JEE Main Exam 2022 - Session 1

28 June 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2 r t^2$, where k is a constant. The power delivered to the particle by the force acting on it is -

A) $2\pi m k^2 r^2$

B) $m k^2 r^2 t$

C) $\frac{(m k^4 r^2 t^5)}{3}$

D) Zero

Answer: $m k^2 r^2 t$

Solution: $a_c = k^2 r t^2$

Or $\frac{v^2}{r} = k^2 r t^2$ Or $v = krt$

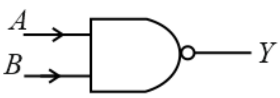
Therefore, tangential acceleration, $a_t = \frac{\mathrm{d}v}{\mathrm{d}t} = kr$

Therefore, tangential force, $F_t = m a_t = mkr$

Only tangential force does work.

$\text{Power} = F_t v = (mkr)(krt) \Rightarrow \text{Power} = m k^2 r^2 t$

Q.2. For the following gate the output Y is given by



A) $A + B$

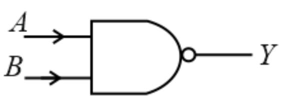
B) $\overline{A + B}$

C) $A \cdot B$

D) $\overline{A \cdot B}$

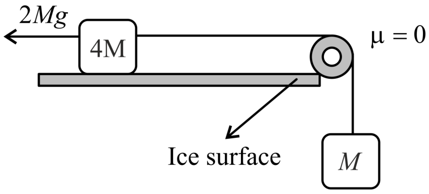
Answer: $\overline{A \cdot B}$

Solution:



This gate is a NAND Gate. It is the combination of two basic logic gates, the AND gate and the NOT gate connected in series. The logic NAND function is given by the Boolean expression $Y = \overline{A \cdot B}$.

Q.3. In the diagram shown, if tension in string is $\frac{x}{5}Mg$. Find the value of x.



A) 6

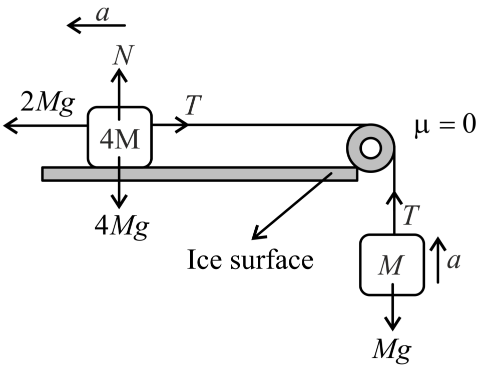
B) 7

C) 8

D) 9

Answer: 6

Solution:



Equation for $4M$: $2Mg - T = 4Ma \quad \ldots(1)$

Equation for M : $T - Mg = Ma \quad \ldots(2)$

Adding both equation we get,

$Mg = 5Ma \Rightarrow a = \frac{g}{5}$

Now, tension $T = Mg + Ma = \frac{6Mg}{5}$

Therefore, $x = 6$

Solving both above two equation we get:

$a = \frac{g}{5}, T = \frac{6Mg}{5}$

$x = 6$

Q.4. The work function of metal surface is 6.63×10^{-19} J. Find the maximum wavelength which can eject photoelectron from this surface.

A) 300 nm

B) 350 nm

C) 400 nm

D) 450 nm

Answer: 300 nm

Solution: Here, Work function $\phi = 6.63 \times 10^{-19}$ J.

Now, threshold wavelength, $\lambda_0 = \frac{hc}{\phi} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{6.63 \times 10^{-19}}$

$\Rightarrow \lambda_0 = 300$ nm

Q.5. Resolving power of a telescope for the aperture 24.4 cm for the wavelength $\lambda = 2440 \mathring{A}$ is

A) 2.5×10^{-5}

B) 8.2×10^5

C) 5.0×10^{-4}

D) 7.5×10^6

Answer: 8.2×10^5

Solution: Given,

The diameter of the objective lens, $D = 24.4$ cm

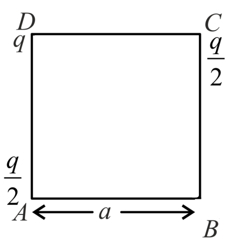
The wavelength of the light, $\lambda = 2440 \ \mathring{A}$

The resolving power of a telescope is given by,

$R.P = \frac{D}{1.22\lambda} \Rightarrow R.P = \frac{24.4 \times 10^{-2}}{1.22 \times 2440 \times 10^{-10}} = 8.2 \times 10^5$

Hence, the resolving power of a telescope is 8.2×10^5.

Q.6. Charges are arranged on the corners of a square of side length a as shown below. Find the electric field at corner B.



A) $\left(\sqrt{2}+1\right)\frac{kq}{a^2}$

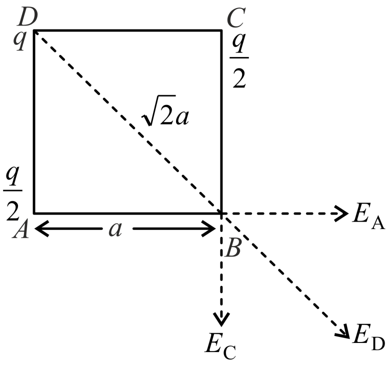
B) $\left(\sqrt{2}+2\right)\frac{kq}{a^2}$

C) $\left(\sqrt{2}+1\right)\frac{kq}{2a^2}$

D) None of these

Answer: $\left(\sqrt{2}+1\right)\frac{kq}{2a^2}$

Solution:



$$E_A=\frac{k\left(\frac{q}{2}\right)}{a^2}=\frac{kq}{2a^2},\ E_C=\frac{k\left(\frac{q}{2}\right)}{a^2}=\frac{kq}{2a^2}\text{ and }E_D=\frac{k(q)}{\left(\sqrt{2}a\right)^2}=\frac{kq}{2a^2}$$

As we can see from diagram, E_C & E_A are perpendicular.

Therefore, net electric field will be,

$$=\frac{kq}{2a^2}+\sqrt{\left(\frac{kq}{2a^2}\right)^2+\left(\frac{kq}{2a^2}\right)^2}=\frac{kq}{2a^2}+\frac{kq}{\sqrt{2}a^2}$$
$$=\left(\sqrt{2}+1\right)\left(\frac{kq}{2a^2}\right)$$

Q.7. Water drop $2\ \mathrm{cm}$ diameter gets divided into 64 equal droplets, the surface tension of the water is $0.075\ \mathrm{N\ m^{-1}}$. Find gain in surface energy

A) $1.9\times10^{-3}\ \mathrm{J}$

B) $2.8\times10^{-4}\ \mathrm{J}$

C) $3.23\times10^{-3}\ \mathrm{J}$

D) $1.51\times10^{-4}\ \mathrm{J}$

Answer: $2.8\times10^{-4}\ \mathrm{J}$

Solution: By conserving the total volume $\frac{4}{3}\pi R^3=64\times\left(\frac{4}{3}\pi r^3\right)\Rightarrow R=4r$

Change in surface energy is

$$\Delta U=T\times\Delta A,=T\left[64\times4\pi r^2-4\pi R^2\right]$$

$$=4\pi TR^2\left[64\times\frac{1}{16}-1\right]$$
$$=12\times3.14\times0.075\times10^{-4}$$
$$\approx2.8\times10^{-4}\ \mathrm{J}$$

Q.8. x & y co-ordinate of a particle is given below as a function of time. Find the shape of the path covered.

$$x=4\sin\left(\frac{\pi}{2}-\omega t\right)$$

$$y=4\sin(\omega t)$$

A) Circle

B) Helix

C) Parabola

D) Ellipse

Answer: Circle

Solution: $x = 4\sin\left(\frac{\pi}{2} - \omega t\right) = 4\cos(\omega t)$

$y = 4\sin(\omega t)$

Clearly,
$x^2 + y^2 = (4\cos(\omega t))^2 + (4\sin(\omega t))^2$
$\Rightarrow x^2 + y^2 = 16$

Hence, the path covered by particle is a circle.

Q.9. A man of $60\ \mathrm{kg}$ is running towards the stationary trolley of $120\ \mathrm{kg}$. The man jumps up on the trolley. The velocity of the trolley becomes $2\ \mathrm{m\,s^{-1}}$. What was the original velocity of man?

A) $4\ \mathrm{m\,s^{-1}}$

B) $5\ \mathrm{m\,s^{-1}}$

C) $6\ \mathrm{m\,s^{-1}}$

D) $8\ \mathrm{m\,s^{-1}}$

Answer: $6\ \mathrm{m\,s^{-1}}$

Solution: Applying conservation of the momentum,

$m_m u = (m_m + m_t)v \Rightarrow 60u = (60 + 120) \times 2 \Rightarrow u = 6\ \mathrm{m\,s^{-1}}$

Q.10. Match The following:

	Column I		Column II
A.	TV signal	I.	$6\ \mathrm{MHz}$
B.	Radio signal	II.	$2\ \mathrm{MHz}$
C.	Human voice	III.	$3\ \mathrm{kHz}$
D.	Music voice	IV.	$20\ \mathrm{kHz}$

A) $A - I,\ B - II,\ C - III,\ D - IV$

B) $A - II,\ B - I,\ C - III,\ D - IV$

C) $A - I,\ B - II,\ C - IV,\ D - III$

D) $A - II,\ B - III,\ C - I,\ D - IV$

Answer: $A - I,\ B - II,\ C - III,\ D - IV$

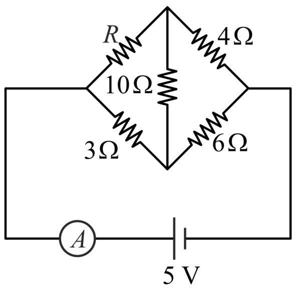
Solution: TV signal $\rightarrow 6\ \mathrm{MHz}$

Radio signal $\rightarrow 2\ \mathrm{MHz}$

Human voice $\rightarrow 3\ \mathrm{kHz}$

Music voice $\rightarrow 20\ \mathrm{kHz}$

Q.11. For what value of R the current in $10\ \Omega$ resistor will be zero?



A) $10\ \Omega$

B) $2\ \Omega$

C) $3\ \Omega$

D) $4\ \Omega$

Answer: $2\ \Omega$

Solution: This is a Wheatstone circuit. For the current to be zero the Wheatstone circuit should be balanced. Therefore,

$\frac{R}{3}=\frac{4}{6} \Rightarrow R=2\ \Omega$

Q.12. Assertion: Product of pressure and time has the same dimensions as coefficient of viscosity.

Reason: Coefficient of viscosity is expressed as $\eta=\frac{\text{Force}}{\text{Velocity gradient}}$.

A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

B) Both Assertion and Reason are true but Reason is NOT the correct explanation of Assertion.

C) Assertion is true but Reason is false.

D) Assertion is false but Reason is true.

Answer: Assertion is true but Reason is false.

Solution: Viscous force is given by $F=\eta A\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)$, here, A is area and $\frac{\mathrm{d}v}{\mathrm{d}t}$ is velocity gradient.

Thus, $\eta=\frac{F}{A\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)}$.

Now, $Pt=\frac{F}{A}t=\eta\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)t$

Then, $[Pt]=ML^{-1}T^{-1}=[\eta]$

Thus, assertion is true but reason is false.

Q.13. **Statement** 1: n-p-n transistor conducts more current than p-n-p transistor

Statement 2: In n-p-n transistor, electrons have more mobility than holes.

A) Both statements are true

B) S_1 is true and S_2 is false

C) S_1 is false and S_2 is true

D) Both statements are false

Answer: Both statements are true

Solution: A n-p-n transistor has more electrons than holes, and p-n-p transistor has more holes than electrons. When a voltage is applied, then the movement of electrons is greater than holes. Therefore, conduction current in n-p-n transistor is more than that in p-n-p transistor.

Both statements are true.

Q.14. If a positive charged particle is moved against an electric field then its

A) Potential energy will increase

B) Potential energy will decrease

C) Potential energy will remain same

D) Behaviour of potential energy is unpredictable

Answer: Potential energy will increase

Solution: If a positive charge is moved against the Coulomb force of an electric field, then the negative work is done by the electric field. Energy is used from some outside source which provides positive work. The change in the potential energy is defined as the negative of the work done by the electrostatic force.

$\Delta U = -(-W_E) = W_E$. Hence, the potential energy of the charged particle will increase.

Q.15. Two planets revolve around Sun such that their time periods of revolution T_A and T_B are related by $T_A = 2T_B$. If their radii of revolution are r_A and r_B, then:

A) $4r_A^3 = r_B^3$

B) $r_A^3 = 4r_B^3$

C) $4r_A^2 = r_B^2$

D) $r_A^2 = 4r_B^2$

Answer: $r_A^3 = 4r_B^3$

Solution: From Kepler's law of periods,

$T^2 \propto R^3$

Therefore,

$\Rightarrow \left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$

$\Rightarrow 2^2 = \frac{r_A^3}{r_B^3}, \Rightarrow r_A^3 = 4r_B^3$

Q.16. The angle of minimum deviation for a prism having refractive index $\mu = \cot\left(\frac{A}{2}\right)$, where A is the angle of prism, is

A) $180^\circ - 2A$

B) $180^\circ - 3A$

C) $90^\circ - A$

D) $180^\circ - 4A$

Answer: $180^\circ - 2A$

Solution:

As we know, angle of minimum deviation is given by, $\Rightarrow \mu = \frac{\sin \frac{(\delta_m + A)}{2}}{\sin \frac{A}{2}}$

$\Rightarrow \cot\left(\frac{A}{2}\right) = \frac{\sin \frac{(\delta_m + A)}{2}}{\sin \frac{A}{2}}$

$\Rightarrow \frac{\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin \frac{(\delta_m + A)}{2}}{\sin \frac{A}{2}}$

$\Rightarrow \sin\left(90^\circ - \frac{A}{2}\right) = \sin\left(\frac{\delta_m + A}{2}\right)$

Therefore, $90^\circ - \frac{A}{2} = \frac{\delta_m + A}{2} \Rightarrow \delta_m = 180^\circ - 2\mathrm{A}$

Q.17. The internal energy of 2 mol of ideal monatomic gas at 300 K is

A) 8314 J

B) 12450 J

C) 7470 J

D) 0 J

Answer: 7470 J

Solution: The internal energy of an ideal gas is a state function and is given by, $U = nC_vT$.

For monatomic gas, $C_v = \frac{3}{2}R$.

Therefore, $U = 2 \times \frac{3}{2} \times 8.3 \times 300 = 7470 \text{ J}$

Q.18. If λ_p and λ are wavelengths of a proton & a photon respectively, having same energy E. Then the ratio of wavelength will be: (Given mass of proton$= m$)

A) $\frac{1}{c}\sqrt{\frac{E}{2m}}$

B) $\sqrt{\frac{E}{2m}}$

C) $\frac{1}{2c}\sqrt{\frac{E}{2m}}$

D) $\frac{1}{3c}\sqrt{\frac{E}{2m}}$

Answer: $\frac{1}{c}\sqrt{\frac{E}{2m}}$

Solution: As we know, $\lambda_p = \frac{h}{mv} \Rightarrow \lambda_p = \frac{h}{\sqrt{2mE}} \quad \ldots(1)$

Also for photon, $\lambda = \frac{hc}{E} \quad \ldots(2)$

Therefore, $\frac{\lambda_p}{\lambda} = \frac{\frac{h}{\sqrt{2mE}}}{\frac{hc}{E}} = \frac{1}{c}\sqrt{\frac{E}{2m}}$

Q.19. Two waves having wavelengths $4.08\ \mathrm{m}$ and $4.16\ \mathrm{m}$ produce 40 beats in $12\ \mathrm{s}$ find velocity of sound in the medium

A) $707\ \mathrm{m\,s^{-1}}$

B) $330\ \mathrm{m\,s^{-1}}$

C) $505\ \mathrm{m\,s^{-1}}$

D) $1028\ \mathrm{m\,s^{-1}}$

Answer: $707\ \mathrm{m\,s^{-1}}$

Solution: If the velocity of the sound in the medium is v then, $f_1 = \frac{v}{4.08}$ and $f_2 = \frac{v}{4.16}$

Now beat frequency,

$$f_b = \Delta f = |f_1 - f_2| = \frac{40}{12} = \frac{10}{3}\ \mathrm{Hz}$$
$$\Rightarrow v\left[\frac{1}{4.08} - \frac{1}{4.16}\right] = \frac{10}{3}\ \mathrm{Hz} \Rightarrow 707.2\ \mathrm{m\,s^{-1}}$$

Q.20. A pendulum of length $250\ \mathrm{cm}$ is released from rest when string makes angle of 60° with vertical. Find its maximum velocity.

A) $5\ \mathrm{m\,s^{-1}}$

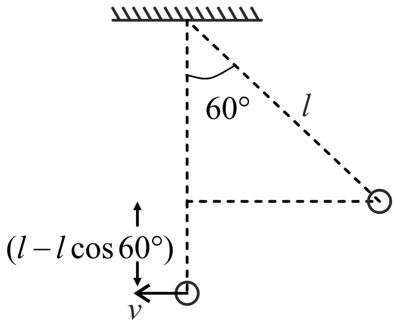
B) $6\ \mathrm{m\,s^{-1}}$

C) $7\ \mathrm{m\,s^{-1}}$

D) $8\ \mathrm{m\,s^{-1}}$

Answer: $5\ \mathrm{m\,s^{-1}}$

Solution:



Velocity will be maximum at the bottom point when potential energy of the pendulum will be zero(considering bottom point as reference for potential energy).

$$\frac{1}{2}mv^2 = mg\left(l - l\cos 60^\circ\right)$$
$$v = \sqrt{gl} = \sqrt{10 \times 2.5} = 5\ \mathrm{m\,s^{-1}}$$

Q.21. The position vector and velocity vector of a particle of mass $m=1\ \text{kg}$ is given by $\overrightarrow{r}=\left(3\hat{\imath}+\hat{\jmath}\right)\ \text{m}$ and $\overrightarrow{v}=\left(3\hat{\jmath}-\widehat{\mathrm{k}}\right)\ \text{m s}^{-1}$. Find x when angular momentum is $\left|\overrightarrow{L}\right|=\sqrt{x}\ \text{N m s}^{-1}$.

A) 19

B) 81

C) 89

D) 71

Answer: 19

Solution: Angular momentum is given by $\overrightarrow{L}=\overrightarrow{r}\times\overrightarrow{p}$, where, $\overrightarrow{p}$ is linear momentum.

Then, $\overrightarrow{L}=m\left(\overrightarrow{r}\times\overrightarrow{v}\right)$

$=1\left(3\hat{\imath}+\hat{\jmath}\right)\times\left(3\hat{\jmath}-\widehat{\mathrm{k}}\right)$

$=-3\widehat{\mathrm{k}}+3\hat{\jmath}-1\hat{\imath}$

$\left|\overrightarrow{L}\right|=\sqrt{19}\ \text{N m s}^{-1}$

Section B: Chemistry

Q.1. The charge required for reduction of 1 mole of $Cr_2O_7^{2-}$ ions to Cr^{3+} is

A) $96500\ C$

B) $6 \times 96500\ C$

C) $3 \times 96500\ C$

D) $2 \times 96500\ C$

Answer: $6 \times 96500\ C$

Solution: 1 faraday of charge is the magnitude of charge of one mole of electrons.

According to the given reaction:
$Cr_2O_7^{2-} + 6e^- \longrightarrow 2Cr^{3+}$

Reduction of $1\ mol$ of $Cr_2O_7^{2-}$ to Cr^{3+} requires 6 moles of electrons. Hence, charge required $= 6 \times 96500\ C$

Q.2. $CuO \stackrel{H_2}{\rightarrow} (P)$ Product P is:

A) $Cu(OH)_2$

B) $Cu(s)$

C) Cu_2O

D) All of these

Answer: $Cu(s)$

Solution: $CuO \stackrel{H_2}{\rightarrow} Cu + H_2O$

Copper oxide undergoes reduction to form elemental copper and water. This is a redox reaction. Since oxygen is removed from the CuO it is undergoing reduction. And hydrogen is getting oxidised.

Q.3. Which of the following will show minimum synergic bonding?

A) $[Mn(CO)_5]$

B) $[Mn_2(CO)_{10}]$

C) $[Cr(CO)_6]$

D) $[Fe(CO)_5]$

Answer: $[Cr(CO)_6]$

Solution: Synergic bonding involves transference of electrons from ligands to metal. The transference of electrons takes place from filled metal orbitals to anti-bonding orbitals of ligands. It is a bond between a carbonyl group acting as a ligand and a metal. Synergic bonding means self strengthening bond. More the number of d electrons on the central metal ion more the synergic bonding. In metal carbonyls' oxidation state of the metal is zero.

In the complex $\left[\mathrm{Mn}\left(\mathrm{CO}\right)_5\right]$, the electronic configuration of Mn is $3\mathrm{d}^5 4\mathrm{s}^2$. As carbon monoxide is a strong field, pairing of electrons occurs. Hence, the new electronic configuration of Mn is $3\mathrm{d}^7$.

In the case of $\left[\mathrm{Mn}_2\left(\mathrm{CO}\right)_{10}\right]$ also electronic configuration after pairing of electrons is $3\mathrm{d}^7$.

In the complex $\left[\mathrm{Cr}\left(\mathrm{CO}\right)_6\right]$, the electronic configuration of Cr is $3\mathrm{d}^5 4\mathrm{s}^1$. As carbon monoxide is a strong field, pairing of electrons occurs. Hence, the new electronic configuration of Cr is $3\mathrm{d}^6$

In the case of $\left[\mathrm{Fe}\left(\mathrm{CO}\right)_5\right]$, the electronic configuration of iron after pairing of electrons is $3\mathrm{d}^8$.

.

Q.4. Which is not a copolymer?

A) Neoprene

B) Buna-N

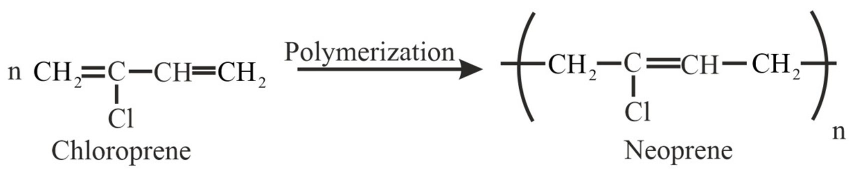
C) Buna-S

D) Nylon-$6,\ 6$

Answer: Neoprene

Solution: A copolymer is a polymer formed when two (or more) different types of monomers are linked in the same polymer chain, as opposed to a homopolymer where only one monomer is used.

Neoprene is a homopolymer



Q.5. $20\ \mathrm{L}$ of an ideal gas is allowed to expand isothermally against vacuum until the total volume becomes $40\ \mathrm{L}$. The amount of heat absorbed in this expansion (in L atm) is:

A) 0

B) 100

C) 10

D) 1

Answer: 0

Solution: In case of expansion, work is done by system.

Now, as we know that,

$W = -P_{ext.}\ \Delta V$

As the gas is expanding into vacuums which has no pressure, i.e.,

$P_{ext.} = 0$

$\therefore\ W = 0$

Hence, no work is done.

From first law of thermodynamics,

$\Delta U = q + W$

As the system is working at constant temperature, i.e., isothermally.

$\therefore \Delta U = 0$

Hence $q = -W = 0$

Q.6. The hybridization of PF_5 is $sp^x d^y$. The value of y is:

A) 0

B) 1

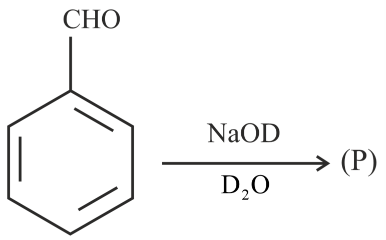
C) 2

D) 3

Answer: 1

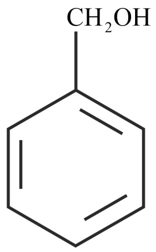
Solution: The hybridization is $sp^3 d$ hybridization and phosphorous atom forms five $sp^3 d$ hybrid orbitals. Five hybrid orbitals will be used to form bonds with five fluorine atoms. There are 5 sigma bonds in this compound.

Q.7.

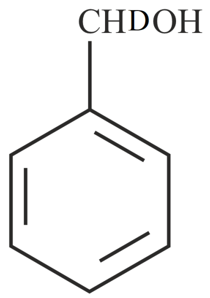


Product P is:

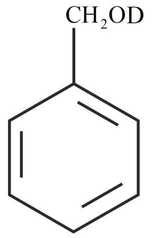
A)



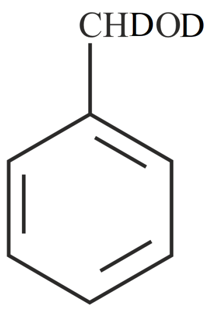
B)



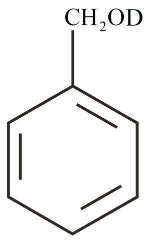
C)



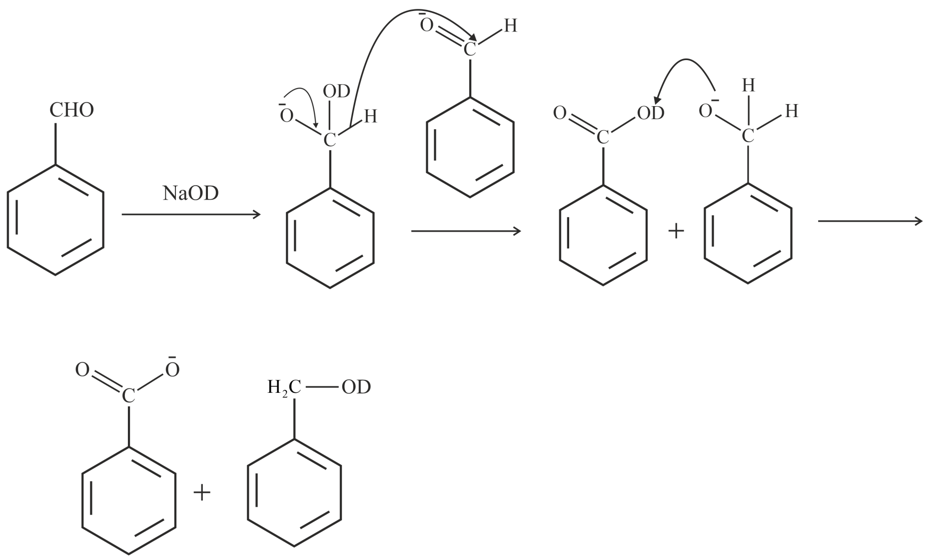
D)



Answer:

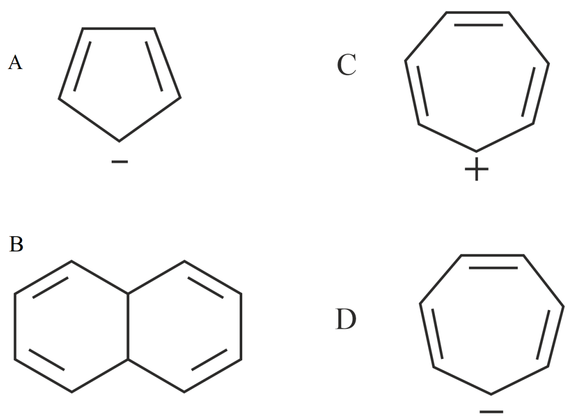


Solution:



Product P

Q.8. Which of the following are aromatic?



A) A,B

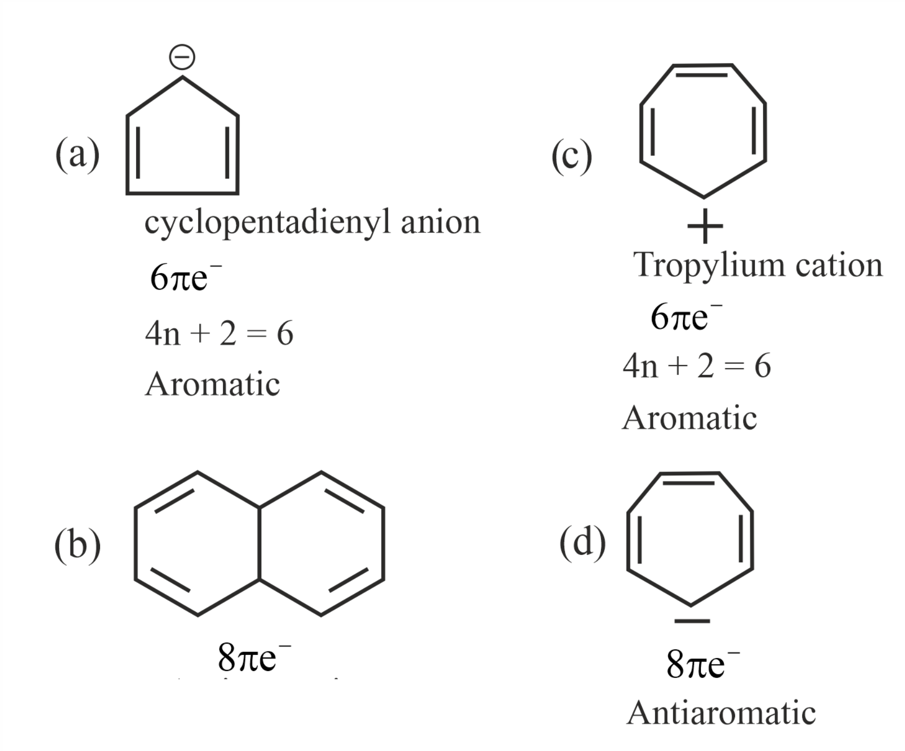
B) A, C

C) A, D

D) B, D

Answer: A, C

Solution: Species which obey Huckel's rule are considered to be aromatic in nature



Q.9. Which of the following is not a pesticide?

A) DDT

B) Dieldrin

C) Organophosphate

D) Sodium Arsenite

Answer: Sodium Arsenite

Solution: Dichlorodiphenyltrichloroethane (DDT) is an insecticide used in agriculture. Dieldrin a decades-old chlorocarbon insecticide that has long been banned from use in most of the world. It was introduced in 1948 by the now-defunct J. Organophosphates are a group of human-made chemicals that poison insects and mammals. Organophosphates are the most widely used insecticides today. They are used in agriculture, the home, gardens, and veterinary practice.

These days, the pesticide industry has shifted its attention to herbicides such as sodium chlorate $(NaClO_3)$, sodium arsenite (Na_3AsO_3) and many others.

Q.10. An element E belongs to group 16 and 4^{th} period of the periodic table. Find out the valence shell electronic configuration of the element which is just above 'E'.

A) $2s^2 2p^4$

B) $5s^2 5p^4$

C) $3s^2 3p^4$

D) $4s^2 4p^4$

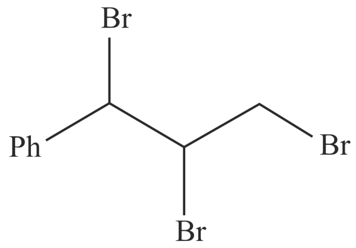
Answer: $3s^2 3p^4$

Solution: General outer electronic configuration of 16^{th} group elements is $ns^2 np^4$

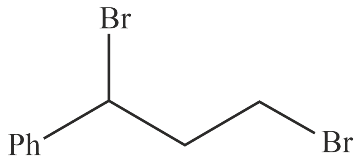
16^{th} group element belonging to 4^{th} period is Se and its outer electronic configuration is $4s^2 4p^4$. The element present above selenium is sulphur and the outer electronic configuration of sulphur is $3s^2 3p^4$.

Q.11. The major product in the below reaction is:

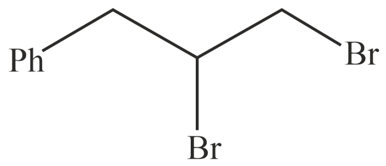
A)



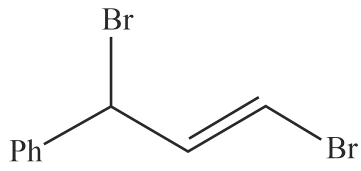
B)



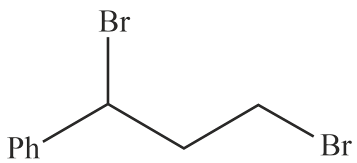
C)



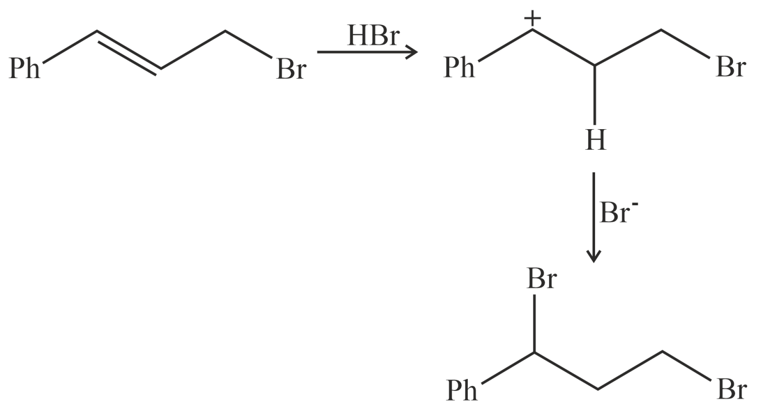
D)



Answer:

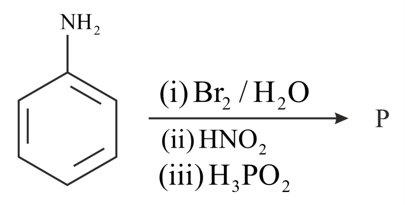


Solution: Electrophilic addition reaction will occur according to Markovnikov addition rule.



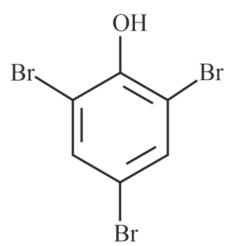
Benzyllic position, carbocation is formed as an intermediate which is stable due to resonance.

Q.12.

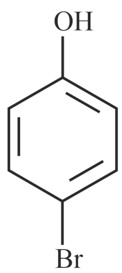


Product P is?

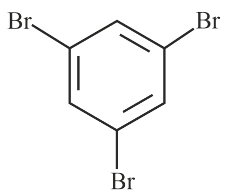
A)



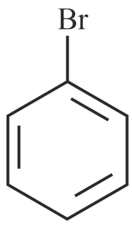
B)



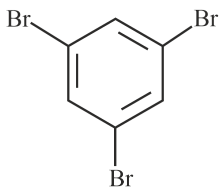
C)



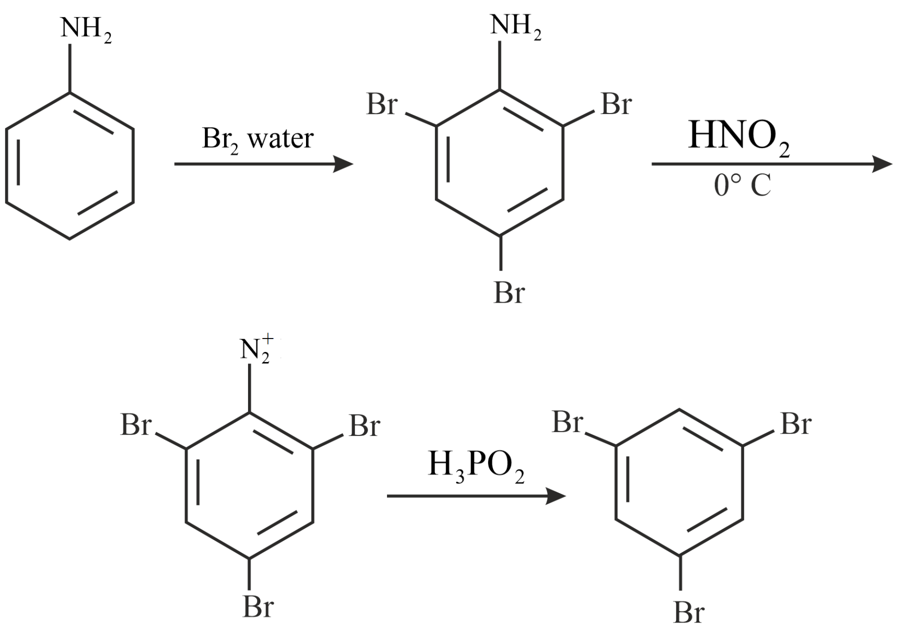
D)



Answer:



Solution:



Q.13. Decomposition of which of the following compounds gives N_2:

A) $NaNO_2$

B) $NaNO_3$

C) $Ba(N_3)_2$

D) $Ba(NO_3)_2$

Answer: $Ba(N_3)_2$

Solution: $NaNO_3 \xrightarrow{\Delta} NaNO_2 + \frac{1}{2}O_2$

(Thermally stable and does not undergo decomposition further)

$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

$2Ba(NO_3)_2 \xrightarrow{\Delta} 2BaO + 4NO_2 + O_2$

Q.14. The zeta potential is a property of colloidal particles for

A) Colour

B) Brownian movement

C) Charge on surface of colloidal particle

D) Tyndall effect

Answer: Charge on surface of colloidal particle

Solution: The combination of the two layers of opposite charges around the colloidal particle is called Helmholtz electrical double layer. According to modern views, the first layer of ions is firmly held and is termed fixed layer while the second layer is mobile which is termed diffused layer. Since separation of charge is a seat of potential, the charges of opposite signs on the fixed and diffused parts of the double layer results in a difference in potential between these layers in the same manner as potential difference is developed in a capacitor. This potential difference between the fixed layer and the diffused layer of opposite charges is called the electrokinetic potential or zeta potential.

Q.15. Most stable lanthanoid in divalent form:

A) Eu^{+2}

B) Sm^{+2}

C) Yb^{+2}

D) Ce^{+2}

Answer: Yb^{+2}

Solution: $Ce^{+2} : (xe)4f^2$

$Eu^{+2} : (xe)4f^7$

$Sm^{+2} : (xe)4f^6$

$Yb^{+2} : (xe)4f^{14}$

As Yb^{+2} has completely filled $4f$ orbital, so it would be the most stable.

Q.16. Following sequence of reaction is provided.

$$\text{Primary amine} \xrightarrow[273^\circ C]{HNO_2(aq)} A \xrightarrow[H_2O]{283^\circ C} B$$

What is B ?

A) Alcohol

B) Amine

C) Nitro compound

D) Alkyl nitride

Answer: Alcohol

Solution:

$$R-NH_2 \xrightarrow[273^\circ C]{HNO_2(aq)} RN_2^+ \xrightarrow[H_2O]{283^\circ C} R-OH+N_2$$

Q.17. Which defect will not affect density?

A) Frenkel defect

B) Schottky defect

C) Interstitial defect

D) Vacancy defect

Answer: Frenkel defect

Solution: Frenkel Defect: This defect is shown by ionic solids. The smaller ion (usually cation) is dislocated from its normal site to an interstitial site. It creates a vacancy defect at its original site and an interstitial defect at its new location. Frenkel defect is also called dislocation defect. It does not change the density of the solid. In all other defects there is a change in the density.

Q.18. Statement I: $[Ni(CN)_4]^{2-}$ and $[Ni(CO)_4]$ are paramagnetic,

Statement II: $[Ni(CN)_4]^{2-}$ and $[Ni(CO)_4]$ are having same geometry and same structure.

The correct statements are:

A) Both I and II

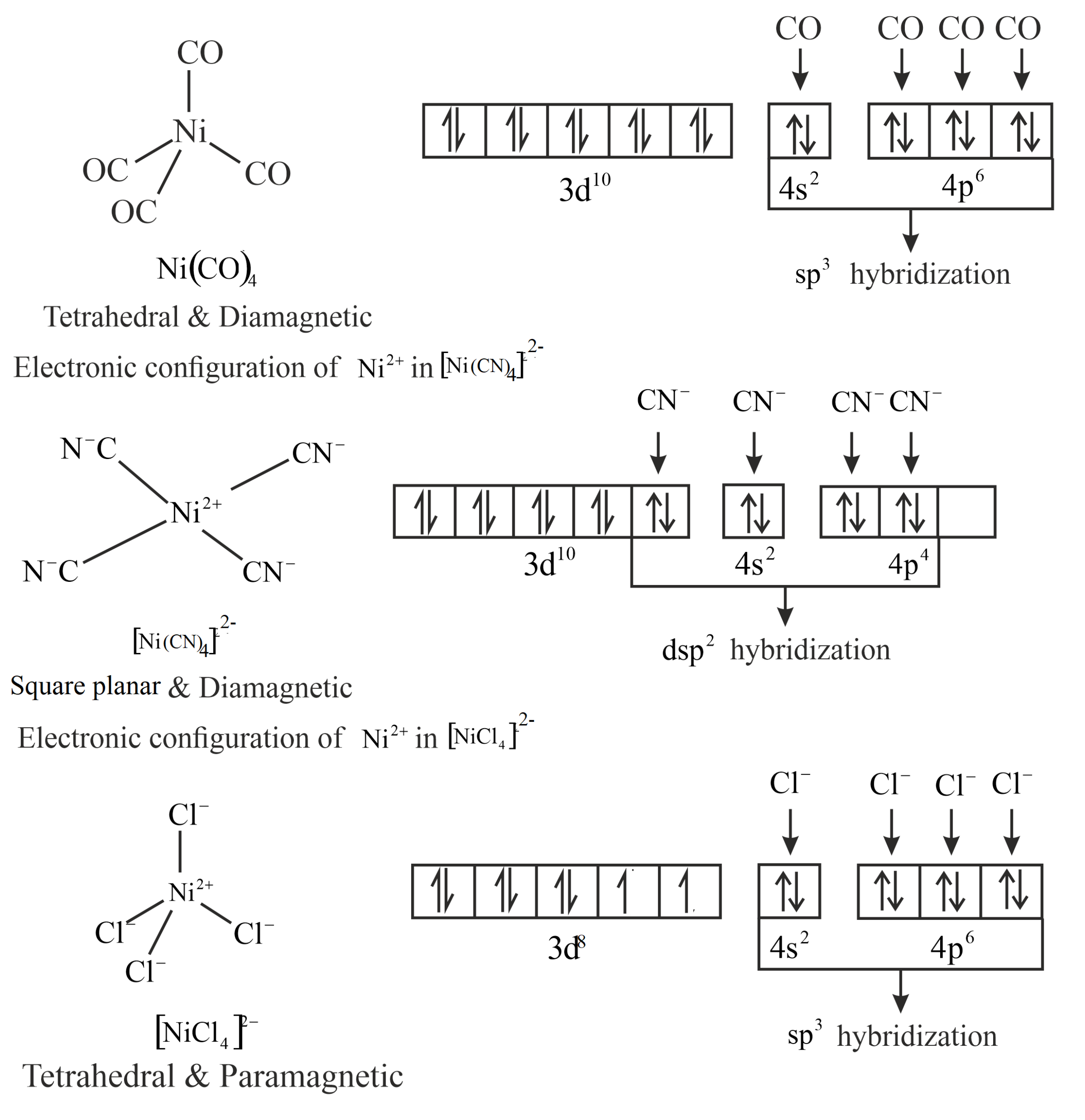
B) Only I

C) Only II

D) Neither I nor II

Answer: Neither I nor II

Solution: Electronic configuration of Ni atom in $Ni(CO)_4$



Q.19. An organic sample of 0.5 g contains Br, it yields 0.4 g of $AgBr$ in a certain reaction. What is % weight of Br in the sample approximately?

(Atomic weight of $Ag = 108\ g\ mol^{-1}$; $Br = 80\ g\ mol^{-1}$)

A) 34

B) 45

C) 54

D) 26

Answer: 34

Solution: The estimation of halogens in organic compounds is usually done by carius method.

Percentage of Bromine $=\frac{\text{Atomic mass of Br}}{\text{Molecular mass of AgBr}}\times\frac{\text{Mass of AgBr}\times 100}{\text{Mass of the organic compound}}$

$=\frac{80\times 0.4\times 100}{188\times 0.5}$

$=34\%$

Q.20. Assertion (A): Purple colour obtained on treatment of Lassaigne's extract with a reagent is a test of sulphur.

Reason (R): Sodium nitroprusside is the reagent that gives purple coloured complex $[Fe(CN)_5NOS]^{4-}$

A) Both (A) and (R) are true and (R) is the correct explanation of (A)

B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

C) (A) Is true but (R) is false.

D) (A) Is false but (R) is true.

Answer: Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution: $\underset{\text{Lassaigne's extract}}{Na_2S}+Na_2[Fe(CN)_5NO]\rightarrow\underset{\text{Purple colour}}{Na_4[Fe(CN)_5NOS]}$

On treating sodium fusion extract with sodium nitroprusside, appearance of a violet colour further indicates the presence of sulphur.

Q.21. Statement I: E_2O_5 is less acidic than E_2O_3.

Statement II: In group 15, acidic nature of E_2O_3 decreases down the group.

The correct statements are:

A) Both I and II

B) Only I

C) Only II

D) Neither I nor II

Answer: Only II

Solution: Reactivity towards oxygen: All these elements from two types of oxides: E_2O_3 and E_2O_5. The oxide in the higher oxidation state of the element is more acidic than that of lower oxidation state. Their acidic character decreases down the group. The oxides of the type E_2O_3 of nitrogen and phosphorus are purely acidic, that of arsenic and antimony are amphoteric and those of bismuth predominantly basic.

Q.22. The stability of $\alpha-$helix structure of proteins is due to

A) Hydrogen bonding

B) Vanderwaal forces

C) Disulphide linkages

D) Ion-dipole forces

Answer: Hydrogen bonding

Solution: The secondary structure of protein refers to the shape in which a long polypeptide chain can exist. They are found to exist in two different types of structures viz. α-helix and β-pleated sheet structure. These structures arise due to the regular folding of the backbone of the polypeptide chain due to hydrogen bonding between

$$\overset{\mathrm{O}}{\overset{\|}{-\mathrm{C}-}}$$

and $-\mathrm{NH}-$ groups of the peptide bond.

α-helix Is one of the most common ways in which a polypeptide chain forms all possible hydrogen bonds by twisting into a right handed screw (helix).

Q.23. Two liquids A and B follow Raoult's law over entire range of concentration. If a solution of A and B has mole fraction of A as 0.3. Then $\mathrm{y_A} = \frac{\mathrm{x}}{17}$. The value of x is:

(Given: $\mathrm{P^\circ{}_A} = 50\ \mathrm{torr}$ and $\mathrm{P^\circ{}_B} = 100\ \mathrm{torr}$)

A) 3

B) 4

C) 5

D) 6

Answer: 3

Solution: Since,

$$\mathrm{y_A} = \frac{\mathrm{p^\circ{}_A \times x_A}}{\mathrm{p^\circ{}_A \times x_A + p^\circ{}_B \times x_B}}$$

and $\mathrm{x_A} = 0.3$ and $\mathrm{x_B} = 0.7$

$$\mathrm{y_A} = \frac{50 \times 0.3}{50 \times 0.3 + 100 \times 0.7}$$

$$\mathrm{y_A} = \frac{15}{85}$$

$$\frac{15}{85} = \frac{\mathrm{x}}{17};\ \mathrm{x} = 3.$$

Q.24. For first order reaction, $\mathrm{t_{67\%}} = \mathrm{x} \times \mathrm{t_{50\%}}$, the value of x will be

A) 3.2

B) 1.6

C) 0.56

D) 1.13

Answer: 1.6

Solution: For first order reaction, $t = \frac{2.303}{k} \log \left[\frac{A_0}{A_t}\right]$

Now, $t_{67\%} = \frac{2.303}{k} \log \left[\frac{A}{0.33A}\right]$

$\Rightarrow t_{67\%} = \left(\frac{2.303}{k}\right) \times 0.48 \ldots (1)$

& $t_{50} = \frac{2.303}{k} \log \left[\frac{A}{0.5A}\right]$

$\Rightarrow t_{50\%} = \left(\frac{2.303}{k}\right) 0.30 \ldots (2)$

Dividing (1) by (2) : $\frac{t_{67\%}}{t_{50\%}} = \frac{0.48}{0.30} = 1.6$

Q.25. Statement I: Mg reduces Al_2O_3 below 1350^o C and Al reduces MgO after 1350^o C

Statement II: Boiling point and melting point of Mg is lower than that of Al

The correct statement(s) is/are:

A) Statement I only

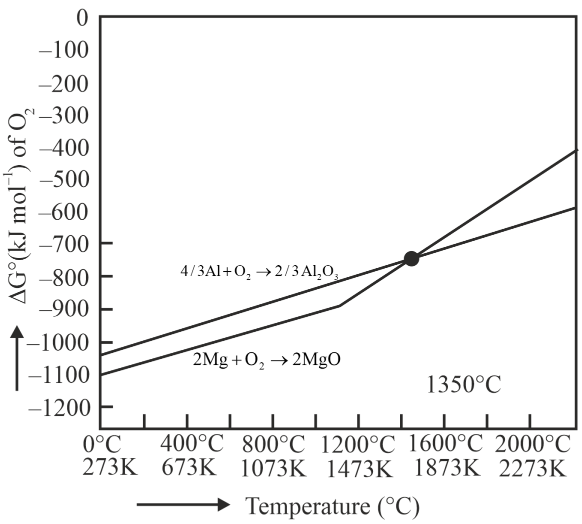
B) Both statement I and statement II

C) Statement II only

D) Neither statement I nor statement II

Answer: Both statement I and statement II

Solution:



	Mg	Al
Melting point	924 K	933 K
Boiling point	1363 K	2740 K

it is true that magnesium (Mg) can reduce aluminium oxide (Al_2O_3) and aluminium can also reduce magnesium oxide. According to the Elingham diagram, we find that below 1350^o C, Mg can reduce aluminium oxide and when the temperature is above 1350^o C, Al can reduce magnesium oxide.

Q.26. The work function of a given metal is 6.63×10^{-19} J. Find the wavelength (in nm) of incident light, if kinetic energy of the ejected electron is zero.

Take $\mathrm{h}=6.63 \times 10^{-34} \mathrm{Js}$

A) 350

B) 450

C) 300

D) 400

Answer: 300

Solution: $\mathrm{E}_{\text{incident}}=\mathrm{W}_{0}+\mathrm{K.E.}$

As K.E is zero

$\mathrm{E}_{\text{incident}}=\mathrm{W}_{0}$

$6.63 \times 10^{-19}=\frac{\mathrm{hc}}{\lambda}=\frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$

$\lambda=3 \times 10^{-7} \mathrm{~m}=300 \mathrm{~nm}.$

Section C: Mathematics

Q.1. Suppose a matrix A of order 3×3 and $|A| = 2$ is given, then $\left||A|(\text{adj}\,(\text{adj}\,A))^3\right|$ is equal to

A) 2^{13}

B) 2^{14}

C) 2^{15}

D) 2^{12}

Answer: 2^{15}

Solution: Given $|A| = 2$

Now $\left||A|(\text{adj}\,(\text{adj}\,A))^3\right| = \left|2(\text{adj}\,(\text{adj}\,A))^3\right|$

$= 2^3 \cdot \left|(\text{adj}\,(\text{adj}\,A))^3\right| = 2^3\left(|A|^{2^2}\right)^3$

$= 2^3 \cdot 2^{12} = 2^{15}$

Q.2. The number of real solution(s) of equation $4x^7 + 3x^3 + 5x + 1 = 0$ are

A) 0

B) 1

C) 2

D) 3

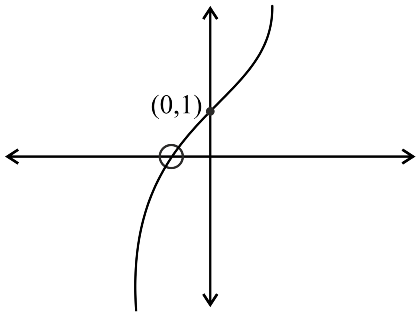
Answer: 1

Solution: Let $f(x) = 4x^7 + 3x^3 + 5x + 1$

$\Rightarrow f'(x) = 28x^6 + 9x^2 + 5 > 0$

$\Rightarrow f(x)$ is increasing function in $\mathbb{R}$

And range of $f(x)$ is $(-\infty, \infty)$ as $f(x)$ is odd degree function.



$\Rightarrow$ Number of real solution(s) is 1

Q.3. Area of region $S : y^2 \leq 8x, y \geq \sqrt{2}, x \leq 1$.

A) $\frac{7}{3\sqrt{2}}$

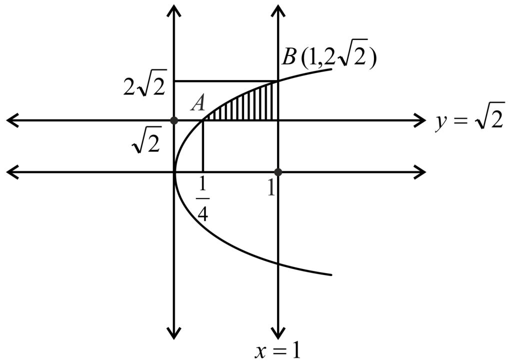
B) $\frac{3}{2\sqrt{2}}$

C) $\frac{6}{\sqrt{2}}$

D) $\frac{5}{6\sqrt{2}}$

Answer: $\frac{5}{6\sqrt{2}}$

Solution: Plotting the region for the given inequalities $y^2 \leq 8x,\ y \geq \sqrt{2},\ x \leq 1$, we get



Required area $= \int_{\frac{1}{4}}^{1} \left(\sqrt{8x} - \sqrt{2}\right) dx = \left[\frac{2\sqrt{2}x^{\frac{3}{2}}}{\frac{3}{2}}\right]_{\frac{1}{4}}^{1} - \left[\sqrt{2}x\right]_{\frac{1}{4}}^{1} = \frac{4\sqrt{2}}{3}\left(1 - \frac{1}{8}\right) - \sqrt{2}\left(1 - \frac{1}{4}\right) = \frac{7}{3\sqrt{2}} - \frac{3}{2\sqrt{2}} = \frac{5}{6\sqrt{2}}$

Q.4. What is the probability of selecting a three digit number with at least two odd digits in the number ?

A) $\frac{19}{36}$

B) $\frac{16}{36}$

C) $\frac{19}{33}$

D) $\frac{16}{30}$

Answer: $\frac{19}{36}$

Solution: Total No of 3-digit Number $= 900$

Case -1 When there are 2 odd digit and 1 even digit

(i) Even odd odd $= 4 \times 5 \times 5 = 100$

{As 0 cannot come at 1^{st} place}

(ii) Odd even odd $= 5 \times 5 \times 5 = 125$

(iii) Odd odd even $= 5 \times 5 \times 5 = 125$

Case-2

All 3 odd digit $= 5 \times 5 \times 5 = 125$

So total number of selecting 3-digit number having atleast 2 odd digit will be addition of both cases which is $= 100 + 125 + 125 + 125 = 475$

So probability $= \frac{475}{900} = \frac{19}{36}$

Q.5. From the set of numbers $\{1,2,3,5,6,7\}$, how many 5 digit numbers which are multiple of 6 can be formed without repetition?

A) 72

B) 48

C) 24

D) 144

Answer: 72

Solution: Given set $\{1,2,3,5,6,7\}$

Here sum is $1+2+3+5+6+7=24$

So there will be two cases

Case (i) When numbers are $\{1,2,5,6,7\}$

Arrangement will be $\_\_\_\_\,2$ & $\_\_\_\_\,6$

In unit place 2 & 6 can take place because for multiple of 6 number should be even & sum of digits should be multiple of 3.

So total way in which remaining number can be taken will be $4!\times 2=48$ ways

Case (ii) When numbers are $\{1,2,3,5,7\}$

Arrangement will be $\_\_\_\_\,2$

So total way in which remaining number can be taken will be $4!=24$ ways

Adding case (i) & case (ii), we get

required 5-digit numbers $=24+48=72$

Q.6. Consider 15 observations whose mean is 8 and standard deviation is 3. But an incorrect observation of 5 was taken. If the correct observation is 20, then the new variance will be

A) 17

B) 12

C) 19

D) 21

Answer: 17

Solution: Old mean$=\frac{x_1+x_2+\ldots x_{14}+5}{15}=8$ (given)

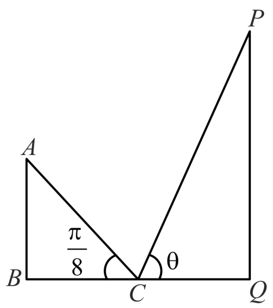
$\Rightarrow x_1+x_2+\ldots x_{14}=115$

Correct mean$=\frac{x_1+x_2+\ldots x_{14}+20}{15}=\bar{x}_{\text{new}}\Rightarrow \bar{x}_{\text{new}}=9$

Given, old variance$=9\Rightarrow \frac{{x_1}^2+{x_2}^2+\ldots {x_{14}}^2+5^2}{15}-8^2=9\Rightarrow {x_1}^2+{x_2}^2+\ldots {x_{14}}^2=1070$

Now $\text{var}_{\text{new}}=\frac{{x_1}^2+{x_2}^2+\ldots {x_{14}}^2+20^2}{15}-9^2=\frac{1070+400}{15}-81=17$

Q.7. Two poles AB, PQ are $160 \mathrm{~m}$ apart such that $PQ = 2AB$ and C is the mid-point of BQ on the ground, then $\tan\theta$ is equal to



A) $2\left(\sqrt{2}+1\right)$

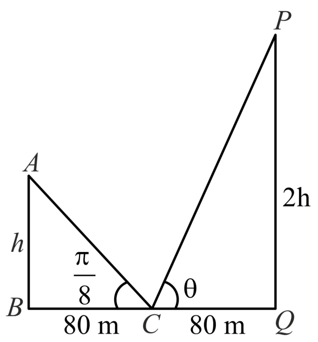
B) $2\left(\sqrt{2}-1\right)$

C) $4\left(3+2\sqrt{2}\right)$

D) $4\left(3-2\sqrt{2}\right)$

Answer: $2\left(\sqrt{2}-1\right)$

Solution:



Let $AB = h$ then $PQ = 2h$.

Now $\tan\theta = \frac{2h}{80}$ and $\tan\frac{\pi}{8} = \frac{h}{80}$

$\therefore \frac{\tan\theta}{\tan\frac{\pi}{8}} = 2$

$\therefore \quad \tan\theta = 2\left(\sqrt{2}-1\right)$

Q.8. A hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ whose eccentricity is $\frac{\sqrt{5}}{2}$and length of latusrectum is $\frac{\sqrt{3}}{2}$. If the equation of tangent to hyperbola is $y = 2x + c$, then the value of c is

A) $\pm\frac{3\sqrt{5}}{2}$

B) $\pm\frac{5\sqrt{3}}{2}$

C) $\pm\frac{3\sqrt{2}}{5}$

D) $\pm\frac{5\sqrt{2}}{3}$

Answer: $\pm\frac{3\sqrt{5}}{2}$

Solution: Given, $e=\frac{\sqrt{5}}{2}$

Length of latusrectum $\frac{2b^2}{a}=\frac{\sqrt{3}}{2} \quad \ldots(1)$

Now replacing b^2 with $a^2\left(e^2-1\right)$ we get,

$\frac{2a^2\left(e^2-1\right)}{a}=\frac{\sqrt{3}}{2}$

$\Rightarrow \quad 2a\left(\frac{5}{4}-1\right)=\frac{\sqrt{3}}{2}\Rightarrow a=\sqrt{3}$

$\Rightarrow \quad b^2=\frac{3}{4} \quad [\text{From}\,(1)]$

Equation of tangent to hyperbola is given by $y=mx\pm\sqrt{a^2m^2-b^2}$

$\Rightarrow \quad y=2x\pm\sqrt{3\times4-\frac{3}{4}}$

$\Rightarrow \quad c=\pm\sqrt{12-\frac{3}{4}}=\pm\frac{3\sqrt{5}}{2}$

Q.9. The value of $\sum_{k=1}^{31}{}^{31}C_k\,{}^{31}C_{k-1}-\sum_{k=1}^{30}{}^{30}C_k\,{}^{30}C_{k-1}$ is equal to

A) ${}^{62}C_{32}-{}^{59}C_{31}$

B) ${}^{62}C_{32}-{}^{60}C_{31}$

C) ${}^{62}C_{31}-1$

D) ${}^{62}C_{32}$

Answer: ${}^{62}C_{32}-{}^{60}C_{31}$

Solution: $(1+x)^{31}={}^{31}C_0x^0+{}^{31}C_1x+{}^{31}C_2x^2+\cdots\cdots+{}^{31}C_{31}x^{31}$

$(x+1)^{31}={}^{31}C_0x^{31}+{}^{31}C_1x^{30}+{}^{31}C_2x^{29}+\cdots+{}^{31}C_{31}x^0$

Multiplying the above two expansions and comparing the coefficients of x^{32}, we get

${}^{62}C_{32}=\sum_{k=1}^{31}{}^{31}C_k\,{}^{31}C_{k-1}$

Similarly

$(1+x)^{30}={}^{30}C_0+{}^{30}C_1x+{}^{30}C_2x^2+\cdots$

$(x+1)^{30}={}^{30}C_0x^{30}+{}^{30}C_1x^{29}+{}^{30}C_2x^{28}+\cdots$

${}^{60}C_{31}=\sum_{k=1}^{30}{}^{30}C_k\,{}^{30}C_{k-1}$

$\therefore \quad \sum_{k=1}^{31}{}^{31}C_k\,{}^{31}C_{k-1}-\sum_{k=1}^{30}{}^{30}C_k\,{}^{30}C_{k-1}={}^{62}C_{32}-{}^{60}C_{31}$

Q.10. If a differential equation is $x\left(1-x^2\right)\frac{dy}{dx}-\left(3x^2y-y-4x^3\right)=0$ and $y(1)=-2$, then the value of $y(3)$ will be

A) $\frac{53}{24}$

B) $\frac{52}{20}$

C) $\frac{50}{22}$

D) $\frac{52}{18}$

Answer: $\frac{53}{24}$

Solution: Given, $x\left(1-x^2\right)\frac{dy}{dx}-\left(3x^2y-y-4x^3\right)=0$

$\Rightarrow\ \frac{dy}{dx}-\frac{\left(3x^2-1\right)y}{x\left(1-x^2\right)}=\frac{-4x^3}{x\left(1-x^2\right)}$

$\Rightarrow$ **I.F.** $=e^{\int\frac{-\left(3x^2-1\right)}{x\left(1-x^2\right)}dx}$

Now let $x\left(1-x^2\right)=t$ or $x-x^3=t\ \Rightarrow\left(1-3x^2\right)dx=dt$

So **I.F.** $=e^{\int\frac{dt}{t}}=e^{\log|t|}=t\quad\Rightarrow\ x-x^3$

Now solution

$y\times\mathrm{I.F}=\int\frac{-4x^3}{x\left(1-x^2\right)}\times\mathrm{I.F}$

$y\times\left(x-x^3\right)=\int\frac{-4x^3}{x\left(1-x^2\right)}\times x\left(1-x^2\right)$

$y\times\left(x-x^3\right)=\int-4x^3\ \Rightarrow y\left(x-x^3\right)=-x^4+c$

Now given $y(2)=-2$ putting the value we get,

$-2\left(2-2^3\right)=-2^4+c\ \Rightarrow\ c=28$

So, $y=\frac{28-x^4}{x\left(1-x^2\right)}$ and $y(3)=\frac{28-3^4}{3\left(1-3^2\right)}=\frac{53}{24}$

Q.11. The equation of circle is given as $x^2+y^2-2x-4y=0$, if O represents origin, co-ordinate of point P is $\left(1+\sqrt{5},2\right)$ and R is the point of intersection of the tangent drawn at P and O, then the area of ΔOPR is

A) $\frac{\sqrt{5}}{2}\left(1+\sqrt{5}\right)^2$

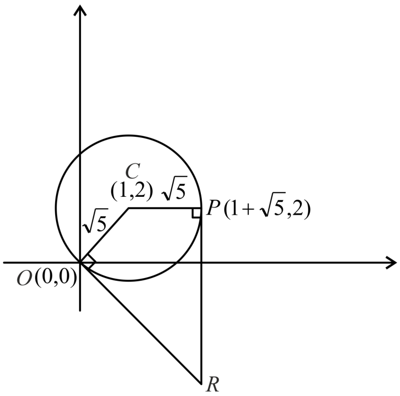
B) $\frac{\sqrt{5}}{4}\left(2+\sqrt{5}\right)^2$

C) $\frac{\sqrt{5}}{8}\left(4+\sqrt{5}\right)^2$

D) $\frac{\sqrt{5}}{12}\left(3+\sqrt{5}\right)^2$

Answer: $\frac{\sqrt{5}}{2}\left(1+\sqrt{5}\right)^2$

Solution: Plotting the diagram we get,



Slope of $CO=2$

So slope of $OR=\frac{-1}{2}$

Now R & P will share same x-co-ordinate as they lie on same line so abscissa of R is $1+\sqrt{5}$

And equation of $OR \Rightarrow y=\frac{-x}{2}$

So $y=-\frac{\left(1+\sqrt{5}\right)}{2}$

Now $R\equiv\left(1+\sqrt{5},\frac{-\left(1+\sqrt{5}\right)}{2}\right)$

Now Area of ΔOPR

$$=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 1+\sqrt{5} & 2 & 1\\ 1+\sqrt{5} & \frac{-\left(1+\sqrt{5}\right)}{2} & 1\end{vmatrix}$$

$$=\frac{1}{2}\left(1+\sqrt{5}\right)\begin{vmatrix}0 & 0 & 1\\ 1 & 2 & 1\\ 1 & \frac{-\left(1+\sqrt{5}\right)}{2} & 1\end{vmatrix}$$

$$=\frac{1}{2}\left(1+\sqrt{5}\right)\left(\frac{1+\sqrt{5}}{2}+2\right)\Rightarrow\frac{1}{2}\left(1+\sqrt{5}\right)\frac{\left(\sqrt{5}+5\right)}{2}$$

$$=\frac{1\times\sqrt{5}}{2}\left(1+\sqrt{5}\right)^2=\frac{\sqrt{5}}{2}\left(1+\sqrt{5}\right)^2$$

Q.12. Given that the three vectors $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}, \vec{b}=3\hat{i}+3\hat{j}+\hat{k}$ & $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ are coplanar. Also $\vec{a}\cdot\vec{c}=5$ & $\vec{b}$ is perpendicular to $\vec{c}$. Then the value of $122\left(c_1+c_2+c_3\right)$ is

A) 150

B) 210

C) 180

D) 270

Answer: 150

Solution:

As the given vectors are coplanar, so $\begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$\Rightarrow 8c_1 - 7c_2 - 3c_3 = 0 \quad \ldots(i)$

Also, $\vec{a} \cdot \vec{c} = 5 \Rightarrow 2c_1 + c_2 + 3c_3 = 5 \quad \ldots(ii)$

and $\vec{b} \cdot \vec{c} = 0 \Rightarrow 3c_1 + 3c_2 + c_3 = 0 \quad \ldots(iii)$

Solving the above three equations using Cramer's rule, we get,

$$\Delta = \begin{vmatrix} 8 & -7 & -3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{vmatrix} = -122$$

$$\Delta_1 = \begin{vmatrix} 0 & -7 & -3 \\ 5 & 1 & 3 \\ 0 & 3 & 1 \end{vmatrix} = -10$$

$$\Delta_2 = \begin{vmatrix} 8 & 0 & -3 \\ 2 & 5 & 3 \\ 3 & 0 & 1 \end{vmatrix} = 85$$

$$\Delta_3 = \begin{vmatrix} 8 & -7 & 0 \\ 2 & 1 & 5 \\ 3 & 3 & 0 \end{vmatrix} = -225$$

Hence, $122\left(c_1 + c_2 + c_3\right) = 122\left(\frac{10}{122} - \frac{85}{122} + \frac{225}{122}\right) = 150$

Q.13.

For a given differential equation $x\left(\frac{x}{\sqrt{x^2-y^2}} + e^{\frac{y}{x}}\right)\frac{dy}{dx} = y\left(\frac{x}{\sqrt{x^2-y^2}} + e^{\frac{y}{x}}\right) + x$, let $y = y(x)$ be the solution and such that $y(1) = 0$. If $y(2\alpha) = \alpha$, then value of α will be

A) $e^{\frac{\pi}{6}+\sqrt{e}-1}$

B) $e^{\frac{\pi}{6}+\sqrt{e}}$

C) $e^{\sqrt{e}-1}$

D) $e^{\frac{\pi}{6}-1}$

Answer: $e^{\frac{\pi}{6}+\sqrt{e}-1}$

Solution: Given $x\left(\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right)\frac{dy}{dx}=y\left(\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right)+x$

Taking x common & cancelling them we get,

$$\frac{dy}{dx}\times\left(\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}+e^{\frac{y}{x}}\right)=\frac{y}{x}\left(\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}+e^{\frac{y}{x}}\right)+1$$

Let $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\left(v+x\frac{dv}{dx}\right)\left(\frac{1}{\sqrt{1-v^2}}+e^v\right)=v\left(\frac{1}{\sqrt{1-v^2}}+e^v\right)+1$$

$$v+x\frac{dV}{dx}=v+\frac{1}{\left(\frac{1}{\sqrt{1-v^2}}+e^v\right)}$$

$$x\frac{dv}{dx}=\frac{1}{\left(\frac{1}{\sqrt{1-v^2}}+e^v\right)}\Rightarrow\left(\frac{1}{\sqrt{1-v^2}}+e^v\right)dv=\frac{dx}{x}$$

Integrating both side we get,

$$\Rightarrow\int\left(\frac{1}{\sqrt{1-v^2}}+e^v\right)dv=\int\frac{dx}{x}$$

$$\Rightarrow\quad\sin^{-1}v+e^v=\ln x+c\Rightarrow\sin^{-1}\left(\frac{y}{x}\right)+e^{\frac{y}{x}}=\ln x+c$$

Now $y(1)=0$

$$\Rightarrow\sin^{-1}\left(\frac{0}{1}\right)+e^0=\ln 1+c$$

$$\Rightarrow c=1$$

$\Rightarrow\sin^{-1}\left(\frac{y}{x}\right)+e^{\frac{y}{x}}=\ln x+1$ ....(i)

Now $y(2\alpha)=\alpha$ putting in equation (i) we get,

$$\Rightarrow\sin^{-1}\left(\frac{\alpha}{2\alpha}\right)+e^{\frac{\alpha}{2\alpha}}=\ln\alpha+1$$

$$\Rightarrow\quad\frac{\pi}{6}+e^{\frac{1}{2}}=\ln\alpha+1\Rightarrow\quad\alpha=e^{\frac{\pi}{6}+\sqrt{e}-1}$$

Q.14. Which of the following is correct?

$S_1: p\rightarrow(r\vee q)$

$S_2: (\sim p\vee q)\vee(\sim p\vee r)$

A) If S_1 is true, then S_2 is true.

B) If S_1 is false, then S_2 is true.

C) If S_1 is true, then S_2 is false.

D) None of these

Answer: If S_1 is true, then S_2 is true.

Solution: Simplifying both the expression we get

$S_1 : p \rightarrow (r \vee q) \equiv \sim p \vee (r \vee q)$

$\equiv (\sim p \vee r) \vee (\sim p \vee q) = S_2$

So, S_1 and S_2 are equivalent.

$\therefore$ If S_1 is true, then S_2 is true.

Q.15. If a_n be n^{th} term of $G.P$ such that $a_1 \times a_3 \times a_5 \times a_7 = \frac{1}{1296}$ and $a_2 + a_4 = \frac{7}{36}$, then the value of $a_6 + a_8 + a_{10}$ is

A) 43

B) 86

C) 68

D) 48

Answer: 43

Solution: Given

$a_1 a_3 a_5 a_7 = \frac{1}{1296}$

Expanding using $a_n = a_1 r^{n-1}$ formula we get,

$\Rightarrow (a_1)\left(a_1 r^2\right)\left(a_1 r^4\right)\left(a_1 r^6\right) = \frac{1}{1296}$

$\Rightarrow a_1^4 r^{12} = \frac{1}{1296} \Rightarrow a_1 r^3 = \frac{1}{6} \quad \ldots \text{(i)}$

Also given ,

$a_2 + a_4 = \frac{7}{36} \Rightarrow a_1 r + a_1 r^3 = \frac{7}{36}$

$\Rightarrow a_1 r + \frac{1}{6} = \frac{7}{36}$

$\Rightarrow a_1 r = \frac{1}{36} \quad \ldots \text{(ii)}$

Dividing equation (i) from equation (ii) we get, $r^2 = 6$

Now $a_6 + a_8 + a_{10} = a_1 r^5 + a_1 r^7 + a_1 r^9$

$= a_1 r^3 \left[r^2 + r^4 + r^6\right] = \frac{1}{6}[6 + 36 + 216] = 43$

Q.16.

Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined as $f(n) = \begin{cases} 2n \text{ if } n = 2, 4, 6 \cdots \\ n - 1 \text{ if } n = 3, 7, 11, \cdots \\ \frac{n+1}{2} \text{ if } n = 1, 5, 9 \cdots \end{cases}$ then f is

A) One-one and onto

B) One-one but not onto

C) Onto but not one-one

D) Neither one-one nor onto

Answer: One-one and onto

Solution: For one value of n we will get only one corresponding value of $f(n)$, so $f(n)$ is one-one

Now, for $n=2,4,6\cdots;\ f(n)=4,8,12\ldots$

for $n=3,7,11\cdots;\ f\ (n)=2,6,10\ldots$

for $n=1,5,9\cdots;\ f(n)=1,3,5,7\ldots$

So range of $f(n)$ is N

Hence $f(n)$ is onto

Q.17. If the term independent of x in $\left(2x^3+\frac{3}{x^k}\right)^{12}$ is of the form $2^8\times l$, where l is an odd natural number, then the number of values of k is

A) 2

B) 3

C) 4

D) 5

Answer: 2

Solution: For $\left(2x^3+\frac{3}{x^k}\right)^{12}$

$T_{r+1}={}^{12}C_r\left(2x^3\right)^{12-r}\cdot\left(\frac{3}{x^k}\right)^r={}^{12}C_r\cdot 2^{12-r}\cdot 3^r\cdot x^{36-3r-kr}$

For the term to be independent of x,

$36-3r-kr=0\Rightarrow r=\frac{36}{k+3}$

Here k can be $0,1,3,6,9$ for r to be a whole number.

But for ${}^{12}C_r\cdot 2^{12-r}\cdot 3^r$ to be in the form of $2^8\cdot l$

Only $k=3,\ 6$ satisfies the above relation.