset{H_2/Pd}{\overset{O_2(\text{ air })}{\rightleftarrows}} H_2O_2+2-\text{ethylanthraquinone}$$



H_2O_2 structure in gas phase, dihedral angle is 111.5°. H_2O_2 structure in solid phase at 110 K, dihedral angle is 90.2°.

Q.22. An organic compound when reacts with dil. HNO_3 produces two isomers A and B. A possess intramolecular hydrogen bonding and B posses intermolecular hydrogen bonding. When the same compound reacts with conc. HNO_3, it produces a strong acid D. Find the number of oxygen atoms in D.

A) 1

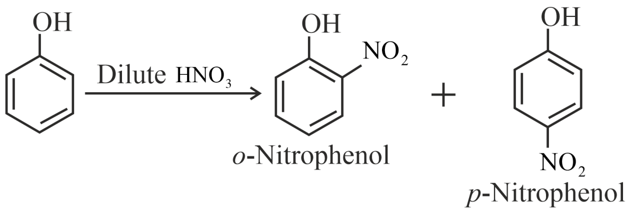
B) 6

C) 3

D) 7

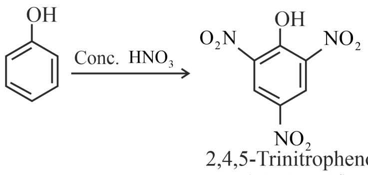
Answer: 7

Solution: With dilute nitric acid at low temperature $(298 \mathrm{~K})$, phenol yields a mixture of ortho and para nitrophenols.



The ortho and para isomers can be separated by steam distillation. o-Nitrophenol is steam volatile due to intramolecular hydrogen bonding while p-nitrophenol is less volatile due to intermolecular hydrogen bonding which causes the association of molecules.

With concentrated nitric acid, phenol is converted to $2,\ 4,\ 6$-trinitrophenol. The product is commonly known as picric acid. The yield of the reaction product is poor.



2,4,5-Trinitrophenol
(Picric acid)

Hence, the number of oxygen atoms in picric acid is 7.

Section C: Mathematics

Q.1. If a biased coin is tossed 5 times and probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting at most 2 heads is

A) $\frac{46}{6^4}$

B) $\frac{275}{6^5}$

C) $\frac{41}{5^5}$

D) $\frac{36}{5^4}$

Answer: $\frac{46}{6^4}$

Solution: $P(4\ \text{heads}) = P(5\ \text{heads})$

${}^5C_4 p^4 q = {}^5C_5 p^5 \Rightarrow p = 5q \quad \ldots(\mathrm{i})$

Also, $p + q = 1 \Rightarrow q = \frac{1}{6}, p = \frac{5}{6}$ (from (i))

So, P (at most 2 heads) $= P(x=0) + P(x=1) + P(x=2)$

$= q^5 + {}^5C_1 pq^4 + {}^5C_2 p^2 q^3$

$= \left(\frac{1}{6}\right)^5 + 5\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^4 + 10\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^3 = \frac{46}{6^4}$

Q.2. $\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1}x) - x}{1 - \tan(\cos^{-1}x)} =$

A) $\frac{1}{\sqrt{2}}$

B) $\frac{-1}{\sqrt{2}}$

C) 1

D) -1

Answer: $\frac{-1}{\sqrt{2}}$

Solution: Let $\cos^{-1}x = t$ i.e. $x = \cos t$

So $\lim_{t \to \frac{\pi}{4}} \frac{\sin t - \cos t}{1 - \tan t} = \lim_{t \to \frac{\pi}{4}} (-\cos t) = \frac{-1}{\sqrt{2}}$

Q.3. The vertex of an equilateral triangle is $(3, 7)$ and equation of its opposite side is $x + y = 5$, then the area of the equilateral triangle is

A) $\frac{25}{\sqrt{3}}$

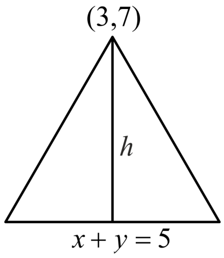
B) $\frac{25}{2\sqrt{3}}$

C) $25\sqrt{3}$

D) 25

Answer: $\frac{25}{2\sqrt{3}}$

Solution:



$h =$ perpendicular distance from $(3, 7)$ to the line $x + y - 5 = 0$

i.e. $h = \frac{|3+7-5|}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

For an equilateral triangle,

length of side $= \frac{2}{\sqrt{3}}h = \frac{2}{\sqrt{3}} \times \frac{5}{\sqrt{2}} = 5\sqrt{\frac{2}{3}}$

and area of triangle $= \frac{\sqrt{3}}{4} \times (\text{length of side})^2$

$= \frac{\sqrt{3}}{4} \times \left(5\sqrt{\frac{2}{3}}\right)^2 = \frac{25}{2\sqrt{3}}$

Q.4. If A is 3×3 matrix and $|\text{adj}\,(24A)| = |\text{adj}\,3\,(\text{adj}\,2A)|$, then the value $|A|^2$ is :

A) 64

B) 8

C) 512

D) 72

Answer: 64

Solution: We know that $|\text{adj}\,A| = |A|^{n-1}$

$|KA| = K^n |A|$

where n is order of matrix

Given

$|\text{adj}\,(24A)| = |\text{adj}\,3\,(\text{adj}\,2A)|$

$\Rightarrow |24A|^2 = |3\,\text{adj}\,(2A)|^2$

$\Rightarrow |24A| = |3\,\text{adj}\,(2A)|$

$\Rightarrow 24^3 |A| = 3^3 |\text{adj}\,2A| \Rightarrow 24^3 |A| = 3^3 |2A|\,|2A|$

$\Rightarrow 24^3 |A| = 3^3 2^3 2^3 |A|\,|A|$

$\Rightarrow |A| = \frac{24^3}{2^3 2^3 3^3} = \frac{(2^3.3)^3}{2^6 3^3} = 8$

$\Rightarrow |A|^2 = 64$

Q.5. From a group of 10 boys $B_1, B_2, \cdots, B_{10}$ and 5 Girls $G_1, G_2, \cdots, G_5$. What will be the number of ways of selection of a group of 3 boys and 3 girls such that B_1 and B_2 are not together in the group?

A) 1120

B) 1200

C) 1600

D) 1180

Answer: 1120

Solution: Given

Total number of boys is 10 and total number of girls is 5

So total number of selections $= {}^{10}C_3 \cdot {}^{5}C_3 = 1200$

No. of selections in which B_1 & B_2 both are there in group $= {}^{8}C_1 \cdot {}^{5}C_3 = 80$

Required no. of selections $= 1200 - 80 = 1120$

Q.6. If $(2021)^{2023}$ is divided by 7, then the remainder is

A) 2

B) 3

C) 4

D) 5

Answer: 5

Solution: The remainder when 2021 divided by 7 is -2. Hence, the problem reduces to finding the remainder when $(-2)^{2023}$ is divided by 7.

$= \frac{(-2)^{2022} \times (-2)}{7}$

$= -\frac{2 \times 2^{2022}}{7}$

$= \frac{-2 \times (2^3)^{674}}{7}$

$= \frac{-2 \times (8)^{674}}{7}$

$= \frac{-2 \times (1+7)^{674}}{7}$

Using binomial theorem, we get

$= \frac{-2 \times (1+7k)}{7}$ where k is an integer.

$= \frac{-2-14k}{7}$

Clearly, the remainder when $-14k$ is divisible by 7 is 0.

$= \frac{-2}{7}$

$= \frac{-2-5+5}{7} = \frac{5}{7}$

Hence, the remainder is 5.

Q.7. If the mean of 5 elements $a, b, 8, 5, 10$ is 6 and variance is 6.8 and mean deviation about mean is M, then the value of $25M$ is

A) 60

B) 12

C) 50

D) 75

Answer: 60

Solution: Mean $\bar{x} = \frac{a+6+8+5+10}{5} = 6$ (given)

$\Rightarrow a + b = 7 \quad \ldots(\text{i})$

Variance $= \frac{\sum x_i^2}{n} - (\bar{x})^2 = \frac{a^2+b^2+8^2+5^2+10^2}{5} - 6^2 = 6.8$ (given)

$\Rightarrow a^2 + b^2 = 25 \quad \ldots(\text{ii})$

on solving equations (i) & (ii), we get $a = 3,\ b = 4$

Now, Mean deviation about Mean $M = \frac{\sum|x_i - \bar{x}|}{n}$

$\Rightarrow \frac{|3-6|+|4-6|+|8-6|+|5-6|+|10-6|}{5}$

$\Rightarrow \frac{3+2+2+1+4}{5} = \frac{12}{5}$

$\therefore \quad M = \frac{12}{5} \Rightarrow 25M = 60.$

Q.8. If $f(x) = \text{Max}\{|x+1|, |x+2|, |x+3|, |x+4|, |x+5|, |x+6|\}$, then find $\int_{-6}^{0} f(x) dx$

A) $\frac{97}{4}$

B) $\frac{235}{4}$

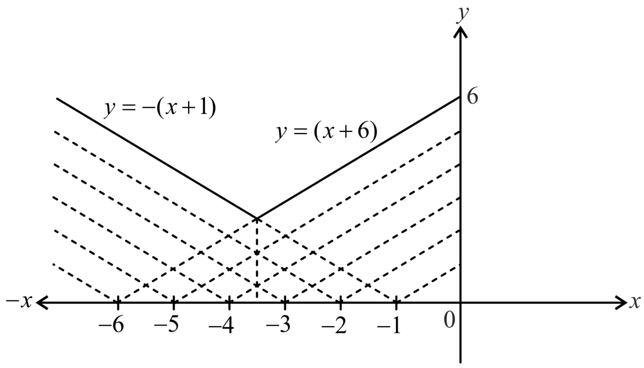
C) $\frac{219}{8}$

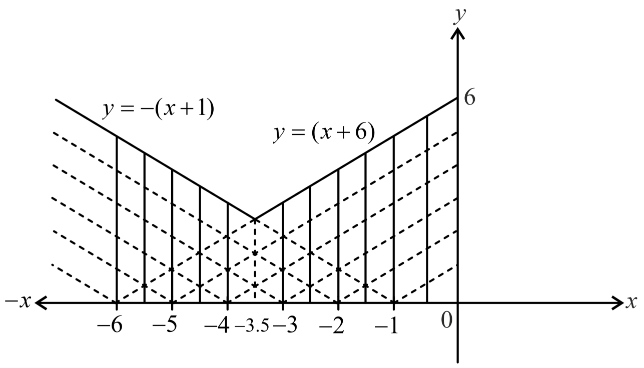
D) $\frac{275}{8}$

Answer: $\frac{97}{4}$

Solution:

Ploting the graph of $f(x)=\operatorname{Max}\{|x+1|,|x+2|,|x+3|,|x+4|,|x+5|,|x+6|\}$we get





So by Graph

Area $=\int_{-6}^{-3.5}-(x+1)dx+\int_{-3.5}^{0}(x+6)dx$

$=\frac{75}{8}+\frac{119}{8}=\frac{194}{8}=\frac{97}{4}$

Q.9. If $\frac{x}{a}+\frac{y}{b}=2$ is tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at (a,b) such that $n\in S$, then $S\in$

A) ϕ

B) $\{1\}$

C) $2k$, where $k\in\mathbb{N}$

D) $\mathbb{N}$

Answer: $\mathbb{N}$

Solution: $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$

Differentiating, we get,

$n\left(\frac{x}{a}\right)^{n-1}\cdot\frac{1}{a}+n\cdot\left(\frac{y}{b}\right)^{n-1}\cdot\frac{1}{b}\cdot\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-\frac{1}{a}\left(\frac{x}{a}\right)^{n-1}}{\frac{1}{b}\left(\frac{y}{b}\right)^{n-1}}=\frac{-b}{a}\left(\frac{xb}{ya}\right)^{n-1}\Rightarrow\left.\frac{dy}{dx}\right|_{(a,b)}=\frac{-b}{a}\left(\frac{ab}{ba}\right)^{n-1}=\frac{-b}{a}$

So, equation of the tangent to the curve at (a,b) will be $y-b=\frac{-b}{a}(x-a)\ \Rightarrow\frac{y}{b}-1=-\frac{x}{a}+1$

$\Rightarrow\frac{x}{a}+\frac{y}{b}=2$

i.e. $S\in N$

Q.10. If a normal to parabola $y^2=6x$ at P, passes through $(5,-8)$, then the coordinates of point of intersection of directrix & tangent at P are

A) $\left(\frac{-3}{2},\frac{-9}{4}\right)$

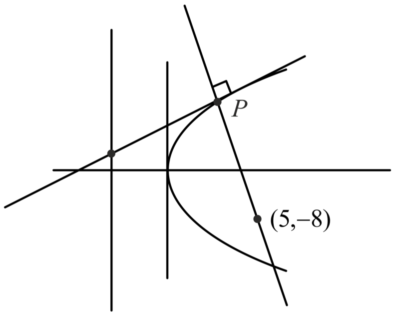
B) $\left(\frac{-1}{2},\frac{-9}{4}\right)$

C) $\left(\frac{-3}{2},\frac{-11}{2}\right)$

D) $\left(\frac{-1}{2},\frac{-7}{2}\right)$

Answer: $\left(\frac{-3}{2},\frac{-9}{4}\right)$

Solution:



Given $y^2 = 6x$ comparing with $y^2 = 4a \rightarrow a = \frac{3}{2}$

Now normal at P is given by $P\left(\frac{3}{2}t^2, 3t\right)$

$y + tx = 3t + \frac{3}{2}t^3$

Now this normal is passing through $(5, -8)$

So $-8 + 5t = 3t + \frac{3}{2}t^3$

$\Rightarrow -16 + 10t = 6t + 3t^3$

$\Rightarrow 3t^3 - 4t + 16 = 0$

Now solving above equation we get $t = -2$

So tangent at P is given by $ty = x + at^2$

putting a & t we get $-2y = x + \frac{3}{2} \times 4$

$-2y = x + 6$

Now tangent is intersecting at directrix which is $x = \frac{-3}{2}$

So $-2y = -\frac{3}{2} + 6 \Rightarrow y = \frac{3}{4} - 3 = -\frac{9}{4}$

So point of intersection is $\left(\frac{-3}{2}, \frac{-9}{4}\right)$

Q.11. If A & B two sets are given such that $A = \{x : \text{HCF}\{x, 45\} = 1\}$ and $B = \{x = 2k; 1 \leq k \leq 100\}$, then the value of $n(A \cap B)$ is

A) 53

B) 40

C) 35

D) 33

Answer: 53

Solution: Given, $A=\{x: \mathrm{HCF}\{x, 45\}=1\}$

Prime factor of $45=3^2 \times 5$

$\dot{B}=\{x=2k: 1 \leqslant k \leqslant 100\}$

So $B=\{2,4,6,8 \ldots .200\}$

Now $n(A)$ will be those whose H.C.F with be 45 will be 1

so we will subtract multiple of 3 or 5 from set B to get required result

So, $n(A \cap B)=$ total elements in set $B-$ (multiple of 3 or 5 in set B)

$=100-(20+33-6)=53$

Q.12. If $A=\sum_{i=1}^{10} \sum_{j=1}^{10} \min \ \{i, j\}$ and $B=\sum_{i=1}^{10} \sum_{j=1}^{10} \max \ \{i, j\}$, then $A+B=$

A) 1100

B) 1000

C) 2200

D) 2000

Answer: 1100

Solution:

We can interpret all the ordered pairs of $\{i, j\}$ where $1 \leq i, j \leq 10$ through this matrix $\begin{bmatrix}(1,1) & (1,2) & \ldots & (1,10) \\ (2,1) & \ldots & \ldots & (2,10) \\ \vdots & & & \\ (10,1) & \ldots & \ldots & (10,10)\end{bmatrix}$

Now, $A=(1+1+\ldots 10 \text{ times})+(2+2+\ldots 10 \text{ times})+\ldots .(10+\ldots .10 \text{ times})$

Similarly $B=(1+1+\ldots 10 \text{ times})+(2+2+\ldots 10 \text{ times})+\ldots .(10+\ldots .10 \text{ times})$

So, $A+B=20(1+2+\ldots+10)=1100$

Q.13. If $f(x)=\frac{x+1}{x-1}$, $f^{n+1}(x)=f\left(f^n(x)\right)$, then the value of $f^7(7)+f^6(6)$ is equal to

A) $\frac{22}{3}$

B) $\frac{20}{3}$

C) $\frac{22}{5}$

D) $\frac{19}{3}$

Answer: $\frac{22}{3}$

Solution:

$f^1(x)=\frac{x+1}{x-1} \Rightarrow f^2(x)=f(f(x))=\frac{f(x)+1}{f(x)-1}=\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}=x$

$f^3(x)=f(f(f(x)))=f(x)$

i.e., $f^4(x)=x \ \Rightarrow f^6(x)=x$ and $f^7(x)=f(x)$

i.e., $f^7(7)+f^6(6)=f(7)+6=\frac{22}{3}$

Q.14. The area of the region bounded by the curves $y = |x^2 - 9|$ and $y = 3$ is $16\sqrt{6} + 16\sqrt{12} - k$, then $k =$

A) 24

B) 48

C) 72

D) 96

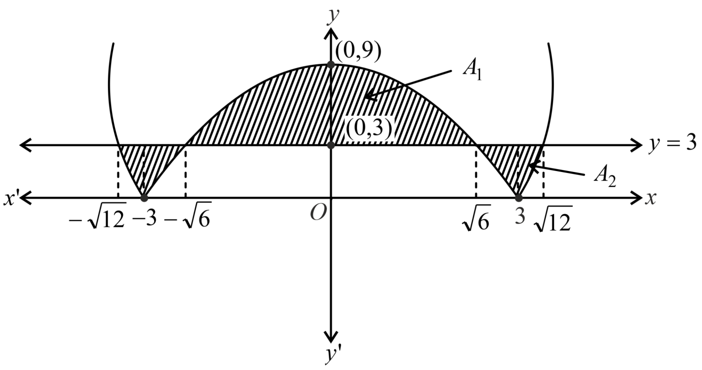
Answer: 72

Solution: We first find the points of intersection of the curves $y = |x^2 - 9|$ and $y = 3$

which gives, $|x^2 - 9| = 3 \Rightarrow x^2 - 9 = \pm 3$

$\Rightarrow x^2 = 12$ (or) $x^2 = 6 \Rightarrow x = \pm\sqrt{12}$ (or) $x = \pm\sqrt{6}$

The graphs of the two curves are shown in figure



We have to find the area of the shaded region.

Required area $= 2A_1 + 2A_2$ ($\because$ The areas A_1 & A_2 are symmetrical about $y-$ axis)

where $A_1 = \int_0^{\sqrt{6}} \left[(9 - x^2) - (3)\right] dx$ $A_1 = \int_0^{\sqrt{6}} (6 - x^2) dx$

$= \left[6x - \frac{x^3}{3}\right]_0^{\sqrt{6}} = 6\sqrt{6} - \frac{6\sqrt{6}}{3} = \frac{12\sqrt{6}}{3} = 4\sqrt{6}$

$A_2 = \int_{\sqrt{6}}^{3} \left[3 - (9 - x^2)\right] dx + \int_{3}^{\sqrt{12}} \left[3 - (x^2 - 9)\right] dx$

$= \int_{\sqrt{6}}^{3} (-6 + x^2) dx + \int_3^{\sqrt{12}} (12 - x^2) dx$

$= \left[-6x + \frac{x^3}{3}\right]_{\sqrt{6}}^{3} + \left[12x - \frac{x^3}{3}\right]_3^{\sqrt{12}}$

$= (-18 + 9) - \left(-6\sqrt{6} + 2\sqrt{6}\right) + \left(12\sqrt{12} - \frac{12\sqrt{12}}{3}\right) - (36 - 9)$

$= -36 + 4\sqrt{6} + 8\sqrt{12}$

Hence, required area $= 2A_1 + 2A_2$

$= 2\left(4\sqrt{6}\right) + 2\left(-36 + 4\sqrt{6} + 8\sqrt{12}\right)$

$= 8\sqrt{6} - 72 + 8\sqrt{6} + 16\sqrt{12}$

$= 16\sqrt{6} + 16\sqrt{12} - 72$

Q.15. If $\vec{a}$, $\vec{b}$ & $\vec{c}$ are non-zero vectors, $\vec{a}\cdot\vec{b}=1$, $\vec{b}\cdot\vec{c}=2$ and $\vec{c}\cdot\vec{a}=3$, then $\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right)\ \vec{b}\times\left(\vec{c}\times\vec{a}\right)\ \vec{c}\times\left(\vec{b}\times\vec{a}\right)\right]=$

A) 0

B) 2

C) 3

D) 1

Answer: 0

Solution: We know that

$\vec{a}\times\left(\vec{b}\times\vec{c}\right)$, $\vec{b}\times\left(\vec{c}\times\vec{a}\right)$ and $\vec{c}\times\left(\vec{a}\times\vec{b}\right)$ are all coplanar as $\vec{a}\times\left(\vec{b}\times\vec{c}\right)+\vec{b}\times\left(\vec{c}\times\vec{a}\right)+\vec{c}\times\left(\vec{a}\times\vec{b}\right)=0$.

Also, the scalar triple product of 3 vectors which are coplanar is 0.

Hence, $\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right)\ \vec{b}\times\left(\vec{c}\times\vec{a}\right)\ \vec{c}\times\left(\vec{a}\times\vec{b}\right)\right]=0$

Q.16. If $\sin^2 10^\circ\times\sin 20^\circ\times\sin 40^\circ\times\sin 50^\circ\times\sin 70^\circ=\alpha-\frac{\sin 10^\circ}{16}$, then α

A) $\frac{1}{64}$

B) $\frac{3}{64}$

C) $\frac{5}{64}$

D) $\frac{7}{64}$

Answer: $\frac{1}{64}$

Solution: We have, $\sin^2 10^\circ \times \sin 20^\circ \times \sin 40^\circ \times \sin 50^\circ \times \sin 70^\circ$

$= (\sin 10^\circ \times \sin 50^\circ \times \sin 70^\circ)(\sin 10^\circ \times \sin 20^\circ \times \sin 40^\circ)$

Using, $\sin\theta = \cos(90^\circ - \theta)$

$= [\cos 80^\circ \times \cos 40^\circ \times \cos 20^\circ][\sin 10^\circ \times \sin(30^\circ - 10^\circ) \times \sin(30^\circ + 10^\circ)]$

We know that $\cos\theta \cos 2\theta \cos 2^2\theta \ldots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta}$

and $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B \Rightarrow \left(\frac{\sin 2^3(20^\circ)}{2^3 \sin 20^\circ}\right)\left[\sin 10^\circ \times \left(\frac{1}{4} - \sin^2 10^\circ\right)\right]$

$\Rightarrow \left(\frac{\sin 160^\circ}{2^3 \sin 20^\circ}\right)\left(\frac{\sin 10^\circ - 4\sin^3 10^\circ}{4}\right)$

$\Rightarrow \left(\frac{\sin(180^\circ - 20^\circ)}{8 \sin 20^\circ}\right)\left(\frac{3\sin 10^\circ - 4\sin^3 10^\circ - 2\sin 10^\circ}{4}\right)$

$\Rightarrow \left(\frac{\sin 20^\circ}{8 \sin 20^\circ}\right)\left(\frac{\sin 30^\circ - 2\sin 10^\circ}{4}\right)$

$\Rightarrow \frac{1}{8}\left(\frac{\frac{1}{2} - 2\sin 10^\circ}{4}\right)$

$\Rightarrow \frac{1}{8}\left(\frac{1 - 4\sin 10^\circ}{8}\right) = \alpha - \frac{\sin 10^\circ}{16}$ (given)

$\Rightarrow \frac{1 - 4\sin 10^\circ}{64} = \frac{16\alpha - \sin 10^\circ}{16}$

$\Rightarrow 1 - 4\sin 10^\circ = 64\alpha - 4\sin 10^\circ$

$\Rightarrow \alpha = \frac{1}{64}.$

Q.17. Find the value of integration $\left(\frac{48}{\pi^4}\right)\int_0^\pi \left(\frac{3\pi}{2}x^2 - x^3\right)\frac{\sin x}{1+\cos^2 x}dx$

A) 6

B) 5

C) 2

D) 7

Answer: 6

Solution: Let $I=\frac{48}{\pi^4}\int_0^\pi\left(\frac{3\pi}{2}x^2-x^3\right)\frac{\sin x}{1+\cos^2 x}dx \qquad \ldots(1)$

Using $\int_a^b f(x)dx=\int_a^b f\Big(a+b-x\Big)dx$

We get $I=\frac{48}{\pi^4}\int_0^\pi\left(\frac{3\pi}{2}(\pi-x)^2-(\pi-x)^3\right)\frac{\sin(0+\pi-x)}{1+\cos^2(0+\pi-x)}dx$

$I=\frac{48}{\pi^4}\int_0^\pi\left(\frac{3\pi^3}{2}-3\pi^2x+\frac{3\pi x^2}{2}-\pi^3+x^3+3\pi^2x-3\pi x^2\right)\frac{\sin x}{1+\cos^2 x}dx$ ....equation 2

Now adding equation (1) + equation (2)

$2I=\frac{48}{\pi^4}\int_0^\pi\left(\frac{3\pi^3}{2}-\pi^3\right)\frac{\sin x}{1+\cos^2 x}dx$

$2I=\frac{48}{\pi^4}\times\frac{\pi^3}{2}\int_0^\pi\frac{\sin x}{1+\cos^2 x}dx$

$2I=\frac{24}{\pi}\times 2\times\int_0^{\frac{\pi}{2}}\frac{\sin x}{1+\cos^2 x}dx$

Let $\cos x=t\Rightarrow-\sin x dx=dt$

So $I=\frac{24}{\pi}\times\int_1^0\frac{-dt}{1+t^2}$

$\Rightarrow I=\frac{-24}{\pi}\left[\tan^1 t\right]_1^0$

$\Rightarrow I=-\frac{24}{\pi}\left[0-\frac{\pi}{4}\right]=6$