EMBIBE

JEE Main Exam 2022 - Session 1

25 June 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. Dielectric constant of a material is 4 and relative permeability is 1 of a medium, then what is the value of the critical angle?

A) 10°

B) 20°

C) 30°

D) 60°

Answer: 30°

Solution: Relative refractive index is $\mu=\sqrt{\mu_r\varepsilon_r}=\sqrt{4\times1}=2$

Now, for complete reflection, critical angle is $\sin\theta_c=\frac{1}{\mu}=\frac{1}{2}$

$\Rightarrow\theta_c=\sin^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow\theta_c=30^\circ$

Q.2. If $\vec{a}$ & $\vec{b}$ are unit vectors with angle θ, then find relation between $\left|\vec{a}+\vec{b}\right|$ and $\left|\vec{a}-\vec{b}\right|$.

A) $\frac{\left|\vec{a}-\vec{b}\right|}{\left|\vec{a}+\vec{b}\right|}=\tan\left(\frac{\theta}{2}\right)$

B) $\frac{\left|\vec{a}-\vec{b}\right|}{\left|\vec{a}+\vec{b}\right|}=\cot\left(\frac{\theta}{2}\right)$

C) $\left|\vec{a}-\vec{b}\right|=\left|\vec{a}+\vec{b}\right|$

D) None of these

Answer: $\frac{\left|\vec{a}-\vec{b}\right|}{\left|\vec{a}+\vec{b}\right|}=\tan\left(\frac{\theta}{2}\right)$

Solution: We know,

$$1-\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$$

$$1+\cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$$

Now,

$$\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=\left|\vec{a}\right|^2+2\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2$$

$$\Rightarrow\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=(1)^2+2\times1\times1\times\cos\theta+(1)^2$$

$$\Rightarrow\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=2(1+\cos\theta)$$

$$\Rightarrow\left|\vec{a}+\vec{b}\right|^2=4\cos^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow\left|\vec{a}+\vec{b}\right|=2\cos\left(\frac{\theta}{2}\right)\ \ldots(1)$$

Similarly,

$$\left(\vec{a}-\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=\left|\vec{a}\right|^2-2\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2$$

$$\Rightarrow\left(\vec{a}-\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=(1)^2-2\times1\times1\times\cos\theta+(1)^2$$

$$\Rightarrow\left(\vec{a}-\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=2(1-\cos\theta)$$

$$\Rightarrow\left|\vec{a}-\vec{b}\right|^2=4\sin^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow\left|\vec{a}-\vec{b}\right|=2\sin\left(\frac{\theta}{2}\right)\ \ldots(2)$$

Therefore,

$$\frac{\left|\vec{a}-\vec{b}\right|}{\left|\vec{a}+\vec{b}\right|}=\tan\left(\frac{\theta}{2}\right)$$

Q.3. Wattless current flows in AC circuit, then the circuit consists of,

A) L only

B) RC only

C) RLC

D) R only

Answer: L only

Solution: The current in an AC circuit containing only either a capacitor or an inductor is said to be wattless as the average power is zero. As wattless current has phase difference of $\phi=90^\circ$ with voltage, therefore power factor $\cos\phi$ will be zero.

Q.4. Two waves of intensities $I_1=9I$ and $I_2=I$ coming from coherent sources interfere at point P have the phase difference as $\frac{\pi}{2}$ and at point Q as π. Then the difference between intensities at point P and point Q is:

A) $6I$

B) $9I$

C) $8I$

D) $3I$

Answer: $6I$

Solution: Resultant intensity is given by, $I_{\text{resultant}} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

At P, $I_P = 9I + I + 2\sqrt{9I \times I}\cos\left(\frac{\pi}{2}\right) = 10I$

At Q, $I_Q = 9I + I + 2\sqrt{9I \times I}\cos(\pi) = 10I - 2 \times 3I = 4I$

Now, difference between intensities is $10I - 4I = 6I$

Q.5. $\vec{A}$ is a non zero vector then which of the following is correct?

A) $\vec{A} \cdot \vec{A} = 0$

B) $\vec{A} \times \vec{A} = 0$

C) $\vec{A} \times \vec{A} < 0$

D) $\vec{A} \times \vec{A} > 0$

Answer: $\vec{A} \times \vec{A} = 0$

Solution: Since both the vectors are same, they will be parallel to each other. We know that vector product of two parallel vectors is zero.

Hence,

$\vec{A} \times \vec{A} = 0$ is the correct answer.

Q.6. If a mass travels (initial velocity is zero) $2\ \text{m}$ in its first second then the distance travelled by it in 9^{th} second is,

A) $34\ \text{m}$

B) $36\ \text{m}$

C) $38\ \text{m}$

D) $40\ \text{m}$

Answer: $34\ \text{m}$

Solution: Distance travelled in n^{th} second is given by,

$S_n = u + \frac{a}{2}(2n - 1)$

Given, $u = 0$.

Now,

$S_1 = u + \frac{a}{2}(2 \times 1 - 1)$

$\Rightarrow 2 = 0 + \frac{a}{2}$

$\Rightarrow a = 4 \text{ m s}^{-2}$

Again,

$S_9 = 0 + \frac{4}{2}(2 \times 9 - 1) = 34 \text{ m}$

Q.7. Find the torque(in N m) of the force, $\vec{F} = \left(3\hat{i} - 4\hat{j} + 2\hat{k}\right) \text{ N}$ acting at the point, $\vec{r} = \left(2\hat{i} + 2\hat{j} + 1\hat{k}\right) \text{ m}$ about the origin.

A) $8\hat{i} - \hat{j} - 14\hat{k}$

B) $8\hat{i} + \hat{j} - 14\hat{k}$

C) $8\hat{i} - \hat{j} + 14\hat{k}$

D) $8\hat{i} + \hat{j} + 14\hat{k}$

Answer: $8\hat{i} - \hat{j} - 14\hat{k}$

Solution: Torque in vector form is expressed as vector multiplication of position vector and force.

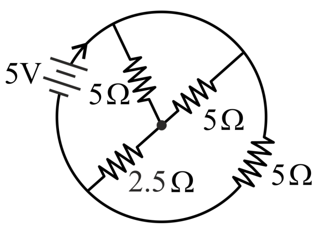
$\vec{\tau} = \vec{r} \times \vec{F}$

$$\Rightarrow \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 3 & -4 & 2 \end{vmatrix}$$

$\Rightarrow \vec{\tau} = (4 + 4)\hat{i} - (4 - 3)\hat{j} + (-8 - 6)\hat{k}$

$\Rightarrow \vec{\tau} = \left(8\hat{i} - \hat{j} - 14\hat{k}\right) \text{ N m}$

Q.8. Find the current supplied by battery in the given circuit.



A) 2 A

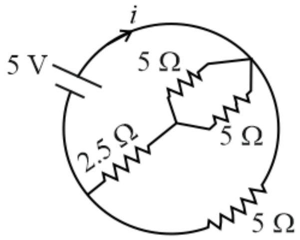
B) 3 A

C) 4 A

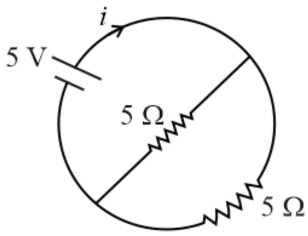
D) 5 A

Answer: $2 \mathrm{~A}$

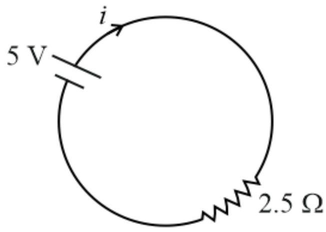
Solution:



As in this case, 5Ω & 5Ω are in parallel hence, their combined resistance value will be 2.5Ω. Now, this 2.5Ω with existing 2.5Ω are in series and hence can be combined to write 5Ω.

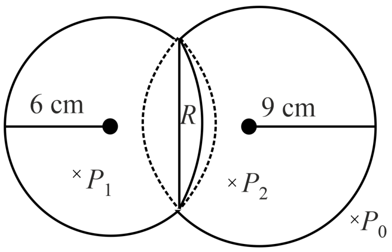


In the final diagram, we can write equivalent resistance across the battery as 2.5Ω.



Hence, current through the battery is, $i=\frac{5 \mathrm{~V}}{2.5 \Omega}=2 \mathrm{~A}$

Q.9. Two air bubbles of radius of curvature $6 \mathrm{~cm}$ and $9 \mathrm{~cm}$ touch each other, then the radius of curvature of common interface to both bubbles will be:



A) $16 \mathrm{~cm}$

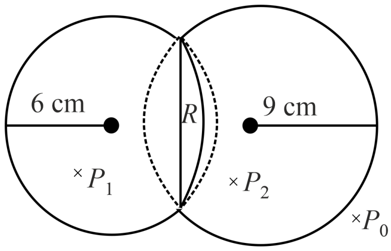
B) $18 \mathrm{~cm}$

C) $20 \mathrm{~cm}$

D) $24 \mathrm{~cm}$

Answer: $18 \mathrm{~cm}$

Solution:



Excess pressure inside air bubble is given as $\Delta P=\frac{4T}{r}$.

$P_1=P_0+\frac{4T}{R_1}$ and $P_2=P_0+\frac{4T}{R_2}$

The interface will have curvature such that,

$P_1-P_2=\frac{4T}{R}$

$\Rightarrow \frac{4T}{R_1}-\frac{4T}{R_2}=\frac{4T}{R}$

$\Rightarrow \frac{1}{6}-\frac{1}{9}=\frac{1}{R} \Rightarrow R=18 \text{ cm}$

Q.10. The terminal velocity v_{T} of the spherical raindrop depends on the radius (r) of the spherical raindrop as

A) r

B) r^2

C) $\frac{1}{r}$

D) $\frac{1}{r^2}$

Answer: r^2

Solution: The terminal velocity of a sphere of radius r and density ρ, immersed in a liquid of density σ and viscosity η is given by $v_{\mathrm{T}}=\frac{2}{9}\frac{(\rho-\sigma)r^2 g}{\eta}$

Hence, for spherical raindrop, $v_{\mathrm{T}} \propto r^2$.

Q.11. A long cylindrical wire having radius of cross-section R carries a steady current I . If the distance from the axis of the wire is r , then the magnetic field B for $(r \ll R)$ varies as

A) $B \propto r$

B) $B \propto \frac{1}{r}$

C) $B \propto r^2$

D) $B \propto \frac{1}{r^2}$

Answer: $B \propto r$

Solution: Using Ampere's law and noting that the steady current is uniformly distributed over the cross-section of the cylindrical wire.

$$B=\begin{bmatrix} \frac{\mu_0 I r}{2\pi R^2}, & r \leq R \\ \frac{\mu_0 I}{2\pi r}, & r > R \end{bmatrix}$$

Thus, magnetic field $B \propto r$ from the centre for $r < R$.

Q.12. Energy produced in 15 sec is 300 J when current flow is 2 A through a resistor, then find energy produced when 3 A current flow for time 10 sec though same resistor.

A) 450 J

B) 350 J

C) 550 J

D) 650 J

Answer: 450 J

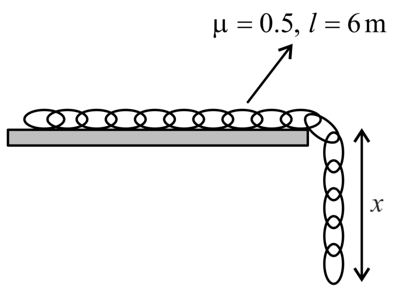
Solution: Energy produced is given by $H = i^2Rt$

When $i = 2$ A, $t = 15$ s, energy is $H = 300 = 2^2 \times R \times 15$

Resistance is $R = 5\ \Omega$

When $i = 3$ A, $t = 10$ s. energy is $H' = 3^2 \times 5 \times 10 = 450$ J

Q.13. In the following figure, x length of a uniform chain of total length 6 m is hanging from the table. What is the maximum value of x for which chain will not slip?



A) 2 m

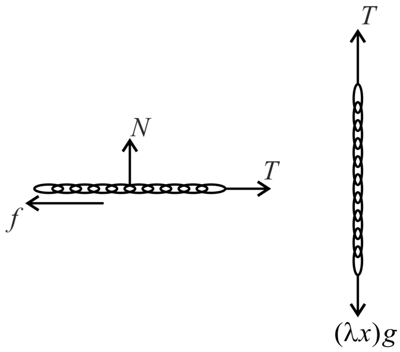
B) 3 m

C) 4 m

D) 5 m

Answer: 2 m

Solution: Let, λ =mass per unit length of the chain and length of mass hanging through the table is x.



Normal force applied by the table on the chain will be, $N=(l-x)\lambda g$.

Tension developed in the chain will balance the gravitational force acting in the downward direction, $T=\lambda x g$.

Maximum limiting force possible for the part of the table, $f_{max}=\mu N=\mu\,(l-x)\lambda g$

For equilibrium,

$f_{max}=T,$

$\Rightarrow \mu\,(l-x)\lambda g=\lambda x g$

$\Rightarrow \frac{1}{2}\times(6-x)=x \;\Rightarrow x=2\text{ m}$

Q.14. A particle starts moving under the influence of force $\vec{F}=\left(10\hat{\imath}+5\hat{\jmath}\right)$ N. If mass of particle is 0.1 kg and its displacement $\left(\vec{s}\right)$ in $t=2$ s is given by $\vec{s}=\left(a\hat{\imath}+b\hat{\jmath}\right)$ m, then the value of $\frac{a}{b}$ is-

A) 2

B) 3

C) 4

D) 5

Answer: 2

Solution: $\vec{a}=\frac{\vec{F}}{m}=\frac{10\hat{\imath}+5\hat{\jmath}}{0.1}=100\hat{\imath}+50\hat{\jmath}$

Displacement covered in vector form can be written as, $\vec{s}=\vec{u}t+\frac{1}{2}\vec{a}t^2$

$\Rightarrow \vec{s}=\frac{1}{2}\left(100\hat{\imath}+50\hat{\jmath}\right)(2)^2$

, $\Rightarrow \vec{s}=200\hat{\imath}+100\hat{\jmath}$

Comparing it with, $\vec{s}=a\hat{\imath}+b\hat{\jmath}$, we get $a=200$ & $b=100$

Therefore, $\frac{a}{b}=2$

Q.15. The ratio of the rms speed to the most probable speed of O_2 gas at a certain temperature is

A) $\frac{\sqrt{3}}{2}$

B) $\frac{\sqrt{2}}{3}$

C) $\sqrt{\frac{3}{2}}$

D) $\sqrt{\frac{2}{3}}$

Answer: $\sqrt{\frac{3}{2}}$

Solution: RMS speed of O_2 gas is $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{32}}$ and most probable speed of O_2 gas is $v_{\text{most probable}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2RT}{32}}$.

Thus, $\frac{v_{\text{rms}}}{v_{\text{most probable}}} = \frac{\sqrt{\frac{3RT}{32}}}{\sqrt{\frac{2RT}{32}}} = \sqrt{\frac{3}{2}}$

Q.16. Photodiode is reverse biased for which of the following reason?

A) To increase the sensitivity

B) To increase the current flow

C) To decrease depletion width

D) To decrease the potential barrier

Answer: To increase the sensitivity

Solution: The photodiode is used in reverse biasing conditions, although the current is less.

In a forward bias p-n junction, the width of depletion region is less and keeps on decreasing as we increase voltage. So there is a small area where photons will break the bonds and less current is generated. Whereas in reverse bias p-n junction, the width of depletion region is more and keeps on increasing as we increase voltage. So the area for photons to work on is more and the large current can be generated.

So, photodiode is reverse biased to increase the sensitivity.

Q.17. If physical quantities A, B, C and D are related as follows $\frac{A^2B^3}{C^4} = D$, then the magnitude of the maximum value of percentage error in D is:

A) $\left(\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}\right) \times 100$

B) $\left(\frac{2\Delta A}{A} - \frac{3\Delta B}{B} + \frac{4\Delta C}{C}\right) \times 100$

C) $\left(\frac{2\Delta A}{A} + \frac{3\Delta B}{B} - \frac{4\Delta C}{C}\right) \times 100$

D) $\left(\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}\right) \times 100$

Answer: $\left(\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}\right) \times 100$

Solution: Given: $D=\frac{A^2B^3}{C^4}$

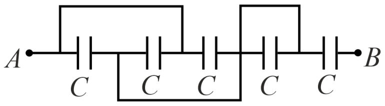
Relative error is given by

$\frac{\Delta D}{D}=\left[2\times\frac{\Delta A}{A}+3\times\frac{\Delta B}{B}+4\times\frac{\Delta C}{C}\right]$

Now, maximum percentage error in D is

$\frac{\Delta D}{D}\times 100=\left[2\times\frac{\Delta A}{A}+3\times\frac{\Delta B}{B}+4\times\frac{\Delta C}{C}\right]\times 100$

Q.18. Find the equivalent capacitance between A and $B.(C=8\ \mu\mathrm{F})$



A) $6\ \mu\mathrm{F}$

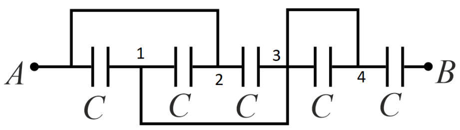
B) $8\ \mu\mathrm{F}$

C) $10\ \mu\mathrm{F}$

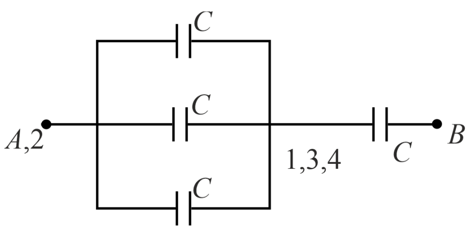
D) $12\ \mu\mathrm{F}$

Answer: $6\ \mu\mathrm{F}$

Solution:



Point 3 and 4 are at same potential, hence we can replace the capacitor with a wire.



Capacitors between points A and 3 are in parallel combination as shown above and their combined capacitance value will be $3C$.

This $3C$ capacitance is in series with C.
The equivalent capacitance of the system between points A and B is,
$C_{\mathrm{AB}}=\frac{3C\times C}{3C+C}=\frac{3C}{4}$
$\Rightarrow C_{\mathrm{AB}}=\frac{3}{4}\times 8\ \mu\mathrm{F}=6\ \mu\mathrm{F}$

Q.19. Find the ratio of speed of e^- in the third orbit of H and He^+ using Bohr's model?

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{1}{5}$

Answer: $\frac{1}{2}$

Solution: The speed of electron is given by $v=2.18\times10^{6}\times\frac{Z}{n}\ \mathrm{m\ s^{-1}}$, where, Z is atomic number and n is number of orbit.

Now, for third orbit, ratio of speed of electron is $\frac{v_{\mathrm{H}}}{v_{\mathrm{He}^{+}}}=\frac{Z_{\mathrm{H}}}{Z_{\mathrm{He}^{+}}}=\frac{1}{2}$

Q.20. The electric field in an electromagnetic wave is $E=56.5\sin(\omega t-kx)$. Find the intensity of the wave.

A) $5.65\ \mathrm{W\ m^{-2}}$

B) $56.5\ \mathrm{W\ m^{-2}}$

C) $4.24\ \mathrm{W\ m^{-2}}$

D) $42.4\ \mathrm{W\ m^{-2}}$

Answer: $4.24\ \mathrm{W\ m^{-2}}$

Solution: Given, $E=56.5\sin(\omega t-kx)$

Comparing it with the general equation of electromagnetic wave,

$E=E_0\sin\ (\omega t-kx)$

We have, $E_0=56.5\ \mathrm{N\ C^{-1}}$

So, the intensity will be $I=\frac{1}{2}\varepsilon_0E_0^2c$

$=\frac{1}{2}\times8.85\times10^{-12}\times(56.5)^2\times3\times10^{8}$

$=4.24\ \mathrm{W\ m^{-2}}$

Q.21. A block of mass $500\ \mathrm{g}$ moving at a speed of $12\ \mathrm{m\ s^{-1}}$ compresses a spring through a distance $30\ \mathrm{cm}$ before its speed is halved. Find the spring constant of the spring.

A) $600\ \mathrm{N\ m^{-1}}$

B) $750\ \mathrm{N\ m^{-1}}$

C) $900\ \mathrm{N\ m^{-1}}$

D) $1050\ \mathrm{N\ m^{-1}}$

Answer: $600\ \mathrm{N\ m^{-1}}$

Solution: Given: Mass of the block $m=500\ \mathrm{g}$, initial velocity of the block $u=12\ \mathrm{m\ s^{-1}}$, and final velocity of the block $v=\frac{u}{2}=6\ \mathrm{m\ s^{-1}}$.

As non-conservative forces are not acting on the system we can conserve energy.

Thus, loss in kinetic energy of block $=$ Gain in potential energy of spring

$\frac{1}{2}mu^2-\frac{1}{2}mv^2=\frac{1}{2}kx^2$

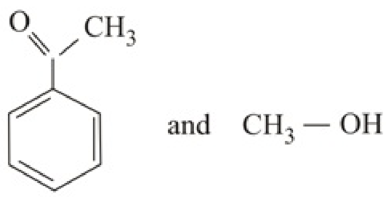
$\frac{1}{2}\times0.5\times12^2-\frac{1}{2}\times0.5\times6^2=\frac{1}{2}k\times0.3^2$

$k=600\ \mathrm{N\ m^{-1}}$

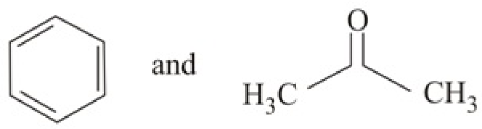
Section B: Chemistry

Q.1. What are the products formed in the reaction of cumene with O_2 followed by treatment with dil. HCl ?

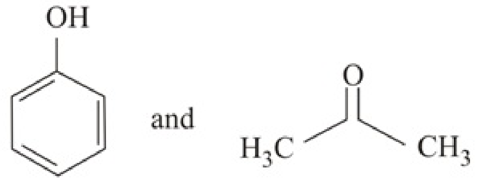
A)



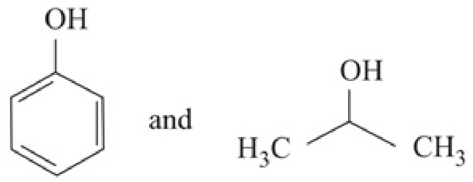
B)



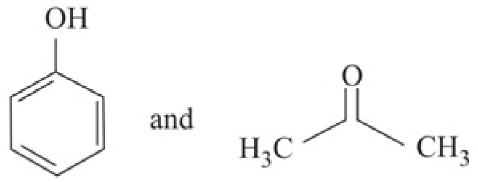
C)



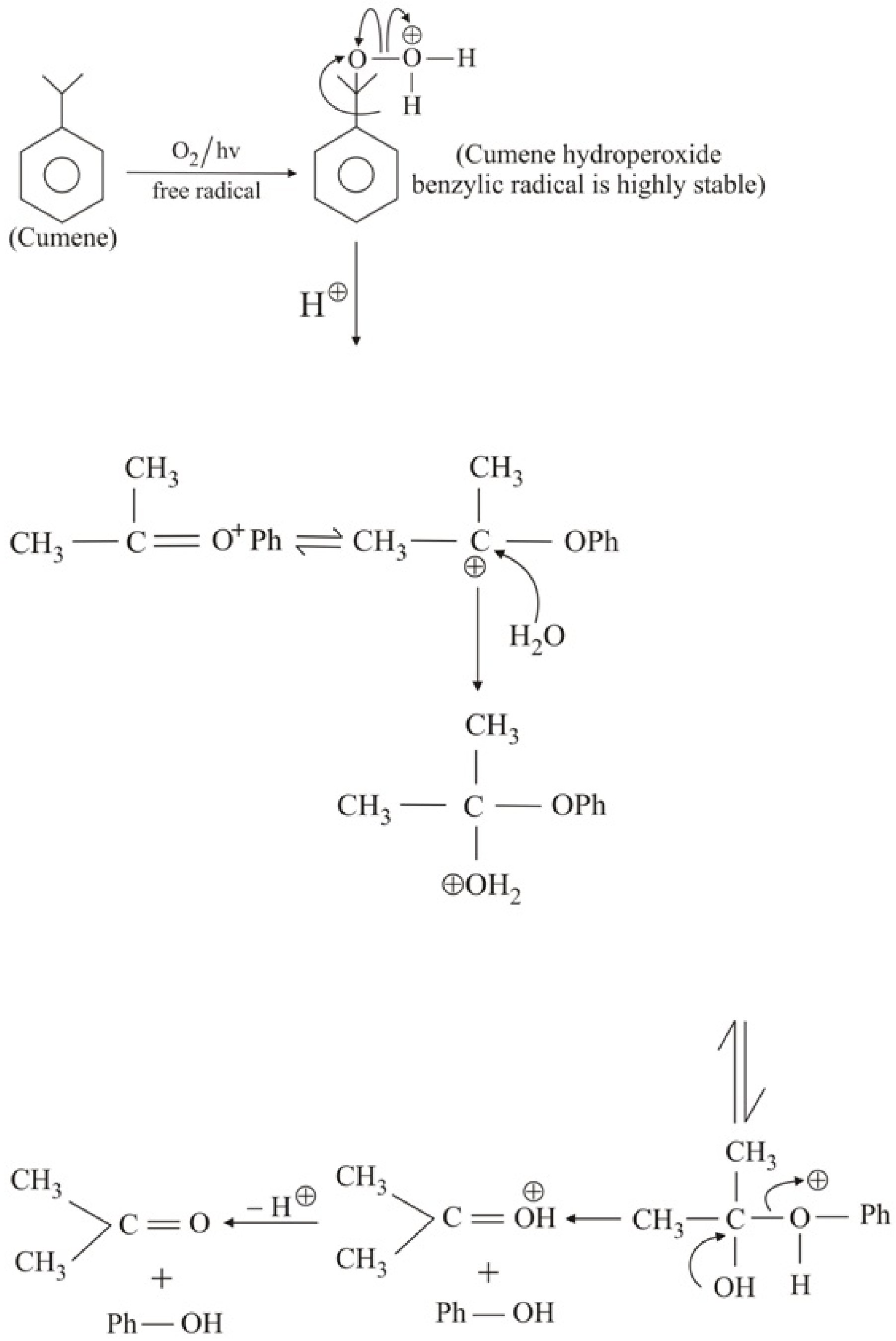
D)



Answer:



Solution:



Q.2. Which of the following is formed by free radical polymerisation?

A) Terylene

B) Melamine

C) Nylon$-6,\ 6$

D) Teflon

Answer: Teflon

Solution: Addition polymerisation takes place via free radical mechanism. Out of the given polymers, Teflon is an addition polymer. Teflon is manufactured by heating tetrafluoroethene with a free radical or persulphate catalyst at high pressures.

$$\underset{\text{(Tetrafluoroethene)}}{nCF_2 = CF_2} \rightarrow \underset{\text{(Teflon)}}{-(CF_2 - CF_2)_n-}$$

Q.3. Reimer-Tiemann reaction involves

A) carbonium ion intermediate.

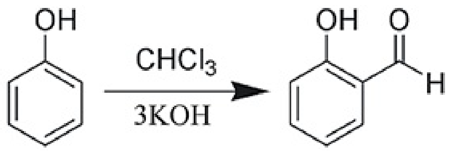
B) carbene intermediate.

C) carbanion intermediate.

D) free radical intermediate.

Answer: carbene intermediate.

Solution: Reimer-Tiemann reaction is the reaction of phenol with chloroform in the basic medium. The product is salicylaldehyde.



Intermediate dichloro carbene is formed by the reaction of chloroform and KOH with $1, 1$ elimination reaction.

$$CHCl_3 + OH^- \leftrightharpoons \overset{\ominus}{C}Cl_3 + H_2O$$

$$\overset{\ominus}{C}Cl_3 \rightleftharpoons :CCl_2 + Cl^-$$

Dichlorocarbene is an electron deficient species.

Q.4. Which of the following alkene on hydration would give tert-butyl alcohol?

A) Ethylene

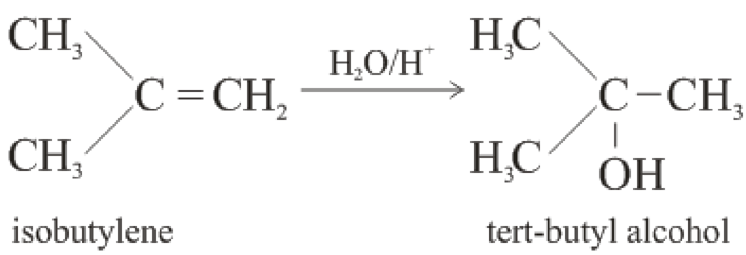
B) Isobutylene

C) Propylene

D) n$-$Butylene

Answer: Isobutylene

Solution: When isobutylene reacts with water in the presence of an acid catalyst, it forms tert-butyl alcohol.



Q.5. Extraction of gold (Au) involves the formation of complex ion 'X' and 'Y'.

Gold ore $\xrightarrow{CN^-, H_2O, O_2}$ $HO^- + ' X ' \xrightarrow{Zn} ' Y ' + Au$

X and Y are, respectively_____.

A) $[Au(CN)_2]^-$ and $[Zn(CN)_4]^{2-}$

B) $[Au(CN)_4]^{3-}$ and $[Zn(CN)_4]^{2-}$

C) $[Au(CN)_3]^-$ and $[Zn(CN)_6]^{4-}$

D) $[Au(CN)_4]^-$ and $[Zn(CN)_3]^-$

Answer: $[Au(CN)_2]^-$ and $[Zn(CN)_4]^{2-}$

Solution: Extraction of gold is carried out by leaching process.

Sodium cyanide $(NaCN)$ has been used as leaching reagent for gold.

Metal is converted into the soluble cyanide complex on reaction with sodium cyanide.

$$4Au + 8CN^- + 2H_2O + O_2 \rightarrow 4\underset{X}{[Au(CN)_2]^-} + 4OH^-$$

Silver is later recovered by displacement with Zn.

$$2[Au(CN)_2]^- + Zn \xrightarrow{\text{Displacement}} \underset{Y}{[Zn(CN)_4]^{2-}} + 2Au$$

X and Y are $[Au(CN)_2]^-$ and $[Zn(CN)_4]^{2-}$, respectively.

Q.6. BOD values (in ppm) for clean water (A) and polluted water (B) are expected respectively as:

A) $A < 5, B > 17$

B) $A > 50, B < 27$

C) $A > 15, B > 47$

D) $A > 25, B < 17$

Answer: $A < 5, B > 17$

Solution: Clean water would have BOD a value of less than, $5\ ppm$ whereas highly polluted water could have a BOD value of $17\ ppm$ or more.

Q.7. The ion having the highest mobility in aqueous solution is

A) Ba^{2+}

B) Mg^{2+}

C) Ca^{2+}

D) Be^{2+}

Answer: Ba^{2+}

Solution: Larger the size of cation, lesser it gets hydrated. Hence, show higher mobility in aqueous solution. Among the given species Ba^{2+} ion has the largest size, so hydrate to smaller extent and show the highest mobility.

Q.8. Electron deficient species among the following is:

A) B_2H_6

B) H_2O

C) CH_4

D) NH_3

Answer: B_2H_6

Solution: A molecule is said to be electron deficient because there are not enough valence electrons to form the expected number of covalent bonds. For example, in B_2H_6, there are only 12 electrons, three from each boron atoms and six from hydrogen, while ethane possesses 14 such electrons.

Q.9. Which of the following molecule stabilizes by removal of electron?

A) C_2

B) O_2

C) N_2

D) H_2

Answer: O_2

Solution: In all these molecules O_2, is stabilized by removal of electrons because it is having electrons in the anti-bonding orbital. And thus removal of electron from the anti bonding orbital , increases the bond order of O_2. And thus is becomes more stable after removal of electrons. Bond order of $O_2^+$is 2.5 and it is more than that of O_2 which is having bond order as 2.

Hence option B is correct.

Q.10. IUPAC name of ethylidene chloride is

A) $1, 1$-dichloroethane

B) $1, 2$-dichloroethane

C) $1, 1$-dichloroethene

D) $1, 2$-dichloroethyne

Answer: $1, 1$-dichloroethane

Solution: 1,1-dichloroethane

Ethylidene chloride is a geminal dihalide with two carbon atoms. In common name system, gem-dihalides are named as alkylidene halides and vic-dihalides are named as alkylene dihalides. The structure of ethylidene chloride is:

$CH_3 - CHCl_2$.

Q.11. The strongest oxidising agent is:

A) Mn^{3+}

B) Ti^{3+}

C) Fe^{3+}

D) Cr^{3+}

Answer: Mn^{3+}

Solution: The substance which will have the highest value of reduction potential will be the strongest oxidising agent. The values of reduction potential are as follow

$$E^0_{Mn^{3+}/Mn^{2+}} = +1.57 \text{ V}$$

$$E^0_{Ti^{3+}/Ti^{2+}} = -0.37 \text{ V}$$

$$E^0_{Fe^{3+}/Fe^{2+}} = +0.77 \text{ V}$$

$$E^0_{Cr^{3+}/Cr^{2+}} = -0.41 \text{ V}$$

As we can see Mn^{+3} has the highest value of standard reduction potential, so it will be the strongest oxidising agent.

Q.12. Product formed on reaction of $AgCl$ with aq. NH_3

A) $[Ag(NH_3)_4]Cl$

B) $[Ag(NH_3)_2Cl_2]$

C) $[Ag(NH_3)_2]Cl$

D) $[Ag(NH_3)Cl]$

Answer: $[Ag(NH_3)_2]Cl$

Solution: The white precipitate silver chloride is soluble in ammonia and forms the complex with coordination number 2. The complex formation is shown below.

$$AgCl(s) + 2NH_3(aq) \rightarrow [Ag(NH_3)_2]Cl(aq)$$

It is ammoniacal silver chloride.

Q.13. Which of the following is artificial sweetener

A) Bithional

B) Alitame

C) Lactose

D) Salvarsan

Answer: Alitame

Solution: Bithional is added to soaps to impart antiseptic properties. Salvarsan is an antimicrobial drug, Alitame is high potency sweetener, although it is more stable than aspartame, the control of sweetness of food is difficult while using it. Lactose is a disaccharide, it is milk sugar.

Q.14. Which of the following are isoelectronic?

A) HF and H_2O

B) CH_4 and SF_6

C) O_2 and O_3

D) H_2 and F_2

Answer: HF and H_2O

Solution: Isoelectronic species are those which have equal number of electrons. Among the given options, in option A both HF and H_2O have 10 electrons each, So, they are isoelectronic.

Q.15. Which of the following is incorrect about Tyndall effect?

A) Greater difference in refractive index

B) Wavelength of dispersed particles is less than the wavelength of incident light

C) Wavelength of dispersed particles is almost same as the wavelength of incident light.

D) This phenomena is shown by colloid solution.

Answer: Wavelength of dispersed particles is less than the wavelength of incident light

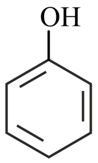
Solution: Tyndall effect is observed only when the following two conditions are satisfied.

(i) The diameter of the dispersed particles is not much smaller than the wavelength of the light used; and

(ii) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.

This is a characteristic property of the colloid solutions.

Q.16.



$+ HNO_3 \rightarrow A + B$

A and B can be separated by:

A) Chromatography

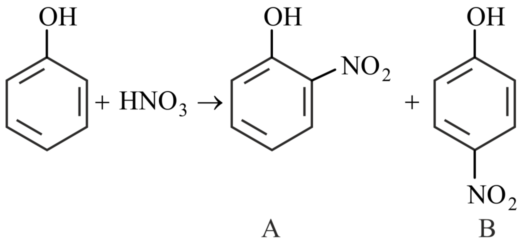
B) Steam distillation

C) Fractional distillation

D) NMR

Answer: Steam distillation

Solution:



The compound A is steam volatile. It is due to the presence of intramolecular hydrogen bonding in the compound A. Hence, the above mixture can be separated by steam distillation.

Q.17. Find intermediate in the following reaction

$$R-CONH_2 + Br_2 \xrightarrow{NaOH} RNH_2$$

A) R—CN

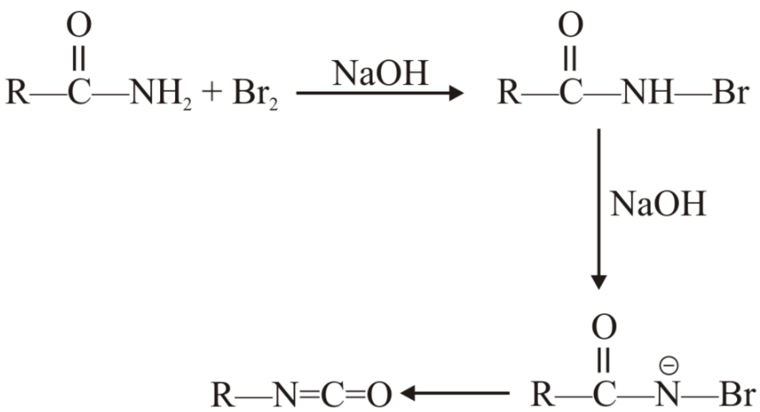
B) R–NC

C) R–NCO

D) R–COOH

Answer: R–NCO

Solution: The given reaction follows Hoffman bromamide degradation mechanism for the preparation of primary amines. In this reaction alkyl or aryl isocynate is formed as an intermediate. The formation of intermediate is shown below.



Q.18. In $681\,\mathrm{mg}$ of $C_7H_5N_3O_6$ the number of nitrogen atoms are $x \times 10^{21}$. What is the value of x?

A) 6

B) 5.4

C) 2.7

D) 1.8

Answer: 5.4

Solution: Given mass of compounds $= 681\,\text{mg}$

Molar mass of $C_7H_5N_3O_6 = 12 \times 7 + 5 \times 1 + 14 \times 3 + 6 \times 16$

$= 84 + 5 + 42 + 96$

$= 227$

Number of moles of compound $= \frac{681 \times 10^{-3}}{227} = 3 \times 10^{-3}$ moles

Number of moles of $N = 3 \times$ number of moles of compound

$= 3 \times 3 \times 10^{-3} = 9 \times 10^{-3}$ moles

Number of atoms of N $= 9 \times 10^{-3} \times 6 \times 10^{23}$

$= 5.4 \times 10^{21}$

So $x = 5.4$

Q.19. $(CH_3)C - Cl$ with $(CH_3)_3C - O^{-}K^{+}$ will give which of the following reaction.

A) SN_1

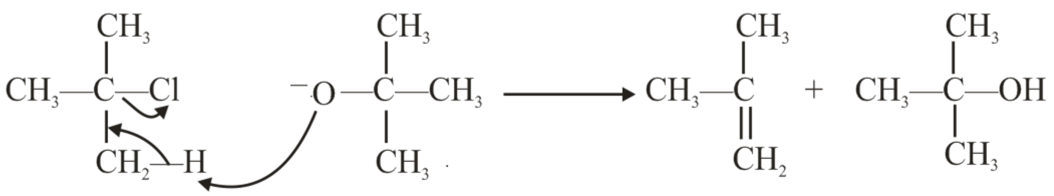
B) SN_2

C) E_1

D) E_2

Answer: E_2

Solution: $(CH_3)_3C - O^{-}K^{+}$ is a very strong base and a weak nucleophile due to its bulkiness. So it will give E_2 elimination reaction as follows.



Q.20. What is the ratio of speeds of electrons in 3^{rd} orbit of H—atom to 3^{rd} orbit of He^{+} ion?

A) $3:2$

B) $1:2$

C) $2:3$

D) $1:4$

Answer: $1:2$

Solution: The velocity of electron in the Bohr's orbit is given by $v=\frac{2\pi me^2 Z}{nh}$

Or $v \propto \frac{Z}{n}$

$\frac{v_1}{v_2}=\left(\frac{Z_1}{Z_2}\right)\left(\frac{n_2}{n_1}\right)$

$\frac{v_1}{v_2}=\left(\frac{1}{2}\right)\left(\frac{3}{3}\right)=\frac{1}{2}$

Q.21. What is eutrophication?

A) Increase in biodiversity

B) Loss in biodiversity

C) Break down of organic matter

D) Oxygen concentration in water.

Answer: Loss in biodiversity

Solution: The addition of phosphates in water enhances algae growth. Such profuse growth of algae, covers the water surface and reduces the oxygen concentration in water. This leads to anaerobic conditions, commonly with accumulation of abnoxious decay and animal death. Thus, bloom-infested water inhibits the growth of other living organisms in the water body. This process in which nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as Eutrophication.

Q.22. Statement 1: Davison Germer Experiment established wave nature of electron.

Statement 2: If electron has wave nature they show interference and diffraction.

A) S_1 is true S_2 is false

B) S_1 is false S_2 is true

C) S_1 and S_2 both true

D) S_1 and S_2 both false

Answer: S_1 and S_2 both true

Solution: The Davisson and Germer experiment showed that electron beams can undergo diffraction when passed through the atomic crystals. This shows that the wave nature of electrons as waves can exhibit interference and diffraction.

Q.23. Density of $NaCI$ solid is $43.1\ g/cm^3$ and distance between Na^+ and Cl^- ions is $[X]\times 10^{-10}\ m$, then value of X is: [Given: $N_A=6\times 10^{23}$] (Report your answer to nearest integer).

A) 1

B) 2

C) 4

D) 9

Answer: 1

Solution: For NaCl $Z = 4$ and $M = 58.5$ gram

$d = \frac{Z \times M}{N_A \times Volume}$

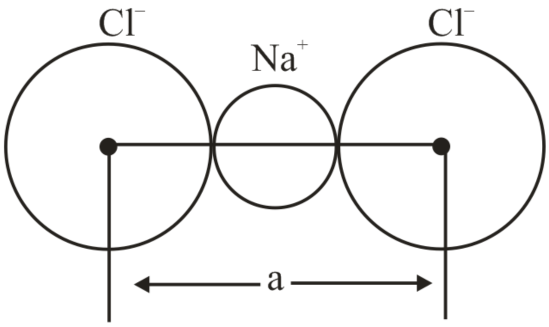
$43.1 = \frac{4 \times 58.5}{6 \times 10^{23} \times [a]^3}$

$a^3 = \frac{4 \times 58.5}{6 \times 43.1} \times 10^{-23}$

$= 0.9 \times 10^{-23}$

$= 9 \times 10^{-24}$

$a = 2.08 \times 10^{-8}$ cm



$d_{Na^+ + Cl^-} = \frac{a}{2} = \frac{2.08 \times 10^{-10}}{2}$ m

Q.24. $N_2O_4(g)$ dissociates to NO_2 according to following reaction.

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

ΔG^o of reaction at 298 K and 1 atm pressure when 50% of N_2O_4 is dissociated at equilibrium is:

A) -684.7 J

B) 684.7 J

C) -342.35 J

D) 342.35 J

Answer: -684.7 J

Solution: $N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$1 - \alpha \quad 2\alpha$

$K_p = \frac{4\alpha^2 p}{1-\alpha^2} = \frac{4 \times (0.5)^2 \times 1}{1-(0.5)^2} = \frac{1}{0.75}$

$K_p = \frac{4}{3}$

$\Delta G^o = -2.303\ RT\ \log K_p$

$= -2.303 \times 8.314 \times 298 \left[\log \frac{4}{3}\right]$

$= -684.7$ J

Q.25. Ce^{4+} act as:

A) Strong oxidising agent

B) Strong reducing agent

C) Not show redox

D) Act as oxidising and reducing agent

Answer: Strong oxidising agent

Solution: Formation of Ce^{IV} is favoured by its noble gas configuration but it is strong oxidant reverting to the $+3$ state. The E^o value for $\frac{Ce^{4+}}{Ce^{3+}}$ is $E^o_{Ce^{4+}/Ce^{3+}} = 1.74$ V is favourable for its oxidising nature.

Q.26. Less amount of soap is not able to clean properly due to:

A) CMC value is higher than the given concentration

B) CMC value is lower than given concentration

C) Macromolecule colloid formation occurs

D) It does not act as electrolyte

Answer: CMC value is higher than the given concentration

Solution: Micelle or associate colloid formation occurs above a certain conc. known as CMC. So, if the concentration of soap is less that means it hasn't reached the CMC value.

Q.27. The E^0_{cell} for the following reaction.

$2Fe^{3+}(aq) + 2I^-(aq) \rightarrow 2Fe^{2+}(aq) + I_2$ is $[X] \times 10^{-2}$ V. The value of X is _____

Given $E^0_{Fe^{3+}/Fe^{2+}} = 0.77$ V and $E^0_{I_2/I^-} = 0.54$ V

A) 23

B) 100

C) 13

D) -23

Answer: 23

Solution: $E^0_{cell} = (E^0_{RP})_C - (E^0_{RP})_A$

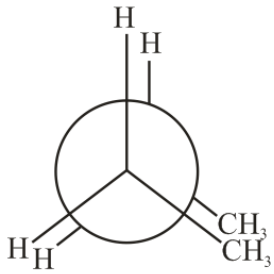
$= 0.77 - 0.54$

$= 0.23$ V

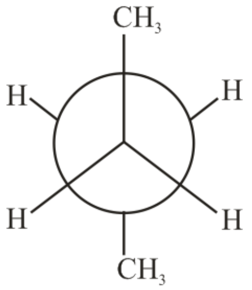
$= 23 \times 10^{-2}$ V

Q.28. Identify the stagged conformation with maximum dihedral angle:

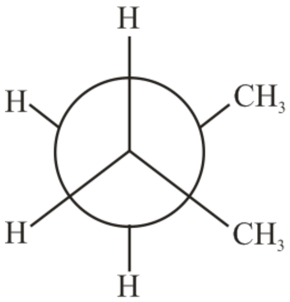
A)


H
H
CH3
CH3
H
H

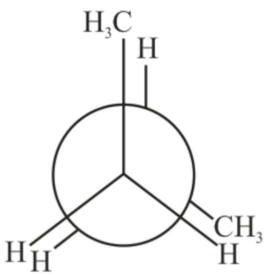
B)


CH3
H
H
H
H
CH3

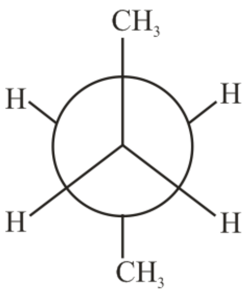
C)


H
H
CH3
H
CH3
H

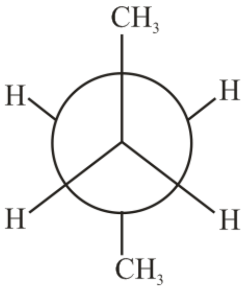
D)


H3C
H
CH3
H
H
H

Answer:


CH3
H
H
H
H
CH3

Solution: A staggered conformation is a chemical conformation of an ethane-like molecule in which the substituents are at the maximum distance from each other. This requires the torsional angle to be 60° .



In anti conformation the dihedral angle is 180^{o} (maximum)

Section C: Mathematics

Q.1. $\frac{1}{2\cdot3^{10}}+\frac{1}{2^2.3^9}+\cdots\cdots+\frac{1}{2^9.3^2}+\frac{1}{2^{10}.3^1}=\frac{k}{2^{10}.3^{10}}$. The remainder when k is divided by 6 is equal to

A) 2

B) 3

C) 4

D) 5

Answer: 5

Solution: We know $x^n-y^n=(x-y)\left(x^{n-1}+x^{n-2}y+\cdots xy^{n-2}+y^{n-1}\right)$

$\frac{1}{2.3^{10}}+\frac{1}{2^2.3^9}+\cdots\cdot\frac{1}{2^9.3^2}+\frac{1}{2^{10}.3}$

$=\frac{2^9+2^8\cdot3+\ \cdots\ 2\cdot3^8+3^9}{2^{10}.3^{16}}=\frac{3^{10}-2^{10}}{2^{10}.3^{10}}$

So, $k=3^{10}-2^{10}$
Since we need the remainder when k is divided by 6

so, $3^{10}=6q_1+3$ and $2^{10}=6q_2+4$
Now k will be of the form $(6q_1+3)-(6q_2+4)$
$=6\left(q_1-q_2\right)-1$
Hence, when k is divided by 6, we get the remainder as $6-1=5$

Q.2.

Find the integration $\int_0^\pi \frac{\sin x\cdot e^{\cos x}}{(1+\cos^2 x)\left(e^{\cos x}+e^{-\cos x}\right)}dx$

A) $\frac{\pi}{2}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{6}$

D) $\frac{\pi}{3}$

Answer: $\frac{\pi}{4}$

Solution:

Given $I=\int_0^{\pi}\frac{\sin x\cdot e^{\cos x}}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx \quad \ldots(\mathrm{i})$

Using property of integral $\int_a^b f(a+b-x)=\int_a^b f(x)$

we get $I=\int_0^{\pi}\frac{\sin x\cdot e^{-\cos x}}{(1+\cos^2 x)(e^{-\cos x}+e^{\cos x})}dx \quad \ldots(\mathrm{ii})$

Adding equation (i) & (ii) we get

$2I=\int_0^{\pi}\frac{\sin x}{(1+\cos^2 x)}\cdot\frac{(e^{\cos x}+e^{-\cos x})}{e^{\cos x}+e^{-\cos x}}dx$

$2I=\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx \Rightarrow 2I=2\int_0^{\frac{\pi}{2}}\frac{\sin x}{1+\cos^2 x}dx$

Let $\cos x=t\Rightarrow -\sin x dx=dt$

$I=-\int_1^0\frac{dt}{1+t^2}\Rightarrow I=-\left[\tan^{-1}t\right]_1^0=-\left[0-\frac{\pi}{4}\right]=\frac{\pi}{4}$

Hence option B is correct answer.

Q.3. If $f(x)=x^3+x-5$ and $f(g(x))=x$, then $g'(63)$ is equal to

A) 49

B) $\frac{1}{49}$

C) $\frac{1}{11908}$

D) $\frac{1}{47}$

Answer: $\frac{1}{49}$

Solution: Here, $g(f(x))=x$ as $f(x)$ and $g(x)$ are inverse of each other.

Now, $g'(f(x))f'(x)=1$

$\Rightarrow g'(f(x))=\frac{1}{f'(x)} \quad \ldots(i)$

Now $f(x)=63 \Rightarrow x^3+x-5=63$

$\Rightarrow x^3+x-68=0$

So $x=4$ satisfies the above equation

$g'(63)=\frac{1}{f'(4)}$ from (i)

$=\frac{1}{3(4)^2+1}=\frac{1}{49}$

Q.4. If a,b,c are sides of ΔABC and if $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}$, then find $\frac{r}{R}=$? [where R is circumradius & r is inradius)

A) $\frac{7}{9}$

B) $\frac{3}{5}$

C) $\frac{2}{5}$

D) $\frac{4}{9}$

Answer: $\frac{2}{5}$

Solution: Let $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}=k$

so $a+b=7k,\ b+c=8k,\ c+a=9k$

Solving this we get $a=4k,\ b=3k,\ c=5k$ and calculating semi-perimeter

$S=\frac{a+b+c}{2}\Rightarrow S=6k$

We know that $\frac{r}{R}=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow\ \frac{r}{R}=4\sqrt{\frac{(s-b)(s-c)}{bc}}\times\sqrt{\frac{(s-a)(s-c)}{ac}}\times\sqrt{\frac{(s-a)(s-b)}{ab}}$

$\Rightarrow \frac{r}{R}=4\cdot\frac{(s-a)(s-b)(s-c)}{abc}$

$\Rightarrow \frac{r}{R}=4\cdot\frac{(2k)(3k)(k)}{(4k)(3k)(5k)}\Rightarrow\frac{r}{R}=\frac{2}{5}$

Q.5. A circle S touches the line $l_1=4x-3y+k_1=0$ and $l_2=4x-3y+k_2=0$ where $k_1,k_2\in R$. If a line passing through the centre of the circle intersect l_1 at $(-1,2)$ and l_2 at $(3,-6)$, then equation of circle is:

A) $x^2+y^2+2x-4y-11=0$

B) $x^2+y^2-2x+4y-11=0$

C) $x^2+y^2-2x-4y+11=0$

D) $x^2+y^2-2x+6y-11=0$

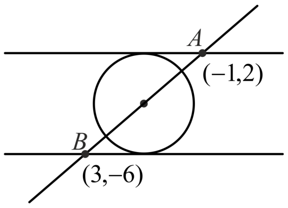
Answer: $x^2+y^2-2x+4y-11=0$

Solution: Given,

$l_1 = 4x - 3y + k_1 = 0$ $l_2 = 4x - 3y + k_2 = 0$

Here line l_1 and l_2 are parallel.

So,



Now point $A(-1, 2)$ will satisfy l_1 so, $-4 - 3 \times 2 + k_2 = 0 \Rightarrow k_2 = 10$

Also point $B(3, -6)$ will satisfy $l_2 = 4x - 3y + k_2$

So $4 \times 3 - 3 \times (-6) + k_2 = 0 \Rightarrow k_2 = -30$

Now Distance between tor parallel line will be Diameter

Diameter $= \left|\frac{k_1 - k_2}{\sqrt{4^2 + 3^2}}\right|$

$= \left|\frac{10+30}{5}\right| = 8$

So Radius $\frac{8}{2} = 4$

Now Mid-point of AB will give us centre of circle by symmetry, so by midpoint formula in AB we get centre $C \equiv (1, -2)$,

Now equation of circle $(x-1)^2 + (y+2)^2 = 4^2$

$x^2 - 2x + 1 + y^2 + 4y + 4 = 16$

$x^2 + y^2 - 2x + 4y - 11 = 0$

Q.6. If $S = \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81}$.... upto 100 terms, then $[S]$ is equal to (where $[.]$ represents greatest integer function)

A) 98

B) 99

C) 97

D) 100

Answer: 98

Solution: Let $S = \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \cdots$ upto 100 terms

$S = \frac{3-2}{3^1} + \frac{3^2-2^2}{3^2} + \frac{3^3-2^3}{3^3} + \ldots$ upto 100 terms

So, $T_n = 1 - \left(\frac{2}{3}\right)^n \Rightarrow S_{100} = 100 - \left[\frac{\frac{2}{3}\left[1-\left(\frac{2}{3}\right)^{100}\right]}{1-\frac{2}{3}}\right]$

$= 98 + 2\left(\frac{2}{3}\right)^{100} \Rightarrow [S_{100}] = 98$

Q.7. Give function $f : N \to R$ such that $f(x+y) = 2f(x) \cdot f(y)$ for natural number x & y, if $f(1) = 2$ and $\sum_{k=1}^{10} f(a+k) = \frac{512}{3}\left(2^{20} - 1\right)$, then find the value of a

A) 3

B) 4

C) 5

D) 6

Answer: 4

Solution: Given $f(x+y)=2f(x)\cdot f(y)$ & $f(1)=2$

Now putting $x=1$& $y=1$ in $f(x+y)=2f(x)\cdot f(y)$ we get

$f(1+1)=2f(1)\cdot f(1)=2\times 2^2=2^3$

So $f(2)=2^3$, Similarly

$f(3)=2^5,\ f(4)=2^7.....$

Now

$\sum_{k=1}^{10} f(a+k)=\sum_{k=1}^{10} 2f(a)\cdot f(k)=\frac{512}{3}\left(2^{20}-1\right)$

$\Rightarrow\ 2f(a)\sum_{k=1}^{10} f(k)=\frac{512}{3}\left(2^{20}-1\right)$

$\Rightarrow\ 2f(a)\{f(1)+f(2)\cdots f(10)\}=\frac{512}{3}\left(2^{20}-1\right)$

$\Rightarrow\ 2f(a)\left(2+2^3+2^5\ldots\right)=\frac{512}{3}\left(2^{20}-1\right)$

$\Rightarrow 2f(a)\frac{2\left(\left(2^2\right)^{10}-1\right)}{2^2-1}=\frac{512}{3}\left(2^{20}-1\right)$

$\Rightarrow 2\times 2f(a)\times\frac{2^{20}-1}{3}=\frac{512}{3}\left(2^{20}-1\right)$

Now comparing both side we get

$4f(a)=512\Rightarrow f(a)=128$

$\Rightarrow f(a)=2^7\Rightarrow a=4$

Hence option B is correct.

Q.8. For two events E_1 and E_2, if $P\left(\frac{E_1}{E_2}\right)=\frac{1}{2}, P\left(\frac{E_2}{E_1}\right)=\frac{3}{4}$ and $P(E_1\cap E_2)=\frac{1}{8}$, then

A) $P\left(E_1'\cap E_2'\right)=P\left(E_1'\right)P(E_2)$

B) $P(E_1\cap E_2)=P\left(E_1{}'\right)P(E_2)$

C) $P(E_1\cap E_2)=P(E_1)P\left(E_2'\right)$

D) $P(E_1\cup E_2)=P(E_1)P(E_2)$

Answer: $P(E_1\cap E_2)=P(E_1)P\left(E_2'\right)$

Solution: $\frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1}{2}, \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{3}{4}, P(E_1 \cap E_2) = \frac{1}{8}$

$\Rightarrow P(E_2) = \frac{1}{4}, P(E_1) = \frac{1}{6}, P(E_1 \cup E_2) = \frac{1}{6} + \frac{1}{4} - \frac{1}{8} = \frac{7}{24}$

$P(E' \cap E_2') = P(E_1 \cup E_2)' = 1 - \frac{7}{24} = \frac{17}{24}$

$P(E_2') = \frac{3}{4} \Rightarrow P(E_1 \cap E_2) = P(E_1)P(E_2')$

Q.9. If $f(x)$ is a polynomial such that $f(x) + f'(x) + f''(x) = x^5 + 64$, then the value of $\lim_{x \to 1} \left(\frac{f(x)}{x-1}\right)$ is equal to

A) -15

B) 15

C) -8

D) -7

Answer: -15

Solution: Let $f(x) = (x-1)(x^4 + ax^3 + bx^2 + cx + d)$

$f(x) + f'(x) + f''(x) = x^5 + (a+4)x^4 + (b+3a+16)x^3 +$

$(9a + 2b + c - 12)x^2 + (d + c + 4b - 6a)x + (c - 2b)$

$\because \; f(x) + f'(x) + f''(x) = x^5 + 64$

$\Rightarrow \; a + 4 = 0, \; b + 3a + 16 = 0, \; 9a + 2b + c - 12 = 0, \; d + c + 4b - 6a = 0$

$\Rightarrow \; a = -4, \; b = -4, \; c = 56, \; d = -64$

So $\lim_{x \to 1} \frac{f(x)}{x-1} = \lim_{x \to 1} (x^4 - 4x^3 - 4x^2 + 56x - 64)$

$= 1 - 4 - 4 + 56 - 64 = -15$

Q.10. If z is a point on circle which is passing through $P : (3+4i), Q : (4+3i)$ & $R : (5i)$. Consider a point T on the circle such that $TP \perp PR$. Find $arg(T)$.

A) $-\frac{\pi}{2}$

B) $\frac{\pi}{2}$

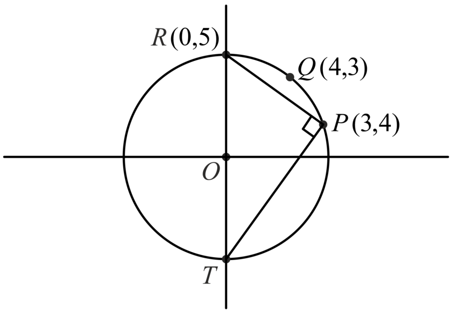
C) π

D) 0

Answer: $-\frac{\pi}{2}$

Solution: Given A circle passing through $P:(3+4i), Q:(4+3i)$ & $R:(5i)$

Now plotting the points on circle.



By diagram we observed that $OR = OP = OQ = \text{radius}$

Now by observation O will be cente of circle which is $(0,0)$

And RT is a diameter as $TP \perp PR$

So by section formula we get $T \equiv (0,-5)$

So $\arg(T) = -\frac{\pi}{2}$ as it lies on negative y-axis

Q.11. $f:\mathbb{R}\to\mathbb{R}, g:\mathbb{R}\to\mathbb{R}$ be two function defined by $f(x) = \log_e(x^2+1) - e^{-x} + 1$ and, $g(x) = \frac{1-2e^{2x}}{e^x}$. If the inequality $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$ holds, then

A) $\alpha \in (0,2)$

B) $\alpha \in (2,3)$

C) $\alpha \in (3,4)$

D) $\alpha \in R$

Answer: $\alpha \in (2,3)$

Solution: $f(x) = \log_e(x^2+1) - e^{-x} + 1 \Rightarrow f'(x) = \frac{2x}{x^2+1} + e^{-x} > 0, \forall x \in R$

So $f(x)$ increasing

$g(x) = e^{-x} - 2e^x \quad \Rightarrow \quad g'(x) = -(e^{-x} + 2e^x) < 0 \ \forall x \in R$

So $g(x)$ decreasing

$\Rightarrow \quad f(g(x))$ is decreasing

$\Rightarrow \quad f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$

$\Rightarrow \quad \frac{(\alpha-1)^2}{3} < \alpha - \frac{5}{3}$

$\Rightarrow \quad (\alpha-2)(\alpha-3) < 0 \quad \Rightarrow \alpha \in (2,3)$

Q.12. How many 3×3 matrices is possible with the elements $\{-1,0,1\}$ such that sum of all elements is 5

A) 512

B) 420

C) 414

D) 520

Answer: 414

Solution:

Given Matrix $3 \times 3 = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}$

Here we can see we have to find nine elements whose sum is 5

Case 1: When five 1's are there & four O's are there

Now by division & Distribution Method

We get $\Rightarrow \frac{9!}{5!4!} = 126 \quad \ldots(\text{i})$

Case 2: when six 1's are there & one $\{-1\}$ & two O's are there.

Again by division & distribution Method

we get $\Rightarrow \frac{9!}{6!2!1!} = 252 \quad \ldots(\text{ii})$

Case 3: When seven 1's are there are two $\{-1\}$ are there,

By division & distribution we get

$\Rightarrow \frac{9!}{7!2!} = 36 \quad \ldots(\text{iii})$

Now adding equation $(\text{i}) + (\text{ii}) + (\text{iii})$

We get $126 + 252 + 36 = 414$

So total 414 ways will be there.

Q.13. If $a_n = 19^n - 12^n$, then the value of $\frac{31a_9 - a_{10}}{57a_8}$ is equal to

A) 4

B) 6

C) 2

D) 5

Answer: 4

Solution: If α and β are roots of the quadratic $ax^2 + bx + c = 0$

then $\alpha^n = 31\alpha^{n-2} - 228\alpha^{n-1}$ and $\beta^n = 31\beta^{n-2} - 228\beta^{n-1}$

So, $\alpha^n - \beta^n = 31\left(\alpha^{n-2} - \beta^{n-2}\right) - 228\left(\alpha^{n-1} - \beta^{n-1}\right)$

Or $a_n - 31a_{n-1} + 228a_{n-2} = 0$ where $a_n = \alpha^n - \beta^n$

Since 19 & 12 are the roots of the quadratic $x^2 - 31x + 228 = 0$

we know, $a_n - 31a_{n-1} + 228a_{n-2} = 0$

Now, for $n = 10, \ \ a_{10} - 31a_3 + 228a_8 = 0$

$\Rightarrow \frac{31a_9 - a_{10}}{57a_8} = 4$

Q.14. If $g:(0,\infty)\rightarrow R$ is a differentiable function and $\int\left[\frac{x\cdot(\cos x-\sin x)}{e^x+1}+g(x)\frac{(e^x+1-xe^x)}{(e^x+1)^2}\right]dx=\frac{x}{e^x+1}g(x)+C$ for all $x>0$, here C is a constant, then

A) $g(x)$ is decreasing in $\left(0,\frac{\pi}{4}\right)$

B) $g(x)$ is increasing in $\left(0,\frac{\pi}{4}\right)$

C) $g+g\,'$ is increasing in $\left(0,\frac{\pi}{2}\right)$

D) $g-g'$ is decreasing in $\left(0,\frac{\pi}{2}\right)$

Answer: $g(x)$ is increasing in $\left(0,\frac{\pi}{4}\right)$

Solution: $\int\frac{x}{e^x+1}(\cos x-\sin x)dx+\int g(x)\frac{e^x+1-xe^x}{(e^x+1)^2}dx$

$=\frac{x}{e^x+1}(\sin x+\cos x)-\int\frac{e^x+1-xe^x}{(e^x+1)^2}(\sin x+\cos x)dx+\int g(x)\frac{e^x+1-xe^x}{(e^x+1)^2}dx$

By comparison, we get, $g(x)=\sin x+\cos x$

$\Rightarrow\ g(x)=\sqrt{2}\left[\sin\left(x+\frac{\pi}{4}\right)\right]$

Since $x\in\left(0,\frac{\pi}{4}\right)$ so, $x+\frac{\pi}{4}\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

So $g(x)$ is increasing in $\left(0,\frac{\pi}{4}\right)$

Q.15. If $\left|\vec{a}\right|=3,\left|\vec{b}\right|=4$ and $\theta\in\left[\frac{\pi}{4},\frac{\pi}{3}\right]$ is the angle between $\vec{a}$ and $\vec{b}$, then $\left|\left(\vec{b}-\vec{a}\right)\times\left(\vec{b}+\vec{a}\right)\right|^2+4\left(\vec{a}\cdot\vec{b}\right)^2$ is equal to

A) 248

B) 576

C) 144

D) 432

Answer: 576

Solution: $\left|\left(\vec{b}-\vec{a}\right)\times\left(\vec{b}+\vec{a}\right)\right|^2+4\left(\vec{a}\cdot\vec{b}\right)^2$

$=\left|\vec{b}\times\vec{b}+\vec{b}\times\vec{a}-\vec{a}\times\vec{b}-\vec{a}\times\vec{a}\right|^2+4\left(\vec{a}\cdot\vec{b}\right)^2$

$=4\left[\left|\vec{a}\times\vec{b}\right|^2+\left(\vec{a}\cdot\vec{b}\right)^2\right]=4\left|\vec{a}\right|^2\left|\vec{b}\right|^2=4\times9\times16=576$

Q.16. If $y=y(x)$ be the solution of given equation $y^2dx+\left(x^2-xy+y^2\right)dy=0$ and this curve also passes through $(1,1)$ and intersect the line at $y=\sqrt{3}x$ at $\left(\alpha,\sqrt{3}\alpha\right)$, then find the value of $\log_e\left(\sqrt{3}\alpha\right)$

A) $\frac{\pi}{2}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{6}$

D) $\frac{\pi}{12}$

Answer: $\frac{\pi}{12}$

Solution: Given $y^2dx+\left(x^2-xy+y^2\right)dy=0$

$\Rightarrow\ \frac{dx}{dy}=-\frac{\left(x^2-xy+y^2\right)}{y^2}\quad \ldots(\mathrm{i})$

Now let $x=vy\ \Rightarrow v+y\frac{dv}{dy}=\frac{dx}{dy}$

Now putting in equation (i) we get

$\Rightarrow v+y\frac{dv}{dy}=-\left(v^2-v+1\right)$

$\Rightarrow\ y\frac{dv}{dy}=-v^2-1$

$\Rightarrow \frac{-dv}{1+v^2}=\frac{dy}{y}$

Now Integrating Both side

$\Rightarrow -\int\frac{dy}{1+v^2}=\int\frac{dy}{y}$

$\Rightarrow -\tan^{-1}v=\log|y|+c\Rightarrow -\tan^{-1}\left(\frac{x}{y}\right)=\log|y|+c$

Also this curve passes through $(1,1)$

$\Rightarrow\ \tan^{-1}1=\log|1|+c\Rightarrow\ \ c=-\frac{\pi}{4}$

$\Rightarrow -\tan^{-1}\frac{x}{y}=\log|y|-\frac{\pi}{4}$

Now putting $y=\sqrt{3}x$ and $x=\alpha$ we get

$=-\tan^{-1}\frac{x}{\sqrt{3}x}=\log\left|\sqrt{3}\alpha\right|-\frac{\pi}{4}=-\tan^{-1}\frac{1}{\sqrt{3}}=\log\left(\sqrt{3}\alpha\right)-\frac{\pi}{4}$

$=-\frac{\pi}{6}+\frac{\pi}{4}=\log\left|\sqrt{3}\alpha\right|$

$\Rightarrow \log\left|\sqrt{3}\alpha\right|=\frac{\pi}{12}$

Q.17. The number of 3-digit numbers which are divisible by 7 is

A) 128

B) 112

C) 127

D) 114

Answer: 128

Solution: The first 3-digit number which is a multiple of 7 is 105

Last 3-digit number which is a multiple of 7 is 994

Now using n^{th} term formula $994 = 105 + (n-1)7$

$\Rightarrow 994 - 105 = (n-1)\ 7$

$\Rightarrow n - 1 = \frac{889}{7} \Rightarrow n - 1 = 127$

So $n = 128$

Q.18. If two common tangents of $y^2 = x$ and $x^2 + y^2 = 2$ has the slopes as m_1 and m_2, then $8\left|m_1 m_2\right|$ is

A) $3\sqrt{2} - 4$

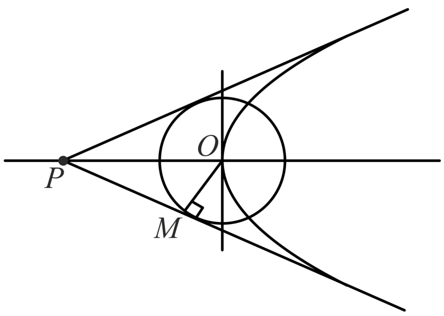
B) $6\sqrt{2} - 4$

C) $\sqrt{2} - 4$

D) $3\sqrt{2}$

Answer: $3\sqrt{2} - 4$

Solution:



Equation of the tangent to the parabola can be written as $y = mx + \left(\frac{1}{4m}\right)$ or $y - mx - \frac{1}{4m} = 0$

Now, the perpendicular distance from $(0, 0)$ to the tangent is equal to the radius of the given circle, so

$OM = \sqrt{2} \Rightarrow \left|\frac{\frac{-1}{4m}}{\sqrt{1+m^2}}\right| = \sqrt{2}$

$\Rightarrow 2\left(16m^2\right)\left(1 + m^2\right) = 1 \quad \Rightarrow 32m^4 + 32m^2 - 1 = 0$

$\Rightarrow \quad m^2 = \frac{-2^5 + 2^3\sqrt{16+2}}{2 \times 2^5}$ Ignoring the negative sign as it is square function.

Now $\left|8m_1 m_2\right| = \left|8m^2\right|$

$= \left|\frac{-2^5 + 2^3\sqrt{16+2}}{2^3}\right| = 3\sqrt{2} - 4$