JEE Main Exam 2022 - Session 1

26 June 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. Dimension of mutual inductance is

A) $\mathrm{ML^2T^{-2}A^{-1}}$

B) $\mathrm{ML^2T^{-2}A^{-2}}$

C) $\mathrm{ML^2T^{-1}A^{-1}}$

D) $\mathrm{ML^2T^{-3}A^{-2}}$

Answer: $\mathrm{ML^2T^{-2}A^{-2}}$

Solution: The formula for mutual inductance is given by, $\varepsilon = M\frac{dI}{dt} \Rightarrow M = \frac{\varepsilon}{\left(\frac{dI}{dt}\right)}$

Dimension of emf or voltage is $\left[\mathrm{M\ L^2\ T^{-3}\ A^{-1}}\right]$,
Dimension of current is $[\mathrm{A}]$,
Dimension of time $[\mathrm{T}]$.

Therefore, the dimension of M is

$$M = \frac{\left[\mathrm{ML^2T^{-3}A^{-1}}\right]}{[\mathrm{A}][\mathrm{T}]^{-1}}$$
$$= \left[\mathrm{ML^2T^{-2}A^{-2}}\right]$$

Q.2. Find the ratio of rotational kinetic energy to the total kinetic energy of a rolling solid sphere?

A) $\frac{7}{5}$

B) $\frac{2}{5}$

C) $\frac{2}{7}$

D) $\frac{5}{7}$

Answer: $\frac{2}{7}$

Solution: Under pure rolling condition: $v=\omega R$

Translational $K.E=\frac{1}{2}mv^2$

Rotational $K.E=\frac{1}{2}\times\frac{2}{5}mR^2\left(\frac{v}{R}\right)^2=\frac{1}{5}mv^2$

Total $K.E=\frac{1}{2}mv^2+\frac{1}{5}mv^2=\frac{7}{10}mv^2$

So, $\frac{\text{Rotational } K.E}{\text{Total } K.E}=\frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2}=\frac{2}{7}$

Q.3. Arrange the EM waves according to increasing order of wavelength

A) $\lambda_{\text{gamma}}<\lambda_{\text{X-ray}}<\lambda_{\text{microwave}}<\lambda_{\text{visible}}$

B) $\lambda_{\text{gamma}}<\lambda_{\text{X-ray}}<\lambda_{\text{visible}}<\lambda_{\text{microwave}}$

C) $\lambda_{\text{X-ray}}<\lambda_{\text{microwave}}<\lambda_{\text{gamma}}<\lambda_{\text{visible}}$

D) $\lambda_{\text{microwave}}<\lambda_{\text{visible}}<\lambda_{\text{X-ray}}<\lambda_{\text{gamma}}$

Answer: $\lambda_{\text{gamma}}<\lambda_{\text{X-ray}}<\lambda_{\text{visible}}<\lambda_{\text{microwave}}$

Solution: We know that $E=\frac{hc}{\lambda}\Rightarrow E\propto\frac{1}{\lambda}$. Also,

$E_{\text{gamma}}>E_{\text{X-ray}}>E_{\text{visible}}>E_{\text{microwave}}$

Therefore,

$\lambda_{\text{gamma}}<\lambda_{\text{X-ray}}<\lambda_{\text{visible}}<\lambda_{\text{microwave}}$

Q.4. Which of the following is the relation of Reynolds number when the velocity(v), diameter(d), viscosity(η) and density of the fluid(ρ) is given?

A) $\frac{\rho v d}{\eta}$

B) $\frac{\rho\eta d}{v}$

C) $\frac{\rho\eta v}{d}$

D) $\frac{\eta v d}{\rho}$

Answer: $\frac{\rho v d}{\eta}$

Solution: Reynolds number is given by, $R=\frac{\rho v d}{\eta}$, where, $v=$flow velocity, $d=$hydraulic diameter of the pipe, $\rho=$density of the fluid & $\eta=$coefficient of viscosity of the fluid.

Q.5. Three capacitors $10\ \mu\text{F}$, $15\mu\text{F}$ and $20\ \mu\text{F}$ in series connected to $13\ \text{V}$ battery. The charge on $15\ \mu\text{F}$ capacitor is:

A) $60\ \mu\text{C}$

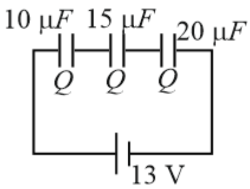
B) $70\ \mu\text{C}$

C) $80\ \mu\text{C}$

D) $90\ \mu\text{C}$

Answer: $60\ \mu\text{C}$

Solution:



As we know, $Q = CV$

$C_{eq} = \left(\frac{1}{10} + \frac{1}{15} + \frac{1}{20}\right)^{-1} = \left(\frac{6+4+3}{60}\right)^{-1} = \frac{60}{13}\ \mu\text{F}$

Therefore, charge $Q = \frac{60}{13} \times 13 = 60\ \mu\text{C}$.

As the capacitors are connected in series, charge on each capacitor wil be same.

Q.6. A ball is thrown up with speed $50\ \text{m s}^{-1}$. After 2 second, another ball is thrown up with the same speed, at time when they will meet, after the first ball thrown.

A) $6\ \text{s}$

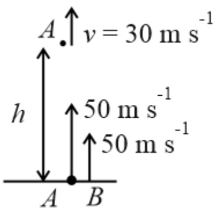
B) $2\ \text{s}$

C) $8\ \text{s}$

D) $4\ \text{s}$

Answer: $6\ \text{s}$

Solution:



Distance covered by ball A is $h = 50\,(2) - \frac{1}{2} \times 10 \times 2^2 = 100 - 20 = 80\ \text{m}$

Now, at time $t = 2$ s, velocity of A is $v_A = 50 - 10 \times 2 = 30\ \text{m s}^{-1}$

Relative velocity of both balls after 2 s is $v_{\text{rel}} = 50 - 30 = 20\ \text{m s}^{-1}$

Here, relative displacement $x_{\text{rel}} = 80\ \text{m s}^{-1}$ and relative acceleration$a_{\text{rel}} = 0$.

Time taken $t = \frac{80}{20} = 4\ \text{s}$

Time at which they will meet is $t = 4 + 2 = 6\ \text{s}$

Q.7. Apparent wavelength of a wave received from a planet is $670.7\ \text{nm}$ and actual wavelength of the wave is $670\ \text{nm}$. Find the speed with which planet is moving away?

A) $2.12 \times 10^5\ \text{m s}^{-1}$

B) $3.13 \times 10^5\ \text{m s}^{-1}$

C) $4.14 \times 10^5\ \text{m s}^{-1}$

D) $6 \times 10^5 \mathrm{~m~s^{-1}}$

Answer: $3.13 \times 10^5 \mathrm{~m~s^{-1}}$

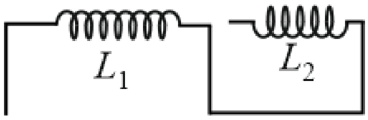
Solution: The apparent change in the wavelength is given by

$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$\Rightarrow v = c\frac{\Delta\lambda}{\lambda} = \frac{3\times10^8\times(670.7-670)}{670} = 3.13 \times 10^5 \mathrm{~m~s^{-1}}$

Hence, velocity of the planet is $3.13 \times 10^5 \mathrm{~m~s^{-1}}$.

Q.8. Two inductors of inductance L_1 and L_2 are connected as shown. If their mutual inductance is M, find the equivalent inductance of the combination.



A) $L_1 + L_2 + 2M$

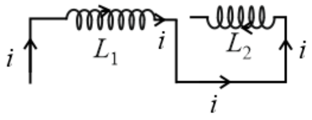
B) $L_1 + L_2 - 2M$

C) $\frac{1}{L_1} + \frac{1}{L_2} + \frac{2}{M}$

D) $L_1 + L_2$

Answer: $L_1 + L_2 - 2M$

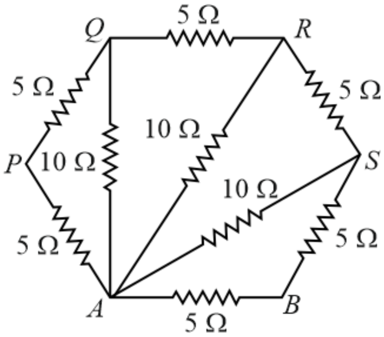
Solution:



As the inductors connected are in series opposition, hence magnetic flux will be opposing each other. Therefore,

$L_{eq} = L_1 + L_2 - 2M$

Q.9. Find the equivalent resistance between point A and B



A) $\frac{10}{3}\ \Omega$

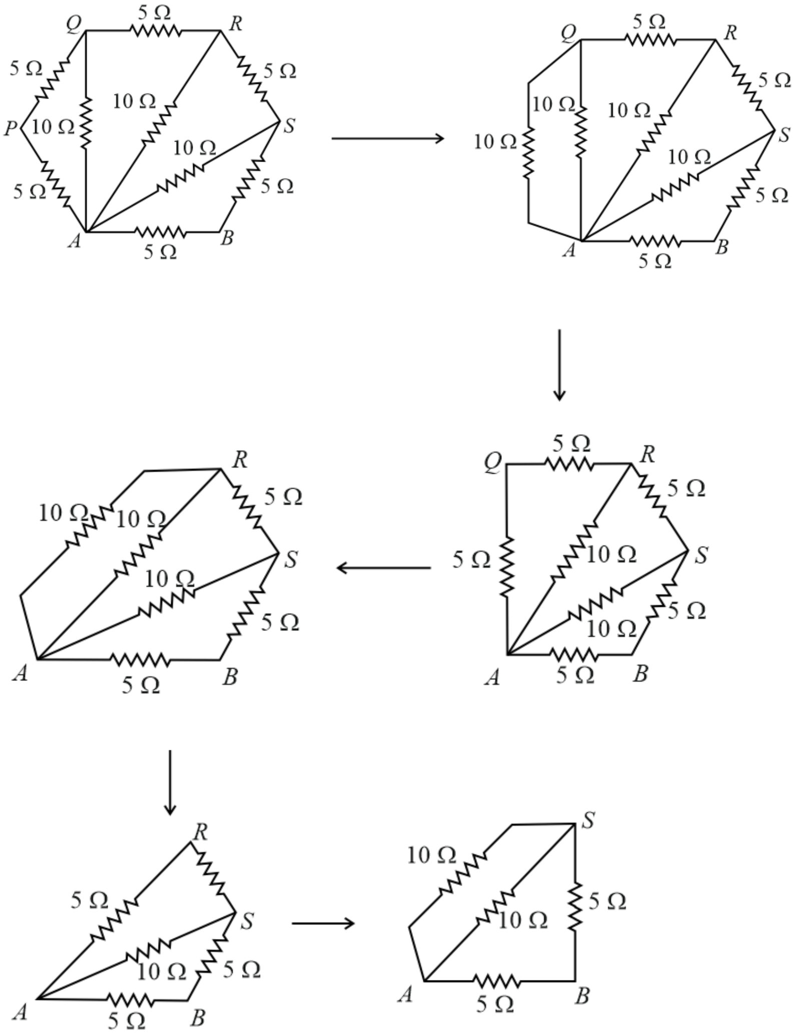
B) $5\ \Omega$

C) $10\ \Omega$

D) $\frac{20}{3}\ \Omega$

Answer: $\frac{10}{3}\ \Omega$

Solution:



Therefore, $R_{AB} = \frac{(R_{AS}+5)\times 5}{(R_{AS}+5)+5} = \frac{(5+5)\times 5}{(5+5)+5} = \frac{10}{3}\ \Omega$

Q.10. In a parallel plate capacitor of capacitance $4\ \mu\text{F}$, if a dielectric plate $(K = 3)$ with width equal to half of the width of the capacitor is introduced, the value of capacitance will become

A) $6\ \mu\text{F}$

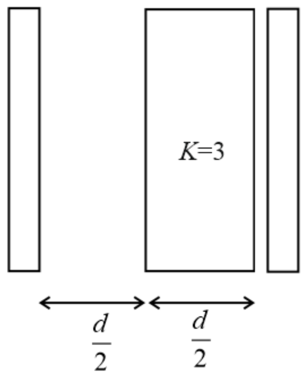
B) $8\ \mu\text{F}$

C) $5\ \mu\text{F}$

D) $3\ \mu\text{F}$

Answer: $6\ \mu\text{F}$

Solution:



This parallel plate capacitor can be divided into two capacitors. One with dielectric (C_1) and other without dielectric(C_2). The two capacitors will be in series.

Initially $C = \frac{\varepsilon_0 A}{d} = 4\ \mu\text{F}$

Now, $C_1 = \frac{k\varepsilon_0 A}{\frac{d}{2}} = \frac{2\times3\times\varepsilon_0 A}{d} = 24\ \mu\text{F}$ and $C_2 = \frac{\varepsilon_0 A}{\frac{d}{2}} = \frac{2\times\varepsilon_0 A}{d} = 8\ \mu\text{F}$

Finally $C' = \frac{C_1 C_2}{C_1+C_2} = \frac{24\times8}{24+8} = 6\ \mu\text{F}$

Q.11. Batsman hits ball of mass $0.4\ \text{kg}$ in the direction of bowler with the same speed bowler throws it at him, which is $15\ \text{m s}^{-1}$. Find impulse.

A) $12\ \text{N s}$

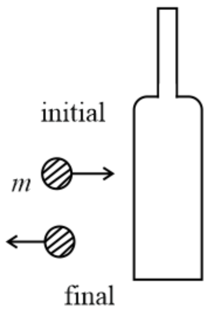
B) $14\ \text{N s}$

C) $16\ \text{N s}$

D) $18\ \text{N s}$

Answer: $12\ \text{N s}$

Solution:



Impulse, $J = \int F\mathrm{d}t$ =Change in momentum

Therefore,
Impulse, $\Delta p = mv - (-mv) = 2mv = 2\times0.4\times15\ = 12\ \text{N s}$

Q.12. A point source is kept at the bottom of a lake with a liquid of refractive index $\frac{4}{3}$ at a depth $\sqrt{7}\ \text{m}$. If the area of circle through which light comes from is $(\alpha\pi)\ \text{m}^2$, then value of α is:

A) 9

B) 7

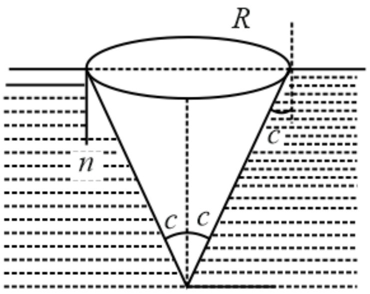
C) 4

D) 3

Answer: 9

Solution: Apply Snell's law, $\frac{\sin 90^{\circ}}{\sin C}=\mu$

$\Rightarrow \sin C=\frac{1}{\mu} \Rightarrow \frac{R}{\sqrt{R^2+h^2}}=\frac{3}{4}$



Point source

Squaring, $16R^2=9R^2+9h^2$

$\Rightarrow 7R^2=9h^2$

$\Rightarrow R=\frac{3}{\sqrt{7}}h=\frac{3}{\sqrt{7}}\times\sqrt{7}=3\ \mathrm{m}$

The area of the circle is, $A=\pi R^2=\pi(3)^2\ \mathrm{m}^2 \Rightarrow \alpha=9$

Q.13. Two parallel wires with magnetic field in which the $5\ \mathrm{A}$ current is flowing. The length of one wire is $10\ \mathrm{cm}$ and other of infinite length. Find the distance between them when force between them is $10^{-5}\ \mathrm{N}$.

A) $5\ \mathrm{cm}$

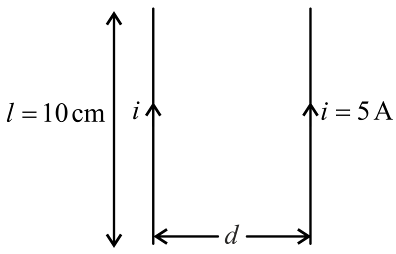
B) $10\ \mathrm{cm}$

C) $6\ \mathrm{cm}$

D) $12\ \mathrm{cm}$

Answer: $5\ \mathrm{cm}$

Solution: Give $F=10^{-5}\ \mathrm{N}$

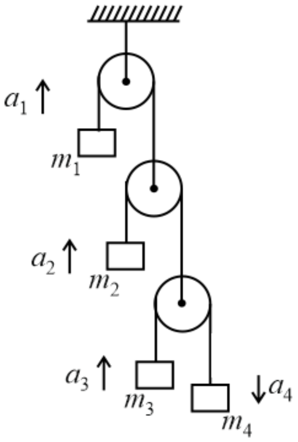


Force between wires $\frac{F}{l}=\frac{\mu_0 i_1 i_2}{2\pi d}$, where, d is separation between them.

$\Rightarrow \frac{10^{-5}}{0.1}=\frac{2\times10^{-7}\times25}{d}$

$\Rightarrow d=50\times10^{-3}\ \mathrm{m}=5\ \mathrm{cm}$

Q.14. Blocks of mass $m_1,\ m_2,\ m_3\ \&\ m_4$ are connected with pulley as shown in the figure. Find the relation between respective accelerations $a_1,\ a_2,\ a_3\ \&\ a_4$ as marked in the diagram.



A) $4a_1 + 2a_2 + a_3 - a_4 = 0$

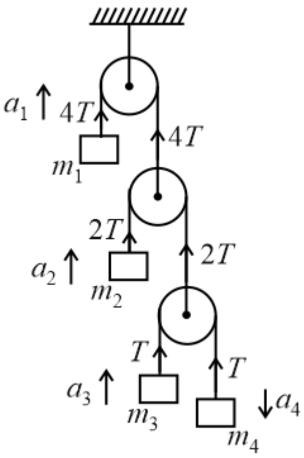
B) $4a_1 + a_2 + a_3 + a_4 = 0$

C) $4a_1 - 2a_2 - a_3 + a_4 = 0$

D) $4a_1 + a_2 - a_3 - a_4 = 0$

Answer: $4a_1 + 2a_2 + a_3 - a_4 = 0$

Solution:



$\sum \overrightarrow{T} \cdot \overrightarrow{a} = 0$

$\Rightarrow (4T)a_1 + (2T)a_2 + Ta_3 - Ta_4 = 0$

$\Rightarrow 4a_1 + 2a_2 + a_3 - a_4 = 0$

Q.15. A light of wavelength $4500\ \overset{\circ}{A}$ causes a photoelectron emission. These electrons are then sent to a magnetic field of $2\ \mathrm{mT}$ perpendicular to the velocity of the electron. The maximum radius in which the electron revolve is $2\ \mathrm{mm}$. Find the work function of the metal.

A) $2.3\ \mathrm{eV}$

B) $4.3\ \mathrm{eV}$

C) $1.3\ \mathrm{eV}$

D) 3.2 eV

Answer: 1.3 eV

Solution: From the Einstein's photoelectric equation $K = \frac{hc}{\lambda} - \phi \Rightarrow \phi = \frac{hc}{\lambda} - K$

Now $R = \frac{mv}{Bq} \Rightarrow v = \frac{RBq}{m} \Rightarrow K = \frac{R^2B^2q^2}{2m}$

Therefore,

$\phi = \frac{hc}{\lambda} - \frac{R^2B^2q^2}{2m} = \frac{6.63\times10^{-34}\times3\times10^{8}}{4500\times10^{-10}} - \frac{(2\times10^{-3}\times2\times10^{-3}\times1.6\times10^{-19})^2}{2\times9.1\times10^{-31}}$

$\Rightarrow \phi = (4.42 - 2.25)\times10^{-19}\text{ J} = \frac{2.27}{1.6}\text{ eV} = 1.3\text{ eV}$

Q.16. Temperature of cold reservoir was 324 K and heat given by the hot reservoir was 300 J. If heat given to the sink was 180 J, then find the temperature of hot reservoir.

A) 540 K

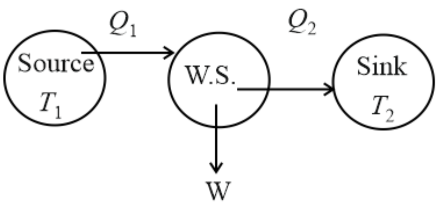
B) 440 K

C) 624 K

D) 354 K

Answer: 540 K

Solution:



Using the relation, $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

Or $\frac{180}{300} = \frac{324}{T_1} \Rightarrow T_1 = \frac{324\times300}{180} = 540 \text{ K}$

Q.17. 20 tuning forks are arranged in increasing order of frequency such that every tuning fork produces 4 beats with previous one. If frequency of last tuning fork is double the first, then the frequency of last tuning fork is?

A) 152 Hz

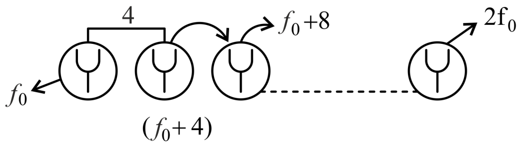
B) 176 Hz

C) 126 Hz

D) 142 Hz

Answer: 152 Hz

Solution:



Each fork produces 4 beats per second with the previous means each fork has frequency $4\ \mathrm{Hz}$ more than the previous.

Using relation, $f_{\text{last}}=f_{\text{first}}+(N-1)x$, here, N is the number of tuning fork in series and x is beat frequency between two successive forks.

$2f_0=f_0+\ (20-1)4$

$\Rightarrow 2f_0=f_0+76$

$\Rightarrow f_0=76\ \mathrm{Hz}$

Frequency of last tuning fork is $152\ \mathrm{Hz}$.

Q.18. Work done in rotating a magnetic dipole of dipole moment $M=14\times10^{-5}\ \mathrm{A\ m^2}$ in a uniform magnetic field $B=2\times10^5\ \mathrm{T}$ by an angle $\theta=60^\circ$ (initially dipole is aligned with the field) is

A) $7\ \mathrm{J}$

B) $14\ \mathrm{J}$

C) $28\ \mathrm{J}$

D) $21\ \mathrm{J}$

Answer: $14\ \mathrm{J}$

Solution: Work done by the external force is equal to the change in the potential energy,

$W_{ext}=\Delta U=U_f-U_i$

$\Rightarrow W_{ext}=-MB\cos\theta_2+MB\cos\theta_1$

$\Rightarrow W_{ext}=MB(\cos\theta_1-\cos\theta_2)$

$\Rightarrow W_{ext}=14\times10^{-5}\times2\times10^5(\cos0^\circ-\cos60^\circ)$

$\Rightarrow W_{ext}=14\times2\times\frac{1}{2}=14\ \mathrm{J}$

Q.19. 64 small balls each of radius $2\ \mathrm{cm}$ having surface charge density $5\ \mu\mathrm{C\ m^{-2}}$ each merged to form a big ball then find the ratio of surface charge density of big ball by small ball

A) $4:1$

B) $2:1$

C) $16:1$

D) $8:1$

Answer: $4:1$

Solution: Total volume will be constant. Therefore, $n\left(\frac{4\pi}{3}r^3\right)=\frac{4\pi}{3}R^3\Rightarrow 64^{\frac{1}{3}}r=R\Rightarrow R=4r$

Final surface charge density $\sigma'=\frac{n\sigma_0 4\pi r^2}{4\pi R^2}=\frac{64\times\sigma_0 r^2}{16r^2}\Rightarrow\frac{\sigma'}{\sigma_0}=\frac{4}{1}$

Q.20. Assertion : As we move from pole to equator magnitude of gravitational acceleration is same and always pointed towards centre of earth.

Reason: At the equator, the acceleration due to gravity is pointed towards centre of earth.

A) Assertion and reason both are correct and reason is correct explanation of assertion.

B) Assertion and reason both are correct but reason is not correct explanation of assertion.

C) Assertion is true and reason is false

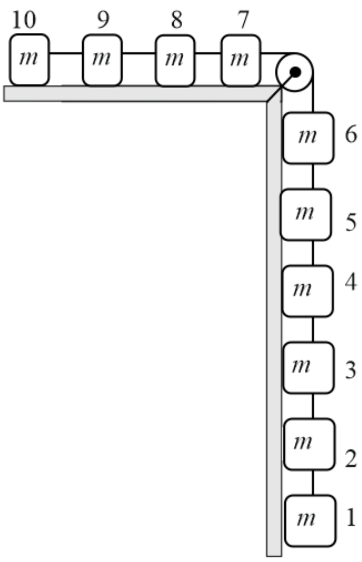
D) Assertion is false and reason is true

Answer: Assertion is false and reason is true

Solution: Value of effective g increases as we move from the equator to pole because on the equator its value is less due to earth's rotational motion and consequent centrifugal force.
Moreover, the equator of the earth is at a larger distance from the centre of the earth as compared to the poles. This is another reason why g is greater on the pole than the equator.

At the equator, direction of gravitational pull and centrifugal force are in same line, hence net force is towards the centre of the Earth.

Q.21. Find the tension between 7^{th} & 8^{th} blocks connected by string placed on a frictionless table as shown in figure below. (Given, $m = 2\ \text{kg}$ & $g = 10\ \text{m s}^{-2}$).



A) $36\ \text{N}$

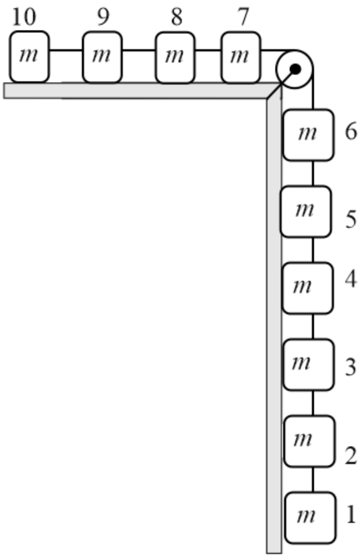
B) $38\ \text{N}$

C) $40\ \text{N}$

D) None of these

Answer: $36\ \text{N}$

Solution:



Acceleration, $a=\frac{\text{Net pulling force}}{\text{Total mass}}$

$a=\frac{6mg}{10m}=\frac{6g}{10}=6\ \mathrm{m\ s^{-2}}$

Considering block $8, 9$ & 10 as system, tension between block 7 & 8 can be written as,

$T_{7,8}=(3m)\times a$

$=3\times 2\times 6=36\ \mathrm{N}$

Section B: Chemistry

Q.1. Which one of the following will not give flame test?

A) Ca

B) Ba

C) Sr

D) Be

Answer: Be

Solution: The electrons in beryllium and magnesium are too strongly bound to get excited by flame. Hence, these elements do not impart any colour to the flame. The flame test for Ca, Sr and Ba is helpful in their detection in qualitative analysis and estimation by flame photometry.

Q.2. Which of the following water soluble vitamin cannot be excreted easily?

A) B_1

B) B_2

C) B_6

D) B_{12}

Answer: B_{12}

Solution: B group vitamins and vitamin C are soluble in water, so they are grouped together. Water-soluble vitamins must be supplied regularly in diet because they are readily excreted in urine and cannot be stored (except vitamin B_{12}) in our body.

Means vitamin B_{12}can not be excreted easily.

Q.3. 6.1 g of CNG gas is supplied with 208 g of oxygen gas. CO_2 and H_2O is produced along with a lot of heat. How much CO_2 (in grams) gas is produced in gram?
(Consider CNG as methane)

A) 34 g

B) 17 g

C) 6 g

D) 12 g

Answer: 17 g

Solution: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O + \text{heat}$

Moles of $CH_4 = \frac{6.1}{16} = 0.38$

Moles of $O_2 = \frac{208}{32} = 6.5$

CH_4 is limiting reagent. So, moles of CO_2 formed $= \frac{6.1}{16}$

Weight of CO_2 formed $= \frac{6.1}{16} \times 44 = 16.775 \approx 17$ g

Q.4. A nucleus has 2 types of radioactive decays. The half life of first is 3 hours and for the seconds is 4.5 hours. Calculate the overall half life of nucleus in hours..

A) 0.56 hours

B) 3.75 hours

C) 2.23 hours

D) 1.80 hours

Answer: 1.80 hours

Solution: $\lambda = \frac{0.693}{t_{\frac{1}{2}}}$

For a parallel reaction.

$\lambda = \lambda_1 + \lambda_2$

$\frac{1}{t_{\frac{1}{2}}} = \frac{1}{\left(t_{\frac{1}{2}}\right)_1} + \frac{1}{\left(t_{\frac{1}{2}}\right)_2}$

$= \frac{1}{3} + \frac{1}{4.5}$

$\frac{1}{t_{\frac{1}{2}}} = \frac{7.5}{3 \times 4.5}$

$t_{\frac{1}{2}} = \frac{9}{5}$

$= 1.8$ hour

Q.5. Select the nitrogen atom having the odd number of electrons.

A) N_2O_5

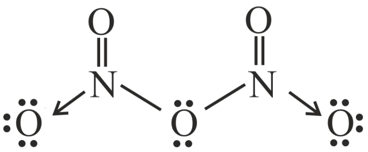
B) NO_2

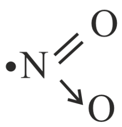
C) N_2O

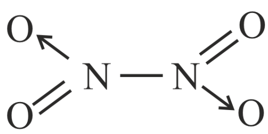
D) N_2O_4

Answer: NO_2

Solution:







As we can see only NO_2 is having an odd electron.

Q.6. Number of molecules having two lone pairs on the central atom among the following is:

CH_4, SF_4, XeF_4, H_2O

A) 1

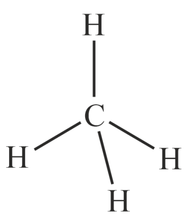
B) 2

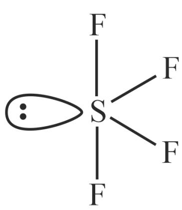
C) 3

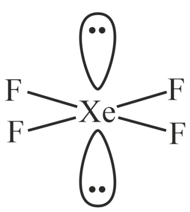
D) 4

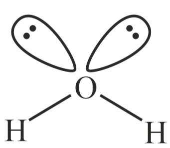
Answer: 2

Solution:









As we can see from the structures XeF_4, $H_2O$are having two lone pairs on the central atom.

Q.7. The Sum of radial nodes and angular nodes in $4s$ orbital is:

A) 1

B) 3

C) 2

D) 4

Answer: 3

Solution: Radial nodes $= \mathrm{n}-\mathrm{l}-1$

Angular nodes $= \mathrm{l}$

So, total nodes $= \mathrm{n}-1=4-1=3$

There would be only 3 radial nodes in 4s orbital there won't be any angular nodes in this orbital.

Q.8. Which of the following is a metalloid?

A) Bi

B) Sc

C) Te

D) Hg

Answer: Te

Solution: Te

The six commonly recognised metalloids are boron, silicon, germanium, arsenic, antimony, and tellurium.

All other options are metals.

Q.9. 40% HI decomposes in to $\mathrm{H_2}$ and $\mathrm{I_2}$ at $300\ \mathrm{K}$, calculate the value of $\Delta \mathrm{G}^\circ$ in Joules

A) 5483

B) 3645

C) 5240

D) 8430

Answer: 5483

Solution: $2\mathrm{HI} \rightleftarrows \mathrm{H_2}+\mathrm{I_2}$

$t=0$

$\mathrm{t}=\mathrm{t} \quad 1-\alpha \quad \frac{\alpha}{2} \quad \frac{\alpha}{2}$

$\mathrm{K_c}=\frac{[\mathrm{H_2}][\mathrm{I_2}]}{[\mathrm{HI}]^2}=\frac{\frac{\alpha}{2}\cdot\frac{\alpha}{2}}{(1-\alpha)^2}$

40% decomposition occurs, So, $\alpha=0.4$

$\mathrm{K_c}=\frac{\frac{0.4}{2}\cdot\frac{04}{2}}{(1-0.4)^2}=\frac{(0.2)^2}{(0.6)^2}=0.11$

$\Delta \mathrm{G}^\circ=-2.303\ \mathrm{RT}\ \log\ \mathrm{k_C}$

$=-2.303\times 8.314\times 300\times \log\ 0.11=5483.3\ \mathrm{Joules}$

Q.10. Which of the following samples of water are polluted:

Sample	BOD value
1	4
2	18
3	21
4	3

A) $1, 2$

B) $2, 3$

C) $2, 4$

D) $3, 4$

Answer: $2, 3$

Solution: The amount of oxygen required by bacteria to break down the organic matter present in a certain volume of a sample of water, is called Biochemical Oxygen Demand (BOD). The amount of BOD in the water is a measure of the amount of organic material in the water, in terms of how much oxygen will be required to break it down biologically. Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

Among the given samples, sample number 2 and 3 are polluted.

Q.11. Toluene can be easily converted into benzaldehyde by which of the following reagents?

A) $CO, HCl, \text{Anhyd. } AlCl_3$

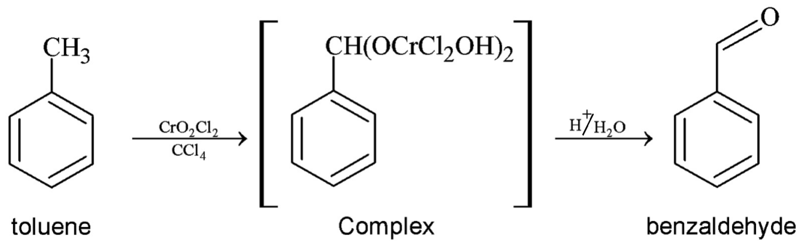
B) Acetic acid, CS_2

C) (i) CCl_4, Chromyl chloride (ii) H_3O^+

D) $H_2, Pd/BaSO_4$

Answer: (i) CCl_4, Chromyl chloride (ii) H_3O^+

Solution: Toluene is oxidised to benzaldehyde in presence of chromyl chloride. This reaction is called Etard's reaction.



Q.12. Which is not correct with benzenesulfonyl chloride.

A) It is hinsberg's reagent

B) It forms a ppt with secondary amine which is soluble with alkali

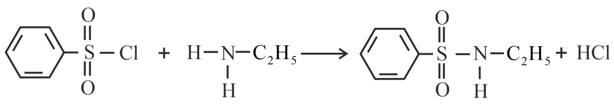
C) It is used to distinguish primary, secondary and tertiary amines.

D) It does not react with tertiary amine.

Answer: It forms a ppt with secondary amine which is soluble with alkali

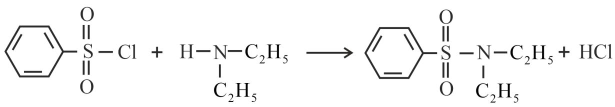
Solution: Benzenesulphonyl chloride $(C_6H_5SO_2Cl)$, which is also known as Hinsberg's reagent, reacts with primary and secondary amines to form sulphonamides.

The reaction of benzenesulphonyl chloride with primary amine yields N-ethylbenzenesulphonyl amide.



N-Ethylbenzenesulphonamide
(soluble in alkali)

In the reaction with secondary amine, N, N-diethylbenzenesulphonamide is formed.



N,N-Diethylbenzenesulphonamide

Tertiary amines do not react with benzenesulphonyl chloride.

Q.13. Which of the following element is most likely to deviate from $+3$ oxidation state?

A) La

B) Ce

C) Lu

D) Gd

Answer: Ce

Solution:

Electronic configuration of $Ce = [Xe]\underbrace{4f^15d^16s^2}\xrightarrow{-4e^-}\underbrace{[Xe]}$

Cerium in $+4$ oxidation state due to stable xenon configuration. La, Lu and Gd are stable in +3 oxidation state.

Q.14. A solid A_xB_y has ccp structure. A forms ccp and B is present in all the octahedral voids. If atoms 'A' are removed from two opposite faces then x will be:

A) 2

B) 3

C) 4

D) 6

Answer: 3

Solution: A forms ccp structure that means number of A atoms per unit cell $\rightarrow 4$

B atoms per unit cell $\rightarrow 4$ (Octahedral voids), as number of octahedral voids will be equal to number of atoms per unit cell.

A removed $= 2 \times \frac{1}{2} = 1$

A left $= 4 - 1 = 3$

Formula becomes A_3B_4

$x = 3.$

Q.15. Match the following:

	Enzyme		Function
i	Invertase	a	Starch to maltose
ii	Maltase	b	Maltose to glucose
iii	Zymase	c	Sugar to ethanol
iv	Diastase	d	Inversion of cane sugar

A) i-d, ii-b, iii-c, iv-a

B) i-b, ii-a, iii-d, iv-c

C) i-c, ii-d, iii-a, iv-b

D) i-d, ii-a, iii-c, iv-b

Answer: i-d, ii-b, iii-c, iv-a

Solution: Invertase helps in the conversion of sucrose to glucose and fructose.

Zymase catalyses the conversion of glucose or fructose to ethyl alcohol.

Diastase is a starch hydrolysing enzyme that breaks down the complex carbohydrate (polysaccharides, i.e. starch) into simple carbohydrates (monosaccharides, i.e. simple sugar like glucose).

Maltase catalyses the conversion of maltose to glucose.

Q.16. Number of electrons in t_{2g} orbital of compound formed by reacting $\left[Co\left(H_2O\right)_6\right]^{2+}$ with excess NH_3 in the presence of air is:

A) 4

B) 6

C) 3

D) 2

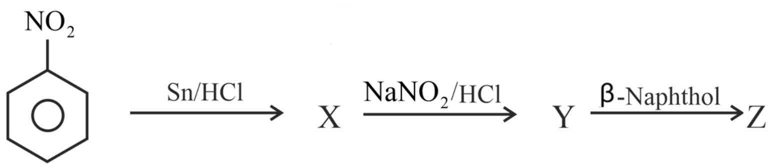
Answer: 6

Solution: $\left[Co\left(H_2O\right)_6\right]^{2+} + 6NH_3 \xrightarrow{air} \left[Co\left(NH_3\right)_6\right]^{3+} + 6H_2O + e^-$

Electronic configuration of $\left[Co\left(NH_3\right)_6\right]^{3+} = t_{2g}^6 e_g^0$

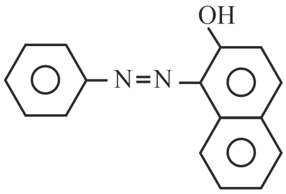
Air oxides the Co(II) to Co(III)

Q.17.

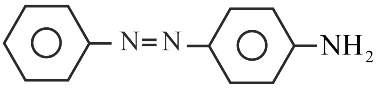


Identify 'Z' among the following

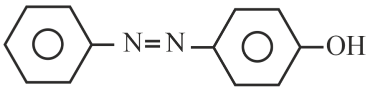
A)



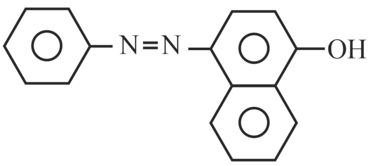
B)



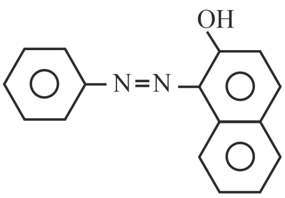
C)



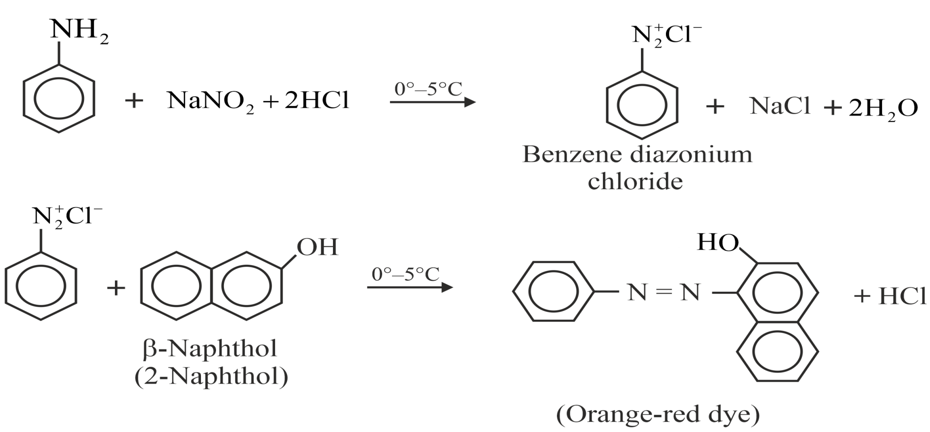
D)



Answer:



Solution: First step involves the reduction of nitrobenzene to aniline in presence of Sn in HCl. Second step is a diazotization reaction where aniline is converted to benzenediazonium chloride in presence of nitrous acid, and third step involves the reaction between benzene diazonium chloride and beta-naphthol to form 1-phenylazo-2-naphthol an orange red dye.



Q.18. Boiling of hard water produces:

A) $CaCO_3$ and $Mg(OH)_2$

B) $Ca(OH)_2$ and $MgCO_3$

C) $CaCO_3$ and $MgCO_3$

D) $Ca(OH)_2$ and $Mg(OH)_2$

Answer: $CaCO_3$ and $Mg(OH)_2$

Solution: During boiling $Mg(HCO_3)_2$ is converted to insoluble $Mg(OH)_2$

$Mg(HCO_3)_2 \xrightarrow{\text{Heating}} Mg(OH)_2 \downarrow + 2CO_2$

$Ca(HCO_3)_2 \xrightarrow{\text{Heating}} CaCO_3 \downarrow + H_2O + CO_2$

$Ca(HCO_3)_2$ changes to insoluble $CaCO_3$

Q.19. In an electrochemical cell $E^o_{A^{2+}/A} = -0.33\ V$, $E^o_{B/B^{2+}} = 0.50\ V$. Find the value of ΔG^o

A) $-0.20\ F$

B) $-0.34\ F$

C) $-0.02\ F$

D) $-0.04\ F$

Answer: $-0.34\ F$

Solution: $E^o_{B^{2+}/B} = -E^o_{B/B^{2+}} = -0.5\ V$

$E^o_{cell} = E^o_{A^{2+}/A} - E^o_{B^{2+}/B}$

$= -0.33 - (-0.50) = 0.17\ V$

$\Delta G^o = -nFE^o_{cell}$

$\Delta G^o = \text{Standard gibbs free energy change}$

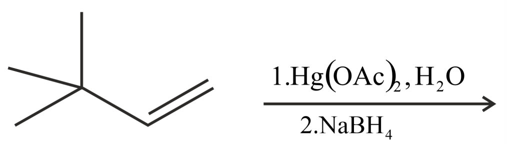
n =Number of electrons transferred=2

F =Faraday's constant

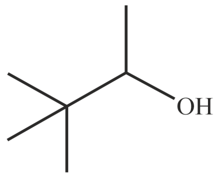
$\Delta G^o = -nFE^o_{cell}$

$= -2 \times F \times 0.17 = -0.34\ F$

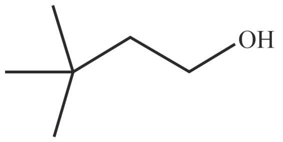
Q.20.



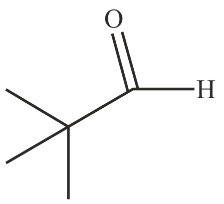
A)



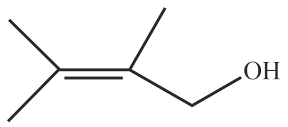
B)



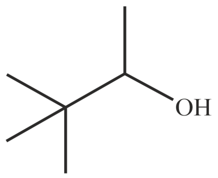
C)



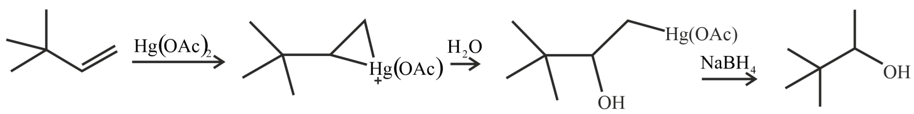
D)



Answer:



Solution:



It is oxymercuration–demercuration reaction. In this an alkene is treated with mercury(II) acetate, and the product is treated with sodium borohydride. The net result is a Markovnikov addition product without rearrangement.

Q.21. A metal is irradiated with the light of wavelength 6640 Å and its stopping potential is $0.4\ \mathrm{V}$. The threshold frequency (v_0) of the metal is $3.55 \times 10^{\mathrm{x}}\ \mathrm{Hz}$. The value of x is:

A) 12

B) 14

C) 15

D) 19

Answer: 14

Solution: $\frac{hc}{\lambda} = h\nu_0 + qV$

$\nu_0 = \text{Threshold frequency}$

$V = \text{Stopping potential}$

$c = \text{Velocity}$

$\lambda = \text{wave length}$

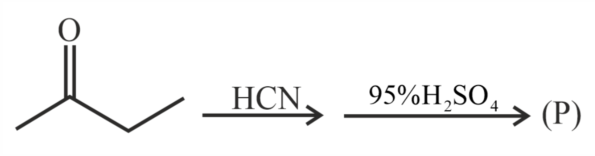
$h = \text{planck's constant.}$

$$\nu_0 = \frac{\frac{hc}{\lambda} - qV}{h}$$

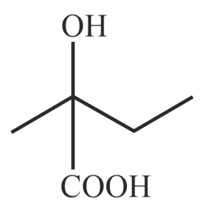
$$\nu_0 = \frac{\frac{6.626\times10^{-34}\times3\times10^{8}}{6640\times10^{-10}} - 1.6\times10^{-19}\times0.4}{6.626\times10^{-34}}$$

$\nu_0 = 3.55 \times 10^{14}$ Hz.

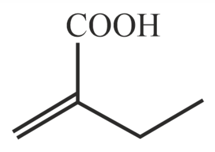
Q.22. Identify the major product (P) in the below sequence of reaction.



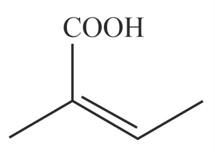
A)



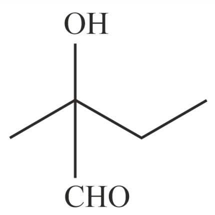
B)



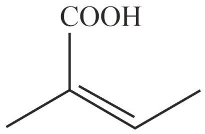
C)



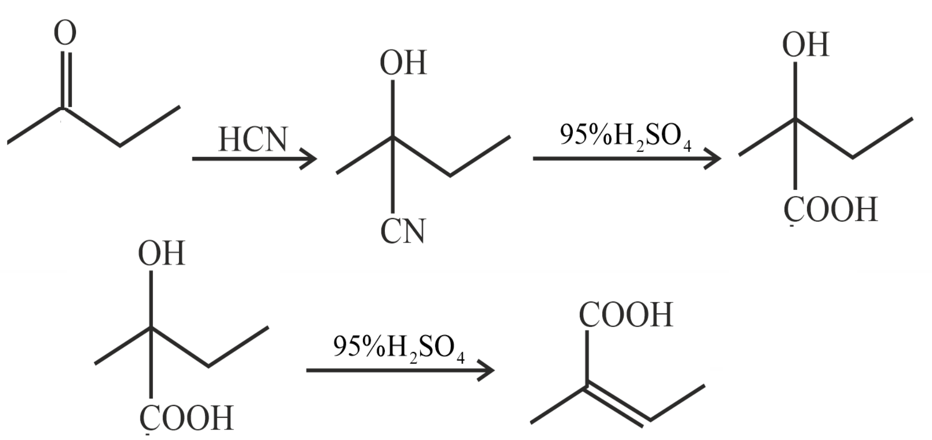
D)



Answer:



Solution: Butan-2-one undergoes nucleophilic addition reaction with HCN to form a cyanohydrin which on hydrolysis forms 2-hydroxy-2methyl butanoic acid which undergoes dehydration to form 2-methyl but-2-enoic acid



Q.23. Which of the following is not a synthetic detergent?

A) Sodium lauryl sulphate

B) Sodium dodecyl benzene sulphonate

C) Cetyl trimethyl ammonium bromide

D) Sodium stearate

Answer: Sodium stearate

Solution: Synthetic detergents are cleansing agents which have all the properties of soaps, but which actually do not contain any soap. These can be used both in soft and hard water as they give foam even in hard water. Some of the detergents give foam even in ice cold water.

Sodium stearate is an example of soap.

Q.24. Find the osmotic pressure (in atm) of a solution in which $2 \mathrm{~g}$ of a protein having molar mass $6 \mathrm{~kg}$ is present in $2 \mathrm{~mL}$ of solution at $27^{\circ} \mathrm{C}$

A) 8

B) 4

C) 6

D) 12

Answer: 4

Solution: Osmotic pressure $(\pi) = \mathrm{iCRT}$

$$\pi = 1 \times \frac{\frac{2}{6000}}{\frac{2}{1000}} \times 0.0821 \times 300$$

$$\pi \approx 4 \text{ atm}$$

Section C: Mathematics

Q.1. The value of $\lim_{x \to 0} \frac{\cos(\sin x)-\cos x}{x^4}$ is equal to

A) $\frac{1}{5}$

B) $\frac{1}{6}$

C) $\frac{1}{4}$

D) $\frac{1}{2}$

Answer: $\frac{1}{6}$

Solution:

Consider $\lim_{x \to 0} \frac{\cos(\sin x)-\cos x}{x^4} \quad \left(\frac{0}{0} \text{ form}\right)$

$$\lim_{x \to 0} \frac{2\sin\left(\frac{\sin x+x}{2}\right) \cdot \sin\left(\frac{x-\sin x}{2}\right)}{x^4}$$

$$= \lim_{x \to 0} 2\left[\frac{\sin\left(\frac{\sin x+x}{2}\right)}{\left(\frac{\sin x+x}{2}\right)}\right]\left[\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\right] \times \left(\frac{\sin x+x}{2}\right) \times \left(\frac{x-\sin x}{2}\right) \times \frac{1}{x^4}$$

$$\lim_{x \to 0} 2\left[\frac{\sin\left(\frac{\sin x+x}{2}\right)}{\left(\frac{\sin x+x}{2}\right)}\right]\left[\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\right] \times \left(\frac{x^2-\sin^2 x}{4x^4}\right)$$

$$\lim_{x \to 0} 2 \times \left(\frac{x^2-\sin^2 x}{4x^4}\right) \quad \left(\frac{0}{0} \text{ form}\right) \quad \left(\because \lim_{t \to 0} \frac{\sin t}{t} = 1\right)$$

Applying L'Hospital's Rule,

$$\lim_{x \to 0} 2 \times \left(\frac{2x-2\sin x\cos x}{4 \cdot 4x^3}\right) = \lim_{x \to 0} \left(\frac{2x-\sin 2x}{8x^3}\right) \quad \left(\frac{0}{0} \text{ form}\right)$$

$$\lim_{x \to 0} \left(\frac{2-2\cos 2x}{24x^2}\right) \quad \left(\frac{0}{0} \text{ form}\right)$$

$$\lim_{x \to 0} \left(\frac{4\sin 2x}{48x}\right) = \frac{1}{6}\lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right) = \frac{1}{6}$$

Q.2. If ${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \cdots + {}^{60}C_{20} = \frac{m}{n} \times {}^{60}C_{20}$ where m & n are co-prime, then $m+n$ is equal to

A) 102

B) 100

C) 104

D) 96

Answer: 102

Solution: We know ${}^{n}C_{r-1}+{}^{n}C_{r}={}^{n+1}C_{r}$

Also ${}^{40}C_{0}={}^{41}C_{0}$

So ${}^{41}C_{0}+{}^{41}C_{1}+{}^{42}C_{2}+\ldots.{}^{60}C_{20}$

$={}^{42}C_{1}+{}^{42}C_{2}+{}^{43}C_{3}\ldots{}^{60}C_{20}={}^{43}C_{2}+{}^{43}C_{3}+\ldots{}^{60}C_{20}$

$={}^{60}C_{19}+{}^{60}C_{20}={}^{61}C_{20}$

Given ${}^{61}C_{20}=\frac{m}{n}{}^{60}C_{20}$

$\Rightarrow \frac{61}{41}=\frac{m}{n} \Rightarrow m+n=102$

Q.3. $\cos^{-1}\left\{\frac{3}{10}\cos\left(\tan^{-1}\frac{4}{3}\right)+\frac{2}{5}\sin\left(\tan^{-1}\frac{4}{3}\right)\right\}=$

A) 0

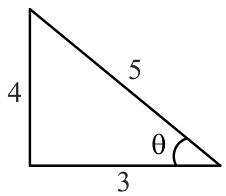
B) $\frac{\pi}{6}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{3}$

Answer: $\frac{\pi}{3}$

Solution: Let $\tan^{-1}\left(\frac{4}{3}\right)=\theta \Rightarrow \tan\theta=\frac{4}{3}$



$\therefore \cos\theta=\frac{3}{5}$ & $\sin\theta=\frac{4}{5}$

So, $\cos^{-1}\left\{\frac{3}{10}\cos\left(\tan^{-1}\frac{4}{3}\right)+\frac{2}{5}\sin\left(\tan^{-1}\frac{4}{3}\right)\right\}$

$=\cos^{-1}\left\{\frac{3}{10}\cos\theta+\frac{2}{5}\sin\theta\right\}$

$=\cos^{-1}\left\{\frac{3}{10}\times\frac{3}{5}+\frac{2}{5}\times\frac{4}{5}\right\}$

$=\cos^{-1}\left\{\frac{9}{50}+\frac{8}{25}\right\}$

$=\cos^{-1}\frac{1}{2}=\frac{\pi}{3}$

Q.4. Suppose l_1 is the tangent to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ and l_2 is a straight line passing through $(0,0)$ and perpendicular to l_1. If the locus of point of intersection of l_1 and l_2 is $\left(x^2+y^2\right)^2=\alpha x^2+\beta y^2$, then the value of $\alpha+\beta$ is equal to

A) 5

B) 7

C) 3

D) 9

Answer: 5

Solution: The equation of tangent to the given hyperbola is $y = mx \pm \sqrt{9m^2 - 4} \quad \ldots(\mathrm{i})$

Hence, $l_1 : y = mx \pm \sqrt{9m^2 - 4}$

Given that, l_2 is a straight line passing through origin and perpendicular to l_1.

So, $l_2 : y = \frac{-1}{m}x \ \Rightarrow m = \frac{-x}{y} \quad \ldots(\mathrm{ii})$

On solving equations (i) & (ii), we get

$y = \left(\frac{-x}{y}\right)x \pm \sqrt{9\left(\frac{-x}{y}\right)^2 - 4} \Rightarrow y = \frac{-x^2}{y} \pm \frac{\sqrt{9x^2-4y^2}}{y}$

$\Rightarrow \left(y^2 + x^2\right)^2 = 9x^2 - 4y^2$

On comparing the above equation with $\left(x^2 + y^2\right)^2 = \alpha x^2 + \beta y^2$, we get $\alpha = 9,\ \beta = -4$

$\therefore \ \alpha + \beta = 5$

Q.5. The area bounded by $y^2 = 8x$ and $y^2 = 16(3 - x)$ is

A) 16

B) 8

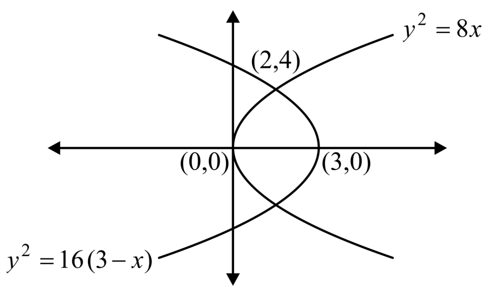
C) 32

D) 64

Answer: 16

Solution: Given curves are $y^2 = 8x$ and $y^2 = 16(3 - x)$

$\Rightarrow 8x = 48 - 16x \Rightarrow x = 2$, so $y = \pm 4$



Required area $= 2\int_0^4 \left(\left(3 - \frac{y^2}{16}\right) - \frac{y^2}{8}\right)dy$

$= 2\left[3y - \frac{y^3}{48} - \frac{y^3}{24}\right]_0^4 = 2\left[12 - \frac{64}{48} - \frac{64}{24}\right] = 16$

Q.6. If p and q are real number such that $p + q = 3,\ p^4 + q^4 = 369$, then the value of $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2}$ is equal to

A) 4

B) $\frac{2}{3}$

C) 6

D) 3

Answer: 4

Solution: $p^4+q^4=369 \Rightarrow (p^2+q^2)^2-2(pq)^2=369$

$\Rightarrow \left((p+q)^2-2pq\right)^2-2(pq)^2=369$

$\Rightarrow (9-2pq)^2-2(pq)^2=369$

$\Rightarrow (pq)^2-18\,(pq)-144=0 \ \Rightarrow pq=24(\text{rejected}),-6$

Now, $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}=\frac{(pq)^2}{(p+q)^2}=\frac{(pq)^2}{9}=4$

Q.7. If $z^2+z+1=0, z\in\mathbb{C}$, then the value of $\left|\sum_{k=1}^{15}\left(z^k+\frac{1}{z^k}\right)^2\right|$ is equal to

A) 30

B) 20

C) 40

D) 50

Answer: 30

Solution: Given $z^2+z+1=0$

So $z=-\frac{1\pm\sqrt{1-4}}{2\times 1} \Rightarrow z=\frac{-1\pm\sqrt{3}i}{2}$

So z roots are ω & ω^2

Now $\left|\sum_{k=1}^{15}\left(z^k+\frac{1}{z^k}\right)^2\right|$

$=\left(z^1+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2\cdots\left(z^{15}+\frac{1}{z^{15}}\right)^2$

Now $z^{3k}+\frac{1}{z^{3k}}=\omega^{3k}+\frac{1}{\omega^{3k}}=2$ as $\omega^3=1$

so $\left(z^{3k}+\frac{1}{z^{3k}}\right)^2=2^2=4$

$\sum_{k=1}^{5}\left(z^{3k}+\frac{1}{z^{3k}}\right)^2=4\times 5=20$ ...equation (i)

Now $z=\omega$ & $\frac{1}{z}=\omega^2$

so $z+\frac{1}{z}=\omega+\omega^2=-1 \Rightarrow \left(z+\frac{1}{z}\right)^2=1$

Similarly $\left(z^{3k-1}+\frac{1}{z^{3k-1}}\right)^2=1$ & $\left(z^{3k-2}+\frac{1}{z^{3k-2}}\right)^2=1$

So $\sum_{k=1}^{5}\left(z^{3k-1}+\frac{1}{z^{3k-1}}\right)^2=1\times 5=5$ ...equation (ii)

And $\sum_{k=1}^{5}\left(z^{3k-2}+\frac{1}{z^{3k-2}}\right)^2=1\times 5=5$ ....equation (iii)

Final answer will be addition of all equation $(i),\ (ii)\ \&\ (iii) 20+5+5=30$

Q.8. If function $f(x)=x-1$ and $g(x)=\frac{x^2}{x^2+1}$, then $fog(x)$ is

A) One-one and onto

B) One-one but not onto

C) Onto but not one-one

D) Neither one-one nor onto

Answer: Neither one-one nor onto

Solution: $f\circ g(x)=f(g(x))\ =f\left(\frac{x^2}{x^2+1}\right)$

$=\frac{x^2}{x^2+1}-1\ =\frac{x^2-x^2-1}{x^2+1}=\frac{-1}{x^2+1}$

We know that, $0\le x^2<\infty,\ \forall x\in R$

$\Rightarrow 1\le x^2+1<\infty,\ \forall x\in R \Rightarrow 1\ge\frac{1}{x^2+1}>0,\ \forall x\in R\ \Rightarrow -1\le\frac{-1}{x^2+1}<0,\ \forall x\in R$

So, range of $fog(x)$ is $[-1,\ 0)\subset R$.

Hence, the function $fog(x)$ is into function and $f\circ g(-x)=f(g(-x))=\frac{-1}{(-x)^2+1}=\frac{-1}{x^2+1}=f(g(x))$

$\therefore\ \ fog(x)$ is an even function. So, it is a many one function.

Hence, $fog(x)$ is neither one-one nor onto function.

Q.9. The value of $16\sin 20^\circ \sin 40^\circ \sin 80^\circ$ is equal to

A) $2\sqrt{3}$

B) $8\sqrt{3}$

C) $\sqrt{3}$

D) $4\sqrt{3}$

Answer: $2\sqrt{3}$

Solution: We know that $\sin\theta\sin(60-\theta)\sin(60+\theta)=\frac{1}{4}\sin 3\theta$

Now given $16\sin 20^\circ \sin 40^\circ \sin 80^\circ$

Comparing with above formula $\theta=20^\circ$

we get $16\sin 20^\circ \sin 40^\circ \sin 80^\circ = 16\times\frac{1}{4}\times\sin(3\times 20^\circ)$

$=16\times\frac{1}{4}\times\sin 60^\circ = 4\times\frac{\sqrt{3}}{2}=2\sqrt{3}$

Q.10. $\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{2-x^2}{(2+x^2)\left(\sqrt{4+x^4}\right)}dx=$

A) 3

B) 2

C) 1

D) $\frac{1}{2}$

Answer: 3

Solution: $I=\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{2-x^2}{(2+x^2)\left(\sqrt{4+x^4}\right)}dx$

$=\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{2-x^2}{x^2\left(\frac{2}{x}+x\right)\sqrt{\left(\frac{4}{x^2}+x^2\right)}}dx$

$=\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{\frac{2}{x^2}-1}{\left(\frac{2}{x}+x\right)\sqrt{\left(\frac{2}{x}+x\right)^2-4}}dx$

Let $\frac{2}{x}+x=t,\ \left(-\frac{2}{x^2}+1\right)dx=dt$

$I=-\frac{24}{\pi}\int_{\infty}^{2\sqrt{2}}\frac{dt}{t\sqrt{t^2-4}}=-\frac{12}{\pi}\int_{\infty}^{2\sqrt{2}}\frac{2tdt}{t^2\sqrt{t^2-4}}$

Let $t^2-4=z^2,\ 2tdt=2zdz$

$I=-\frac{12}{\pi}\int_{\infty}^{2}\frac{2zdz}{z(z^2+4)}=-\frac{24}{\pi}\int_{\infty}^{2}\frac{dz}{(z^2+4)}=-\frac{24}{\pi}\left(\frac{1}{2}\tan^{-1}\frac{z}{2}\right)_{\infty}^{2}$

$=-\frac{24}{\pi}\left(\frac{\pi}{8}-\frac{\pi}{4}\right)=3$

Q.11. If $x\frac{dy}{dx}+2y=xe^x$ and $y(1)=0$, then the value of local maximum of the function $z(x)=x^2y(x)-e^x;\ x\in R$ is

A) $\frac{4}{e} - e$

B) $\frac{4}{e} + e$

C) $\frac{2}{e} - e$

D) $\frac{2}{e} + e$

Answer: $\frac{4}{e} - e$

Solution: $\frac{dy}{dx} + \frac{2y}{x} = e^x$

$I.F. = e^{\int \frac{2}{x} dx} = e^{2 \ln x} = x^2$

$\therefore$ General solution: $y\left(x^2\right) = \int e^x \left(x^2\right) dx$

We know that

$\int e^x f(x) dx == e^x \left[f(x) - f'(x) + f''(x) - f'''(x) + \cdots + (-1)^n f^n(x)\right] + C$

So, $y\left(x^2\right) = e^x \left[x^2 - 2x + 2\right] + C \quad \ldots(\mathrm{i})$

Given $y(1) = 0 \Rightarrow (0)(1) = e^1 [1 - 2(1) + 2] + C$

$\Rightarrow C = -e$

$\therefore y = \frac{e^x}{x^2} \left[x^2 - 2x + 2\right] - e$ (from eq (i))

Hence, $z\left(x\right) = x^2 \left(\frac{e^x}{x^2}\right) \left[x^2 - 2x + 2\right] - e - e^x$

$= e^x \left[x^2 - 2x + 2\right] - e - e^x$

$z(x) = e^x \left[x^2 - 2x + 1\right] - e$

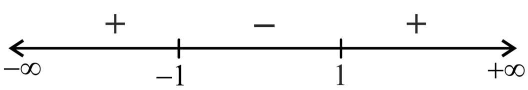
$z'(x) = e^x (2x - 2) + \left(x^2 - 2x + 1\right) e^x$

$z'(x) = e^x \left[x^2 - 1\right]$

To find local maxima, put $z'(x) = 0$

$\Rightarrow e^x \left(x^2 - 1\right) = 0 \Rightarrow \quad (x-1)(x+1) = 0 \quad (\because \quad e^x > 0, \forall x \in R)$

$\Rightarrow x = 1, -1$



It is clear from the sign scheme method, $z(x)$ has local maximum value at $x = -1$ and has local minimum value at $x = 1$.

$\therefore$ The local maximum value is

$z(-1) = e^{-1} \left[(-1)^2 - 2(-1) + 1\right] - e$

$= \frac{1}{e} [1 + 2 + 1] - e = \frac{4}{e} - e$

Q.12. If $\frac{dy}{dx} + e^x \left(x^2 - 2\right) y = \left(x^2 - 2x\right)\left(x^2 - 2\right) e^{2x}$ and $y(0) = 0$, then the value of $y(2)$ is

A) 0

B) 2

C) 1

D) 4

Answer: 0

Solution: Given

$\frac{dy}{dx}+e^{x}\left(x^{2}-2\right)y=\left(x^{2}-2x\right)\left(x^{2}-2\right)e^{2x}$

It is linear differential equation so,

$IF=e^{\int\left(x^{2}-2\right)e^{x}dx}$

$=e^{x^{2}e^{x}-2\int xe^{x}dx-2e^{x}}=e^{x^{2}e^{x}-2\left[xe^{x}-e^{x}\right]-2e^{x}}$

$IF=e^{\left(x^{2}-2x\right)e^{x}}$

Now solution is given by

$y\times e^{\left(x^{2}-2x\right)e^{x}}=\int e^{\left(x^{2}-2x\right)e^{x}}\times\left(x^{2}-2x\right)\left(x^{2}-2\right)e^{2x}$

$=\int e^{\left(x^{2}-2x\right)e^{x}}\times\left(x^{2}-2x\right)e^{x}\left(x^{2}-2\right)e^{x}$

Now let $\left(x^{2}-2x\right)e^{x}=t$

We get $e^{x}\left(x^{2}-2x\right)+e^{x}(2x-2)dx=dt$

$e^{x}\left(x^{2}-2x+2x-2\right)dx=dt$

$e^{x}\left(x^{2}-2\right)dx=dt$

$y\times e^{\left(x^{2}-2x\right)e^{x}}=\int e^{t}\times t\times dt$

$y\times e^{\left(x^{2}-2x\right)e^{x}}=e^{t}(t-1)+c$

$y\times e^{\left(x^{2}-2x\right)e^{x}}=e^{\left(x^{2}-2x\right)e^{x}}\left(\left(x^{2}-2x\right)e^{x}-1\right)+c$

Now at $x=0\;\; y=0$

$0\times e^{0}=e^{0}\left(0xe^{x}-1\right)+c$

$0=-1+c\;\; c=1$

Putting the value of $c=1$ we get the equation as

$\Rightarrow y\times e^{\left(x^{2}-2x\right)e^{x}}=e^{\left(x^{2}-2x\right)e^{x}}\left(\left(x^{2}-2x\right)e^{x}-1\right)+1$

Now putting $x=2$ in equation we get

$y\times e^{(4-4)e^{2}}=e^{(4-4)e^{2}}\left((4-4)e^{2}-1\right)+1$

$\Rightarrow y\times e^{0}=e^{0}\left((0)e^{2}-1\right)+1$

$\Rightarrow y=-1+1=0$

Q.13. If $A=\sum_{n=1}^{\infty}\frac{1}{(3+(-1)^{n})^{n}}$ and $B=\sum_{n=1}^{\infty}\frac{(-1)^{n}}{(3+(-1)^{n})^{n}}$, then the value of $\frac{A}{B}$ is equal to

A) $-\frac{11}{9}$

B) $-\frac{11}{3}$

C) $-\frac{11}{6}$

D) -11

Answer: $-\frac{11}{9}$

Solution: $A=\sum_{n=1}^{\infty}\frac{1}{(3+(-1)^n)^n}=\frac{1}{2}+\frac{1}{4^2}+\frac{1}{2^3}+\frac{1}{4^4}+\cdots$

$=\left(\frac{1}{2}+\frac{1}{2^3}+\cdots\right)+\left(\frac{1}{4^2}+\frac{1}{4^4}+\cdots\right)=\frac{\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{16}}{1-\frac{1}{16}}$

$=\frac{2}{3}+\frac{1}{15}=\frac{11}{15}$

$B=\sum_{n=1}^{\infty}\frac{(-1)^n}{(3+(-1)^n)^n}=-\frac{1}{2}+\frac{1}{4^2}-\frac{1}{2^3}+\frac{1}{4^4}-\cdots$

$=-\left(\frac{1}{2}+\frac{1}{2^3}+\cdots\right)+\left(\frac{1}{4^2}+\frac{1}{4^4}+\cdots\right)$

$=-\frac{\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{16}}{1-\frac{1}{16}}=-\frac{2}{3}+\frac{1}{15}=-\frac{9}{15}$

Hence, $\frac{B}{A}=-\frac{11}{9}$

Q.14. If $\int\frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx=g(x)+c$, then the value of $g\left(\frac{1}{2}\right)$

A) $\ln\left(2-\sqrt{3}\right)-\frac{\pi}{6}$

B) $\ln\left(2+\sqrt{3}\right)-\frac{\pi}{3}$

C) $\ln\left(2+\sqrt{3}\right)-\frac{\pi}{6}$

D) $\ln\left(2-\sqrt{3}\right)-\frac{\pi}{3}$

Answer: $\ln\left(2-\sqrt{3}\right)-\frac{\pi}{6}$

Solution: Let $x=\sin\theta,\theta\in\left(0,\frac{\pi}{2}\right)$

$dx=\cos\theta d\theta$

$=\int\frac{1}{\sin\theta}\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}\cos\theta d\theta$

$=\int\frac{1-\sin\theta}{\sin\theta\cdot\cos\theta}\cos\theta d\theta$

$=\int(\operatorname{cosec}\theta-1)d\theta$

$=\ln(\operatorname{cosec}\theta-\cot\theta)-\theta+c$

Since, $x=\sin\theta\Rightarrow\operatorname{cosec}\theta-cot\theta=\frac{1-\sqrt{1-x^2}}{x}$

$=\ln\left(\frac{1-\sqrt{1-x^2}}{x}\right)-\sin^{-1}x+c$

$g(x)=\ln\left(\frac{1-\sqrt{1-x^2}}{x}\right)-\sin^{-1}x$

$\therefore g\left(\frac{1}{2}\right)=\ln\left(2-\sqrt{3}\right)-\frac{\pi}{6}$

Q.15. The sides of a cuboid are given as $2x,4x,5x$ and there is a closed hemisphere of radius r such that the sum of their surface area is a constant k. The ratio of $x:r$ such that the sum of their volume is maximum is equal to

A) $\frac{19}{45}$

B) $\frac{45}{19}$

C) $\frac{19}{24}$

D) $\frac{24}{7}$

Answer: $\frac{19}{45}$

Solution: Let sum of surface area

$S=3\pi r^2+76x^2$

$\because$ S is constant so $\frac{dS}{dx}=0$

$\Rightarrow\ 6\pi r\frac{dr}{dx}+2\times 76x=0$

$\Rightarrow\ \frac{dr}{dx}=-\frac{76x}{3\pi r}\quad\cdots(1)$

Now total volume $V=\frac{2}{3}\pi r^3+40x^3$

For maximum volume $\frac{dV}{dx}=0$

$\Rightarrow 2\pi r^2\frac{dr}{dx}+120x^2=0\quad\cdots(2)$

$\Rightarrow\ 2\pi r^2\left(-\frac{76x}{3\pi r}\right)+120x^2=0$

$\Rightarrow\ 8x\left[-\frac{19r}{3}+15x\right]=0$

$\Rightarrow\ \frac{x}{r}=\frac{19}{45}$

Q.16. If the system of equations $\alpha x+y+z=5,\ x+2y+4z=4$ and $x+3y+5z=\beta$ has infinitely many solutions, then the value of α and β are

A) $0,9$

B) $-1,-3$

C) $-1,3$

D) $1,-3$

Answer: $0,9$

Solution: For infinitely many solutions, $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \alpha(-2) - 1(1) + 1(1) = 0 \Rightarrow \alpha = 0$$

$$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 4 \\ \beta & 3 & 5 \end{vmatrix} = 0$$

$$\Rightarrow 5(-2) - 1(20 - 4\beta) + 1(12 - 2\beta) = 0$$

$$\Rightarrow -10 - 20 + 4\beta + 12 - 2\beta = 0$$

$$\Rightarrow -18 + 2\beta = 0$$

$$\Rightarrow \beta = 9$$

$$\therefore \alpha = 0,\ \beta = 9.$$

Q.17. If the function $f(x) = \min\{1, 1 + x\sin x\}, x \in [0, \pi]$, then the nature of $f(x)$ is

A) Continuous & differentiable everywhere

B) Discontinuous at $x = \frac{\pi}{2}$

C) Continuous but not differentiable at $\frac{\pi}{2}$

D) None

Answer: Continuous & differentiable everywhere

Solution: Given,

$f(x) = \min\{1, 1 + x\sin x\}$

Now for $x \in [0, \pi]$ x is positive and

$\sin x$ is also positive in $[0, \pi]$

So, $1 + x\sin x \geqslant 1$

So, $f(x) = \min\{1, 1 + x\sin x\} \Rightarrow f(x) = 1$

which is a constant function

So it is Continuous & differentiable everywhere.

Q.18. Let the mean of 50 observations is 15 and the standard deviation is 2. However, one observation was wrongly recorded. The sum of the correct and incorrect observations is 70. If the mean of the correct set of observations is 16, then the variance of the correct set is equal to

A) 43

B) 45

C) 47

D) 49

Answer: 43

Solution: We have, Mean$=\frac{\sum x_i}{50}=15 \Rightarrow \sum x_i=750$

Variance$=\frac{\sum x_i^2}{50}-15^2=2^2 \Rightarrow \sum x_i^2=11450$

$\Rightarrow \underset{(\text{new})}{\sum x_i}=50\times 16=800$

So, $\underset{(\text{new})}{\sum x_i}-\sum x_i=800-750=50$

Hence, if wrong observation was x then corrected one is $x+50$.
$\Rightarrow x+(x+50)=70 \Rightarrow x=10$

Therefore, the correct observation $=60$.

Now, $\underset{(\text{new})}{\sum x_i{}^2}=11450-(10)^2+(60)^2=14950$

Therefore, variance of the correct set $=\frac{14950}{50}-(16)^2=299-256=43$.

Q.19. If a 3 digit number is randomly formed, then the probability that its common divisor with 36 is only 2 is

A) $\frac{1}{6}$

B) $\frac{1}{2}$

C) $\frac{1}{3}$

D) $\frac{1}{4}$

Answer: $\frac{1}{6}$

Solution: The total 3-digit numbers are $9\times 10\times 10=900$

Favourable cases $=$ Three-digit even numbers

$-$ Three-digit numbers which are multiples of 4

$-$ Three-digit numbers which are multiples of 6

$+$ Three-digit numbers which are multiples of 12

$=450-225-150+75=150$

Hence, required probability $=\frac{150}{990}=\frac{1}{6}$