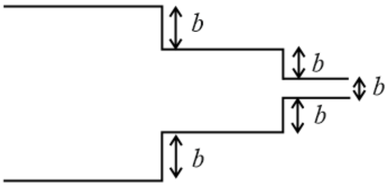
JEE Main Exam 2022 - Session 1

27 June 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. Six capacitor plates are arranged as shown. The area of each plate is A. The capacitance of the arrangement is



A) $\frac{15}{28}\left(\varepsilon_0 \frac{A}{b}\right)$

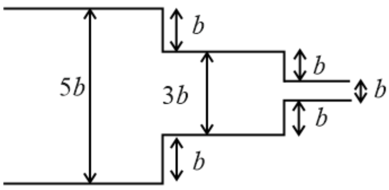
B) $\frac{23}{15}\left(\varepsilon_0 \frac{A}{b}\right)$

C) $\frac{15}{22}\left(\varepsilon_0 \frac{A}{b}\right)$

D) $\frac{17}{23}\left(\varepsilon_0 \frac{A}{b}\right)$

Answer: $\frac{23}{15}\left(\varepsilon_0 \frac{A}{b}\right)$

Solution:



Here the system can be considered as three capacitors in parallel.

So, net capacitance is

$C = \frac{\varepsilon_0 A}{5b} + \frac{\varepsilon_0 A}{3b} + \frac{\varepsilon_0 A}{b}$

$C = \frac{\varepsilon_0 A}{b}\left[\frac{1}{5} + \frac{1}{3} + \frac{1}{1}\right]$

$C = \frac{23\varepsilon_0 A}{15b}$

Q.2. Deuteron and proton enter a magnetic field perpendicularly having equal kinetic energy. Find $\frac{R_d}{R_p}$, where R_d and R_p are radius of circular trajectories of deuteron and proton respectively.

A) $\sqrt{2}$

B) $\frac{1}{\sqrt{2}}$

C) 2

D) $\frac{1}{2}$

Answer: $\sqrt{2}$

Solution: As we know, radius of circular path followed by charged particle under perpendicular magnetic field is given by,

$R=\frac{mv}{qB}$ & $KE=\frac{1}{2}mv^2$

Therefore,

$\Rightarrow R=\frac{\sqrt{2\mathrm{m}(KE)}}{qB}$ Then, $R\propto\frac{\sqrt{m}}{q}$

$\Rightarrow \frac{R_d}{R_p}=\sqrt{\frac{m_d}{m_p}}\times\frac{q_p}{q_d}=\sqrt{\frac{2}{1}}\times\frac{1}{1}=\sqrt{2}$

Q.3. Transistor works like a switch in:

A) Active region

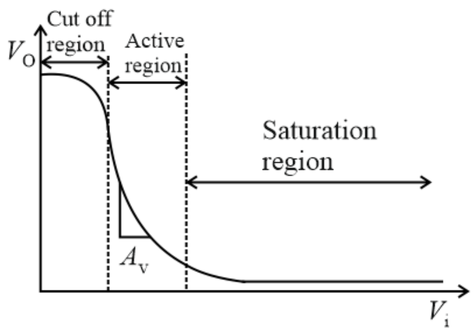
B) Cutoff and saturation region

C) Cutoff region only

D) Saturated region only

Answer: Cutoff and saturation region

Solution:



The saturation zone and cut-off area are known as the transistor switch's working regions. This implies that, by switching between its "top-off" (saturation) and "absolute OFF," the transistor is used as a switch.

Q.4. A wave propagates from one medium to another medium. Out of the parameters: wavelength, frequency and speed of the wave, the parameters that change are

A) Wavelength and frequency

B) Frequency and speed

C) Wavelength and speed

D) All the three

Answer: Wavelength and speed

Solution: In refraction from one medium to another, the speed and wavelength gets changed. The frequency remains unchanged.

Q.5. An ideal diatomic gas is expanded isobarically and work done in the process is $400\ \mathrm{J}$. Find the heat given to the gas in this process.

A) 160 J

B) 700 J

C) 320 J

D) 1400 J

Answer: 1400 J

Solution: For isobaric process: $W = nR\Delta T \Rightarrow 400 = nR\Delta T$

Also: $Q = nC_p\Delta T \ \Rightarrow Q = n\left(\frac{7R}{2}\right)\Delta T$ ($C_p = \frac{7R}{2}$, for diatomic gas)

$Q = \frac{7}{2} \times 400 = 1400 \text{ J}$

Q.6. A block of mass 1 kg hanging vertically on 5 m long rope, if a force of 30 N is applied at the centre of the rope horizontally, what is the angle made by the upper half of the rope with the vertical in equilibrium?

A) $\tan^{-1}(3)$

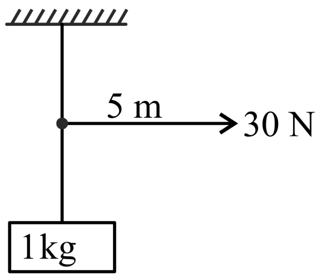
B) $\tan^{-1}(4)$

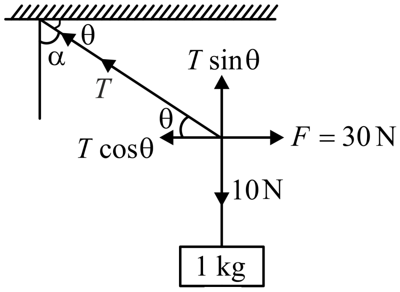
C) $\tan^{-1}(5)$

D) $\tan^{-1}(6)$

Answer: $\tan^{-1}(3)$

Solution:





$\Rightarrow T\sin\theta = 10$ and $T\cos\theta = 30$

Therefore, $\tan\theta = \frac{1}{3}$

Then, $\tan\alpha = 3 \ \Rightarrow \alpha = \tan^{-1}(3)$

Q.7. Four particles of mass m are at the corners of square(side length, a) and one particle of mass M is at the centre. Find gravitational potential energy of system.

A) $-\frac{\left(4+\sqrt{2}\right)Gm^2}{a}-\frac{4\sqrt{2}GMm}{a}$

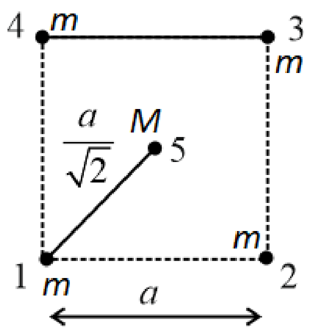
B) $\frac{\left(4+\sqrt{2}\right)Gm^2}{a}+\frac{4\sqrt{2}GMm}{a}$

C) $-\frac{\left(4+\sqrt{2}\right)Gm^2}{a}+\frac{4\sqrt{2}GMm}{a}$

D) $\frac{\left(4+\sqrt{2}\right)Gm^2}{a}-\frac{4\sqrt{2}GMm}{a}$

Answer: $-\frac{\left(4+\sqrt{2}\right)Gm^2}{a}-\frac{4\sqrt{2}GMm}{a}$

Solution:



$$U_{system}=(U_{12}+U_{23}+U_{34}+U_{41})+(U_{13}+U_{24})+(U_{15}+U_{25}+U_{35}+U_{45})$$

$$=-\frac{4Gmm}{a}-\frac{2Gmm}{a\sqrt{2}}-\frac{4\sqrt{2}GMm}{a}$$

$$=-\frac{\left(4+\sqrt{2}\right)Gm^2}{a}-\frac{4\sqrt{2}GMm}{a}$$

Q.8. A particle executing SHM is given by $x=\sin\pi\left(t+\frac{1}{3}\right)$, then find its velocity at $t=1$s.

A) $\frac{-\pi}{2}$

B) $\frac{\pi}{2}$

C) $\frac{\pi}{3}$

D) $\frac{-\pi}{3}$

Answer: $\frac{-\pi}{2}$

Solution: Given, $x=\sin\left(\pi t+\frac{\pi}{3}\right)$

Velocity $v=\frac{\mathrm{d}x}{\mathrm{d}t}=\pi\cos\left(\pi t+\frac{\pi}{3}\right)$

Now, at $t=1$ s, $v=\pi\cos\left(\pi+\frac{\pi}{3}\right)$

$v=\frac{-\pi}{2}$

Q.9. Dimensions of $\text{Pascal}\times\text{sec}$ is

A) $ML^{-1}T^{-3}$

B) MLT^{-3}

C) $ML^{-1}T^{-1}$

D) $ML^{-2}T^{-2}$

Answer: $ML^{-1}T^{-1}$

Solution: Unit of Pascal is $\mathrm{N\ m^{-2}}$.

So, dimension of $\text{Pascal}\times\text{sec}$ is

$\left[\mathrm{N\ m^{-2}}\right]\times[\mathrm{s}]=\frac{MLT^{-2}}{L^2}\times T=ML^{-1}T^{-1}$

Q.10. A lens is cut into two halves horizontally. One part is cut into two equal halves vertically. Let If P_1 be the power of the half lens and P_2 and P_3 be the power of quarter lenses. Find incorrect relation.

A) $P_2=\frac{P_1}{2}$

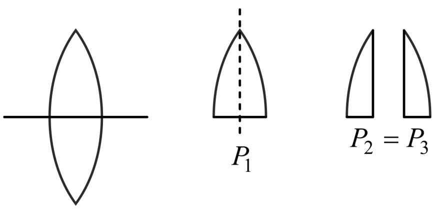
B) $P_1=\frac{P_2}{2}$

C) $P_3=\frac{P_1}{2}$

D) None of these

Answer: $P_1=\frac{P_2}{2}$

Solution:

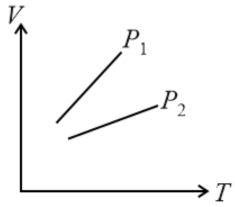


Power of half lens is $P_1=P_2+P_3$.

Here, $P_2=P_3$

So, $P_1=2P_2$ or $P_1=2P_3$

Q.11. From the following $V-T$ graph, we can conclude



A) $P_1 < P_2$

B) $P_1 > P_2$

C) $P_1 = P_2$

D) No relationship can be obtained

Answer: $P_1 < P_2$

Solution: We know that the ideal gas equation of gas is

$PV = nRT$

Keeping the temperature constant, we have, $V \propto \frac{1}{P}$

As seen from graph, $V_1 > V_2$, so $P_1 < P_2$.

Q.12. A spring with spring constant K and length l was attached to a mass m and rotated about its axis at other end (in horizontal plane) with angular velocity ω. Find the elongation in spring.

A) $\frac{K-m\omega^2 l}{m\omega^2}$

B) $\frac{K+m\omega^2 l}{m\omega^2}$

C) $\frac{m\omega^2 l}{K-m\omega^2}$

D) $\frac{m\omega^2 l}{K+m\omega^2}$

Answer: $\frac{m\omega^2 l}{K-m\omega^2}$

Solution: Spring force is providing the required centripetal force.

Therefore,

$Kx = m\omega^2 (l + x)$

$\Rightarrow x = \frac{m\omega^2 l}{K-m\omega^2}$

Q.13. A charge q is placed at centre of non-conducting hemisphere, then the flux through curved surface area is

A) $\frac{q}{\varepsilon_0}$

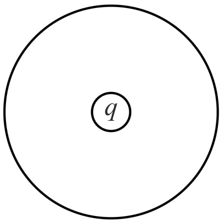
B) $\frac{q}{2\varepsilon_0}$

C) $\frac{2q}{\varepsilon_0}$

D) $\frac{\pi q}{4\varepsilon_0}$

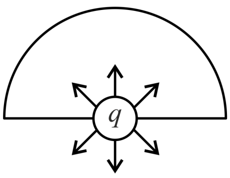
Answer: $\frac{q}{2\varepsilon_0}$

Solution:



Assume a complete sphere, then flux due to a charge placed at the center of sphere is $\phi=\frac{q}{\varepsilon_0}$

Flux through hemisphere surface $\phi=\frac{1}{2}\times\frac{q}{\varepsilon_0}=\frac{q}{2\varepsilon_0}$



Q.14. Which one in not showing the dimension of time

A) $\sqrt{LC}$

B) $\frac{L}{R}$

C) CR

D) $\frac{C}{R}$

Answer: $\frac{C}{R}$

Solution: Dimensions of L is $\left[ML^2T^{-2}A^{-2}\right]$

Dimensions of C is $\left[M^{-1}L^{-2}T^4A^2\right]$

and dimensions of R is $\left[ML^2T^{-3}A^{-2}\right]$

Clearly, dimensions of $\sqrt{LC}=[T]$,

dimensions of $\frac{L}{R}=[T]$,

dimensions of $CR=[T]$ and

dimensions of $\frac{C}{R}=\left[M^{-2}L^{-4}T^7A^4\right]$

Q.15. A rod of length $20\ \mathrm{cm}$ is moving with speed $10\ \mathrm{m\,s^{-1}}$ in uniform magnetic field. Horizontal component of earth's magnetic field is $0.3\mathrm{T}$ and angle of dip is 60°. Find the emf induced across the ends of the rod.

A) $1.039\mathrm{V}$

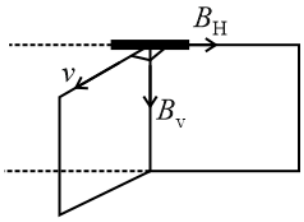
B) $1.545\ \mathrm{V}$

C) $2.169\ \mathrm{V}$

D) $2.653\ \mathrm{V}$

Answer: $1.039\mathrm{V}$

Solution:



Emf produced is $\varepsilon = B_V L_\perp v_\perp$

We know $\frac{B_V}{B_H} = \tan 60^\circ$, so $B_V = \sqrt{3} \times 0.3\ \mathrm{T}$

Then, $\varepsilon = \frac{3\sqrt{3}}{10} \times \frac{1}{5} \times 10 = \sqrt{3} \times 0.6 = 1.039\ \mathrm{V}$

Q.16. A particle is moving in a vertical circle tied to string. Velocity at bottom is u. Magnitude of change in velocity when string becomes horizontal is $v = \sqrt{x\left(u^2 - gl\right)}$, find value of x.

A) 2

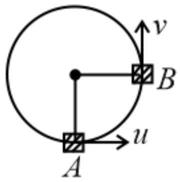
B) 3

C) 4

D) 5

Answer: 2

Solution:



Applying conservation of mechanical energy at the point A and B, we get $\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + mgL$

$\Rightarrow v = \sqrt{u^2 - 2gL}$

Now in vector form. $\vec{v}_i = u\ \hat{\mathrm{i}}$ and $\vec{v}_f = v\ \hat{\mathrm{j}}$

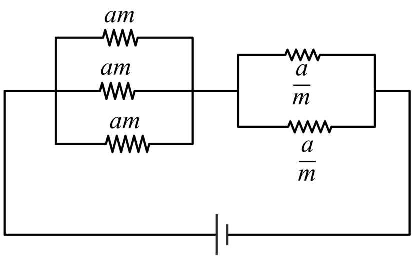
Therefore, change in velocity will be, $\Delta\vec{V} = v\hat{\mathrm{j}} - u\hat{\mathrm{i}}$ and $\left|\Delta\vec{V}\right| = \sqrt{u^2 + v^2}$

$\Rightarrow \Delta V = \sqrt{u^2 + \left(u^2 - 2gL\right)}$

$= \sqrt{2\left(u^2 - gL\right)}$

Hence, $x = 2$

Q.17. A network of resistors is shown below. Find the value of m for minimum resistance of the network



A) $\sqrt{\frac{3}{2}}$

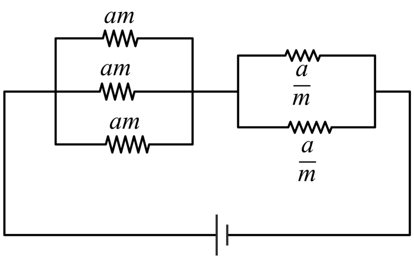
B) $\sqrt{\frac{2}{3}}$

C) $\sqrt{\frac{5}{4}}$

D) $\sqrt{\frac{4}{5}}$

Answer: $\sqrt{\frac{3}{2}}$

Solution:



Left part of the resistors are connected in parallel, hence $R_1 = \frac{am}{3}$. Similarly for the right part of the resistors we can write, $R_2 = \frac{a}{3m}$.

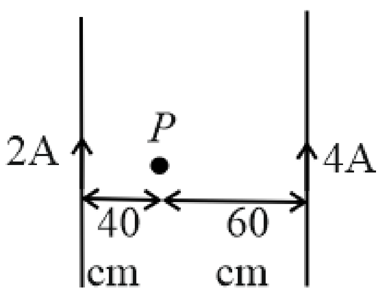
Therefore, equivalent resistance will be,

$R = \frac{am}{3} + \frac{a}{2\mathrm{m}}$

Differentiating both sides w.r.t m to observe critical points, we get $\frac{\mathrm{d}R}{\mathrm{d}m} = \frac{a}{3} + \frac{a(-1)}{2m^2} = 0$

$\Rightarrow \frac{a}{3} = \frac{a}{2\mathrm{m}^2} \Rightarrow m = \sqrt{\frac{3}{2}}$

Q.18. A point charge $q = 2\ \mathrm{C}$ is projected with the velocity $\vec{v} = 2\hat{\imath} + 3\hat{\jmath}$ from point P. The magnetic force acting on the charge at this moment is



A) 2.4×10^{-6} N

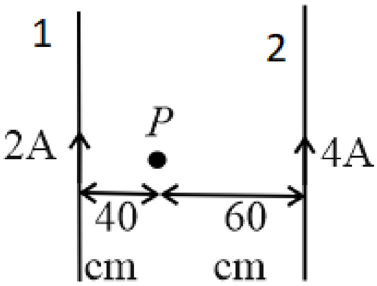
B) 3.2×10^{-6} N

C) 4.2×10^{-6} N

D) 3.6×10^{-6} N

Answer: 2.4×10^{-6} N

Solution:



Magnetic field due to left wire at point P, $\vec{B}_1 = \frac{\mu_0 I_1}{2\pi d_1}\left(-\hat{k}\right)$

Magnetic field due to right wire at point P, $\vec{B}_2 = \frac{\mu_0 I_2}{2\pi d_2}\left(\hat{k}\right)$

$\therefore \vec{B}_P = \vec{B}_1 + \vec{B}_2$

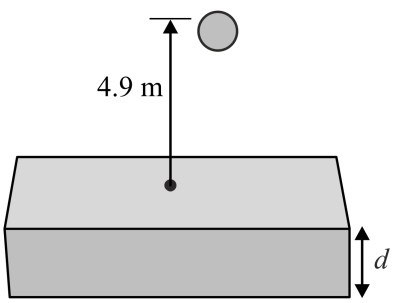
$\Rightarrow \vec{B}_P = \frac{\mu_0}{2\pi}\left(\frac{I_2}{d_2} - \frac{I_1}{d_1}\right)\left(\hat{k}\right)$

$\vec{B}_P = 2 \times 10^{-7}\left(\frac{4}{0.6} - \frac{2}{0.4}\right) = \frac{10}{3} \times 10^{-7}$ T

$F = q(vB) = 2 \times \sqrt{4+9} \times \frac{10}{3} \times 10^{-7}$

$\Rightarrow F_m = 2 \times \sqrt{13} \times \frac{10}{3} \times 10^{-7} \approx 2.4 \times 10^{-6}$ N

Q.19. A particle is released from a height of 4.9 m above the surface of water as shown. The particle enters the water and moves with constant velocity and reaches bottom of the tank in 4 s after the release, the value of d is $\left(g = 9.8 \text{ m s}^{-2}\right)$



A) 34.3 m

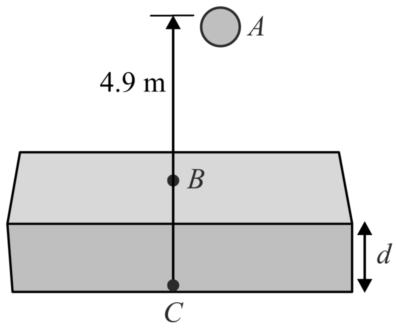
B) 19.8 m

C) 38.2 m

D) 29.4 m

Answer: $29.4 \mathrm{~m}$

Solution:



Using $s=ut+\frac{1}{2}at^2$

Initially, $u=0$, so time taken to reach from A to B is $t=\sqrt{\frac{2s}{g}}=\sqrt{\frac{2\times4.9}{9.8}}=1 \mathrm{~s}$

Now, using $v^2=u^2+2as$, we have

$v=\sqrt{2 \mathrm{gh}}=\sqrt{2\times9.8\times4.9}=9.8 \mathrm{~m~s}^{-1}$

$\because \ t_2=(4-1) \mathrm{~s}=3 \mathrm{~s}$

$\therefore \ \ d=t_2\times v=3\times9.8=29.4 \mathrm{~m}$

Q.20. An electron makes a transition from lower orbit showing energy E_1 to higher orbit having energy E_2 by absorbing a photon of frequency 'f' then

A) $f=\frac{(E_2-E_1)}{h}$

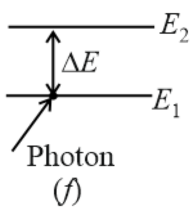
B) $f=\frac{E_1-E_2}{h}$

C) $f=\frac{E_1+E_2}{h}$

D) $f=\frac{E_2+E_1}{h}$

Answer: $f=\frac{(E_2-E_1)}{h}$

Solution:



According to Bohr's model, when electron makes the transition from one orbit to another, its energy can be given as

$\Delta E=E_{\text{final}}-E_{\text{initial}}=hf$, here, h is Planck's constant.

Then, $f=\frac{E_2-E_1}{h}$

Q.21. Three charged particles having charge q each are suspended by the means of thread from a common point. In equilibrium they make an equilateral triangle of side l. The electrostatic force on one of the charges is

A) $\frac{\sqrt{3}q^2}{4\pi\varepsilon_0 l^2}$

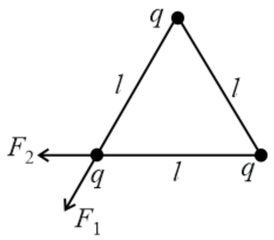
B) $\frac{2q^2}{4\pi\varepsilon_0 l^2}$

C) $\frac{q^2}{8\pi\varepsilon_0 l^2}$

D) $\frac{2\sqrt{3}q^2}{4\pi\varepsilon_0 l^2}$

Answer: $\frac{\sqrt{3}q^2}{4\pi\varepsilon_0 l^2}$

Solution:



We have $F_1 = F_2 = \frac{q^2}{4\pi\varepsilon_0 l^2} = F$

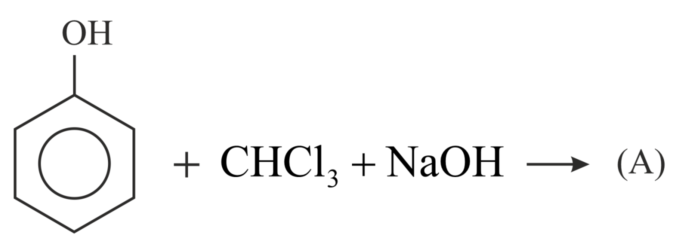
Resultant electrostatics force $F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60^\circ}$

$= \sqrt{F^2 + F^2 + \left(2F^2 \times \frac{1}{2}\right)} = \sqrt{3F^2}$

$\Rightarrow F_R = \sqrt{3} \times \frac{q^2}{4\pi\varepsilon_0 l^2}$

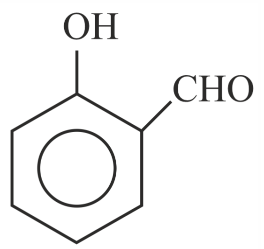
Section B: Chemistry

Q.1.

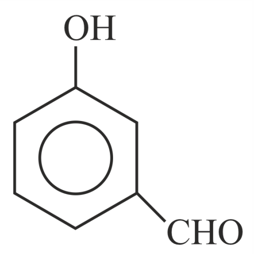


Product (A) is :-

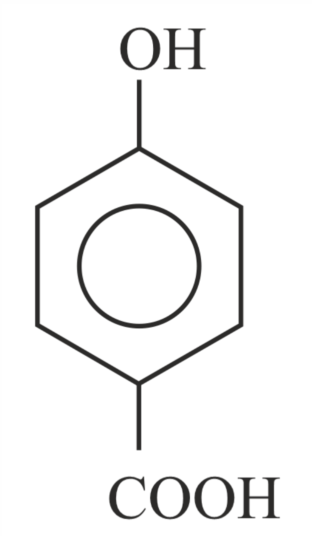
A)



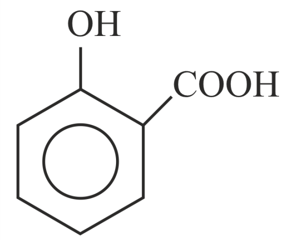
B)



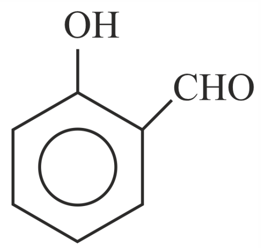
C)



D)



Answer:

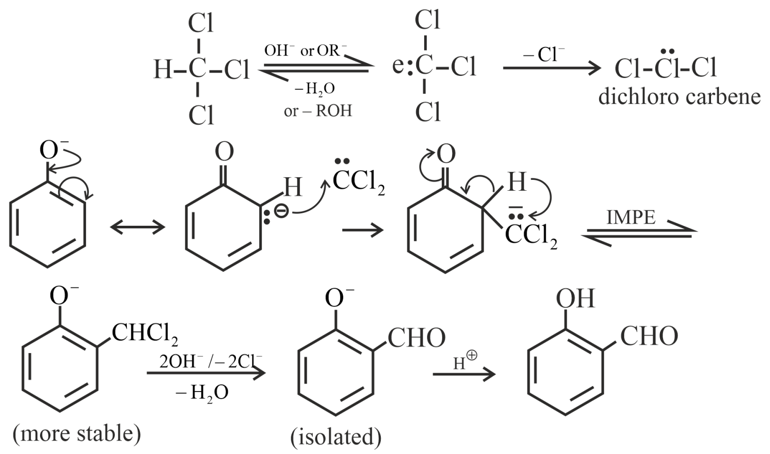


Solution: This reaction is Riemer Tieman reaction in which formylation of phenol at ortho position take place and salicylaldehyde is formed.

When phenol react with chloroform and strong base then salicylaldehyde is formed.

In this reaction, dichloro carbene is formed as intermediate.

This reaction is example of electrophilic substitution, in which dichloro carbene act as electrophile.



Q.2. The correct order of ionic size of $N^{3-}, Na^{+}, F^{-}, Mg^{2+}$ and O^{2-} is :

A) $Mg^{2+} > Na^{+} > F^{-} > O^{2-} < N^{3-}$

B) $N^{3-} < F^{-} > O^{2-} > Na^{+} > Mg^{2+}$

C) $Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

D) $N^{3-} > O^{2-} > F^{-} > Na^{+} < Mg^{2+}$

Answer: $Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

Solution: These are all isoelectronic species. More the negative charge on the species, more is the size of that ion and more the positive charge on the species, less is the size of that ion.

Q.3. Chlorine nitrate on hydrolysis produces X along with HNO_3 and chlorine nitrate on reaction with HCl produces Y along with HNO_3. X and Y respectively, are:

A) $HOCl,\ HClO_2$

B) $HOCl,\ Cl_2$

C) $HCl,\ Cl_2$

D) $HOCl,\ HClO_3$

Answer: $HOCl,\ Cl_2$

Solution: The reactions are:

$ClONO_2(g) + H_2O(g) \rightarrow HOCl(g) + HNO_3(g)$

$ClONO_2(g) + HCl(g) \rightarrow Cl_2(g) + HNO_3(g)$

X and Y are $HOCl$ and Cl_2 respectively.

Q.4. Match the acid radicals present in column I with their characteristic observation in column II.

	Column I		Column II
i	CO_3^{2-}	P	Brisk Effervescence
ii	NO_3^-	Q	White precipitate
iii	SO_4^{2-}	R	Brown ring
iv	S^{2-}	S	Rotten egg smell

A) i-S, ii-R, iii-Q, iv-P

B) i-P, ii-Q, iii-R, iv-S

C) i-P, ii-R, iii-Q, iv-S

D) i-P, ii-R, iii-S, iv-Q

Answer: i-P, ii-R, iii-Q, iv-S

Solution: When small amount of carbonate salt is treated with dilute H_2SO_4 or dilute HCl, A colourless, odourless gas (CO_2) is evolved with brisk effervescence

When small amount of the following salts are treated with dilute H_2SO_4 or dilute HCl the following observations are seen.

Carbonate salt $\left(CO_3^{2-}\right)$	Brisk effervescence of CO_2 gas, $CO_3^{2-} + H_2SO_4 \rightarrow SO_4^{2-} + H_2O + CO_2 \uparrow$
Sulphide salt (S^{-2})	A colourless gas (H_2S) with rotten egg smell

When small amount of the following $Na_2S + H_2SO_4 \rightarrow Na_2SO_4 + H_2S \uparrow$

When salt are treated with conc. H_2SO_4 in a dry test tube the following observation are made

Nitrate $\left(NO_3^-\right)$ salt	A light brown gas having pungent smell is evolved

NO_3^- is confirmed by using brown ring test.

SO_4^{-2} (sulphate) salts on reaction with $BaCl_2$ give white ppt of $BaSO_4$ is formed.

$Na_2SO_4 + BaCl_2 \rightarrow BaSO_4 \downarrow + 2NaCl$

Q.5. Statement 1: In extraction of gold, the oxidation state of gold in the cyanide complex formed is $+3$

Statement 2: When the cyanide complex is treated with zinc, Zn gets oxidised to $+2$ state.

A) Statement 1 and statement 2 both are correct

B) Statement 1 is correct but statement 2 is wrong

C) Statement 1 is wrong but statement 2 is correct

D) Statement 1 and statement 2 both are wrong

Answer: Statement 1 is wrong but statement 2 is correct

Solution: $4\ \mathrm{Au}(\mathrm{s}) + 8\,\mathrm{CN}^-(\mathrm{aq}) + 2\mathrm{H_2O}(\mathrm{aq}) + \mathrm{O_2}(\mathrm{g}) \rightarrow 4\left[\mathrm{Au}(\mathrm{CN})_2\right]^-(\mathrm{aq}) + 4\,\mathrm{OH}^-(\mathrm{aq})$

The oxidation state of gold in the complex is +1.

$2\left[\mathrm{Au}(\mathrm{CN})_2\right]^-(\mathrm{aq}) + \mathrm{Zn}(\mathrm{s}) \rightarrow 2\,\mathrm{Au}(\mathrm{s}) + \left[\mathrm{Zn}(\mathrm{CN})_4\right]^{2-}(\mathrm{aq})$

The oxidation state of zinc in the complex is +2.

Q.6. Arrange the following coordination complexes in the increasing order of their magnetic moment.

(i) $[\mathrm{Fe(CN)_6}]^{3-}$
(ii) $[\mathrm{FeF_6}]^{3-}$
(iii) $[\mathrm{Mn\,Cl_6}]^{3-}$
(iv) $[\mathrm{Mn(CN)_6}]^{3-}$

A) $\mathrm{i} < \mathrm{ii} < \mathrm{iii} < \mathrm{iv}$

B) $\mathrm{i} < \mathrm{iv} < \mathrm{iii} < \mathrm{ii}$

C) $\mathrm{iv} < \mathrm{ii} < \mathrm{i} < \mathrm{iii}$

D) $\mathrm{ii} < \mathrm{i} < \mathrm{iv} < \mathrm{iii}$

Answer: $\mathrm{i} < \mathrm{iv} < \mathrm{iii} < \mathrm{ii}$

Solution: In $[\mathrm{Fe(CN)_6}]^{3-}$ Fe^{+3} is in $3\mathrm{d}^5$ and there is strong field ligand so inner orbital complex will be formed. $\mathrm{n} = 1$ $\mathrm{M} = \sqrt{3}$

In $[\mathrm{Fe(F)_6}]^{3-}$, Fe^{+3} is in $3\mathrm{d}^5$ and there is weak field ligand so outer orbital complex will be formed. $\mathrm{n} = 5$ $\mathrm{M} = \sqrt{35}$

In $[\mathrm{Mn(Cl)_6}]^{3-}$, Mn^{+3} is in $3\mathrm{d}^4$ configuration and Cl is a weak field ligand so outer orbital complex will be formed. $\mathrm{n} = 4$ $\mathrm{M} = \sqrt{24}$

In $[\mathrm{Mn(CN)_6}]^{3-}$, Mn^{3+} is in $3\mathrm{d}^4$ and there is a strong field ligand so inner orbital complex will be formed. $\mathrm{n} = 2$ $\mathrm{M} = \sqrt{8}$

Q.7. The Gas evolved when ammonium chloride react with sodium nitrite?

A) $\mathrm{NH_3}$

B) $\mathrm{N_2}$

C) $\mathrm{Cl_2}$

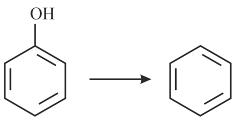
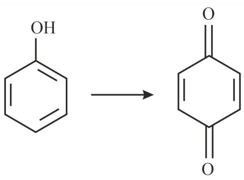
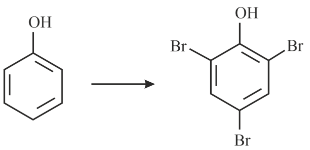
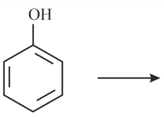
D) $\mathrm{N_2O}$

Answer: $\mathrm{N_2}$

Solution: In the laboratory, dinitrogen is prepared by treating an aqueous solution of ammonium chloride with sodium nitrite.

$$NH_4Cl(aq) + NaNO_2(aq) \rightarrow N_2(g) + 2H_2O(l) + NaCl(aq)$$

Q.8.

	Column I		Column II
(i)		(P)	$Na_2Cr_2O_7/H^+$
(ii)		(Q)	Zn
(iii)		(R)	$FeCl_3$
(iv)	 Violet coloration	(S)	Br_2/H_2O

A) (i)-(Q); (ii)-(P); (iii)-(S); (iv)-(R)

B) (i)-(P); (ii)-(Q); (iii)-(R); (iv)-(S)

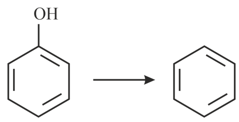
C) (i)-(Q); (ii)-(R); (iii)-(S); (iv)-(P)

D) (i)-(R); (ii)-(S); (iii)-(P); (iv)-(Q)

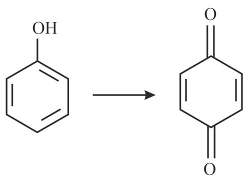
Answer: (i)-(Q); (ii)-(P); (iii)-(S); (iv)-(R)

Solution: (i)-(Q); (ii)-(P); (iii)-(S); (iv)-(R)

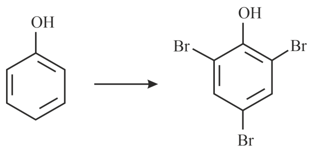
Reduction of phenol take place in presence of Zn metal.



Oxidation of phenol take place in benzoquinone in presence of oxidising agent like $Na_2Cr_2O_7/H^+$.

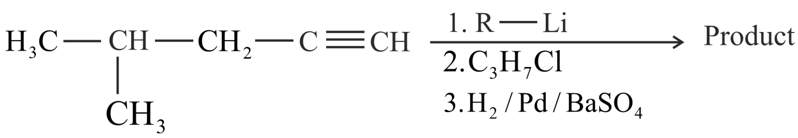


Bromination of phenol take place with Br_2/H_2O to get 2,4,6-tribromophenol.

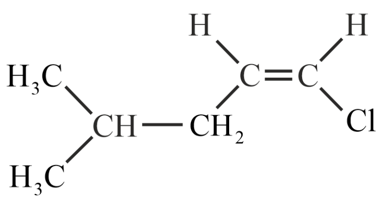


Phenol gives violet colour with $FeCl_3$

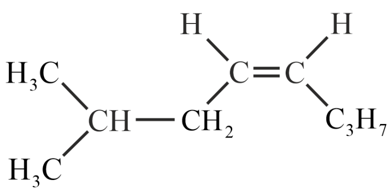
Q.9. The product of the following reaction is:



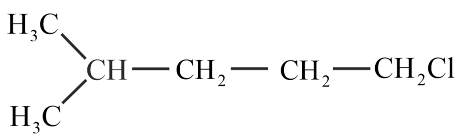
A)



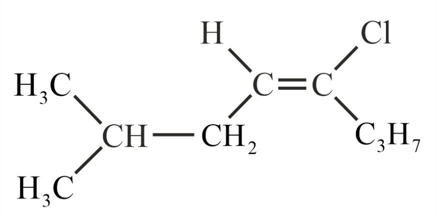
B)



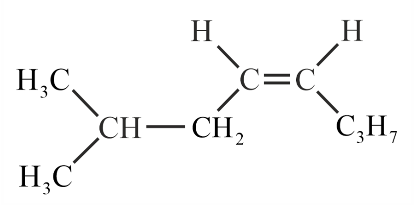
C)



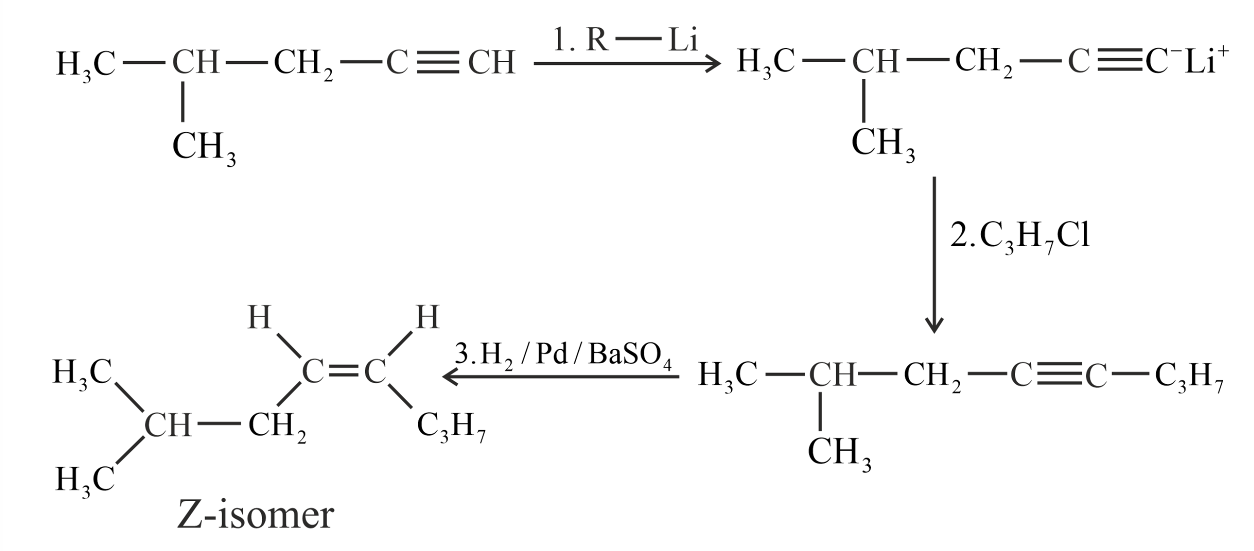
D)



Answer:



Solution:



Catalytic hydrogenation of alkyne is syn addition reaction so Z-isomer is formed.

Q.10. Match the following?

	Column I		Column II
a	Tranquillizer	1	Reduces pain
b	Analgesic	2	Reduces acidity
c	Antacid	3	Reduces fever
d	Antipyretic	4	Reduces stress

A) a-4, b-1, c-2, d-3

B) a-1, b-4, c-3, d-2

C) a-2, b-4, c-1, d-3

D) a-4, b-1, c-3, d-2

Answer: a-4, b-1, c-2, d-3

Solution: Analgesics reduce or abolish pain without causing impairment of consciousness, mental confusion, incoordination, paralysis or some other disturbances of nervous system.

Tranquillizers are a class of chemical compounds used for the treatment of stress, and mild or even severe mental diseases. These relieve anxiety, stress, irritability or excitement by inducing a sense of well-being. They form an essential component of sleeping pills. Treatment for acidity was administration of antacids, such as sodium hydrogen carbonate or a mixture of aluminium and magnesium hydroxide. However, excessive hydrogen carbonate can make the stomach alkaline and trigger the production of even more acid.

An antipyretic is a substance that reduces fever. Antipyretics cause the hypothalamus to override a prostaglandin-induced increase in temperature.

Q.11. Correct order of increasing intermolecular hydrogen bond strength is

A) $CH_4 < NH_3 < HCN$

B) $HCN < H_2O < NH_3$

C) $HCN < CH_4 < NH_3$

D) $CH_4 < HCN < NH_3$

Answer: $CH_4 < NH_3 < HCN$

Solution: Intermolecular hydrogen bond strength is directly proportional to the difference between the electron negativity of element and hydrogen atom.

sp hybrid carbon has more electronegativity than N atom.

Hence the correct order of intermolecular hydrogen bond strength is $CH_4 < NH_3 < HCN$

Q.12. Which of the following statement is correct about Buna-N?

A) Monomers of Buna-N are styrene and butadiene.

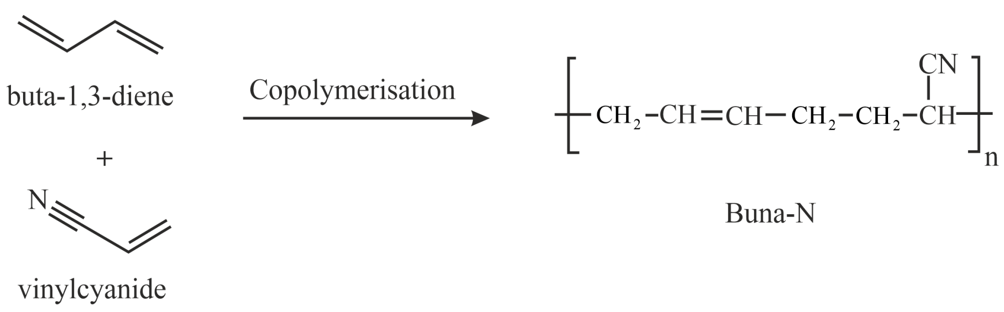
B) Monomers of Buna-N are butadiene and vinylcyanide.

C) Buna-N is a condensation polymerisation.

D) Buna-N is a natural rubber.

Answer: Monomers of Buna-N are butadiene and vinylcyanide.

Solution: Buna-N is formed by the co-polymerization of butadiene and vinylcyanide.



Q.13. Calculate the pH of $0.05\ \mathrm{M}\ \mathrm{NaOH}$ solution.

A) 1.3

B) 12.7

C) 5

D) 9

Answer: 12.7

Solution: Concentration of NaOH given is $0.05 M$

$\left[OH^-\right] = 0.05 = 5 \times 10^{-2}$

$pOH = -\log\left[OH^-\right]$

$= -\log\left(5 \times 10^{-2}\right) = -0.699 + 2$

$= 1.3010$
$pH = 14 - pOH$

$= 14 - 1.3010 = 12.699$

Q.14. The gas released in the following reaction is:

$PCl_5 + NH_4Cl \rightarrow Gas + side\ \ product$

A) NCl_3

B) PCl_3

C) HCl

D) N_2

Answer: HCl

Solution: Phosphorous(V) chloride react with ammonium chloride as follows:

$3 PCl_5 + 3 NH_4Cl \rightarrow (PNCl_2)_3 + 12 HCl$

Phosphorus(V) chloride react with ammonium chloride to produce poly(dichlorophosphazene). This reaction takes place at a temperature near 135°C, in the liquid tetrachloroethane.

Q.15. How many of the following sets of quantum numbers are possible?

	n	l	m
1)	3	3	-2
2)	3	2	$+1$
3)	3	2	-2
4)	3	3	-1

A) $1, 2, 3$

B) $1, 4$

C) $2, 3, 4$

D) $2, 3$

Answer: $2, 3$

Solution: For a given 'n' value l will have 0 to $(n-1)$ values and for a given 'l' value m will have $-m$ to $+m$ values

For $n=3,\ l=3$, is not possible as n and m have same values.

So, set $1,\ 4$ are not possible

If $n=3$, l can have 0 to 2 values and 'm' can have -2 to $+2$ values,

Hence, set 2 and 3 are possible.

Q.16. ASSERTION: Fluorine forms only one oxoacid.

REASON: Fluorine is smallest among all halogens and most electro-negative.

A) Both assertion and reason are correct and reason is the correct explanation of assertion.

B) Both assertion and reason are correct and but reason is not the correct explanation of assertion.

C) Assertion is correct and reason is incorrect

D) Assertion is incorrect and reason is correct

Answer: Both assertion and reason are correct and reason is the correct explanation of assertion.

Solution: Due to high electronegativity and small size, fluorine forms only one oxoacid.The other halogens form several oxoacids.

Q.17. In $4d$ orbital the number of angular and radial nodes are respectively

A) $2,\ 2$

B) $2,\ 1$

C) $1,\ 2$

D) $3,\ 2$

Answer: $2,\ 1$

Solution: Number of Angular nodes for an orbital is equal to its l value,

$l=$ azimuthal quantum number

Number of Radial nodes $=n-l-1$,

$n=$ Principal quantum number

In $4d$ orbital, Angular nodes $=2$

as 'l' value of d orbital $=2$

Radial nodes $=n-l-1$

$=4-2-1=1$

Hence, option B is correct.

Q.18. When BeO reacts with HF in the presence of ammonia, a compound A is formed which on heating forms a compound, B along with ammonium fluoride. The oxidation state of Be in compound B is:

A) $+2$

B) $+1$

C) $+3$

D) 0

Answer: $+2$

Solution: BeO on reaction with HF in the presence of ammonia, $(\mathrm{NH_4})_2\mathrm{BeF_4}$ is formed which on heating forms $\mathrm{BeF_2}$ along with ammonium fluoride.

$\mathrm{BeO} + 2\mathrm{NH_3} + 4\mathrm{HF} \rightarrow (\mathrm{NH_4})_2\mathrm{BeF_4} + \mathrm{H_2O}$

$(\mathrm{NH_4})_2\mathrm{BeF_4} \xrightarrow{\text{heat}} \mathrm{BeF_2} + 2\mathrm{NH_4F}$

In $\mathrm{BeF_2}$ oxidation state of Be is $+2$.

Q.19. Statement I: Maltose is composed of two $\alpha-\mathrm{D}$-glucose units in which $\mathrm{C}-1$ of one glucose is linked to $\mathrm{C}-4$ of another glucose unit.

Statement II: Maltose is composed of $\alpha-\mathrm{D}$-glucose and $\beta-\mathrm{D}$-glucose in which $\mathrm{C}-1$ of $\alpha-\mathrm{D}$-glucose is linked to $\mathrm{C}-6$ of $\beta-\mathrm{D}$-glucose

A) Statement I is correct and statement II is incorrect

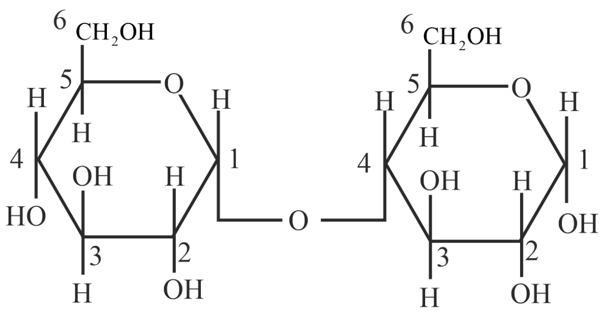
B) Statement I is incorrect and statement II is correct

C) Both the statements are correct

D) Both the statements are incorrect

Answer: Statement I is correct and statement II is incorrect

Solution:



Maltose is composed of two $\alpha-\mathrm{D}$-glucose units in which $\mathrm{C}-1$ of one glucose is linked to $\mathrm{C}-4$ of another glucose unit.

Maltose is a sugar made out of two glucose molecules bound together. It's created in seeds and other parts of plants as they break down their stored energy in order to sprout. Thus, foods like cereals, certain fruits and sweet potatoes contain naturally high amounts of this sugar.

Q.20. Correct statement about $\mathrm{PCl_5}$ is/are:

(a) $\mathrm{PCl_5}$ has Trigonal Bipyramidal geometry.

(b) Axial bonds are stronger than equatorial bonds

(c) All equatorial bonds are in the same plane.

(d) $\mathrm{PCl_5}$ shows $\mathrm{sp^3d}$ hybridization.

A) a, b and c

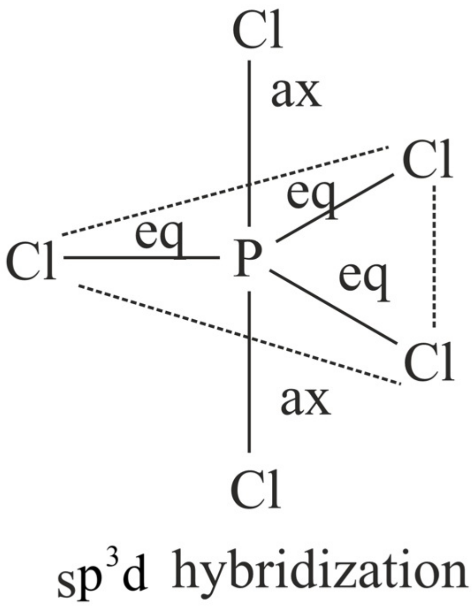
B) a, b and d

C) a, c and d

D) b, c and d

Answer: a, c and d

Solution:



The axial bonds are longer than equatorial bonds. Axial bonds are weaker than equatorial bonds. All equatorial bonds are in same plane.

Q.21. Consider an electrochemical cell, $\mathrm{Pt},\ \mathrm{H_2}\left|\mathrm{H^+}\right|\left|\mathrm{Ag^+}\right|\mathrm{Ag}$

Given, $\mathrm{E^0}_{\mathrm{Ag^+|Ag}} = +0.80\ \mathrm{V}$, the value of $\Delta\mathrm{G}^{\circ}$ for the cell represented above is $-\mathrm{x}\ \mathrm{kJ}$, then the value of x in nearest integer is:

A) 77

B) 47

C) 80

D) 67

Answer: 77

Solution: $\Delta\mathrm{G^0} = -\mathrm{nFE^0}$

$= -1 \times 96500 \times 0.80$

$= -77200\ \mathrm{J}$

$= -77.2$

$= -77\ \mathrm{kJ}$

Q.22. The boiling point of pure water is $373.15\ \mathrm{K}$. It changes to $373.535\ \mathrm{K}$, when $2.5 \times 10^{-3}\,\mathrm{Kg}$ of a non-volatile and non-electrolyte solute has been added to $7.5 \times 10^{-2}\ \mathrm{Kg}$ water. Find the molecular mass of solute in $\mathrm{g/mL}$.
$\mathrm{K_{b(H_2O)}} = 0.52\ \mathrm{K\ Kg\ mol^{-1}}$ [Round off to the nearest integer]

A) 90

B) 3

C) 45

D) 22.5

Answer: 45

Solution: $\Delta T_b = iK_b m$

$m = \frac{2.5}{M} \times \frac{1000}{75}$

$\Delta T_b = 373.535 - 373.15 = 0.385 \text{ K}$

$0.385 = 1 \times 0.52 \times \frac{2.5 \times 1000}{M \times 75}$

$M = 45 \text{ g/ ml.}$

Q.23. In carius method of estimation of halogen, 0.25 g of an organic compound gave 0.40 g of $AgCl$. Find out the percentage of chlorine in the compound.

A) 40

B) 32

C) 60

D) 26

Answer: 40

Solution: Percentage of chlorine in the organic compound= $\frac{\text{atomic mass of chlorine X mass of AgCl formed X 100}}{\text{molecular mass ofAgCl X weight of organic compound}}$

Mass of organic compound $= 0.25$ g

Mass of $AgCl$ formed $= 0.40$ g

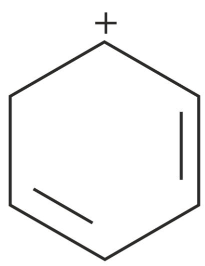
Mass of Cl in organic compound $= \frac{0.40 \times 35.5}{143.5 \times 0.25}$

Percentage of Cl in the compound $= \frac{0.40 \times 35.5}{143.5 \times 0.25} \times 100$

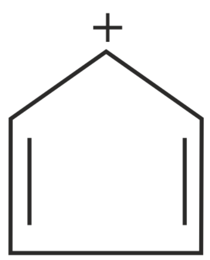
$= 39.58\%$

Q.24. Most stable corbocation among the following is

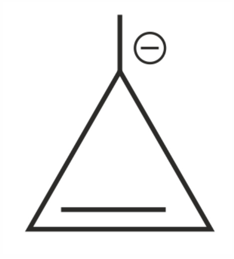
A)



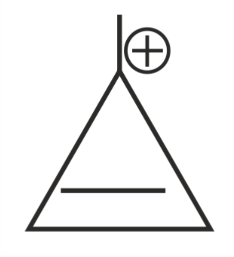
B)



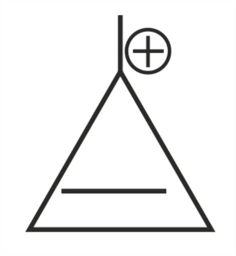
C)



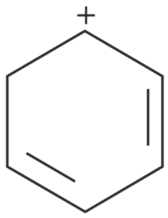
D)



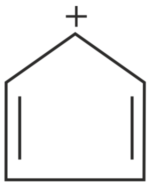
Answer:



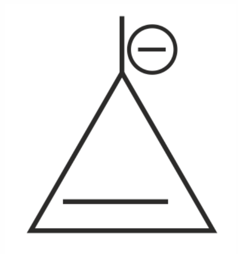
Solution:



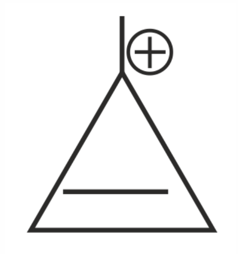
The above structure has 4π electrons, not completely conjugated so, non-aromatic and unstable.



The above structure is planar and completely conjugated 4π electrons so anti-aromatic and highly unstable



The above structure has 4π electrons, conjugated and planar, so the compound is anti-aromatic and highly unstable



The above structure has 2π electrons, conjugated, planar cyclic, satisfies Huckel's rule $(4n+2)\pi$. So, it is Aromatic and highly stable.

Q.25. Which of the following element has the highest value of $E^{o}{}_{M^{2+}/M}$?

A) Ni

B) Mn

C) Cu

D) Fe

Answer: Cu

Solution: $E^{o}_{Cu^{2+}/Cu} = +0.34\ V$

$E^{o}_{Ni^{2+}/Ni} = -0.25\ V$

$E^{o}_{Mn^{2+}/Mn} = -1.18\ V$

$E^{o}_{Fe^{2+}/Fe} = -0.44\ V$

Section C: Mathematics

Q.1. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

A) $\frac{25}{26}$

B) $\frac{3}{25}$

C) $\frac{26}{25}$

D) $\frac{3}{26}$

Answer: $\frac{26}{25}$

Solution: Now $\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)=\sum_{n=1}^{50}\tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$=\sum_{n=1}^{50}\left[\tan^{-1}(n+1)-\tan^{-1}n\right]$

$=\tan^{-1}51-\tan^{-1}1=\tan^{-1}\left(\frac{51-1}{1+51}\right)$

$=\tan^{-1}\left(\frac{50}{52}\right)=\cot^{-1}\left(\frac{26}{25}\right)$

So $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)=\cot\left(\cot^{-1}\frac{26}{25}\right)=\frac{26}{25}$

Q.2. If $S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\cdots$, then the value of $4S$ is equal to

A) $\left(\frac{7}{2}\right)^2$

B) $\left(\frac{7}{3}\right)^3$

C) $\frac{7}{3}$

D) $\left(\frac{7}{3}\right)^4$

Answer: $\left(\frac{7}{3}\right)^3$

Solution: $S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\cdots \quad \ldots(i)$

$\frac{S}{7}=0+\frac{2}{7}+\frac{6}{7^2}+\frac{12}{7^3}+\frac{20}{7^4}+\cdots \quad \ldots(ii)$

On subtracting the equation (ii) from equation (i), we get

$\frac{6S}{7}=2+\frac{4}{7}+\frac{6}{7^2}+\frac{8}{7^3}+\cdots \quad \ldots(iii)$

$\frac{6S}{49}=0+\frac{2}{7}+\frac{4}{7^2}+\frac{6}{7^3}+\cdots \quad \ldots(iv)$

On subtracting the equation (iv) from equation (iii), we get

$\left(\frac{6}{7}-\frac{6}{49}\right)S=2+\frac{2}{7}+\frac{2}{7^2}+\frac{2}{7^3}+\cdots$

$\frac{36}{49}S=2\left(1+\frac{1}{7}+\frac{1}{7^2}+\cdots\infty\right)$

$\Rightarrow \quad S=2\cdot\frac{1}{1-\frac{1}{7}}\cdot\frac{49}{36}$

$\Rightarrow \quad 4S=\left(\frac{7}{3}\right)^3$

Q.3. If the mean and variance of the data: $4,\ 5,\ 6,\ 6,\ 7,\ 8,\ x,\ y$ is 6 and $\frac{9}{4}$ respectively, then the possible value of x^4+y^2 is equal to

A) 320

B) 420

C) 4114

D) 2112

Answer: 320

Solution: Mean $=\frac{4+5+6+6+7+8+x+y}{8}=6$

$\Rightarrow x+y=12 \qquad \ldots(1)$

Variance $=\frac{\sum x_i^2}{n}-(\bar{x})^2$

$\Rightarrow \quad \frac{9}{4}=\frac{4^2+5^2+6^2+6^2+7^2+8^2+x^2+y^2}{8}-(6)^2$

$\Rightarrow \quad x^2+y^2=80 \qquad \ldots(2)$

On solving equations (1) and (2), we get $x^2+(12-x)^2=80$

$\Rightarrow \quad x^2+144+x^2-24x=80$

$\Rightarrow \quad 2x^2-24x+64=0$

$\Rightarrow \quad x^2-12x+32=0$

$\Rightarrow \quad (x-4)(x-8)=0$

$\Rightarrow \quad x=4$ or 8

From equation (2), when $x=4,\ y=8$ and when $x=8,\ y=4$.

$\therefore \quad x^4+y^2=4^4+8^2=256+64=320$

(or) $x^4+y^2=8^4+4^2=4096+16=4112$

Q.4. If $A_1, A_2, A_3 \ldots$ and $B_1, B_2, B_3 \ldots$ are two $\mathrm{A.P}$ and $A_1 = 2, A_{10} = 3$ and $A_1B_1 = 1 = A_{10}B_{10}$ then the value of A_4B_4 is

A) $\frac{28}{27}$

B) $\frac{28}{24}$

C) $\frac{23}{26}$

D) $\frac{22}{23}$

Answer: $\frac{28}{27}$

Solution: Given, $A_1, A_2, A_3 \ldots\ldots \& B, B_2, B_3$.....are in $\mathrm{A.P}$

Also given, $A_1 = 2, A_{10} = 3$

and $A_1B_1 = 1 = A_{10}B_{10}$

So $B_1 = \frac{1}{A_1} = \frac{1}{2}$ and $B_{10} = \frac{1}{A_{10}} = \frac{1}{3}$

Now using n^{th} term formula we get, $A_{10} = A_1 + 9d_A \Rightarrow 3 = 2 + 9d_A$

$\Rightarrow d_A = \frac{1}{9}$

Using same formula we get,

$B_{10} = B_1 + 9d_B \Rightarrow \frac{1}{3} = \frac{1}{2} + 9d_B \Rightarrow d_B = -\frac{1}{54}$

Now calculating $A_4B_4 = (A_1 + 3d_A)(B_1 + 3d_B)$

$= \left(2 + 3 \times \frac{1}{9}\right)\left(\frac{1}{2} + 3 \times \left(\frac{-1}{54}\right)\right)$

$= \frac{7}{3} \times \frac{8}{18} = \frac{56}{54} = \frac{28}{27}$

Q.5. If the curve $(\tan^{-1} y - x)dy = (1 + y^2)dx$ is passing through $(1, 0)$, then the value of x at $y = \tan 1$ is

A) $\frac{1}{e}$

B) $\frac{2}{e}$

C) $\frac{3}{e}$

D) $2e$

Answer: $\frac{2}{e}$

Solution: $(\tan^{-1}y - x)dy = (1+y^2)dx$

$\Rightarrow \frac{dx}{dy} + \left(\frac{1}{1+y^2}\right)x = \frac{\tan^{-1}y}{1+y^2}$

I.F. $= e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

General solution will be

$x\left(e^{\tan^{-1}y}\right) = \int e^{\tan^{-1}y} \times \frac{\tan^{-1}y}{1+y^2}dy$

$\Rightarrow x\left(e^{\tan^{-1}y}\right) = \tan^{-1}y\left(e^{\tan^{-1}y}\right) - e^{\tan^{-1}y} + C$

Since the curve passes through $(1,0) \Rightarrow \quad C = 2$

So, the curve reduces to $x\left(e^{\tan^{-1}y}\right) = \tan^{-1}y\left(e^{\tan^{-1}y}\right) - e^{\tan^{-1}y} + 2$

when $y = \tan 1,\ x(e) = 1(e) - e + 2$

$\Rightarrow \quad x = \frac{2}{e}$

Q.6. Which of the following is a tautology?

A) $(\sim p \wedge q) \vee (p \vee \sim q)$

B) $(p \rightarrow q) \vee q$

C) $(p \leftrightarrow q) \wedge (p \wedge q)$

D) $p \vee (p \leftrightarrow q)$

Answer: $(\sim p \wedge q) \vee (p \vee \sim q)$

Solution: We know that $p \vee \sim p$ is a tautology and $\sim (p \wedge q) \equiv \sim p \vee \sim q$

i.e. $\sim (\sim p \wedge q) \equiv p \vee \sim q$

So,$(\sim p \wedge q) \vee (p \vee \sim q)$ will be a tautology.

Q.7. Suppose a 2×2 matrix A is given whose entries are taken from the set $\{0,1,2,3,4,5,6\}$ such that sum of all entries is a prime number between 2 and 6 (both excluded), then the number of possible matrices A is

A) 76

B) 64

C) 72

D) 48

Answer: 76

Solution: We have to take entries from set $\{0,1,2,3,4,5,6\}$ excluding 2 and 6 so there will be two cases,

Case I : When sum of entries is 3-

Entries will be $(3,0,0,0)$ or $(2,1,0,0)$ or $(1,1,1,0)$.

Number of matrices formed $=\frac{4!}{3!}+\frac{4!}{2!}+\frac{4!}{3!}=20$

Case II: When sum of entries is 5-

Entries will be: $(5,0,0,0),(4,1,0,0),(3,2,0,0),(3,1,1,0),(2,2,1,0),(2,1,1,1)$

Number of matrices formed $=\frac{4!}{3!}+\frac{4!}{2!}+\frac{4!}{2!}+\frac{4!}{2!}+\frac{4!}{2!}+\frac{4!}{3!}=56$

Adding case (I) and case (II) we get $20+56=76$

Q.8. The number of complex numbers z satisfying $|z-(4+3\,i)|=2$ and $|z|+|z-4|=6$ is

A) 1

B) 2

C) 3

D) 4

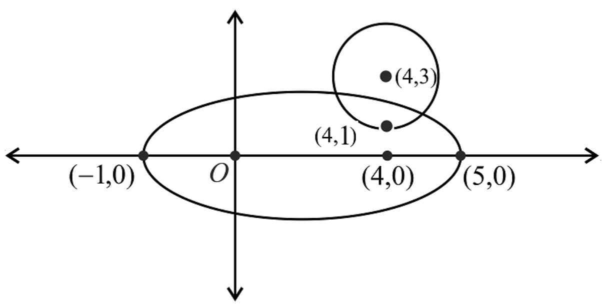
Answer: 2

Solution: Clearly, $|z-(4+3\,i)|=2$ represents a circle with centre $(4,\ 3)$ and radius 2 units and $|z|+|z-4|=6$ represents an ellipse with foci $(0,\ 0)$ and $(4,\ 0)$ and vertices are $(5,\ 0)$ and $(-1,\ 0)$.

Observe that, $z=(4+i)$ lies on circle and $|4+i|+|4+i-4|=1+\sqrt{17}<6$.

Hence, the point $z=(4+i)$ lies inside the ellipse.

So, the graphs of the curves are as shown in figure.



It is clear from the above diagram, there exist two complex numbers which satisfy both the given curves.

Q.9. If A and B are matrices of order 3×3 and $AB=I,|A|=\frac{1}{8}$, then the value of $|\operatorname{adj}(B\cdot\operatorname{adj}2A)|$ is

A) 128

B) 32

C) 64

D) 102

Answer: 64

Solution: Given, $AB = I$ and $|A| = \frac{1}{8}$

Now by using property $\text{adj}\,(PQ) = (\text{adj}\,Q)\,(\text{adj}\,P)$ we get, $|\text{adj}\,(B \cdot \text{adj}\,2A)| = |\text{adj}\,(\text{adj}\,2A)|\,|\text{adj}\,B| \quad \ldots(\text{i})$

Now $|\text{adj}\,(\text{adj}\,2A)| = |\text{adj}\,2A|^2 = |2A|^4$

$= |2^3|^4 |A|^4 = (2^3)^4 |A|^4 = 8^4 |A|^4 \quad \ldots(\text{ii})$

{Property used $|\text{adj}\,A| = |A|^{n-1}$ and $|kA| = k^n |A|$ where n is order of matrix}

Now $|\text{adj}\,B| = |B|^2 \ldots\ldots\ldots\ldots (iii)$

Now putting the value of equation (ii) & (iii) in eq (i)

we get $|\text{adj}\,(B \cdot \text{adj}\,2A)| = 8^4 |A|^4 |B|^2$

Now using $|A| = \frac{1}{|B|}$ as $AB = I$, we get

$|\text{adj}\,(B \cdot \text{adj}\,2A)| = 8^4 |A|^4 \times \frac{1}{|A|^2} = 8^4 \times |A|^2 = 8^4 \times \frac{1}{8^2} = 64$

Q.10. If $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$, then the number of points of local maxima and local minima are respectively

A) $2, 3$

B) $3, 2$

C) $1, 1$

D) $2, 1$

Answer: $2, 3$

Solution: Given $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$

We know that,

$\frac{d}{dx} \int_{u(x)}^{v(x)} f(v) dx = f(v(x)) \frac{d}{dx} v(u) - f(u(x)) \frac{d}{dx} u(x)$

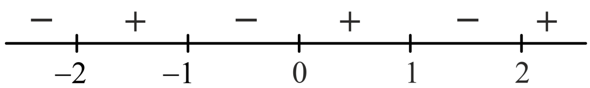
Using above formula we get $f'(x) = \left(\frac{x^4 - 5x^2 + 4}{2 + e^{x^2}}\right) \cdot 2x$

For critical points $f'(x) = 0$

So, $\frac{x(x^2 - 1)(x^2 - 4)}{2 + e^{x^2}} = 0$

$\Rightarrow x(x+1)(x-1)(x+2)(x-2) = 0$

Now by first derivative test we get,



Here points of maxima are $\{-1, 1\}$ & points of minima are $\{-2, 0, 2\}$

Q.11. For which of the following equation $\sin 36^\circ$ is a root?

A) $16x^4 - 20x^2 + 5 = 0$

B) $16x^4 + 20x^2 + 5 = 0$

C) $16x^4 - 20x^2 - 5 = 0$

D) $x^4 + 20x^2 - 5 = 0$

Answer: $16x^4 - 20x^2 + 5 = 0$

Solution: We know $\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$

Let $x = \frac{\sqrt{10-2\sqrt{5}}}{4}$

On squaring both sides, we get

$(4x)^2 = 10 - 2\sqrt{5} \Rightarrow 16x^2 = 10 - 2\sqrt{5}$

Again squaring, we get

$\left(\sqrt{5}\right)^2 = \left(5 - 8x^2\right)^2 \Rightarrow 5 = 25 + 64x^4 - 80x^2$

$\Rightarrow 16x^4 - 20x^2 + 5 = 0$

Q.12. Let $f(x) = \frac{[1+x]+\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$. If $\lim_{x\to 0^-} f(x) = \alpha - \frac{4}{3}$, then the integral value of α is equal to

A) 1

B) 2

C) 3

D) 4

Answer: 3

Solution: $\underset{x\to 0^-}{\mathrm{Lt}} f(x) = \underset{h\to 0}{\mathrm{Lt}} \frac{[1-h]+\alpha^{2[0-h]+\{0-h\}}+[0-h]-1}{2[0-h]+\{0-h\}}$

$= \underset{h\to 0}{\mathrm{Lt}} \frac{1-1+\alpha^{2(-1)+1-h}-1-1}{2(-1)+1-h} = \underset{h\to 0}{\mathrm{Lt}} \frac{\alpha^{-1-h}-2}{-1-h}$

$= \frac{\alpha^{-1}-2}{-1}$

$\therefore \underset{x\to 0^-}{\mathrm{Lt}} f(x) = 2 - \frac{1}{\alpha} = \alpha - \frac{4}{3}$ (given)

$\Rightarrow \alpha + \frac{1}{\alpha} = 2 + \frac{4}{3} \Rightarrow \alpha + \frac{1}{\alpha} = \frac{10}{3} = 3 + \frac{1}{3}$

$\therefore \alpha = 3$

Q.13. If f is a differentiable function such that $\int_{\cos x}^{1} t^2 f(t)dt = \sin^3 x + \cos x$, then the value of $\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right)$ is

A) $6 - \frac{19}{\sqrt{2}}$

B) $6 + \frac{9}{\sqrt{2}}$

C) $6 - \frac{9}{\sqrt{2}}$

D) $3 + \sqrt{2}$

Answer: $6-\frac{9}{\sqrt{2}}$

Solution: $\int_{\cos x}^{1} t^2 f(t)dt=\sin^3 x+\cos x$

On differentiating, we get

$f(\cos x)\sin x\cdot\cos^2 x=3\sin^2 x\cos x-\sin x$

$f(\cos x)=3\tan x-\sec^2 x$

Again differentiating, we get

$-\sin x f'(\cos x)=3\sec^2 x-2\sec^2 x\tan x$

When $\cos x=\frac{1}{\sqrt{3}}$ then $\sec x=\sqrt{3}$, $\tan x=\sqrt{2}$ & $\sin x=\frac{\sqrt{2}}{\sqrt{3}}$

Then $-\frac{\sqrt{2}}{\sqrt{3}}f'\left(\frac{1}{\sqrt{3}}\right)=3\times 3-2\times 3\sqrt{2}$

$\Rightarrow \frac{1}{\sqrt{3}}f'\left(\frac{1}{\sqrt{3}}\right)=6-\frac{9}{\sqrt{2}}$

Q.14. If the equation of the parabola whose vertex is $(5,4)$ and equation of directrix is $3x+y-29=0$ is $x^2+ay^2+bxy+cx+dy+e=0$, then the value of $(a+b+c+d+e)$ is

A) 711

B) -711

C) 576

D) -576

Answer: -576

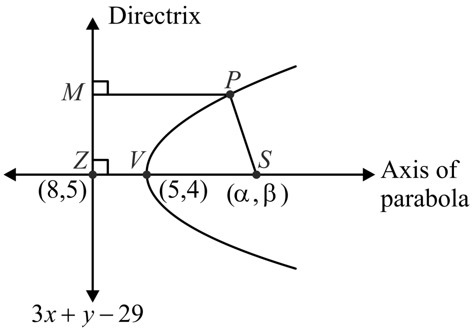
Solution: Given the equation of the parabola whose vertex is $(5,4)$ and equation of directrix is $3x+y-29=0$ is $x^2+ay^2+bxy+cx+dy+e=0$

Let focus be (α,β)

Let equation of ZV which is perpendicular to $3x+y-29=0$ be $x-3y+k=0$ and it passes through point $(5,4)$. So, $k=7$ and equation of ZV become $x-3y+7=0$

Now the intersection of $3x+y-29=0$ & $x-3y+7=0$ will be $Z=(8,5)$

Now plotting the diagram we get,



So, foot of perpendicular from $(5,4)$ on $3x+y-29=0$ is $(8,5)$

Now using the midpoint formula we will find the focus of parabola$\Rightarrow \frac{\alpha+8}{2}=5, \frac{\beta+5}{2}=4 \Rightarrow (\alpha,\beta)=(2,3)$

$\therefore$ Focus is $(2,3)$ and directrix is $3x+y-29=0$

Applying $(PS)^2=(PM)^2$, we get

$(x-2)^2+(y-3)^2=\frac{(3x+y-29)^2}{10}$

$\Rightarrow x^2+9y^2-6xy+134x-2y-711=0$

Comparing with$x^2+ay^2+bxy+cx+dy+e=0$, we get $(a+b+c+d+e)=(9-6+134-2-711)=-576$

Q.15. The shortest distance between the lines $\frac{x-1}{4}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-5}{5}=\frac{y-3}{6}=\frac{z-2}{7}$ is equal to

A) $\sqrt{43}$

B) $\frac{43}{\sqrt{381}}$

C) $\frac{43}{\sqrt{391}}$

D) $\sqrt{381}$

Answer: $\frac{43}{\sqrt{381}}$

Solution: Given lines are $\frac{x-1}{4}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-5}{5}=\frac{y-3}{6}=\frac{z-2}{7}$

Let $\overrightarrow{a_1}=\hat{\imath}+2\hat{\jmath}+3\hat{k},\ \vec{a}=4\hat{\imath}+2\hat{\jmath}+3\hat{k}$

$\overrightarrow{a_2}=5\hat{\imath}+3\hat{\jmath}+2\hat{k},\ \vec{b}=5\hat{\imath}+6\hat{\jmath}+7\hat{k}$

Now, $\overrightarrow{a_2}-\overrightarrow{a_1}=4\hat{\imath}+\hat{\jmath}-\hat{k}$

$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\cdot\left(\vec{a}\times\vec{b}\right)=\begin{vmatrix}4 & 1 & -1\\ 4 & 2 & 3\\ 5 & 6 & 7\end{vmatrix}=-43$$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 4 & 2 & 3\\ 5 & 6 & 7\end{vmatrix}=-4\hat{\imath}-13\hat{\jmath}+14\hat{k}$$

$$\left|\vec{a}\times\vec{b}\right|=\sqrt{16+169+196}=\sqrt{381}$$

$\therefore$ Required shortest distance $=\left|\frac{\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\cdot\left(\vec{a}\times\vec{b}\right)}{\left|\vec{a}\times\vec{b}\right|}\right|=\frac{43}{\sqrt{381}}$

Q.16. If the foot of perpendicular from point $(1,2,4)$ on line $\frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{3}$ is P, then the distance of P from the plane $3x+4y+12z+23=0$ will be,

A) $\frac{63}{13}$

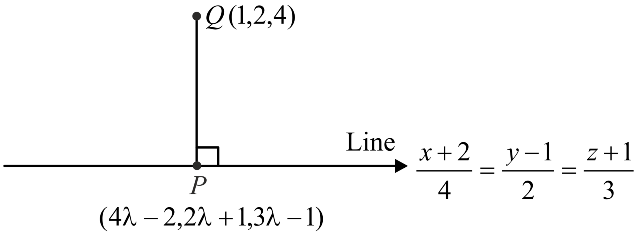
B) $\frac{55}{13}$

C) $\frac{65}{13}$

D) $\frac{44}{13}$

Answer: $\frac{65}{13}$

Solution: Let P be foot of perpendicular of point $Q\,(1,2,4)$



Let $\frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{3}=\lambda$

So $x=4\lambda-2,\ y=2\lambda+1, z=3\lambda-1$

Then coordinates of point P will be $(4\lambda-2,\ 2\lambda+1, 3\lambda-1)$

Now direction ratios of QP

$=<4\lambda-2-1,\ 2\lambda+1-2,\ 3\lambda-1-4>$

$=<4\lambda-3, 2\lambda-1, 3\lambda-5>$

and D.R's of line will be $<4,2,3>$

Now PQ and line are perpendicular

so, $4\,(4\lambda-3)+2\,(2\lambda-1)+3\,(3\lambda-5)=0$

$\Rightarrow\quad \lambda=1$

Putting the value of λ in P we get $P\equiv(2,3,2)$

Now distance of point $P(2,3,2)$ from plane$3x+4y+12z+23=0$ will be $=\left|\frac{3\times2+4\times3+12\times2+23}{\sqrt{9+16+144}}\right|=\frac{65}{13}$

Q.17. For some $\alpha,\beta\in R,\ a=\alpha-i\beta$. If system of equations $4ix+(1+i)y=0$ and $8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)x+\bar{a}y=0$ has more than one solution, then $\frac{\alpha}{\beta}=$

A) $2-\sqrt{3}$

B) $2+\sqrt{3}$

C) $-2+\sqrt{3}$

D) $-2-\sqrt{3}$

Answer: $2-\sqrt{3}$

Solution: The given system of equations has more than one solution, then it must have infinitely many solutions.

So, $\frac{4i}{8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)}=\frac{1+i}{\alpha+i\beta}$ $\left(\because \begin{array}{rl} a & =\alpha-i\beta \\ \Rightarrow \bar{a} & =\alpha+i\beta \end{array}\right)$

$\Rightarrow \frac{4i}{8\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)}=\frac{1+i}{\alpha+i\beta}$

$\Rightarrow \alpha i-\beta=-1-i+\sqrt{3}i-\sqrt{3}$

$\Rightarrow -\beta+\alpha i=\left(-1-\sqrt{3}\right)+\left(-1+\sqrt{3}\right)i$

$\Rightarrow \alpha=-1+\sqrt{3}$ & $-\beta=-1-\sqrt{3}$

$\therefore \frac{\alpha}{\beta}=\frac{-1+\sqrt{3}}{1+\sqrt{3}}=\frac{-1+\sqrt{3}}{1+\sqrt{3}}\times\frac{1-\sqrt{3}}{1-\sqrt{3}}$

$=\frac{-\left(1-\sqrt{3}\right)^2}{1-3}=\frac{1+3-2\sqrt{3}}{2}=2-\sqrt{3}$

Q.18. The value of $\int_0^1 \frac{dx}{7^{\left[\frac{1}{x}\right]}}$ is (where $[\cdot]$ denotes the greatest integer function)

A) $1-6\ln\left(\frac{6}{7}\right)$

B) $1+6\ln\left(\frac{6}{7}\right)$

C) $1-7\ln\left(\frac{6}{7}\right)$

D) $1+7\ln\left(\frac{6}{7}\right)$

Answer: $1+6\ln\left(\frac{6}{7}\right)$

Solution:

$I=\int_0^1 \frac{dx}{7^{\left[\frac{1}{x}\right]}}=\int_0^1 \left(\frac{1}{7}\right)^{\left[\frac{1}{x}\right]}dx$

Let $\frac{1}{7}=k \Rightarrow I=\int_0^1 k^{\left[\frac{1}{x}\right]}dx$

$I=\int_0^1 k^{\left[\frac{1}{x}\right]}dx$

$\Rightarrow I=\int_{\frac{1}{2}}^1 k^{\left[\frac{1}{x}\right]}dx+\int_{\frac{1}{3}}^{\frac{1}{2}} k^{\left[\frac{1}{x}\right]}dx+\int_{\frac{1}{4}}^{\frac{1}{3}} k^{\left[\frac{1}{x}\right]}dx+\cdots$

$\Rightarrow \quad I=k\left(1-\frac{1}{2}\right)+k^2\left(\frac{1}{2}-\frac{1}{3}\right)+k^3\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots$

$\Rightarrow I=\left(k+\frac{k^2}{2}+\frac{k^3}{3}+\cdots\right)-\frac{1}{k}\left(\frac{k^2}{2}+\frac{k^3}{3}+\cdots\right)$

$\Rightarrow I=-\ln(1-k)-\frac{1}{k}\left[-\ln(1-k)-k\right]$

$\Rightarrow \quad I=-\ln\frac{6}{7}-7\left[-\ln\frac{6}{7}-\frac{1}{7}\right]$ $\left(\because k=\frac{1}{7}\right)$

$\therefore I=1+6\ln\frac{6}{7}$

Q.19. If $y(x)=\left(x^x\right)^x, x>0$, then the value of $\frac{d^2y}{dx^2}+20$ at $x=1$ is

A) 24

B) 4

C) 20

D) -24

Answer: 24

Solution: We have $y=(x^x)^x$

$\Rightarrow y=x^{x^2} \quad \ldots$ (i)

$\Rightarrow \log_e(y)=\log_e\left(x^{x^2}\right)$

$\Rightarrow \log_e(y)=x^2\log_e(x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=x^2\left(\frac{1}{x}\right)+\log_e(x)(2x)$

$\Rightarrow \frac{dy}{dx}=y\left[x+2x\log_e(x)\right] \quad \ldots$ (ii)

$\Rightarrow \frac{d^2y}{dx^2}=y\left[1+2\frac{d}{dx}(x\log_e(x))\right]+\left[x+2x\log_e(x)\right]\frac{dy}{dx}$

$=y\left[1+2\left(x\left(\frac{1}{x}\right)+\log_e(x)(1)\right)\right]+\left[x+2x\log_e(x)\right]\frac{dy}{dx}$

$\frac{d^2y}{dx^2}=y\left[1+2(1+\log_e(x))\right]+\left[x+2x\log_e(x)\right]\frac{dy}{dx} \quad \ldots$ (iii)

Now from equation (i) when $x=1,\ y=1$.

Also from equation (ii), $\left(\frac{dy}{dx}\right)_{(1,1)}=1[1+0]=1$

and from equation (iii), $\left(\frac{d^2y}{dx^2}\right)_{(1,1)}=1(1+2+0)+(1+0)1=4$

Therefore, the value of $\frac{d^2y}{dx^2}+20=4+20$ at $x=1$ is equal to 24.

Q.20. Let $\vec{a}$ and $\vec{b}$ be two vectors along the diagonals of a parallelogram having area $2\sqrt{2}$. Also the angle between $\vec{a}$ and $\vec{b}$ is acute, $|a|=1$ and $\left|\vec{a}\cdot\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$. If $\vec{c}=2\sqrt{2}\left(\vec{a}\times\vec{b}\right)-2\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is

A) $\frac{-\pi}{4}$

B) $\frac{5\pi}{6}$

C) $\frac{\pi}{3}$

D) $\frac{3\pi}{4}$

Answer: $\frac{3\pi}{4}$

Solution: $\vec{c}=2\sqrt{2}\left(\vec{a}\times\vec{b}\right)-2\vec{b}$

So, $\vec{b}\cdot\vec{c}=2\sqrt{2}\,\vec{b}\cdot\left(\vec{a}\times\vec{b}\right)-2\vec{b}\cdot\vec{b}=-2\left|\vec{b}\right|^2\ \ldots(i)$

Since $\left|\vec{a}\cdot\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$, so angle between $\vec{a}$ & $\vec{b}$ is $\frac{\pi}{4}$

Now, area of triangle is $2\sqrt{2}$

i.e. $\frac{1}{2}\left|\vec{a}\times\vec{b}\right|=\frac{1}{2}\left|\vec{a}\right|\left|\vec{b}\right|\sin\frac{\pi}{4}=2\sqrt{2}\Rightarrow\left|\vec{b}\right|=8$

From (i), $\vec{b}\cdot\vec{c}=-128$

Now, $\left|\vec{c}\right|^2=\left|2\sqrt{2}\left(\vec{a}\times\vec{b}\right)-2\vec{b}\right|^2$

$=8\left|\vec{a}\times\vec{b}\right|^2+4\left|\vec{b}\right|^2-8\sqrt{2}\left(\vec{a}\times\vec{b}\right)\cdot\vec{b}=8\times32+4\times64=512$

Hence, angle between $\vec{b}$ & $\vec{c}$ will be $\cos^{-1}\left(\frac{\vec{b}\cdot\vec{c}}{\left|\vec{b}\right|\left|\vec{c}\right|}\right)=\cos^{-1}\left(\frac{-128}{8\times\sqrt{512}}\right)=\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)=\frac{3\pi}{4}$