JEE Main Exam 2022 - Session 1

27 June 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. The dimensional formula of self-inductance is

A) $\mathrm{MLT^{-2}}$

B) $\mathrm{ML^2T^{-1}A^{-2}}$

C) $\mathrm{ML^2T^{-2}A^{-2}}$

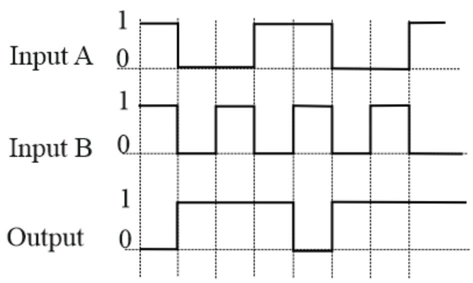
D) $\mathrm{ML^2T^{-2}A^{-1}}$

Answer: $\mathrm{ML^2T^{-2}A^{-2}}$

Solution:

Energy $E = \frac{1}{2}LI^2 \Rightarrow [\mathrm{L}] = \frac{[E]}{[I]^2} = \frac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{A^2}]} = \left[\mathrm{ML^2T^{-2}A^{-2}}\right]$

Q.2. Identify the logic gate for the following output, inputs A and B.



A) NAND

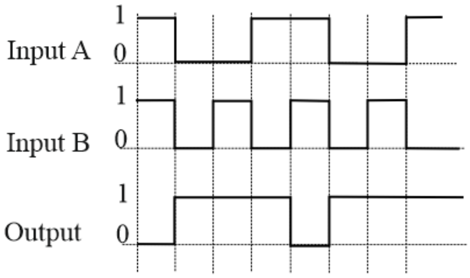
B) AND

C) OR

D) NOR

Answer: NAND

Solution: For the given input and output, truth table is given below.



Input A	Input B	Output
1	1	0
0	1	1
1	0	1
0	0	1

Clearly, it is a NAND gate.

Q.3. **Statement**
1: Law of gravitation hold true for all bodies in universe.

Statement
2: The weight of anybody at centre of the earth becomes zero.

A) Statement 1 and statement 2 both are true.

B) Statement 1 is true and statement 2 is false.

C) Statement 1 and statement 2 both are false.

D) Statement 1 is false and statement 2 is true.

Answer: Statement 1 and statement 2 both are true.

Solution: Gravitational force of attraction exists in all the places in the universe. This is the force of attraction between any two substances that are having a mass of their own.

Every object in the universe attracts every other object with a force which is proportional to the product of their masses and inversely proportional to the square of the distance between them.

The weight of a body placed at the centre of the earth is zero. This is because acceleration due to gravity is zero at the centre and hence weight (mg) is zero at the centre. Thus, both statements are true.

Q.4. Find the ratio of de-Broglie wavelength of α-particle and Carbon-12 having same kinetic energy.

A) $\sqrt{3}:1$

B) $1:\sqrt{3}$

C) $2:\sqrt{3}$

D) $\sqrt{3}:2$

Answer: $\sqrt{3}:1$

Solution: De-Broglie wavelength is stated as $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$, where, K is kinetic energy and m is mass.

Here, $(K)_\alpha = (K)_{\text{C-12}}$.

Now, the ratio $\frac{\lambda_\alpha}{\lambda_{\text{C-12}}} = \frac{\sqrt{12\text{m}}}{\sqrt{4\text{m}}} = \sqrt{3} : 1$

Q.5. A point charge is placed between the two plates of an isolated parallel plate capacitor. The force on the charge is 10N. What will be the force if one plate is removed?

A) 0N

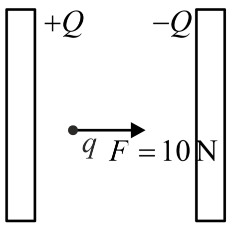
B) 5N

C) 10N

D) 20 N

Answer: 5N

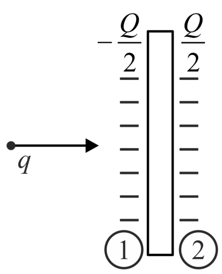
Solution:



Electric field due to each plate, $E_1 = E_2 = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$

Net electric field between the plates, $E_{\text{net}} = E_1 + E_2 = \frac{Q}{A\varepsilon_0}$

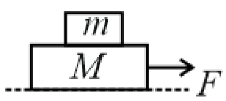
Force on charged particle between the plates, $F_1 = qE_{net} = \frac{qQ}{A\varepsilon_0} = 10\text{ N}$



Now, in second case, the net electric field, $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0} = 5\text{ N}$

Force on charged particle, $F_2 = qE = \frac{qQ}{2A\varepsilon_0} = 5\text{ N}$

Q.6. In the diagram shown, friction is present everywhere with frictional coefficient $\mu = 0.5$, $m = 2\text{ kg}$ & $M = 8\text{ kg}$. The maximum value of F for which both the blocks will move together is:



A) 50 N

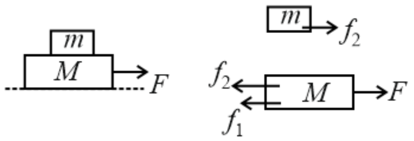
B) $100 \mathrm{~N}$

C) $150 \mathrm{~N}$

D) $200 \mathrm{~N}$

Answer: $100 \mathrm{~N}$

Solution:



Given, $\mu=0.5,\ m=2 \mathrm{~kg}\ \&\ M=8 \mathrm{~kg}$

Maximum friction force possible between M and ground, $\left(f_1\right)_{\max }=0.5 \times(10 \times 10)=50 \mathrm{~N}$

Maximum friction force possible between m and M,

$\left(f_2\right)_{\max }=0.5 \times(2 \times 10)=10 \mathrm{~N}$

For block of mass m, maximum acceleration possible, $a_{\max }=\frac{\left(f_2\right)_{max}}{m}=5 \mathrm{~m} \mathrm{~s}^{-2}$

Now force required is,

$\therefore\ F-\left(f_1\right)_{max}=10 \times 5\ \Rightarrow\ F=100 \mathrm{~N}$

Q.7. Measurements of a wire yields the following results:

Mass $=(0.6 \pm 0.006) \mathrm{~g}$; Radius $=(0.5 \pm 0.005) \mathrm{~mm}$; length $=(4 \pm 0.04) \mathrm{~cm}$

The maximum percentage error in the measurement of its density is:

A) 2%

B) 3%

C) 4%

D) 5%

Answer: 4%

Solution: Volume of the wire is $V=\pi r^2 l$.

Density of the wire is $\rho=\frac{m}{\pi r^2 l}$

Relative error in measurement of density

$\Rightarrow \frac{\Delta \rho}{\rho}=\frac{\Delta m}{m}+2 \times \frac{\Delta r}{r}+\frac{\Delta l}{l}$

$=\frac{0.006}{0.6}+2 \times \frac{0.005}{0.5}+\frac{0.04}{4}$

$=0.01+0.02+0.01=0.04$

$\therefore$ Percentage error in $\rho=0.04 \times 100=4\%$

Q.8. A particle starts from mean position at $t=0$. At time $t=3 \mathrm{~s}$, its displacement becomes half of its initial amplitude. Find the time period of particle.

A) $12 \mathrm{~s}$

B) $18 \mathrm{~s}$

C) $36\ \mathrm{s}$

D) $48\ \mathrm{s}$

Answer: $36\ \mathrm{s}$

Solution: Displacement equation is given by $x = A\sin\omega t$

Now, when amplitude is halved, $\frac{A}{2} = A\sin\omega t$

Or $\omega t = \frac{\pi}{6}$

At, $t = 3\ \mathrm{s}$, $\omega(3) = \frac{\pi}{6} \Rightarrow \omega = \frac{\pi}{18}$

Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{18}} = 36\ \mathrm{s}$

Q.9. Two columns are given below. Column 1 gives a range of electromagnetic spectrum while column 2 gives possible uses. Match column 1 with column 2.

	Column 1		**Column 2**
P	UV	A	Study crystal structure
Q	Microwaves	B	Greenhouse effect
R	Infrared	C	Sterilising surgical instruments
S	X-rays	D	Radio signal

A) $\mathrm{P-C,\ Q-D,\ R-B,\ S-A}$

B) $\mathrm{P-A,\ Q-B,\ R-C,\ S-D}$

C) $\mathrm{P-D,\ Q-B,\ R-C,\ S-A}$

D) $\mathrm{P-C,\ Q-D,\ R-A,\ S-B}$

Answer: $\mathrm{P-C,\ Q-D,\ R-B,\ S-A}$

Solution: X-rays $\rightarrow$ Study crystal structure

Infrared $\rightarrow$ Green house effect

Microwaves $\rightarrow$Communication (Radio signal)

UV $\rightarrow$ For sterilization surgical instruments

Q.10. A $72\ \Omega$ galvanometer is shunted by a resistance of $8\ \Omega$. The percentage of the total current which passes through the galvanometer is

A) 0.1%

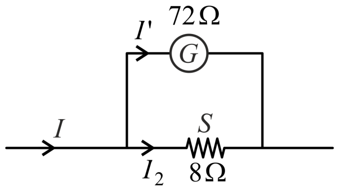
B) 10%

C) 25%

D) 0.25%

Answer: 10%

Solution:



Let G be resistance of galvanometer and I' be the current passing through the galvanometer produces full scale deflection. If I is the maximum current, and since, G and S are in parallel so, the potential difference between them will be same.

Then,

$$I'G = I_2 S$$
$$\Rightarrow 72I' = 8I_2$$
$$\Rightarrow I_2 = 9I'$$
$$\Rightarrow I = I' + I_2 = 10I'$$

So the percentage of total current passes through galvanometer is

$$\frac{I'}{I} \times 100 = \frac{I'}{10I'} \times 100 = 10\%$$

Q.11. Find the wavelength of the light emitted for the transition of electron from 3^{rd} to 1^{st} orbit in Li^{+2}.

A) $114\ \mathring{A}$

B) $141\ \mathring{A}$

C) $411\ \mathring{A}$

D) $444\ \mathring{A}$

Answer: $114\ \mathring{A}$

Solution: Energy released in the transition will be,

$$\Delta E = 13.6 \times Z^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = 13.6 \times 3^2 \times \frac{8}{9}\ \mathrm{eV} = 108.8\ \mathrm{eV}$$

Now energy of photon can be written as,

$$\frac{hc}{\lambda} = 108.8\ \mathrm{eV} \Rightarrow \lambda = \frac{12400}{108.8}\ \mathring{A} \approx 114\ \mathring{A}$$

Q.12. A light incident from air to a surface of refractive index $\sqrt{2}n$. Angle of incidence is twice of angle of refraction, then the angle of incidence is:

A) $\cos^{-1}\left(\frac{\sqrt{n}}{2}\right)$

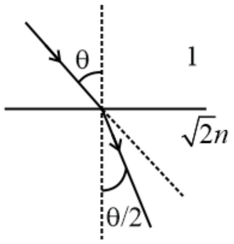
B) $2\cos^{-1}\left(\frac{n}{\sqrt{2}}\right)$

C) $\sin^{-1}\left(\sqrt{n}\right)$

D) $\sin^{-1}\left(\frac{n}{2}\right)$

Answer: $2\cos^{-1}\left(\frac{n}{\sqrt{2}}\right)$

Solution:



Let angle of incidence be θ, then angle of refraction $\frac{\theta}{2}$. From Snell's law, we can write,

$1 \times \sin\theta = \sqrt{2}n \times \sin\frac{\theta}{2}$, $2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = \sqrt{2}n\sin\left(\frac{\theta}{2}\right)$

$\cos\left(\frac{\theta}{2}\right) = \frac{n}{\sqrt{2}} \Rightarrow \theta = 2\cos^{-1}\left(\frac{n}{\sqrt{2}}\right)$

Q.13. A girl is holding an umbrella at 45° and rain was falling on it. Girl starts running with velocity of $15\sqrt{2}\ \mathrm{m\ s^{-1}}$ without umbrella, rain falls vertically on girl. Find the velocity of the rain with respect to ground.

A) $15\sqrt{2}\ \mathrm{m\ s^{-1}}$

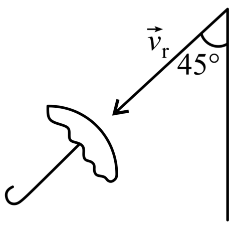
B) $15\ \mathrm{m\ s^{-1}}$

C) $30\ \mathrm{m\ s^{-1}}$

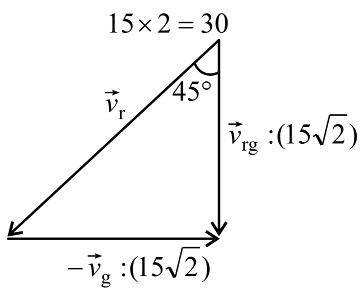
D) $30\sqrt{2}\ \mathrm{m\ s^{-1}}$

Answer: $30\ \mathrm{m\ s^{-1}}$

Solution:



$\vec{v}_{rg} = \vec{v}_r - \vec{v}_g$



Hence, $\left|\vec{v}_r\right| = \sqrt{\left(15\sqrt{2}\right)^2 + \left(15\sqrt{2}\right)^2} = 30\ \mathrm{m\ s^{-1}}$

Q.14. The current flowing through an ac circuit is given by $I = 5\sin(120\pi t)\ \mathrm{A}$. How long will the current take to reach the peak value starting from zero.

A) $\frac{1}{60}\ \mathrm{s}$

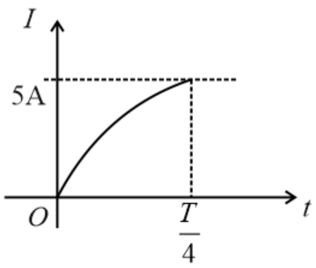
B) $60 \mathrm{~s}$

C) $\frac{1}{120} \mathrm{~s}$

D) $\frac{1}{240} \mathrm{~s}$

Answer: $\frac{1}{240} \mathrm{~s}$

Solution:



If initial value of current is zero, then it will take $\frac{T}{4}$ time to reach the peak value, where $T=$ time period.

As we know, $\omega=120\pi$

Therefore,

$$t=\frac{T}{4}=\frac{\frac{2\pi}{\omega}}{4}=\frac{\pi}{2\omega}=\frac{\pi}{2\times120\pi}=\frac{1}{240} \mathrm{~s}$$

Q.15. Two blocks of mass $10 \mathrm{~kg}$ and $30 \mathrm{~kg}$ lies on x-axis at $(0,\ 0)$ and $(x,\ 0)$ respectively. The $10 \mathrm{~kg}$ block is moved on the same line $6 \mathrm{~cm}$ towards the other block. What distance should the other block cover to keep the centre of mass of the system unchanged?

A) $4 \mathrm{~cm}$ away from $10 \mathrm{~kg}$ block

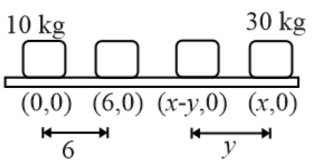
B) $4 \mathrm{~cm}$ towards $10 \mathrm{~kg}$ block

C) $2 \mathrm{~cm}$ away from $10 \mathrm{~kg}$ block

D) $2 \mathrm{~cm}$ towards $10 \mathrm{~kg}$ block

Answer: $2 \mathrm{~cm}$ towards $10 \mathrm{~kg}$ block

Solution:



Initial position of centre of mass: $x_{CM}=\frac{10(0)+30\times x}{40}=\frac{3x}{4}$

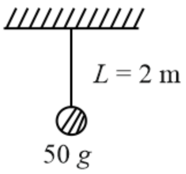
Final position of the centre of mass: $x'_{CM}=\frac{10(6)+30\times(x-y)}{40}=\frac{6+3(x-y)}{4}$

As position of centre of mass remains same,

$$x_{CM}=x'_{CM} \Rightarrow \frac{3x}{4}=\frac{6+3(x-y)}{4} \Rightarrow 3x=6+3x-3y \Rightarrow y=2 \mathrm{~cm}$$

Alternate method, $\Delta x_{\mathrm{COM}}=\frac{m_1\Delta x_1+m_2\Delta x_2}{(m_1+m_2)} \Rightarrow 0=10\times6+30y \Rightarrow y=-2 \mathrm{~cm}$

Q.16. A bullet of mass $75 \mathrm{~g}$ is moving with velocity v strikes a bob of mass $50 \mathrm{~g}$ as shown. If bullet emerges out from bob with velocity $\frac{v}{3}$ and the bob just completes vertical circular motion, then find the velocity v of bullet.



A) $10 \mathrm{~ms}^{-1}$

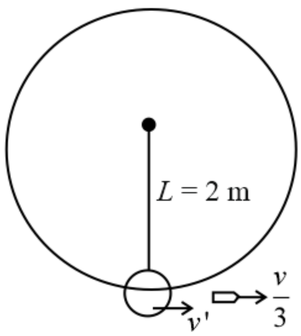
B) $5 \mathrm{~m} \mathrm{~s}^{-1}$

C) $15 \mathrm{~m} \mathrm{~s}^{-1}$

D) $20 \mathrm{~m} \mathrm{~s}^{-1}$

Answer: $10 \mathrm{~ms}^{-1}$

Solution:



Let the velocity of the bob after the collision be v'.

Apply conservation of momentum just before and after the collision

$75 v+0=50 v'+(75) \frac{v}{3}$

$\Rightarrow v=v'$

As the bob just completes the vertical circular motion, so

v' $=\sqrt{5 g L}=\sqrt{5 \times 10 \times 2}=10 \mathrm{~m} \mathrm{~s}^{-1}$

Q.17. For a Carnot engine, source is at temperature $527{ }^{\circ} \mathrm{C}$ and sink is at temperature $200 \mathrm{~K}$. If $12 \mathrm{~kJ}$ of work is done, then heat absorbed is

A) $16 \mathrm{~kJ}$

B) $20 \mathrm{~kJ}$

C) $14 \mathrm{~kJ}$

D) $18 \mathrm{~kJ}$

Answer: $16 \mathrm{~kJ}$

Solution:

Efficiency of Carnot engine is $\eta = 1 - \frac{T_L}{T_H} = \frac{W}{Q}$, where, Q is heat absorbed.

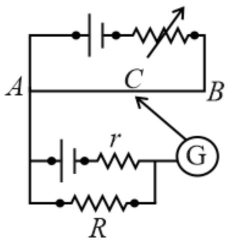
Here, $T_H = 527 + 273 = 800\ \mathrm{K}$

Now, $\eta = 1 - \frac{200}{800} = \frac{12}{Q}$

$\therefore\ \frac{3}{4} = \frac{12}{Q}$

$\therefore\ Q = 16\ \mathrm{kJ}$

Q.18. In an experiment of potentiometer, if $R = 8\ \Omega$, then null point AC is equal to $3\ \mathrm{m}$ and when $R = 4\ \Omega$, then AC is $2\ \mathrm{m}$. Find the value of internal resistance r.



A) $3\ \Omega$

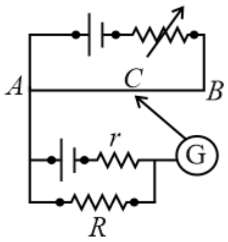
B) $4\ \Omega$

C) $6\ \Omega$

D) $8\ \Omega$

Answer: $8\ \Omega$

Solution:



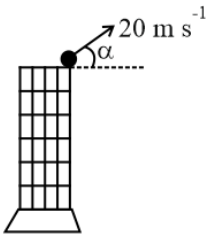
Potential difference across test battery is, $\because\ V = \frac{ER}{R+r} \propto l;\ \frac{R}{R+r} \propto l$

Therefore, we can write, $\frac{\left(\frac{8}{8+r}\right)}{\left(\frac{4}{4+r}\right)} = \frac{3}{2}$

$\Rightarrow 4(4+r) = 3(8+r)\ \Rightarrow 16 + 4r = 24 + 3r\ \Rightarrow r = 8\ \Omega$

Q.19. A projectile is projected (from the top of a tower) with velocity $20 \mathrm{~m} \mathrm{~s}^{-1}$ at an angle α with the horizontal. After $10 \mathrm{~s}$, the inclination of velocity with the horizontal becomes β. Then the value of $(\tan \alpha-\tan \beta)$ is,

(use $g=10 \mathrm{~m} \mathrm{~s}^{-2}$)



A) $5 \cos \alpha$

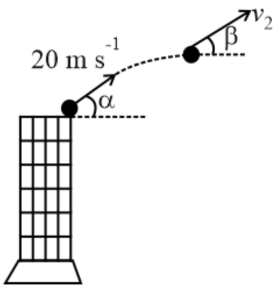
B) $5 \sec \alpha$

C) $6 \sin \alpha$

D) $4 \cot \alpha$

Answer: $5 \sec \alpha$

Solution:



$v_x=20 \cos \alpha=v_2 \cos \beta$

$v_y=20 \sin \alpha-g \times 10=v_2 \sin \beta$

$\Rightarrow \tan \beta=\frac{20 \sin \alpha-100}{20 \cos \alpha}$

$\Rightarrow \tan \beta=\tan \alpha-\frac{5}{\cos \alpha}$

$\Rightarrow \tan \alpha-\tan \beta=\frac{5}{\cos \alpha} \quad=5 \sec \alpha$

Q.20. A transmitter antenna has height of $49 \mathrm{~m}$ and receiver antenna has height $25 \mathrm{~m}$. Find maximum distance between them for satisfactory communication in line of sight mode.

A) $42.93 \mathrm{~km}$

B) $45.86 \mathrm{~km}$

C) $52.73 \mathrm{~km}$

D) $58.64 \mathrm{~km}$

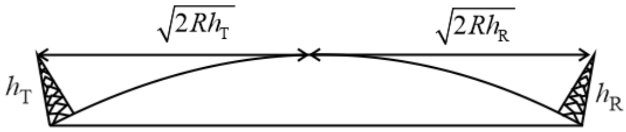
Answer: $42.93 \mathrm{~km}$

Solution: Here,

$h_T = 49\text{m},\ h_R = 25\text{m},\ R = 64 \times 10^5\text{m}$

Maximum distance between two antennae is,

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$
$$= \sqrt{2R(49)} + \sqrt{2R(25)} = 12\sqrt{2R}$$
$$= 12\sqrt{2 \times 64 \times 10^5} = 42.93\ \text{km}$$



Q.21. A capacitor of capacitance C is connected to a source of voltage V. After a long time, it is disconnected and then connected to a capacitor of same capacitance. The loss of energy in the process after connecting with the capacitor is

A) $\frac{1}{2}CV^2$

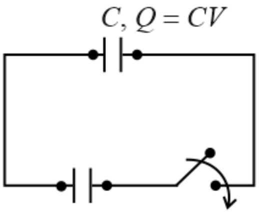
B) $\frac{1}{8}CV^2$

C) $\frac{1}{4}CV^2$

D) CV^2

Answer: $\frac{1}{4}CV^2$

Solution:



Initial charge on first capacitor will be, $Q = CV$

As the second capacitor is identical to the first one, hence potential drop across both capacitor will be equal to $\frac{V}{2}$.

Now, loss of energy, $\Delta H = U_i - U_f$

$$\Rightarrow \Delta H = \frac{1}{2}CV^2 - \frac{1}{2}(2C) \times \left(\frac{V}{2}\right)^2$$

$$\Rightarrow \Delta H = \frac{1}{2}CV^2 - \frac{1}{4}CV^2 \Rightarrow \Delta H = \frac{1}{4}CV^2$$

Q.22. If magnetic susceptibility of a material is 99, then find its magnetic permeability.

A) 12.6×10^{-5}

B) 14.8×10^{-5}

C) 16.4×10^{-5}

D) 18.2×10^{-5}

Answer: 12.6×10^{-5}

Solution: Using the relation of magnetic susceptibility (χ) and relative permeability(μ_r).

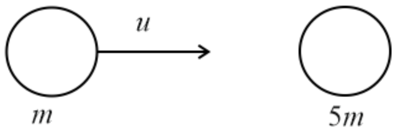
$\mu_r = 1 + \chi = 1 + 99 = 100$

Magnetic permeability, $\mu = \mu_0 \mu_r$

$= 4\pi \times 10^{-7} \times 100$

$\mu = 12.6 \times 10^{-5}$

Q.23. If bodies with masses m and $5\mathrm{m}$ collide as shown in the figure, then find out percentage change in kinetic energy of body m. (coefficient of restitution $e = 1$)



A) 45%

B) 50%

C) 55%

D) 60%

Answer: 55%

Solution: After collision,

Applying momentum conservation,

$mu = mv_1 + 5mv_2 \Rightarrow u = v_1 + 5v_2 \quad \ldots(1)$

Coefficient of restitution, $e = 1 = \frac{v_2 - v_1}{u}$

$\therefore\ u = v_2 - v_1 \quad \ldots(2)$

Adding $2u = 6\mathrm{v}_2 \Rightarrow v_2 = \frac{u}{3}$ and $u = \frac{u}{3} - v_1$

$\therefore\ v_1 = \frac{u}{3} - u = -\frac{2\mathrm{u}}{3}$

Percentage change in kinetic energy $= \frac{\frac{1}{2}\mathrm{mv}_1^2 - \frac{1}{2}\mathrm{mu}^2}{\frac{1}{2}\mathrm{mu}^2} \times 100$

$= \frac{v_1^2 - u^2}{u^2} \times 100 = \frac{\frac{4u^2}{9} - u^2}{u^2} \times 100 = \frac{-5}{9} \times 100 = 55\%$

Section B: Chemistry

Q.1. White P $\xrightarrow{\text{Conc.HNO}_3}$?

A) $H_3PO_3 + N_2$

B) $NO_2 + PH_3$

C) $H_3PO_4 + NO_2$

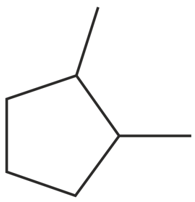
D) $H_3PO_3 + NO_2$

Answer: $H_3PO_4 + NO_2$

Solution: $P_4 + 20HNO_3 \rightarrow 4H_3PO_4 + 20NO_2 + 4H_2O$

The products formed when phosphorus reacts with concentrated nitric acid are phosphoric acid, nitrogen dioxide and water.

Q.2. Find out the number of stereoisomers formed by



A) 4

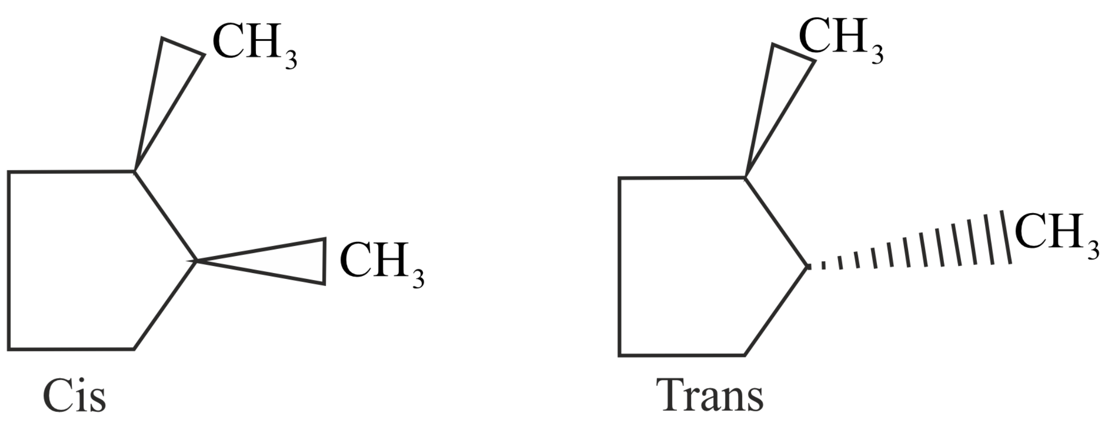
B) 2

C) 3

D) 5

Answer: 3

Solution:



Trans form is optically active it forms 2 enantiomers

So, total stereoisomers $= 1 + 2 = 3$.

Q.3. $Fe_{0.93}O$ has metal deficiency defect. Calculate the percentage of Fe^{2+} ions in $Fe_{0.93}O$ compound. (Round off to the nearest integer)

A) 15

B) 85

C) 19

D) 81

Answer: 85

Solution: In $Fe_{0.93}O$

Let percentage of Fe^{+2} is x and percentage of Fe^{+3} is y

Given, $x + y = 0.93$ ... (1)

Applying charge balance for FeO

$2x + 3y - 2 = 0$

$2x + 3y = 2.00$ ... (2)

Charge balance for $Fe_{0.93}$ is $x + y = 0.93$

$2x + 2y = 1.86$ ... (3)

Subtracting (3) from (2)

we get $y = 0.14$

$\%y = \frac{0.14}{0.93} \times 100 = 15\%$

$x = 100 - 15 = 85\%$

Hence percentage of Fe^{+2} is 85%

Q.4. Calculate $\wedge^{o}{}_{m}$ for AgI given that $\wedge^{o}{}_{m}$ for $AgNO_3,\ KI$ and KNO_3 $13.3,\ 12.7,\ 12\ Scm^2 mol^{-1}$ respectively.

A) 26

B) 13.5

C) 14

D) 28

Answer: 14

Solution: According to kohlrausch's law

$\lambda^{\infty}{}_{AgI} = \lambda^{\infty}{}_{Ag^+} + \lambda^{\infty}{}_{I^-}$

$= \lambda_{AgNO_3} + \lambda_{KI} - \lambda_{KNO_3}$

$= 13.3 + 12.7 - 12$

$= 14\ Scm^2 mol^{-1}$

Q.5. $2\ g$ of solute is dissolved in two different solvent A and B having $200\ g$ mass each. Given that $K_b(A)\ :\ K_b(B) = 1:2$. Calculate the ratio of $\Delta T_b(A)\ :\ \Delta\, T_b(B)$

A) 1

B) 2

C) 4

D) 0.5

Answer: 0.5

Solution: Elevation in boiling point, $\Delta T_b = K_b m$

$K_b =$ molal elevation constant, $m =$ molality

$m = \frac{\text{mass of solute}}{\text{Molarr mass(M) of solute}} \times \frac{1000}{\text{mass of the solvent}}$

$\Delta T_{b_1} = K_{b_1} \times m_1$

$\Delta T_{b_2} = K_{b_2} \times m_2$

Given $\frac{K_{b_1}}{K_{b_2}} = \frac{1}{2}$

$\frac{\Delta T_{b_1}}{\Delta T_{b_2}} = \frac{K_{b_1} \times m_1}{K_{b_2} \times m_2}$

$\frac{m_1}{m_2} = 1$

$= \frac{1}{2} = 0.5$

Q.6. Statement-1: $\Delta T_f = k_f m$

Statement-2: molality is independent of temperature.

A) statement-1 true and statement-2 is false

B) Statement-1 is false and statement-2 is true

C) Both the statements are true

D) Both the statements are false

Answer: Both the statements are true

Solution: The freezing point depression, ΔT_f is equal to the freezing point of the solvent minus the freezing point of the solution.

It is also proportional to the molal concentration of the solute.

$\Delta T_f = K_f m$

Where K_f, the freezing point depression constant, depends only on the solvent. Molality is moles of solute per kg of solvent. So, the concentration of solvent is expressed in terms of mass, and the mass of a substance is not affected by the change in temperature. Hence, molality is independent of temperature.

Q.7. Match the following molecules in column I with their corresponding shapes in column II

Column I	Column II
(i) SF_4	(P) T shaped
(ii) BF_3	(Q) See-saw
(iii) XeF_4	(R) Trigonal planar
(iv) ClF_3	(S) Square planar

A) (i)-(Q); (ii)-(R); (iii)-(S); (iv)-(P)

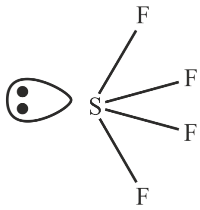
B) (i)-(P); (ii)-(Q); (iii)-(R); (iv)-(S)

C) (i)-(R); (ii)-(P); (iii)-(Q); (iv)-(S)

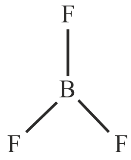
D) (i)-(Q); (ii)-(S); (iii)-(P); (iv)-(R)

Answer: (i)-(Q); (ii)-(R); (iii)-(S); (iv)-(P)

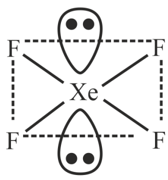
Solution:



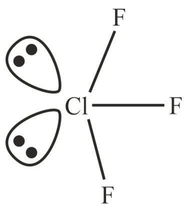
See-Saw



Trigonal planar



Square planar



T-shaped

Q.8. Statement I: Classical smog is formed in cold and humid environment.

Statement II: Photochemical smog contains O_3 and PAN.

The correct statements are:

A) Both I and II

B) Only I

C) Only II

D) Neither I nor II

Answer: Both I and II

Solution: Classical smog occurs in cool humid climate. It is a mixture of smoke, fog and sulphur dioxide. Chemically it is a reducing mixture and so it is also called as reducing smog.

Photochemical smog occurs in warm, dry and sunny climate. The main components of the photochemical smog result from the action of sunlight on unsaturated hydrocarbons and nitrogen oxides produced by automobiles and factories. Photochemical smog has high concentration of oxidising agents and is, therefore, called as oxidising smog.

Ozone is a toxic gas and both NO_2 and O_3 are strong oxidising agents and can react with the unburnt hydrocarbons in the polluted air to produce chemicals such as formaldehyde, acrolein and peroxyacetyl nitrate (PAN).

Q.9. Statement 1 : Mg^{2+} and O^{2-} have same ionic radius

Statement 2 : Mg^{2+} and O^{2-} are isoelectronic species

A) Both statements are true

B) Both statements are false

C) Statement A is correct and B is incorrect

D) Statement A is incorrect and B is correct

Answer: Statement A is incorrect and B is correct

Solution: Both Mg^{2+} and O^{-2} have 10 electrons each and they are isoelectronic species.

But the size of O^{-2} is greater than Mg^{2+} as size $\propto \frac{1}{\frac{z}{e}}$

$\frac{z}{e}$ of $Mg^{2+} = \frac{12}{10} = 1.2$

$\frac{z}{e}$ of $O^{-2} = \frac{8}{10} = 0.8$

$\frac{z}{e} = \frac{\text{Number of protons}}{\text{Number of electrons}}$

Q.10. Angular nodes in $4f$ orbital

A) 1

B) 2

C) 3

D) 4

Answer: 3

Solution: All s-orbitals have 0 angular nodes. All p-orbitals have 1, d-orbitals have 2 and similarly all f-orbitals have 3 angular nodes. So $4f$-orbital will also have 3 angular nodes.

Q.11. Which of the following has maximum CFSE value?

A) $\left[Fe(H_2O)_6\right]^{3+}$

B) $\left[Co(H_2O)_6\right]^{3+}$

C) $\left[Co(CN)_6\right]^{3-}$

D) $\left[Fe(H_2O)_6\right]^{2+}$

Answer: $\left[Co(CN)_6\right]^{3-}$

Solution: $\left[Fe(H_2O)_6\right]^{3+}$; $t_{2g}^3 e_g^2$; CFSE $= 0$

$\left[Co(H_2O)_6\right]^{3+}$; $t_{2g}^6 e_g^0$; CFSE $= [-0.4 \times 6 + 0.6 \times 0]\Delta_o + 2P = -2.4\,\Delta_o + 2P$

$\left[Co(CN)_6\right]^{3-}$; $t_{2g}^6 e_g^0$; CFSE $= [-0.4 \times 6 + 0.6 \times 0]\Delta_o + 2P = -2.4\,\Delta_o + 2P$

$\left[Fe(H_2O)_6\right]^{2+}$; $t_{2g}^4 e_g^2$; CFSE $= [-0.4 \times 4 + 0.6 \times 2]\Delta_o + 2P = -0.4\,\Delta_o$

Since CN^{-1} is a strong field ligand, Δ_o of this complex will be higher than that of $\left[Co(H_2O)_6\right]^{3+}$

Q.12. Match the following in column P with column Q

P	Q
1) Lyophilic colloid	x) Liquid-Liquid colloidal system
2) Emulsion	y) Protective colloids
3) Positively charged colloid	z) $FeCl_3 + NaOH$
4) Negatively charged colloid	w) $FeCl_3 +$ hot water

A) 1-y, 2-x, 3-w, 4-z

B) 1-y, 2-x, 3-z, 4-w

C) 1-x, 2-y, 3-z, 4-w

D) 1-z, 2-w, 3-x, 4-y

Answer: 1-y, 2-x, 3-w, 4-z

Solution: Lyophilic colloids are stable due to charge and solvation, they can be used to protect Lyophobic colloids

Emulsions are liquid-liquid type of colloids

If $FeCl_3$ is added to the excess of hot water, a positively charged sol of hydrated ferric oxide is formed due to adsorption of Fe^{3+} ions. However, when ferric chloride is added to $NaOH$ solution, a negatively charged sol is obtained with adsorption of OH^- ions.

$Fe_2O_3 \cdot xH_2O / Fe^{3+}$
Positively charged

$Fe(OH)_3 / OH^-$
Negatively charged

Q.13. A monochromatic radiation of wavelength λ is incident on a Li^{2+} ion that lifts it to 3^{rd} orbit from ground level. Find the wavelength of incident photon in $\mathring{A}$.

A) 228

B) 114

C) 122.4

D) 244.4

Answer: 114

Solution: As we know that, $\Delta E = \frac{hc}{\lambda}$

Therefore, $\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_3 - E_1}$

$$E_n = -13.6 \times \frac{Z^2}{n^2} eV$$

$$Z = 3 \text{ for } Li^{2+} \text{ ion}$$

$$E_3 = -13.6 \times \frac{9}{9} = -13.6 \, eV$$

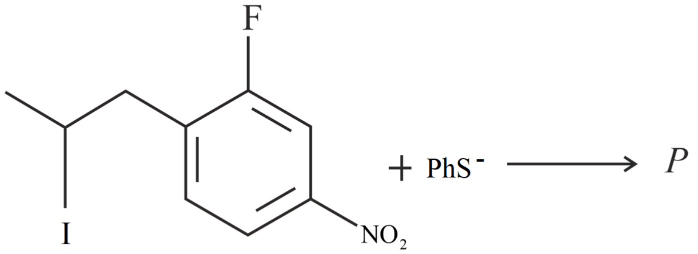
$$E_1 = -13.6 \times \frac{9}{1} = -122.4 \text{ eV}$$

$$\lambda = \frac{hc}{-13.6 - (-122.4) eV} = \frac{hc}{108.8 \times 1.602 \times 10^{-19}}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{108.8 \times 1.602 \times 10^{-19}} = 114 \times 10^{-10} m$$

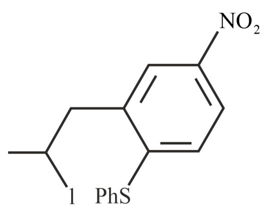
Hence, $\lambda = 114 \mathring{A}$

Q.14.

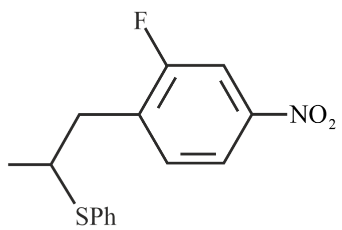


The major product (P) is:

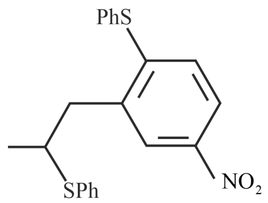
A)



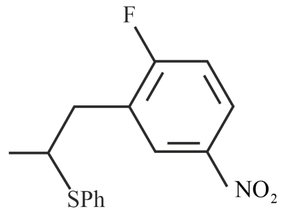
B)



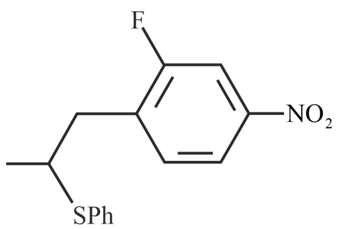
C)



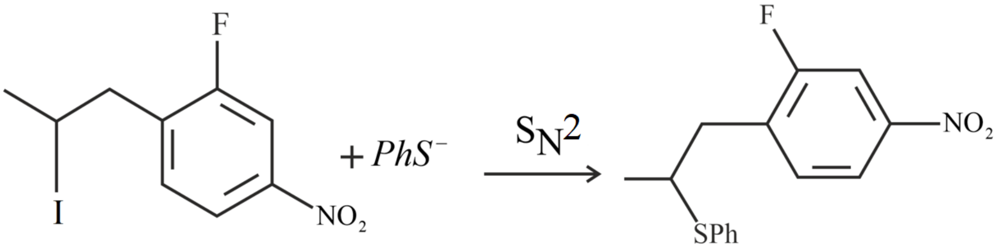
D)



Answer:

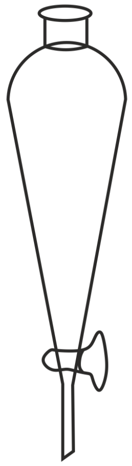


Solution: Bimolecular nucleophilic substitution dominates the aromatic nucleophilic substitution reaction to form product P.

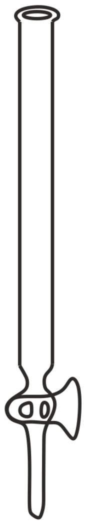


Q.15. Which of the following is a separating funnel?

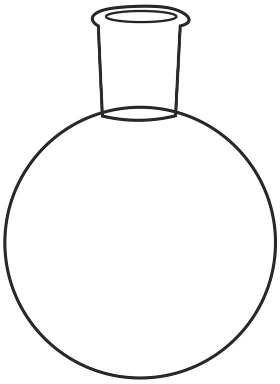
A)



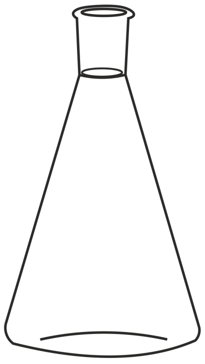
B)



C)

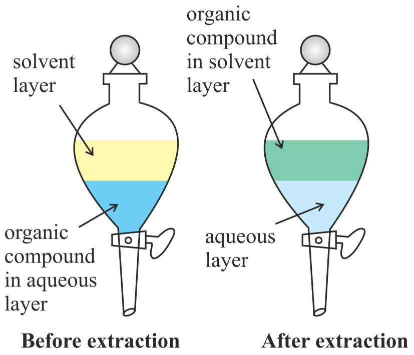


D)



Answer:

Solution: When an organic compound is present in an aqueous medium, it is separated by shaking it with an organic solvent in which it is more soluble than in water. The organic solvent and the aqueous solution should be immiscible with each other so that they form two distinct layers which can be separated by a separatory funnel.



Q.16. Match the following

a)	Bakelite	p)	Paints and lacquers
b)	Glyptal	q)	Water pipes
c)	Polystyrene	r)	Electrical switches
d)	PVC	s)	Television cabinet

A) a-q, b-r, c-s, d-p

B) a-r, b-p, c-s, d-q

C) a-q, b-s, c-r, d-p

D) a-r, b-p, c-q, d-s

Answer: a-r, b-p, c-s, d-q

Solution: Polyvinyl chloride (PVC) : Manufacture of rain coats, hand bags, vinyl flooring, water pipes.

Polystyrene : As insulator, wrapping material, manufacture of toys, radio and television cabinets.

Glyptal : Manufacture of paints and lacquers.

Bakelite : For making combs, electrical switches, handles of utensils and computer discs.

Q.17. Find the spin only magnetic moment (in B.M) of Mn containing species which is formed by $KMnO_4$ in acidic medium.

A) 5.9

B) 1.73

C) 4.8

D) 2.73

Answer: 5.9

Solution: In acidic medium, Mn^{7+} gets reduced to Mn^{2+}

Mn (II) $= [Ar]3d^5$

Number of unpaired electron in $Mn^{2+} = 5$

$\mu = \sqrt{n(n+2)}$ B.M

$\mu = \sqrt{5(5+2)}$ B.M

$\mu = 5.9$ B.M

Q.18. Match the column I having processes and elements with their corresponding ores/reagents or processes used in extraction in column II.

	Column I		Column II
(i)	Blister copper	(P)	Sulphide ore
(ii)	Froth floatation	(Q)	Electrolytic refining
(iii)	Gold extraction	(R)	$[Au(CN)_2]^-$

A) (i)-(P); (ii)-(Q), (P); (iii)-(R), (P)

B) (i)-(Q), (P); (ii)-(P); (iii)-(Q), (R)

C) (i)-(R), (P); (ii)-(Q); (iii)-(P)

D) (i)-(Q), (P); (ii)-(R), (Q); (iii)-(Q), (P)

Answer: (i)-(Q), (P); (ii)-(P); (iii)-(Q), (R)

Solution: i) Copper was smelted in an open furnace. The slightly impure copper had impurities, mostly sulphur dioxide which bubbled up through the copper as it solidified. That gave a blistered surface and the results of the smelting were called blister copper. Copper is purified by electrolytic refining of blister copper.

ii) The principle of froth floatation is that sulphide ores are preferentially wetted by pine oil, whereas the gangue particles are wetted by water. Collectors are added to enhance the non-wettability of the mineral particles. Examples of collectors are pine oil, fatty acids and xanthates.

iii) In the metallurgy of gold, the respective gold ore is leached with a dilute solution of NaCN or KCN in the presence of air (for O_2) from which the gold is obtained later by replacement:

$$4Au(s) + 8CN^-(aq) + 2H_2O(aq) + O_2(g) \rightarrow 4[Au(CN)_2]^-(aq) + 4OH^-(aq)$$

$$2[Au(CN)_2]^-(aq) + Zn(s) \rightarrow [Zn(CN)_4]^{2-}(aq) + 2Au(s)$$

Q.19. Which of the following is correct for general equation for combustion of hydrocarbon.

A) $C_xH_y + (x+y)O_2 \rightarrow \frac{x}{2}CO_2 + \frac{y}{4}H_2O$

B) $C_xH_y + (x+y)O_2 \rightarrow xCO_2 + yH_2O$

C) $C_xH_y + \left(x+\frac{y}{2}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

D) $C_xH_y + \left(x+\frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

Answer: $C_xH_y + \left(x+\frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

Solution: The general equation for the combustion of a hydrocarbon can be written as:

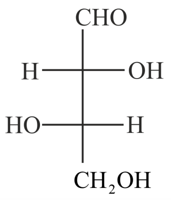
$$C_xH_y + \left(x + \frac{y}{4}\right) O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

For example, the equation for the combustion reaction of propane, C_3H_8 is:

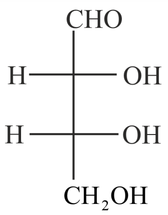
$$C_3H_8(g) + 5O_2(g) \rightarrow 3\,CO_2(g) + 4H_2O(l)$$

Q.20. A compound X, having four carbon atoms, can react with 3 moles of $CH_3\,COOH$ during acetylation reaction. The compound X also gives positive Tollen's test. It reacts with bromine water to form an optically active compound, but reacts with con. HNO_3 to form an optically inactive compound. Compound X is:

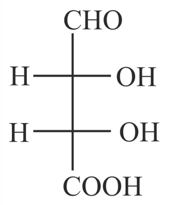
A)



B)

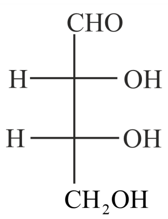


C)

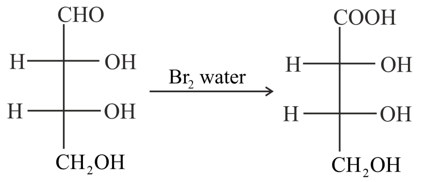


D) None of these

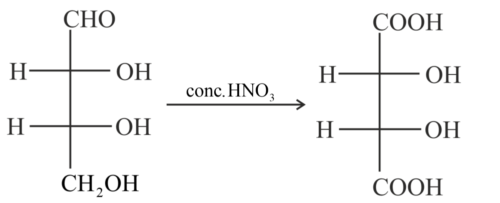
Answer:



Solution:



(X) optically active



(X) optically inactive

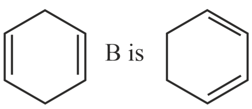
Q.21. The compound A and compound B are respectively:

$$\text{Compound A} \xrightarrow[\text{Zn}/\text{H}_2\text{O}]{\text{O}_3} \text{glyoxal} + \text{butane} - 1,\ 4 - \text{dial}$$

$$\text{Compound B} \xrightarrow[\text{Zn}/\text{H}_2\text{O}]{\text{O}_3} 5 - \text{oxohexanal}$$

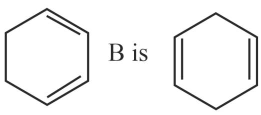
A)

A is B is



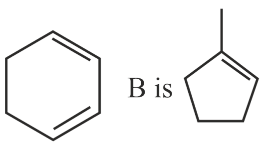
B)

A is B is



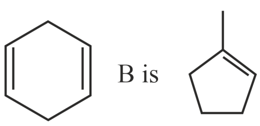
C)

A is B is



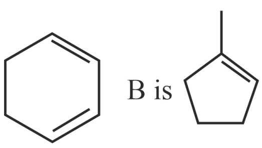
D)

A is B is

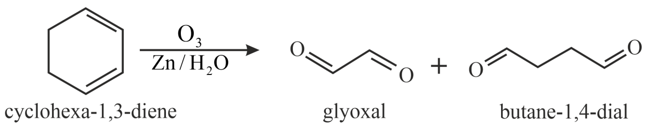


Answer:

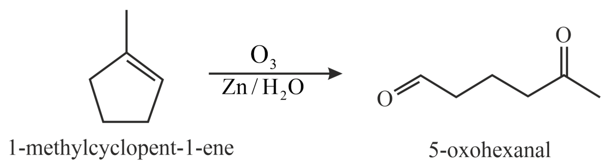
A is B is



Solution: Compound A is cyclohexa-$1,\ 3$-diene



Compound B is 1-methylcyclopent-1-ene



Q.22. Consider the following statements regarding Hoffmann degradation reaction.

Statement I: One alkyl(R) group migrates from carbonyl to N-atom.

Statement II: Migration of alkyl group takes place towards electron deficient N-atom.

The correct statements are:

A) Both I and II

B) Only I

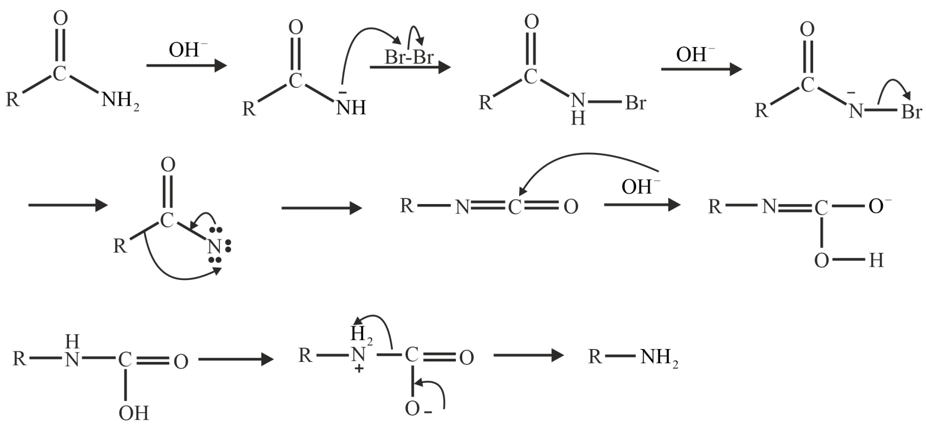
C) Only II

D) Neither I nor II

Answer: Both I and II

Solution:

The mechanism of Hoffmann bromamide degradation is shown below,



In the above mechanism, one alkyl group migrates form carbonyl to nitrogen atom. This migration of alkyl group is taking place towards electron dificient N-atom.

Q.23. Consider the following equilibrium.

$2NOCl \rightleftharpoons 2NO + Cl_2$

If the reaction has started with 2 moles of $NOCl$ in a 1 litre closed container and allowed to attain equilibrium.At equilibrium the moles of NO was found to be 0.4.The equilibrium constant K_C for the reaction is

A) 20×10^{-3}

B) 1.25×10^{-3}

C) 12.5×10^{-3}

D) 25×10^{-3}

Answer: 12.5×10^{-3}

Solution: $2NOCl \rightleftharpoons 2NO + Cl_2$

$t = 0$ 2 -   -

$t = t_{eq}$ $(2 - 2x)$ $2x$ x

$2x = 0.4$

$x = 0.2$

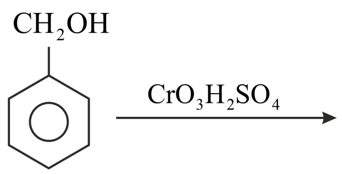
$$K_{eq} = \frac{[NO]^2[Cl_2]}{[NOCl]^2}$$

$$= \frac{(2x)^2(x)}{(2-2x)^2}$$

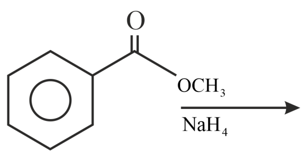
$$= \frac{(0.4)^2 \times 0.2}{(1.6)^2} = \frac{16 \times 0.2}{16 \times 16} = 12.5 \times 10^{-3}$$

Q.24. Which one of the following reactions give Benzaldehyde?

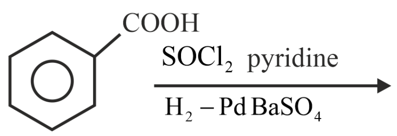
A)



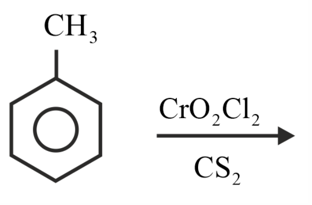
B)



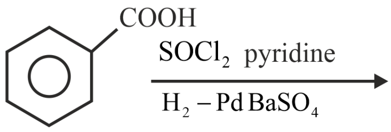
C)



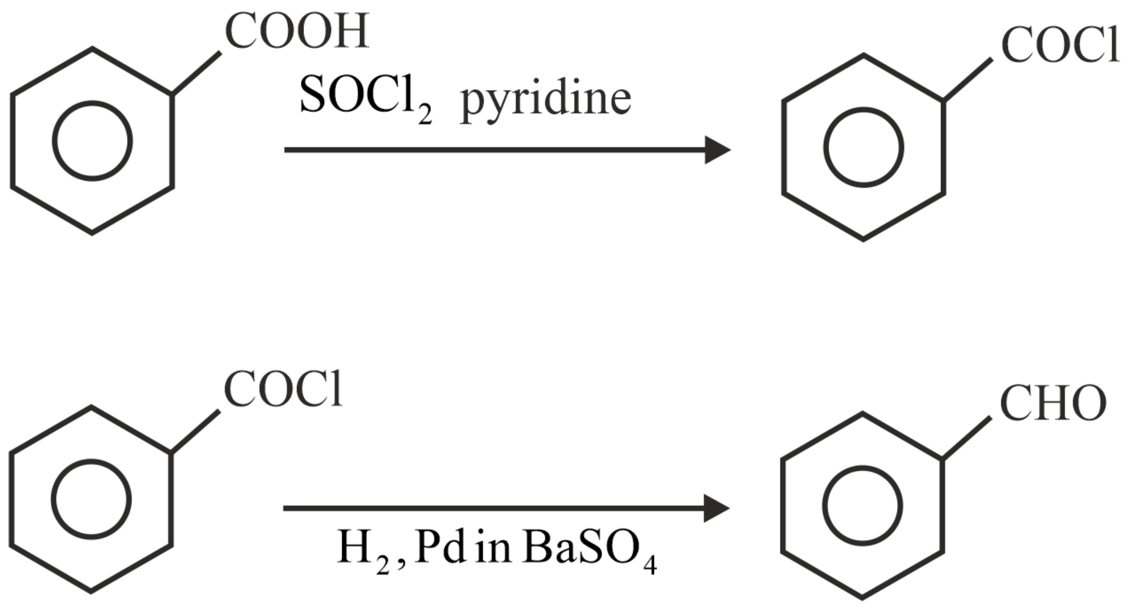
D)



Answer:



Solution:



The above reaction is Rosenmund's reduction reaction.

Q.25. The change in angular momentum during the transition of an electron from the ground state of H-atom to an excited state is? It is given that the electron absorbs $10.2\ \mathrm{eV}$ energy during the transition from ground state to the excited state.

A) $\frac{\mathrm{h}}{\pi}$

B) $\frac{\mathrm{h}}{2\pi}$

C) $\frac{3\mathrm{h}}{2\pi}$

D) $\frac{2\mathrm{h}}{\pi}$

Answer: $\frac{\mathrm{h}}{2\pi}$

Solution: Energy of ground state of H-atom is $-13.6\ \mathrm{ev} \Rightarrow n=1$

Energy of I excited state $=-3.4\ \mathrm{eV} \Rightarrow n_2=2$

Change in angular momentum $\mathrm{mvr}=\frac{\mathrm{nh}}{2\pi}$

Change in angular momentum $=(\mathrm{n}_2-\mathrm{n}_1)\frac{\mathrm{h}}{2\pi}$

$=\left(2-1\right)\frac{\mathrm{h}}{2\pi}$

$=\frac{\mathrm{h}}{2\pi}$

Q.26. Match the following

(a)	Laundry soaps filler	(i)	cetyltrimethylammonium bromide
(b)	Hair conditioner	(ii)	Non-ionic detergent
(c)	Liquid dishwasher	(iii)	Sodium dodecylbenzenesulfonate
(d)	Household detergent	(iv)	Na_2CO_3, sodium rosinate

A) (a)-(iv); (b)-(i); (c)-(ii); (d)-(iii)

B) (a)-(iii); (b)-(iv); (c)-(i); (d)-(ii)

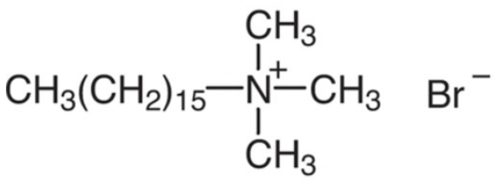
C) (a)-(ii); (b)-(iii); (c)-(iv); (d)-(i)

D) (a)-(iii); (b)-(i); (c)-(iv); (d)-(ii)

Answer: (a)-(iv); (b)-(i); (c)-(ii); (d)-(iii)

Solution: 1. Laundry soaps contain fillers like sodium rosinate, sodium silicate, borax and sodium carbonate.

2. Cationic detergents are quaternary ammonium salts of amines with acetates, chlorides or bromides as anions. Cationic part possess a long hydrocarbon chain and a positive charge on nitrogen atom. Hence, these are called cationic detergents. Cetyltrimethylammonium bromide is a popular cationic detergent and is used in hair conditioners.



3. Non-ionic Detergents: Non-ionic detergents do not contain any ion in their constitution. One such detergent is formed when stearic acid reacts with polyethyleneglycol. Liquid dishwashing detergents are non-ionic type.

4. In anionic detergents, the anionic part of the molecule is involved in the cleansing action. Sodium salts of alkylbenzenesulphonates are an important class of anionic detergents. They are mostly used for household work.

Section C: Mathematics

Q.1. The value of integral $\int_{-2}^{2} \frac{|x^3+x|}{1+e^{x|x|}} dx$ is equal to

A) 6

B) 4

C) 3

D) 12

Answer: 6

Solution: Given

$\int_{-2}^{2} \frac{|x^3+x|}{1+e^{x|x|}} dx$

$I = \int_{-2}^{2} \frac{|x|(x^2+1)}{1+e^{x|x|}} dx$ .....equation (i)

Now using the property $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$ in equation (i) we get

$I = \int_{-2}^{2} \frac{|x|(x^2+1)}{1+e^{-x|x|}} dx$ ......equation (ii)

Adding (i) & (ii),

$2I = \int_{-2}^{2} \frac{|x|(x^2+1)(1+e^{x|x|})}{1+e^{x|x|}} dx$

$\Rightarrow 2I = \int_{-2}^{2} |x^3+x| dx$

$\Rightarrow 2I = 2\int_0^2 \left(x^3+x\right) dx$

$\Rightarrow I = \left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^2 = 4+2 = 6$

Q.2. The value of $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}$ is equal to

A) $\frac{1}{4}$

B) $-\frac{1}{2}$

C) $-\frac{1}{4}$

D) $\frac{1}{8}$

Answer: $\frac{1}{8}$

Solution: $\frac{1}{2\sin\frac{2\pi}{7}} \times 2\sin\frac{2\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7} = \frac{1}{4\sin\frac{2\pi}{7}} \times 2\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}$

$= \frac{1}{8\sin\frac{2\pi}{7}} \times 2\sin\frac{8\pi}{7}\cos\frac{6\pi}{7} = \frac{1}{8\sin\frac{2\pi}{7}}\left(\sin 2\pi + \sin\frac{2\pi}{7}\right) = \frac{1}{8}$

Q.3. The number of real roots of the equation $x^4-4x+1=0$ is

A) 0

B) 1

C) 2

D) 4

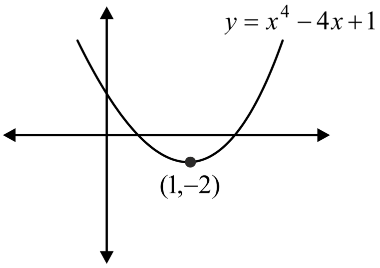
Answer: 2

Solution: Let $f(x) = x^4 - 4x + 1$

$f'(x) = 4x^3 - 4$ and $f'(x) = 0 \Rightarrow x = 1$

So the extrema of the function is at $x = 1$

Plotting the graph we get



$f''(x) = 12x^2 \quad \therefore \; f''(1) > 0$

And at $x = 1, \; f(1) = -2 < 0$

So, graph of $f(x)$ will cut x-axis at two points,

$\therefore$ Number of real roots of $f(x) = 0$ is equal to 2

Q.4. Given a binomial distribution where $n = 7$ and $P(X = 3) = 5 \times P(X = 4)$. The value of sum of mean and variance is

A) $\frac{14}{36}$

B) $\frac{77}{36}$

C) $\frac{31}{36}$

D) $\frac{35}{36}$

Answer: $\frac{77}{36}$

Solution: Let probability of success is p and failure is q
Given $n=7$,

also , $P(X=3)=5\,P(X=4) \Rightarrow {}^7C_3 p^3 q^4 = 5 \cdot {}^7C_4 p^4 q^3$

$\Rightarrow q=5p \quad \ldots(\mathrm{i})$

We know that $p+q=1 \Rightarrow 6p=1$ (from (i))

$\therefore \;\; p=\frac{1}{6}$ and $q=\frac{5}{6}$

$\therefore$ Mean $=np=7\cdot\frac{1}{6}=\frac{7}{6}$

and Variance $=npq=7\cdot\frac{1}{6}\cdot\frac{5}{6}=\frac{35}{36}$

$\therefore$ Sum of mean and variance $=\frac{7}{6}+\frac{35}{36}=\frac{77}{36}$

Q.5. If the sum of the series $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\cdots \;=\frac{m}{n}$ where m & n are co-prime, then the value of $m+n$ is

A) 5

B) 6

C) 12

D) 10

Answer: 5

Solution: Given

$\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\cdots$

Let the r^{th} term be

$t_r=\frac{r}{4r^4+1}$

$\Rightarrow t_r=\frac{r}{4r^4+1+4r^2-4r^2} \;\;\Rightarrow t_r=\frac{r}{(2r^2+1)^2-(2r)^2}$

$\Rightarrow t_r=\frac{1}{4}\times\frac{4r}{(2r^2+1-2r)(2r^2+1+2r)}$

$\Rightarrow t_r=\frac{1}{4}\left(\frac{1}{(2r^2+1-2r)}-\frac{1}{(2r^2+1+2r)}\right)$

$\Rightarrow t_r=\frac{1}{4}\left(\frac{1}{2r^2+1-2r}-\frac{1}{2(r+1)^2-2(r+1)+1}\right)$

$\Rightarrow \sum_{r=1}^{\infty} t_r=\frac{1}{4}\sum_{r=1}^{\infty}\left(\frac{1}{2r^2+1-2r}-\frac{1}{2(r+1)^2-2(r+1)+1}\right)$

$\Rightarrow \sum_{r=1}^{\infty} t_r=\frac{1}{4}\left[\left(\frac{1}{1}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{25}\right)+\ldots\right]$

$\Rightarrow \sum_{r=1}^{\infty} t_r=\frac{1}{4}(1)=\frac{1}{4}=\left(\frac{m}{n}\right)$

$\therefore m+n=1+4=5$

Q.6. If $f(x)=\frac{2e^{2x}}{e^{2x}+e}$, then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+\ldots.f\left(\frac{99}{100}\right)$ is equal to

A) 8

B) 99

C) 100

D) 101

Answer: 99

Solution: $f(x)=\frac{2e^{2x}}{e^{2x}+e}$

$f(1-x)=\frac{2e^{2(1-x)}}{e^{2(1-x)}+e}=\frac{2e}{e+e^{2x}}$

Now, $f(x)+f(1-x)=2$

i.e. $f\left(\frac{1}{100}\right)+f\left(\frac{99}{100}\right)=2$

$\Rightarrow f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+\ldots f\left(\frac{99}{100}\right)=2\times 49+f\left(\frac{50}{100}\right)$

$=98+1=99$

Q.7. The sides of a triangle are $10+x^2, 10+x^2$ and $20-2x^2$. If the area of triangle is maximum for $x=k$, then the value of $3k^2$ is equal to

A) 5

B) $\frac{10}{3}$

C) 10

D) 20

Answer: 10

Solution: Area of triangle, $\Delta=\sqrt{20(20-10-x^2)(20-10-x^2)(20-20+2x^2)}$

$\Delta=2\sqrt{10}x\left(10-x^2\right)$

For maxima, $\frac{d\Delta}{dx}=0\Rightarrow 2\sqrt{10}\left(10-x^2\right)-4\sqrt{10}x^2=0$

$\Rightarrow \quad x^2=\frac{10}{3}$

Here, $\frac{d^2\Delta}{dx^2}<0$

Hence $3k^2=10$

Q.8. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{5\pi}{4}\right)=$

A) $\frac{17\pi}{12}$

B) $\frac{5\pi}{3}$

C) $\frac{17\pi}{4}$

D) $\frac{19\pi}{12}$

Answer: $\frac{17\pi}{12}$

Solution: $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

$=\sin^{-1}\left(\sin\frac{\pi}{3}\right)+\cos^{-1}\left(-\cos\left(\frac{\pi}{6}\right)\right)+\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$=\frac{\pi}{3}+\pi-\cos^{-1}\cos\frac{\pi}{6}+\frac{\pi}{4}\ =\frac{\pi}{3}+\pi-\frac{\pi}{6}+\frac{\pi}{4}$

$=\frac{17\pi}{12}$

Q.9. Suppose an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ satisfies the point $\left(4\sqrt{\frac{2}{5}},3\right)$ & $e=\frac{1}{4}$, then the value of a^2+b^2 is

A) 31

B) 29

C) 30

D) 25

Answer: 31

Solution: Given, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will satisfy $\left(4\sqrt{\frac{2}{5}},3\right)$

So $\frac{16\times\frac{2}{5}}{a^2}+\frac{9}{b^2}=1$ ......equation (i)

Given $e=\frac{1}{4}$ and we know $e=\sqrt{1-\frac{b^2}{a^2}}$

$\Rightarrow\frac{1}{16}=1-\frac{b^2}{a^2}\quad\Rightarrow\frac{b^2}{a^2}=\frac{15}{16}$

$\Rightarrow b^2=\frac{15a^2}{16}$

Now putting value of b^2 in equation (i) we get

$\frac{16\times2}{5a^2}+\frac{9\times16}{15a^2}=1$

$\frac{240}{5a^2}=1$ or $a^2=16$

Now, $b^2=\frac{15a^2}{16}=\frac{15}{16}\times16=15$

So $a^2+b^2=16+15=31$

Q.10. There are 16 spherical ball out of them 11 are blue and rest are red. In how many ways we can arrange these balls in a line so that between two red balls atleast two blue balls are present?

A) ${}^{8}C_3$

B) ${}^{8}C_4$

C) ${}^{9}C_3$

D) ${}^{9}C_4$

Answer: ${}^{8}C_3$

Solution: Given, 4 blue & 5 red spherical ball

Now according to question there should be minimum 2 blue ball in between any two red ball.

So assume a, b, c, d, e, f are number of blue balls,

$\underline{a}$ ◯ $\underline{b}$ ◯ $\underline{c}$ ◯ $\underline{d}$ ◯ $\underline{e}$ ◯ $\underline{f}$

So, $a+b+c+d+e+f=11$ ......(i)

Where $b, c, d, e \geqslant 2$

Now let $b=b'+2$

$c=c'+2$

$d=d'+2$

$e=e'+2$

So equation (i) becomes

$a+b'+2+c'+2+d'+2+e'+2+f=11$

$\Rightarrow \ a+b'+c'+d'+e'+f=3$

Now total ways will be

$={}^{3+6-1}C_{6-1}={}^{8}C_{5}={}^{8}C_{3}$

Q.11. The boolean expression $(\sim(p \wedge q)) \vee q$ is equivalent to

A) $q \rightarrow (p \wedge q)$

B) $p \rightarrow q$

C) $(p \rightarrow q) \rightarrow p$

D) $p \rightarrow (p \vee q)$

Answer: $p \rightarrow (p \vee q)$

Solution: Given $\sim(p \wedge q) \vee q \equiv \sim p \vee \sim q \vee q \equiv \sim p \vee t \equiv t$

Also $p \rightarrow (p \vee q) \equiv \sim p \vee (p \vee q) \equiv \sim p \vee p \vee q \equiv t \vee q \equiv t$

Q.12. The area of the polygon whose vertices are given by the non-real roots of $\bar{z}=iz^2$ is equal to

A) $\frac{3\sqrt{3}}{2}$

B) $\frac{3\sqrt{3}}{4}$

C) $\frac{\sqrt{3}}{4}$

D) $\frac{\sqrt{3}}{2}$

Answer: $\frac{3\sqrt{3}}{4}$

Solution: $\bar{z}=iz^2 \Rightarrow (x-iy)=i(x+iy)^2$

$\Rightarrow\ x-iy=\left(x^2-y^2\right)i-2xy$

i.e., $x=-2xy$ and $-y=x^2-y^2$

$\Rightarrow\ x=0, y=-\frac{1}{2}$

When $x=0$; $y=0,1$

When $y=-\frac{1}{2}$; $x=\pm\frac{\sqrt{3}}{2}$

$(0,0)$ will be rejected as vertices would be non-real roots.

So, the vertices will be $(0,1)$, $\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$ and $\left(-\frac{\sqrt{3}}{2},-\frac{-1}{2}\right)$

Hence, area of $\Delta=\frac{1}{2}\times\sqrt{3}\times\frac{3}{2}=\frac{3\sqrt{3}}{4}$

Q.13. If $X=\sum_{n=0}^{\infty}a^n, Y=\sum_{n=0}^{\infty}b^n$ and $Z=\sum_{n=0}^{\infty}c^n$ where $a,b,c\in(0,1)$ and a,b,c are in A.P., then

A) $X,\ Y,\ Z$ are in A.P

B) $X,\ Y,\ Z$ are in G.P

C) $\frac{1}{X},\frac{1}{Y},\frac{1}{Z}$ are in A.P

D) $\frac{1}{X},\frac{1}{Y},\frac{1}{Z}$ are in H.P

Answer: $\frac{1}{X},\frac{1}{Y},\frac{1}{Z}$ are in A.P

Solution: $X=\sum_{n=0}^{\infty}a^n=\frac{1}{1-a}$

$Y=\sum_{n=0}^{\infty}b^n=\frac{1}{1-b}$

$Z=\sum_{n=0}^{\infty}c^n=\frac{1}{1-c}$

We know $a,\ b,\ c$ are in A.P.

So, $1-a,\ 1-b,\ 1-c$ will also be in A.P. or $\frac{1}{X},\ \frac{1}{Y},\ \frac{1}{Z}$ are in A.P.

Hence, $\frac{1}{1-a},\ \frac{1}{1-b},\ \frac{1}{1-c}$ will be in H.P

Q.14. If x_1,x_2,x_3,x_4,x_5 are numbers from 1 to 18 such that $x_1<x_2<x_3<x_4<x_5$, then the probability that $x_2=7$ and $x_4=11$ is equal to

A) $\frac{3}{68}$

B) $\frac{1}{68}$

C) $\frac{7}{68}$

D) $\frac{5}{68}$

Answer: $\frac{1}{68}$

Solution: We know $x_1 < 7 < x_3 < 11 < x_5$

$x_1 \in \{1, 2, 3, 4, 5, 6\}$

$x_3 \in \{8, 9, 10\}$

$x_5 \in \{12, 13, 14, 15, 16, 17, 18\}$

Required probability $P = \frac{{}^{6}C_1 \times {}^{3}C_1 \times {}^{7}C_1}{{}^{18}C_5} = \frac{1}{68}$

Q.15. In an isosceles triangle ABC, a vertex is $A(6, 1)$, base BC is $2x + y = 4$ and point B lies on the line $x + 3y = 7$. If centroid of the triangle is (α, β), then $15(\alpha + \beta)$ is equal to

A) 51

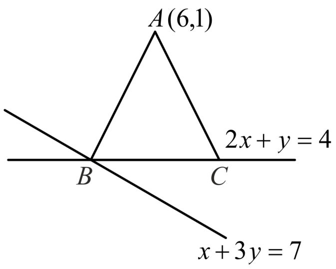
B) 39

C) 41

D) 49

Answer: 51

Solution:



In $\triangle ABC$, $AB = AC$

Solving $2x + y = 4$ & $x + 3y = 7$, we get

$B \equiv (1, 2)$

Let $C \equiv (h, k)$ and as it lies on $2x + y = 4$

so $2h + k = 4$

Now, $AB^2 = AC^2$

$26 = \left(h - 6\right)^2 + (k - 1)^2 \Rightarrow 26 = (h - 6)^2 + (3 - 2h)^2$

$\Rightarrow 26 = 5h^2 - 24h + 45 \Rightarrow (h - 1)(5h - 19) = 0$

$\Rightarrow h = \frac{19}{5}$ (as $h = 1$rejected)

so $k = -\frac{18}{5}$

Hence, centroid $= \left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3}\right) \equiv \left(\frac{18}{5}, -\frac{1}{5}\right)$

i.e. $15(\alpha + \beta) = 15 \times \frac{17}{5} = 51$

Q.16. If $\lim\limits_{x \to 7} \frac{18 - [1 - x]}{[x - 3a]}$ exists, where $[\cdot]$ represents the greatest integer function, then the value of a is equal to (where $a \in \mathbb{I}$)

A) -2

B) -3

C) -6

D) -7

Answer: -6

Solution: Since $\lim_{x \to 7} \frac{18-[1-x]}{[x-3a]}$ exists $(a \in \mathbb{I})$

$\therefore \mathrm{L.H.L} = \mathrm{R.H.L}$

$\Rightarrow \lim_{h \to 0} \frac{18-(-6)}{[7-h-3a]} = \lim_{h \to 0} \frac{18-(-7)}{[7+h-3a]}$

$\Rightarrow \lim_{h \to 0} \frac{24}{(7-3a)+[-h]} = \lim_{h \to 0} \frac{25}{(7-3a)+[h]}$

$\Rightarrow \frac{24}{6-3a} = \frac{25}{7-3a} \Rightarrow a = -6$

Q.17. If $\int \frac{(x^2+1)e^x}{(x+1)^2} = f(x)e^x + C$, then the value of $\frac{d^3 f}{dx^3}$ at $x = 1$ is

A) $\frac{3}{4}$

B) $\frac{3}{8}$

C) $\frac{1}{4}$

D) $\frac{7}{8}$

Answer: $\frac{3}{4}$

Solution: $\int \frac{(x^2+1)e^x dx}{(x+1)^2} = f(x)e^x + C$

$\int e^x\left(\frac{x^2-1}{(x+1)^2} + \frac{2}{(x+1)^2}\right)dx = f(x)e^x + C$

$\int e^x\left(\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right)dx = f(x)e^x + C$

We know that $\int e^x(f(x) + f'(x)) = e^x f(x) + c$

Here $f(x) = \frac{x-1}{x+1}$ & $f'(x) = \frac{2}{(x+1)^2}$

So $\int e^x\left(\left(\frac{x-1}{x+1}\right) + \frac{2}{(x+1)^2}\right)dx = f(x)e^x + C$

$\Rightarrow e^x\left(\frac{x-1}{x+1}\right) + C = e^x f(x) + C$

On comparing both sides we get $f(x) = \frac{x-1}{x+1}$

So $f'(x) = \frac{2}{(x+1)^2}$ & $f''(x) = \frac{-4}{(x+1)^3}$

$f'''(x) = \frac{12}{(x+1)^4}$,

Now $f'''(1) = \frac{12}{(1+1)^4} = \frac{12}{16} = \frac{3}{4}$

Q.18. If $\begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix} = (\operatorname{adj}(\operatorname{adj}(A)))$,then the value of $|A| =$

A) 14

B) 11

C) 10

D) 15

Answer: 14

Solution: Given,

$$\begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix} = (\operatorname{adj}(\operatorname{adj} A))$$

Now we know that $|\operatorname{adj} A| = A^2$ if it is 3×3 matrix

So $|\operatorname{adj}(\operatorname{adj} A)| = |\operatorname{adj} A|^2$

$\Rightarrow |A|^{2\times2} = |A|^4$

Now, $|A|^4 = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$

$\Rightarrow \quad |A|^4 = 14^3 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$

$\Rightarrow |A|^4 = 14^3 \times (14)$

$|A|^4 = (14)^4 \quad \Rightarrow \quad |A| = 14$

Q.19. If the coefficient of x^{10} in the expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k m$, then k is

A) 1

B) 3

C) 5

D) 8

Answer: 5

Solution:

$T_{r+1} = {}^{60}C_r \left(\frac{\sqrt{x}}{5^{\frac{1}{4}}}\right)^{60-r} \left(\frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^r$

For x^{10}, $\frac{60-r}{2} - \frac{r}{3} = 10 \Rightarrow \quad r = 24$

Hence coefficient of x^{10} will be ${}^{60}C_{24} 5^3$

Now, exponent of 5 in ${}^{60}C_{24}$ will be the power of 5 in $\frac{60!}{24!36!}$ i.e. $\frac{5^{14}}{5^4.5^8} = 5^2$

i.e. $k = 5$

Q.20. If the image of (a, b, c) in $3x - 4y + 12z + 19 = 0$ is $(a-6, \beta, \gamma)$, $a + b + c = 5$, then the value of $-7\beta + 9\gamma$ is

A) -137

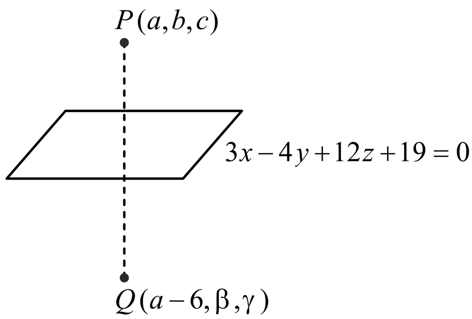
B) 135

C) -135

D) 137

Answer: -137

Solution: Given plane $3x-4y+12z+19=0$ .....(i)



Now direction ratio of plane $= <3,-4,12>$

& D.R of $PQ = <6, b-\beta, c-\gamma>$

As D.R of plane $=$ D.R. of PQ

So $b-\beta=-8 \Rightarrow \beta=b+8$

$c-\gamma=24 \Rightarrow \gamma=c-24$

Now midpoint of $PQ=\left(a-3,\frac{b+\beta}{2},\frac{c+\gamma}{2}\right)$

$=(a-3,\ b+4,\ c-12)$

Now midpoint will lie on plane so putting the values in equation (i) we get,

$3(a-3)+-4(b+4)+12(c-12)+19=0$

$\Rightarrow 3a-4b+12c=150$ ...(ii)

Also given $a+b+c=5$ ....(iii)

Now from equation (ii) $-3\times$ equation (iii), we get

$-7b+9c=135$

$\Rightarrow -7(\beta-8)+9(\gamma+24)=135$

$\Rightarrow -7\beta+56+9\gamma+216=135 \Rightarrow -7\beta+9\gamma=-137$

Q.21. For the equation $e^{2x}-11e^{x}-\frac{45}{e^{x}}+\frac{81}{2}=0$ if the sum of roots is $\ln P$, then P is

A) 45

B) 40

C) 36

D) 18

Answer: 45

Solution: Given $e^{2x}-11e^{x}-\frac{45}{e^{x}}+\frac{81}{2}=0$

$e^{3x}-11e^{2x}+\frac{81}{2}e^{x}-45=0$

Let $e^{x}=t$, so $t^{3}-11t^{2}+\frac{81}{2}t-45=0$

Product of roots $t_1t_2t_3=45$

i.e. $e^{x_1}\times e^{x_2}\times e^{x_3}=45$

$e^{x_1+x_2+x_3}=45$

$x_1+x_2+x_3=\log_2 45$

Given sum of root $=\ln|P|$

So $P=45$