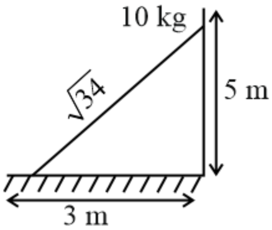
JEE Main Exam 2022 - Session 1

28 June 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. A 10 kg ladder of length $\sqrt{34} \text{ m}$ lean against frictionless wall 3 m away from wall. Find the ratio of normal force by floor and wall.



A) $\frac{10}{3}$

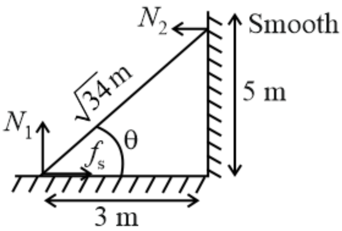
B) $\frac{20}{3}$

C) $\frac{13}{\sqrt{109}}$

D) 0

Answer: $\frac{10}{3}$

Solution:



Applying force balance in vertical direction

$N_1 = mg = 100 \text{ N}$

Applying torque balance at the bottom contact point,

$mg \times \frac{3}{2} = N_2 \times 5 \Rightarrow N_2 = 30 \text{ N}$

Therefore, $\frac{N_1}{N_2} = \frac{10}{3}$

Q.2. A particle moves along the straight line such that it moves $\left(\frac{1}{3}\right)^{\text{rd}}$ distance with speed v_1, the next $\left(\frac{1}{3}\right)^{\text{rd}}$ distance with speed v_2 and remaining $\left(\frac{1}{3}\right)^{\text{rd}}$ distance with speed v_3. Then its average speed throughout motion is

A) $\frac{2(v_1v_2+v_2v_3+v_3v_1)}{v_1+v_2+v_3}$

B) $\frac{(v_1+v_2+v_3)}{3}$

C) $\frac{v_1+v_2}{2}+\frac{v_2+v_3}{2}+\frac{v_3+v_1}{2}$

D) $\frac{3(v_1v_2v_3)}{v_1v_2+v_2v_3+v_3v_1}$

Answer: $\frac{3(v_1v_2v_3)}{v_1v_2+v_2v_3+v_3v_1}$

Solution: Let the total distance be d.

Time taken to cover $\left(\frac{1}{3}\right)^{\text{rd}}$ of total distance is $t_1=\frac{d}{3v_1}$.

Time taken to cover $\left(\frac{1}{3}\right)^{\text{rd}}$ of total distance is $t_2=\frac{d}{3v_2}$.

Time taken to cover $\left(\frac{1}{3}\right)^{\text{rd}}$ of total distance is $t_3=\frac{d}{3v_3}$.

Average speed$=\frac{\text{Total distance}}{\text{Total time}}$

$v_{avg}=\frac{d}{\frac{d}{3v_1}+\frac{d}{3v_2}+\frac{d}{3v_3}}$

$\Rightarrow \mathrm{v}_{avg}=\frac{3(v_1v_2v_3)}{v_1v_2+v_2v_3+v_3v_1}$

Q.3. Half life of radioactive material is 200 days. Find percent of nuclei remaining after 83 days.

A) 75%

B) 50%

C) 25%

D) 12.5%

Answer: 75%

Solution: As we know, $\lambda=\frac{\ln 2}{t_{\frac{1}{2}}}$

Also number of nuclei at anytime is given by, $N=N_0e^{-\lambda t}$

Percentage of nuclei after 83 day will be, $=\frac{N}{N_0}\times 100=e^{-\left(\frac{\ln 2}{200}\right)\times 83}\times 100=e^{-0.287}\times 100\approx 75\%$

Q.4. A $2\ \mathrm{kg}$ block has velocity $4\ \mathrm{m\ s^{-1}}$ enters a rough surface at $x=0.5\ \mathrm{m}$ to $x=1.5\ \mathrm{m}$, where $f=-kx$, ($k=12\ \mathrm{N\ m^{-1}}$). Find speed when it comes out of the surface.

A) $2\ \mathrm{m\ s^{-1}}$

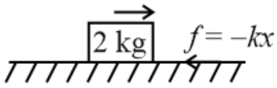
B) $3\ \mathrm{m\ s^{-1}}$

C) $4\ \mathrm{m\ s^{-1}}$

D) $1\ \mathrm{m\ s^{-1}}$

Answer: $2\ \mathrm{m\ s^{-1}}$

Solution:



Acceleration of the block $a=\frac{f}{m}=-\frac{12x}{2}=-6x$ (Here the block is retarding)

Now, using, $a=v\frac{\mathrm{d}v}{\mathrm{d}x}$
$\Rightarrow v\ \mathrm{d}v\ =\ a\ \mathrm{d}x$
on integrating, we get

$\int_4^v v\,\mathrm{d}\,v=-6\int_{0.5}^{1.5}x\,\mathrm{d}\,x$

$\Rightarrow \frac{v^2-4^2}{2}=-6\left(\frac{(1.5)^2-(0.5)^2}{2}\right)$

$\Rightarrow v^2=16-12\Rightarrow v=2\ \mathrm{m\ s^{-1}}$

Q.5. Water falls at a rate of $600\ \mathrm{kg\ s^{-1}}$ from a height of $60\ \mathrm{m}$. How many bulbs of capacity $100\ \mathrm{W}$ each will glow from the energy produced at the bottom of the fall? Assume full conversion of energy of falling water.

A) 600

B) 2400

C) 3000

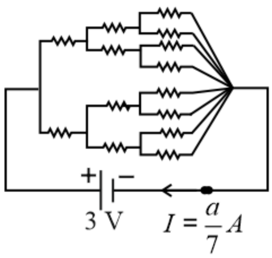
D) 3600

Answer: 3600

Solution: Since 100% efficiency is given, all the energy of the falling water will be used by the bulbs. Therefore,

$\frac{\Delta mgh}{\Delta t}=n\times P\Rightarrow 600\times 10\times 60=100n\Rightarrow n=3600$

Q.6. In the circuit shown below, if all the resistances are $r=1\ \Omega$, then the value of a is ______.



A) 24

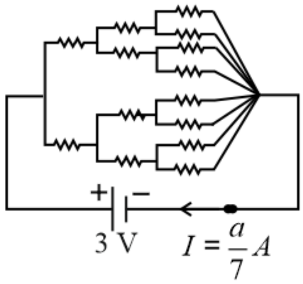
B) 12

C) 6

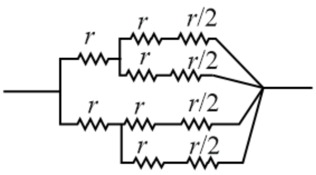
D) 3

Answer: 24

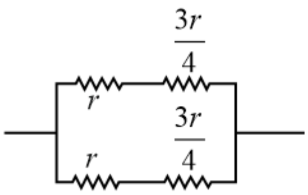
Solution:



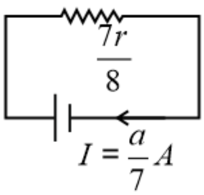
Each pair on the right side of the circuit is in parallel connection, hence it can be replaced by a resistance of $\frac{r}{2}$ as shown.



Now, r & $\frac{r}{2}$ in each branch on the right side is in series, hence combined resistance value will be, $\frac{3r}{2}$. Upper branch having resistance $\frac{3r}{2}$ is in parallel with another $\frac{3r}{2}$, hence it can be written as $\frac{3r}{4}$.



Further, simplified circuit will have net resistance of $\frac{7r}{8}$.



Therefore,

$$\mathrm{I} = \frac{3}{\frac{7r}{8}} = \frac{24}{7} \Rightarrow a = 24$$

Q.7. In a $YDSE$, a slab $(\mu = 1.5)$ of thickness $x\lambda$ is kept in front of one of the slits. If the intensity at the central maxima remains unchanged, what is the value of x?

A) $x = 2$

B) $x = 1$

C) $x = 0.5$

D) $x = 1.5$

Answer: $x = 2$

Solution: Since the intensity at the central maxima remains unchanged, the change in the path difference should be an integral multiple of the wavelength. Therefore,

$\Delta x=(\mu-1)t=n\lambda \Rightarrow (1.5-1)x\lambda=n\lambda \Rightarrow x=\frac{n}{0.5}=2n$

For $n=1$

$x=2$

Q.8. A short-circuit coil is placed in time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns are halved and the wire radius doubled, then the electrical power dissipated would be

A) Doubled

B) Same

C) Quadruple

D) Half

Answer: Doubled

Solution:

Power in the coil is $P=\frac{V^2}{R}$,
Induced voltage V in a coil is depends on number of turns N, area of coil A, and time varying magnetic field B.
$V=\frac{d\phi}{dt}=\frac{d(NBA)}{dt}$
And $R=\frac{\rho l}{\pi r^2}$, where ρ is resistivity, l is length of wire and r is radius of wire.

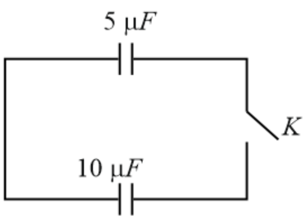
So power dissipated power in coil is
$P=\frac{\pi r^2}{\rho l}\left[\frac{d(NBA)}{dt}\right]^2=\frac{\pi r^2 N^2}{\rho l}\left[\frac{d(BA)}{dt}\right]^2$
$P\propto\frac{N^2 r^2}{l}$

Now, when $N'=\frac{N}{2}$, $r'=2r$, $l'=\frac{l}{2}$,

$\frac{P_2}{P_1}=\left(\frac{N_2}{N_1}\right)^2\left(\frac{r_2}{r_1}\right)^2\left(\frac{l_1}{l_2}\right)=\left(\frac{1}{2}\right)^2\left(\frac{2}{1}\right)^2(2)=2$

Power becomes doubled.

Q.9. In the given circuit the capacitor of $5\ \mu\mathrm{F}$ is initially charged to $30\ \mathrm{V}$ and the $10\ \mu\mathrm{F}$ capacitor is uncharged. What will be the charge on the $10\ \mu\mathrm{F}$ capacitor when the key K is closed?



A) $150\ \mu\mathrm{C}$

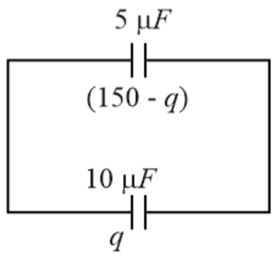
B) $100\ \mu\mathrm{C}$

C) $50\ \mu\mathrm{C}$

D) $250\ \mu\mathrm{C}$

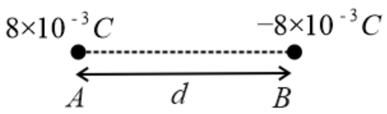
Answer: $100\ \mu\mathrm{C}$

Solution: The initial charge on the $5\ \mu\mathrm{F}$ capacitor will be $q = CV = 150\ \mu\mathrm{C}$. If the charge on the $10\ \mu\mathrm{F}$ capacitor is q after the key is closed then,



$\frac{150-q}{5} = \frac{q}{10} \Rightarrow q = 100\ \mu\mathrm{C}$

Q.10. Two opposite charges are placed at a distance d as shown. Electric filed strength at mid-point is $6.4 \times 10^4\ \mathrm{N\ C^{-1}}$. Then the value of d is,



A) $42.1\ \mathrm{m}$

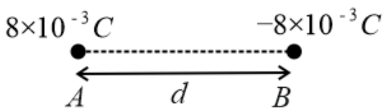
B) $94.86\ \mathrm{m}$

C) $72.2\ \mathrm{m}$

D) $62.8\ \mathrm{m}$

Answer: $94.86\ \mathrm{m}$

Solution:



Direction of electric field at mid-point due to both charge will be in same direction, which is towards right.

Therefore, E at mid point $= \frac{kq}{\left(\frac{d}{2}\right)^2} + \frac{kq}{\left(\frac{d}{2}\right)^2}$

$\Rightarrow 6.4 \times 10^4 = 2 \times \left[9 \times 10^9 \times \frac{(8\times10^{-3})}{d^2} \times 4\right]$

$\Rightarrow d = 30\sqrt{10}\ \mathrm{m} \approx 94.86\ \mathrm{m}$

Q.11. In an AC circuit, the voltage across the inductor and capacitor is 2 times that of resistance. The supply voltage is $220\ \mathrm{V},\ 50\,\mathrm{Hz}$ and resistance is $5\ \Omega$. If the inductance is $\frac{1}{k\pi}\ \mathrm{H}$, find the value of k.

A) 8

B) 10

C) 7

D) 5

Answer: 10

Solution: Since the voltage across the inductor and capacitor is same, the circuit is in resonance. Therefore, the current in the circuit will be,

$i=\frac{V}{R}=\frac{220}{5}=44\ \mathrm{A}.$

The voltage across the inductor is double that across the resistor. Therefore, $V_L=2V_R=440\ \mathrm{V}$.

We know that $V_L=iX_L=i\times 2\pi fL\Rightarrow L=\frac{440}{44\times 100\pi}=\frac{1}{10\pi}\ \mathrm{H}$

Therefore, $k=10$.

Q.12. Consider two particles of equal mass and at separation r. How many times will the force between them become when the mass of one of the particles becomes three times maintaining the same separation?

A) 9

B) 3

C) 1.5

D) $\sqrt{3}$

Answer: 3

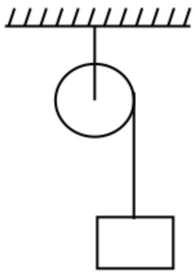
Solution: The gravitational force between particles is given by, $F=\frac{Gm_1m_2}{r^2}$

$\because\ F_1=\frac{Gm^2}{r^2}$ and

$F_2=\frac{Gm(3m)}{r^2}=\frac{3Gm^2}{r^2}$

$\Rightarrow F_2=3F_1$

Q.13. A block of mass m and a pulley of mass m are arranged as shown. The string connecting the block and the pulley does not slip on the pulley as the block moves down. Find the tension in the string.



A) $\frac{mg}{4}$

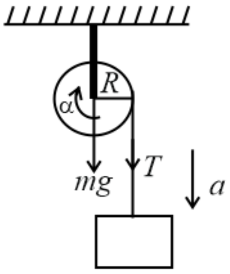
B) $\frac{mg}{2}$

C) $\frac{mg}{3}$

D) $\frac{2mg}{3}$

Answer: $\frac{mg}{3}$

Solution:



Torque about centre $\tau = TR = I\alpha$, where, angular acceleration $\alpha = \frac{a}{R}$ and I is moment of inertia.

$\Rightarrow TR = \frac{mR^2\alpha}{2} = \frac{mR^2a}{2R} = \frac{maR}{2} \Rightarrow T = \frac{ma}{2}$

Now, using Newton's equation of motion,

$mg - T = ma$

$mg - \frac{ma}{2} = ma \Rightarrow a = \frac{2g}{3}$

So, tension in string is $T = \frac{ma}{2} = \frac{m}{2}\left[\frac{2g}{3}\right] = \frac{mg}{3}$

Q.14. Time period of simple pendulum of length l when placed in a lift which is accelerating upwards with the acceleration $\frac{g}{6}$ is

A) $2\pi\sqrt{\frac{6l}{7g}}$

B) $2\pi\sqrt{\frac{7l}{6g}}$

C) $2\pi\sqrt{\frac{3l}{2g}}$

D) $2\pi\sqrt{\frac{5l}{g}}$

Answer: $2\pi\sqrt{\frac{6l}{7g}}$

Solution: When the lift is accelerating upward with acceleration a the effective acceleration due to gravity can be taken as $g' = g + a$

$\Rightarrow T = 2\pi\sqrt{\frac{l}{g+a}} \Rightarrow T = 2\pi\sqrt{\frac{l}{g+\frac{g}{6}}}$

$\Rightarrow T = 2\pi\sqrt{\frac{6l}{7g}}$

Q.15. de-Broglie wavelength of two identical particles are related as $\lambda_1 = 3\lambda_2$, then the kinetic energy K_1 and K_2 of the particles are related as

A) $K_2 = 3K_1$

B) $K_2 = 9K_1$

C) $K_1 = 3K_2$

D) $K_1 = 2K_2$

Answer: $K_2 = 9K_1$

Solution: As we know, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

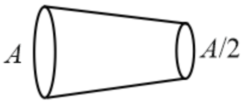
Hence, we can write $\lambda_1 = \frac{h}{\sqrt{2mK_1}}$ & $\lambda_2 = \frac{h}{\sqrt{2mK_2}}$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}}$

$\Rightarrow 3 = \sqrt{\frac{K_2}{K_1}}$

$\Rightarrow K_2 = 9K_1$

Q.16. Water is flowing through a frustum like section of a pipe as shown in the diagram. Pressure difference across the ends is $4000\ \mathrm{N\ m^{-2}}$. Area of cross-section $A = \sqrt{6}\ \mathrm{m^2}$. Find the volume flow rate through the pipe.



A) $1\ \mathrm{m^3\ s^{-1}}$

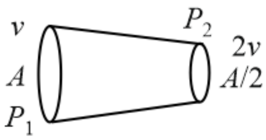
B) $2\ \mathrm{m^3\ s^{-1}}$

C) $4\ \mathrm{m^3\ s^{-1}}$

D) $8\ \mathrm{m^3\ s^{-1}}$

Answer: $4\ \mathrm{m^3\ s^{-1}}$

Solution:



From the equation of continuity, $v_2 = 2v_1$.

Applying the Bernoulli's Theorem,

$P_1 + \frac{\rho v^2}{2} = P_2 + \frac{\rho 4(v)^2}{2} \Rightarrow P_1 - P_2 = \frac{3\rho v^2}{2}$

$\Rightarrow 4000 = \frac{3\times 1000\times v^2}{2} \quad \Rightarrow v = \sqrt{\frac{8}{3}}$

Now volume flow rate, $Q = Av = \sqrt{6} \times \sqrt{\frac{8}{3}} = 4\ \mathrm{m^3\ s^{-1}}$

Q.17. A drop of water of radius $1\,\mathrm{mm}$ is falling through air. Find the terminal speed of the drop knowing that density of air is negligible as compared to density of water. $\left(\eta_{\text{air}} = 2 \times 10^{-3}\mathrm{Nsm^{-2}}, g = 10\ \mathrm{ms^{-2}}\right)$

A) $2.2\,\mathrm{ms^{-1}}$

B) $1.1\,\mathrm{ms^{-1}}$

C) $1.6\,\mathrm{ms^{-1}}$

D) $2.8\,\mathrm{ms^{-1}}$

Answer: $1.1\,\mathrm{ms^{-1}}$

Solution: Terminal velocity is given by $v_T=\frac{2r^2(\rho-\sigma)g}{9\eta}$, where, r is radius of falling body, ρ is density of falling body and σ is density of fluid.

$\Rightarrow v_T=\frac{2\times10^{-6}\times10^3\times10}{9\times2\times10^{-3}}$

$\Rightarrow v_T=\frac{10}{9}=1.1\ \mathrm{m\ s^{-1}}$

Q.18. The temperature of a sample of gaseous $\mathrm{O_2}$ is doubled such that $\mathrm{O_2}$ disassociates into O. Find the ratio of new v_{rms} to old v_{rms}

A) 2

B) $\sqrt{2}$

C) 4

D) $\frac{1}{2}$

Answer: 2

Solution: The RMS velocity of a gas is given by, $v_{RMS}=\sqrt{\frac{3RT}{M}}$

When $\mathrm{O_2}$ dissociates into O its molecular mass becomes $\frac{M}{2}$. Therefore,

Ratio of RMS speed, $\frac{v_1}{v_2}=\sqrt{\frac{T_1}{T_2}\times\frac{M_2}{M_1}}\Rightarrow\frac{v_1}{v_2}=\sqrt{\frac{T}{2T}\times\frac{M/2}{M}}\Rightarrow\frac{v_2}{v_1}=2$

Q.19. For an amplitude modulated wave given by $y(t)=10\left[1+0.4\cos\left(2\pi\times10^4t\right)\right]\cos\left(2\pi\times10^7t\right)$. Find the bandwidth.

A) $10\,\mathrm{kHz}$

B) $20\,\mathrm{MHz}$

C) $20\,\mathrm{kHz}$

D) $10\,\mathrm{MHz}$

Answer: $20\,\mathrm{kHz}$

Solution: Expression for modulation is given by,

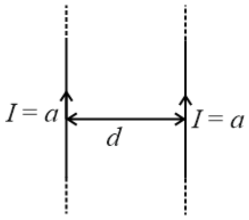
$s(t)=A_c[1+\mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$, where f_m and f_c are the frequency of the modulating signal and the carrier signal respectively.

Comparing it with the given expression in question, we get,

$f_m=10^4\ \mathrm{Hz}$

Now, Bandwidth $=2f_m=20\ \mathrm{kHz}$

Q.20. Two parallel wires carry same magnitude current that is $1 \mathrm{~A}$ the distance between two wires is given as $d=4 \mathrm{~cm}$. The force per unit length experienced by the wires (in $10^{-7} \mathrm{~N} \mathrm{~m}^{-1}$) is equal to ______.



A) 50

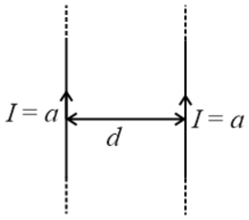
B) 25

C) 100

D) 75

Answer: 50

Solution:



The force per unit length is given by,

$$\left(\frac{F}{l}=\frac{\mu_0 I_1 I_2}{2 \pi d}\right) \Rightarrow \frac{F}{l}=\frac{\left(2 \times 10^{-7}\right) \times 1 \times 1}{0.04}=50 \times 10^{-7} \mathrm{~N} \mathrm{~m}^{-1}$$

Section B: Chemistry

Q.1. Consider the following oxides of nitrogen:
N_2O, N_2O_2, N_2O_3, N_2O_4, N_2O_5
How many of them consist of $N-N$ bond?

A) Four

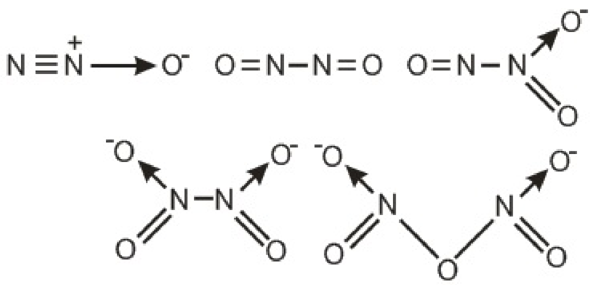
B) Five

C) Two

D) One

Answer: Four

Solution: The structures of the given compounds are given below.



Only four of the above have $N-N$ bond. N_2O, N_2O_2, N_2O_3 and N_2O_4 have $N-N$ type bond.

Q.2. Using the following standard reduction potential (E^o) :
$E^o\left(Sn^{4+}/Sn^{2+}\right) = 0.15\ V$ and $E^o\left(Sn^{2+}/Sn\right) = -0.14\ V$; E^o for Sn^{4+}/Sn would be

A) 0.005 V

B) 0.01 V

C) –0.005 V

D) 0.002 V

Answer: 0.005 V

Solution: E^o is an intensive thermodynamic property.

	E^o	$\Delta G^o = nE^o F$
$Sn^{4+} + 2e^- \longrightarrow Sn^{2+}$	0.15 V	– 0.30 F
$Sn^{2+} + 2e^- \longrightarrow Sn$	– 0.14 V	+ 0.28 F

Adding : $Sn^{4+} + 4e^- \longrightarrow Sn,$

$\Delta G^0_{Reaction} = \Delta G^0_{Product} - \Delta G^0_{Reactant}$
$\Delta G^o = -nF E^0_{Cell}$
$\Delta G^o = -4E^o F = -0.02F$

$\Rightarrow$ $E^o = 0.005\ V$

Q.3. Photochemical smog contains

A) N_2

B) O_3

C) SF_4

D) F_2

Answer: O_3

Solution: Photochemical smog is formed due to photochemical oxidation (due to action of sunlight) of hydrocarbons and nitrogen oxides. UV light is absorbed by NO_2. This highly energised molecule.

The photochemical smog always contains PAN and O_3. It is a type of air pollution derived from vehicular emission from internal combustion engines and industrial fumes.

(NO_2) decomposes into nitric oxide and atomic oxygen. Atomic oxygen reacts quickly with oxygen to form ozone.

Hence, the photochemical smog contains O_3.

Q.4. Consider the following complexes $[Fe(CN)_6]^{3-}$, $[Ni(CN)_4]^{2-}$ and $[Fe(CN)_6]^{4-}$
How many complex(es) is/are paramagnetic?

A) 0

B) 1

C) 2

D) 3

Answer: 1

Solution: $[Fe(CN)_6]^{3-} = Fe\left(III\right) = 3d^5$ (1 unpaired electron)

$[Ni(CN)_4]^{2-} = Ni\left(II\right) = 3d^8$ (No unpaired electron)

$[Fe(CN)_6]^{4-} = Fe\left(II\right) = 3d^6$ (No unpaired electron)

Q.5. Which of the following is a basic oxide?

A) Al_2O_3

B) SiO_2

C) Na_2O

D) NO_2

Answer: Na_2O

Solution: Oxides of metals are generally basic in nature but some are amphoteric in nature.

Oxides of non-metals are acidic in nature but some of them are neutral.

NO_2 is acidic in nature it reacts with bases.

Al_2O_3 is amphoteric in nature as it reacts with both acids and bases.

SiO_2 is acidic in nature.

Na_2O is a basic oxide and it reacts with acids readily.

Q.6. X reacts with Br_2/H_2O to give gluconic acid and reacts with HNO_3 to give saccharic acid. Name X

A) Maltose

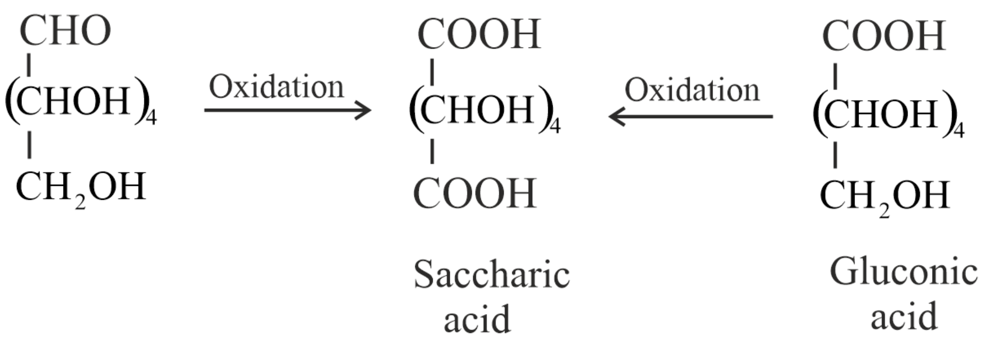
B) Starch

C) Fructose

D) Glucose

Answer: Glucose

Solution: On oxidation with nitric acid, glucose as well as gluconic acid both yield a dicarboxylic acid, saccharic acid. This indicates the presence of a primary alcoholic $(-OH)$ group in glucose.



Q.7. The isotopes of Hydrogen differ in the following property

A) Electronic configuration

B) Number of protons

C) Atomic number

D) Atomic mass

Answer: Atomic mass

Solution: Isotopes are the atoms of the same element with different mass number. They have the same atomic number. So, number of protons and electrons are same, Hence their electronic configuration are same, but number of neutrons are different.

Hydrogen has three isotopes: protium, ${}_{1}^{1}H$, deuterium, ${}_{1}^{2}H$ or D and tritium, ${}_{1}^{3}H$ or T. These isotopes differ from one another in respect of the presence of neutrons. Ordinary hydrogen, protium, has no neutrons, deuterium (also known as heavy hydrogen) has one and tritium has two neutrons in the nucleus.

Q.8. Nitration of Aniline in presence of conc. HNO_3 and conc. H_2SO_4 gives

A) o-nitroaniline as the major product

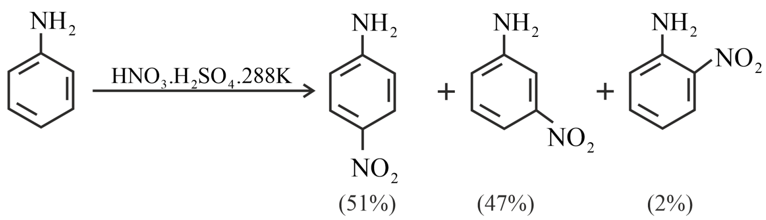
B) m-nitroaniline as the major product

C) p-nitroaniline as the major product

D) $2, 4$-dinitroaniline as the major product

Answer: p-nitroaniline as the major product

Solution: Nitration: Direct nitration of aniline yields tarry oxidation products in addition to the nitro derivatives. Moreover, in the strongly acidic medium, aniline is protonated to form the anilinium ion which is meta directing. That is why besides the ortho and para derivatives, significant amount of meta derivative is also formed.



Q.9. $FeO + SiO_2 \rightarrow FeSiO_3$

Considering the extraction of copper, SiO_2 and $FeSiO_3$ are respectively:

A) Flux and slag

B) Slag and flux

C) Gauge and flux

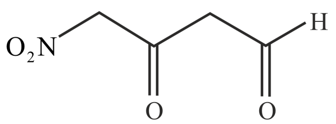
D) Gauge and slag

Answer: Flux and slag

Solution: During extraction of copper, iron oxide present as impurity is removed by adding SiO_2 and converting it into $FeSiO_3$

$$FeO + \underset{\text{Flux}}{SiO_2} \rightarrow \underset{\text{Slag}}{FeSiO_3}$$

Q.10. The correct IUPAC name of the given compound is:



A) 1-formyl-4-nitrobutanal

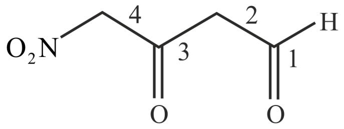
B) 4-nitro-3-oxobutanal

C) 4-oxo-3-nitrobutanal

D) 3-oxo-4-nitropropanal

Answer: 4-nitro-3-oxobutanal

Solution:



4-nitro-3-oxobutanal

Aldehyde is the main functional group and ketone group and nitro groups will act as substituent groups. Numbering will be done from the aldehyde side.

Q.11. In which of the following compounds does sulphur shows two different oxidation states?

A) $H_2S_2O_3$

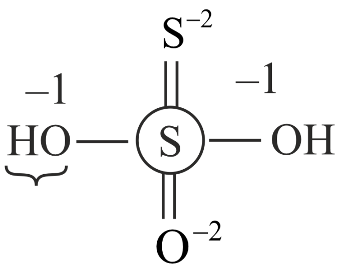
B) $H_2S_2O_6$

C) $H_2S_2O_7$

D) $H_2S_2O_8$

Answer: $H_2S_2O_3$

Solution:



In this molecule both sulphur has different oxidation number one has -2 oxidation number and other has +6 oxidation number. Average oxidation number of sulphur will be +2.

Q.12. Which of the following set of quantum number is valid?

A)

n	l	m	s
4	3	0	$\frac{1}{2}$

B)

n	l	m	s
2	1	−2	$\frac{1}{2}$

C)

n	l	m	s
3	3	2	$\frac{1}{2}$

D)

n	l	m	s
1	1	0	$\frac{1}{2}$

Answer:

n	l	m	s
4	3	0	$\frac{1}{2}$

Solution: In an atom possible values of l, m and s for a given value of n are

$l = 0, \ldots\ldots (n-1)$

$m = -l,\ 0,\ +l = (2l+1)$

$s = +\frac{1}{2}$ or $-\frac{1}{2}$

The possible values of given quantum number are

n	l	m	s
4	3	0	$\frac{1}{2}$

Q.13. Find the empirical formula of a compound which contains 74% C, 17.3% N, 8.7% H by mass.

A) C_4H_6N

B) C_5H_7N

C) C_3H_5N

D) $C_4H_5N_2$

Answer: C_5H_7N

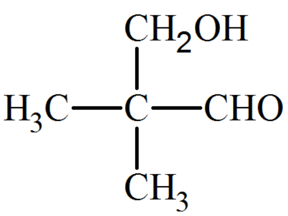
Solution:

Elements	% by mass	Moles	Whole number ratio
C	74	$\frac{74}{12} = 6.17$	5
H	8.7	$\frac{8.7}{1} = 8.7$	7
N	17.3	$\frac{17.3}{14} = 1.24$	1

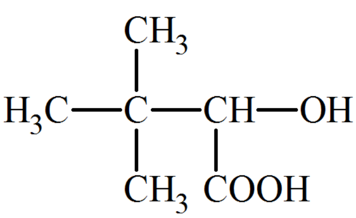
Q.14. Isobutaldehyde $\xrightarrow[K_2CO_3]{HCHO} A \xrightarrow{CN^-} B \xrightarrow{H_3O^+} P$

The product P is:

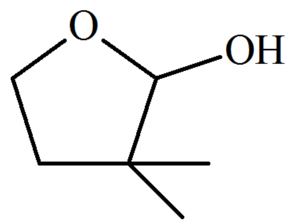
A)



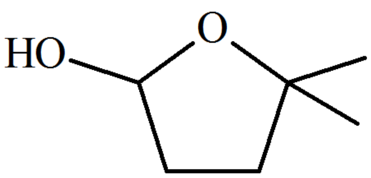
B)



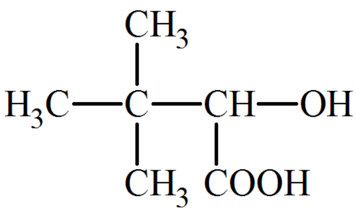
C)



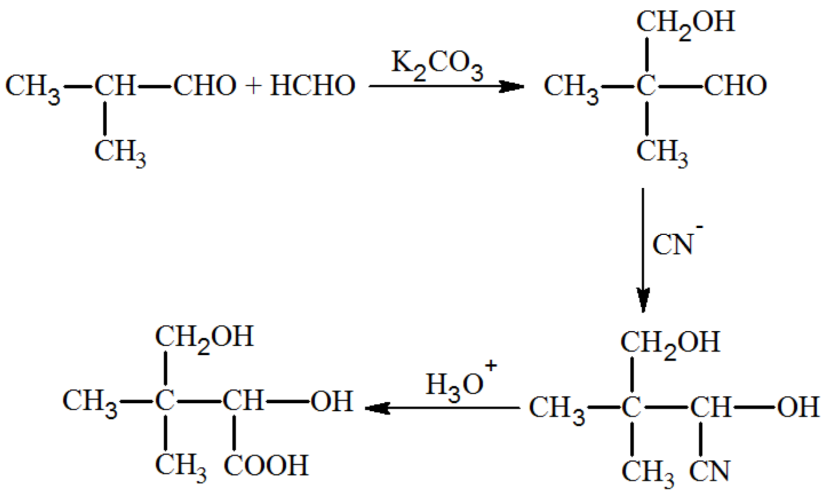
D)



Answer:



Solution:

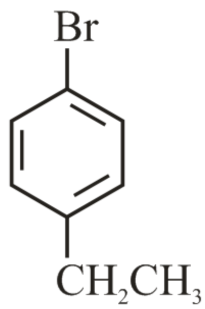


Q.15. Consider the following reaction,

$\mathrm{A} \xrightarrow[\text{2. KCN}]{\text{1. } Cl_2/hv} {}^{\text{3. } H_3O^+/\Delta}$ $4-\text{bromophenylacetic acid}$

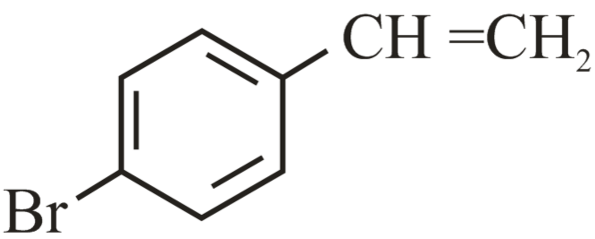
What is A in the above reaction?

A)

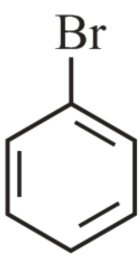


B)

C)

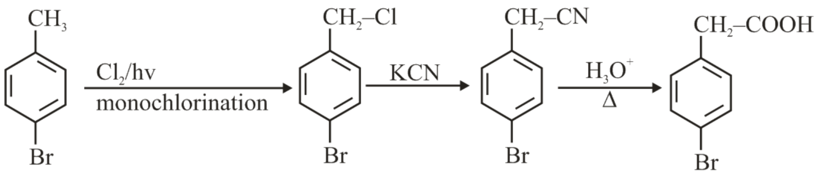


D)



Answer:

Solution:



4-Bromophenylacetic acid

Q.16. The pH of a buffer solution of acetic acid is 4. Find the value of $\frac{[CH_3COO^-]}{[CH_3COOH]}$

Given K_a of acetic acid is 1.3×10^{-5}.

A) 2.3

B) 10.2

C) 0.13

D) 1.5

Answer: 0.13

Solution:

$$pH = pK_a + \log\left(\frac{[CH_3COO^-]}{[CH_3COOH]}\right)$$

$$4 = 5 - \log\left(1.3\right) + \log\left(\frac{[CH_3COO^-]}{[CH_3COOH]}\right)$$

$$\log\left(1.3 \times 10^{-1}\right) = \log\left(\frac{[CH_3COO^-]}{[CH_3COOH]}\right)$$

$$\frac{[CH_3COO^-]}{[CH_3COOH]} = 0.13$$

Q.17. Nature of colloidal solution of $Fe(OH)_3$ is:

A) Neutral

B) Positive

C) Negative

D) Amphoteric

Answer: Positive

Solution: Nature of colloidal solution of $Fe(OH)_3$ is obtained by hydrolysis of $FeCl_3$

$$FeCl_3 \rightarrow Fe^{3+} + 3Cl^-$$

$$Fe^{3+} + H_2O \rightarrow H^+ + Fe(OH)_3$$

$$Fe(OH)_3 + Fe^{3+} \rightarrow Fe(OH)_3/Fe^{3+}$$

$Fe(OH)_3$ sol absorbs Fe^{3+} ions to become positive colloid.

Q.18. Consider the structure of SF_4, the number of lone pair(s), position of lone pair(s) and number of lone pair-bond pair repulsions respectively are:

A) 1, equatorial position, 4

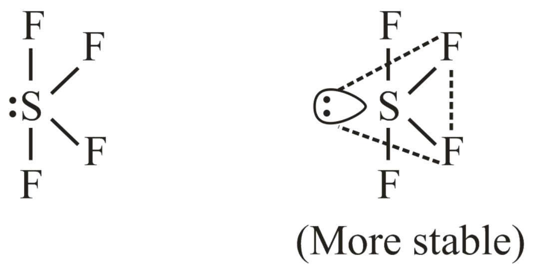
B) 1, axial position, 4

C) 1, axial position, 3

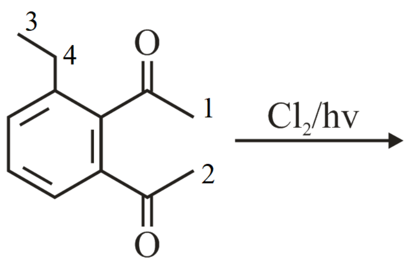
D) 1, equatorial position, 6

Answer: 1, equatorial position, 4

Solution: The lp is in an equatorial position, and there are two lp—bp repulsions. Hence, arrangement (b) is more stable. The shape shown in (b) is described as a distorted tetrahedron, a folded square or a see-saw.



Q.19. Position of C atoms to which Cl is attached in the product:



A) 3

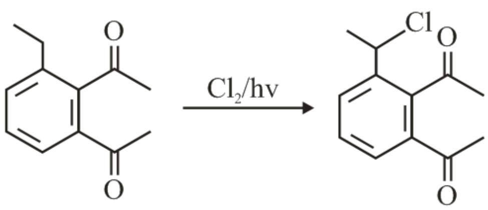
B) 2

C) 4

D) 1

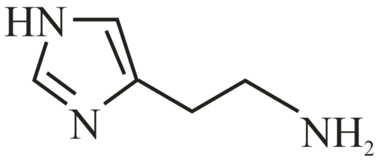
Answer: 4

Solution: In presence of sunlight free radical substitution reaction will take place. Among the given positions, the free radical is most stable at the benzyl position. So, major product will be formed accordingly.

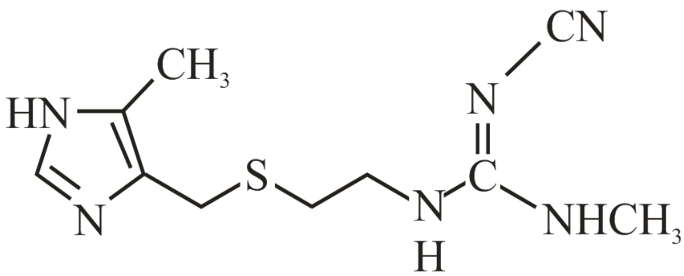


Q.20. The structure of Tagamet (Cimetidine) is:

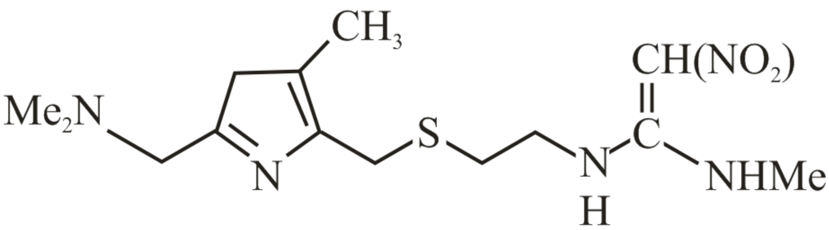
A)


HN
N
NH2

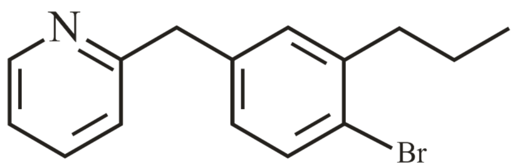
B)


CN
CH3
HN
N
S
N
C
N
H
NHCH3

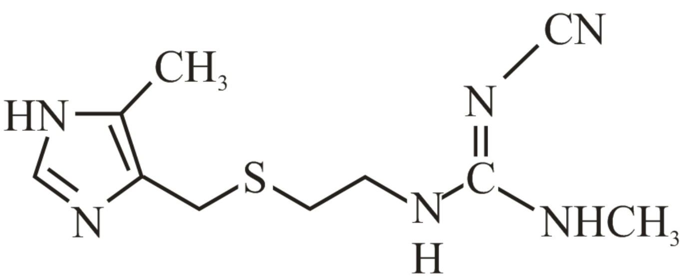
C)


CH3
CH(NO2)
Me2N
S
N
C
N
H
NHMe

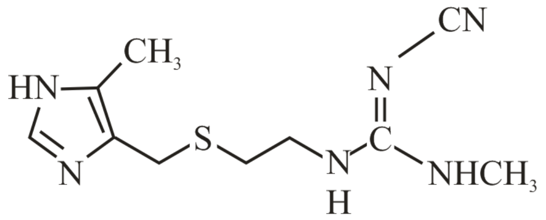
D)


N
Br

Answer:


CN
CH3
HN
N
S
N
C
N
H
NHCH3

Solution: Cimetidine is a member of the class of guanidines that consists of guanidine carrying a methyl substituent at position 1, a cyano group at position 2 and a $2-\{[(5-\text{methyl}-1\text{H}-\text{imidazol}-4-\text{yl})\,\text{methyl}]\,\text{sulfanyl}\}\,\text{ethyl}$ group at position 3.



Q.21. The half-life of a substance is 200 days. Find the $\%$ activity of remaining substance after 83 days, if it decays through first order kinetics.

[Round off to the nearest integer]

A) 25%

B) 50%

C) 75%

D) 83%

Answer: 75%

Solution: For first order reaction,

$\mathrm{K}=\frac{2.303}{\mathrm{t}}\log\frac{\mathrm{N_o}}{\mathrm{N_t}}$ ... (1)

Also, $\mathrm{K}=\frac{0.693}{\mathrm{t}_{\frac{1}{2}}}$ ... (2)

Equating (1) and (2)

Hence, $\frac{2.303}{\mathrm{t}}\log\frac{\mathrm{N_o}}{\mathrm{N_t}}=\frac{0.693}{\mathrm{t}_{\frac{1}{2}}}$

$\frac{2.303}{83}\log\frac{\mathrm{N_o}}{\mathrm{N_t}}=\frac{0.693}{200}$

$\frac{\mathrm{N_o}}{\mathrm{N_t}}=\frac{4}{3}$

$\%\frac{\mathrm{N_t}}{\mathrm{N_0}}=0.75\times 100$

$=75\%$

Q.22. V ml of $0.01\ \mathrm{M}\ \mathrm{KMnO_4}$ solution is titrated with 20 ml of $0.05\ \mathrm{M}$ Mohr's salt, calculate the value of V_2.

A) 10 ml

B) 5 ml

C) 20 ml

D) 15 ml

Answer: 20 ml

Solution: $N_1V_1 = N_2V_2$

n factor for $KMnO_4 = 5$, n factor for Mohr's salt $= 1$

$0.01 \times 5 \times \frac{V}{1000} = 1 \times 0.05 \times \frac{20}{1000}$

$V = \frac{1 \times 0.05 \times 20}{0.01 \times 5} = 20 \text{ ml}$

$2KMnO_4 + 10FeSO_4(NH_4)_2SO_4.6H_2O + 8H_2SO_4 \rightarrow K_2SO_4 + 2MnSO_4 + 5Fe_2(SO_4)_3 + 10(NH_4)_2SO_4 + 68H_2O$

Section C: Mathematics

Q.1. $6\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\frac{1}{r^2+3r+3}\right)=$

A) 3

B) 4

C) 6

D) 8

Answer: 3

Solution: $T_r=\tan^{-1}\frac{1}{r^2+3r+2+1}=\tan^{-1}\left(\frac{(r+2)-(r+1)}{1+(r+1)(r+2)}\right)$

$=\tan^{-1}(r+2)-\tan^{-1}(r+1)$

$T_1=\tan^{-1}3-\tan^{-1}2$

$T_2=\tan^{-1}4-\tan^{-1}3$

.

.

$T_n=\tan^{-1}(n+2)-\tan^{-1}(n+1)$

$\Rightarrow\sum_{r=1}^{n}\tan^{-1}\frac{1}{r^2+3r+3}=\tan^{-1}(n+2)-\tan^{-1}2$

i.e. $6\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\frac{1}{r^2+3r+3}\right)=6\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(n+2)-\tan^{-1}2\right]\right)$

$=6\tan\left(\frac{\pi}{2}-\tan^{-1}2\right)$

$=6\tan\left(\tan^{-1}\frac{1}{2}\right)=3$

Q.2. The term independent of x in the expansion of $\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ is equal to

A) $-\frac{43}{200}$

B) $\frac{17}{100}$

C) $-\frac{17}{200}$

D) $\frac{33}{200}$

Answer: $\frac{33}{200}$

Solution: General term of $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ is

$T_{r+1}={}^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(-\frac{1}{5x^2}\right)^r={}^{11}C_r(-1)^r\cdot\frac{5^{11-2r}}{2^{11-r}}\cdot x^{33-5r}$

The term independent of x in the expansion of $\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ will be the coefficient of x^0 in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}-$ coefficient of x^{-2} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}+3\times$ coefficient of x^{-3} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

$=-{}^{11}C_7(-1)^7\cdot\frac{5^{-3}}{2^4}=\frac{330}{5^3.2^4}=\frac{33}{200}$

Q.3. If vertex of parabola is $(2,-1)$ and equation of its directrix is $4x-3y=21$, then the length of latus rectum is

A) 2

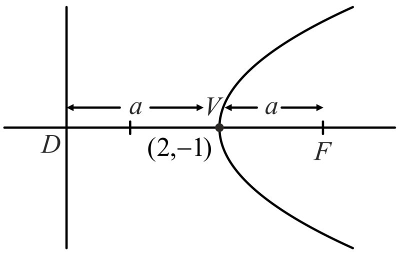
B) 8

C) 3

D) 4

Answer: 8

Solution:



We know that, in any parabola, the length of the latus rectum $=4a$ and perpendicular distance from vertex to the directrix $=a$

So, $a=\left|\frac{8+3-21}{\sqrt{4^2+(-3)^2}}\right|$

$\Rightarrow a=\left|\frac{-10}{5}\right|=2$

Therefore, the length of the latus rectum $4a=4\times 2=8$

Q.4. The area enclosed by x-axis & the curve $y=3-|x+1|-\left|x-\frac{1}{2}\right|$ is

A) $\frac{27}{8}$

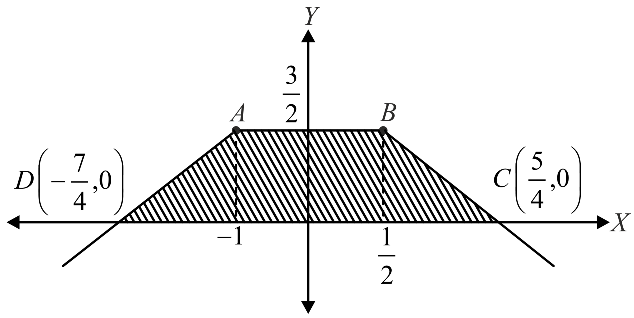
B) $\frac{23}{8}$

C) $\frac{25}{8}$

D) $\frac{27}{4}$

Answer: $\frac{27}{8}$

Solution: Plotting the graph of the function $y=3-|x+1|-\left|x-\frac{1}{2}\right|$ and x-axis, we get



Now applying the formula of area of trapezium which is $\frac{1}{2}\times\text{height}\times(\text{sum of parallel sides})$ we get,

Required area of $\operatorname{Trap}(ABCD)=\frac{1}{2}\times\frac{3}{2}\times\left(3+\frac{3}{2}\right)=\frac{27}{8}$

Q.5. If $\cot\alpha=-1, \sec\beta=-\frac{5}{3}$, where $\alpha\in\left(\frac{\pi}{2},\pi\right)$ and $\beta\in\left(\pi,\frac{3\pi}{2}\right)$, then $\tan(\alpha+\beta)$ is

A) $\frac{1}{7}$

B) $-\frac{1}{3}$

C) $\frac{1}{3}$

D) $-\frac{1}{7}$

Answer: $\frac{1}{7}$

Solution: $\cot\alpha=-1\Rightarrow\tan\alpha=-1$ and $\sec\beta=-\frac{5}{3}\Rightarrow\tan\beta=\frac{4}{3}$

$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{-1+\frac{4}{3}}{1+\frac{4}{3}}=\frac{1}{7}$

Q.6. If 30 identical candies are distributed among 4 students S_1, S_2, S_3 and S_4 such that S_2 can get at least 4 and at most 7, S_3 can get at least 2 and at most 6 and no restrictions on S_1 and S_4, then number of ways in which the candies can be distributed are

A) 430

B) 520

C) 640

D) 330

Answer: 430

Solution: Given, $S_1+S_2+S_3+S_4=30$

Now it has given that $4 \leq S_2 \leq 7$ & $2 \leq S_3 \leq 6$

and S_1 & S_4 can take any value.

Finding coefficient of x^{30} in

$\left(x^0+x^1+\ldots\ldots x^{30}\right)\left(x^4+x^5+\ldots\ldots x^7\right)\left(x^2+\ldots x^6\right)\left(x^0+\ldots . x^{30}\right)$

$=\left(\frac{1-x^{31}}{1-x}\right)^2 x^4\left(\frac{1-x^4}{1-x}\right) x^2\left(\frac{1-x^5}{1-x}\right)$

{Ignoring higher power more than 30}

$=\left(\frac{1}{1-x}\right)^2 x^4\left(\frac{1-x^4}{1-x}\right) x^2\left(\frac{1-x^5}{1-x}\right)$

$=x^6\left(1-x^4\right)\left(1-x^5\right)(1-x)^{-4}=x^6\left(1-x^5-x^4+x^9\right)(1-x)^{-4}$

$=\left(x^6-x^{11}-x^{10}+x^{15}\right)(1-x)^{-4}$

Required coefficient $={ }^{4+24-1} C_{24}-{ }^{4+19-1} C_{19}-{ }^{20+4-1} C_{20}+{ }^{15+4-1} C_{15}$

$={ }^{27} C_{24}-{ }^{22} C_{19}-{ }^{23} C_{20}+{ }^{18} C_{15}=430$

Q.7. The equation of plane passing through the point $(2,-1,0)$ and perpendicular to planes $2x-3y+z=0$ and $2x-y-3z=0$ is

A) $5x+4y+2z=0$

B) $2x-y+z=3$

C) $5x+4y+2z-6=0$

D) $2x+y-z=3$

Answer: $5x+4y+2z-6=0$

Solution: Let the required equation of the plane be $a\left(x-x_1\right)+b\left(y-y_1\right)+c\left(z-z_1\right)=0$

So $2a-3b+c=0$ and $2a-b-3c=0$

i.e. $\frac{a}{5}=\frac{b}{4}=\frac{c}{2}$

Hence, equation of required plane is $5(x-2)+4(y+1)+2(z-0)=0 \Rightarrow 5x+4y+2z-6=0$

Q.8. If the $\lim\limits_{x \rightarrow 1}\left(\frac{\sin \left(3x^2-4x+1\right)-x^2+1}{2x^3-7x^2+ax+b}\right)=-2$, then the value of $a-b$ is

A) 11

B) 21

C) 17

D) 7

Answer: 11

Solution: Given $\lim_{x \to 1} \left(\frac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b} \right) = -2 \quad \ldots(\text{i})$

Putting $x = 1$ in limit we get

$\Rightarrow \lim_{x \to 1} \left(\frac{\sin 0-1+1}{2-7+a+b} \right) = -2 \Rightarrow \lim_{x \to 1} \left(\frac{0}{a+b-5} \right) = -2$

For limit to exist $a+b-5$ should be zero

so $a+b-5=0 \Rightarrow \quad a+b=5 \quad \ldots(\text{ii})$

Now Using L-Hospital in equation (i) we get

$\lim_{x \to 1} \frac{\cos(3x^2-4x+1)(6x-4)-2x}{6x^2-14x+a} = -2$

Again putting $x = 1$ we get

$\Rightarrow \lim_{x \to 1} \frac{(\cos 0)\times(6-4)-2}{6-14+a} = -2$

$\Rightarrow \lim_{x \to 1} \frac{0}{a-8} = -2$

Again for limit to exist $a-8=0 \Rightarrow a=8$

So from equation (ii) we get

$8+b=5 \Rightarrow b=-3$

So $a-b=8-(-3)=11$

Q.9. If n arithmetic means are inserted between a and 100, then ratio of first arithmetic mean and n^{th} arithmetic mean is 1 : 7 and $a+n=33$, then the value of n is

A) 21

B) 22

C) 23

D) 24

Answer: 23

Solution: Let the A.P. be $a,\ A_1,\ A_2 \ldots A_n,\ 100$

Here, common difference, $d = \frac{100-a}{n+1}$

Given $\frac{A_1}{A_n} = \frac{1}{7} \Rightarrow \frac{a+d}{100-d} = \frac{1}{7} \quad \ldots(i)$

Also $a+n=33$

From options, when $n=23,\ a=10$ and $d = \frac{90}{24} = \frac{15}{4}$

from (i) $\frac{10+\frac{15}{4}}{100-\frac{15}{4}} = \frac{55}{385} = \frac{1}{7}$

Q.10. If two sets be $A=\{a,b,c,d\}$ & $B=\{1,2,3,4,5\}, f: A \to B$ be one-one function, then find the probability when $f(a)+2\,f(b)-f(c)=f(d)$

A) $\frac{1}{20}$

B) $\frac{1}{30}$

C) $\frac{1}{40}$

D) $\frac{1}{50}$

Answer: $\frac{1}{20}$

Solution: Total cases will be $^5C_4 \times 4!$

Now favourable cases for $2f(b) = f(c) + f(d) - f(a)$ will be,

Case (I) if $f(b) = 1$ then $f(c),\ f(d),\ f(a)$ can take the value $3, 4, 5$ & $4, 3, 5$ respectively

Case (II) if $f(b) = 2$ then $f(c),\ f(d),\ f(a)$ can take value $3, 5, 4\ \&\ 5, 3, 4$ respectively

Case (III) if $f(b) = 3$ then $f(c),\ f(d),\ f(a)$ can take value $2, 5, 1\ \&\ 5, 2, 1$ respectively

So total favourable case will be 6

So probability$= \frac{\text{favourable cases}}{\text{total outcomes}} = \frac{6}{5 \times 4!} = \frac{1}{20}$

Q.11. Let $f(x)$ be a quadratic expression such that $f(-2) + f(3) = 0$. If one root of $f(x) = 0$ is -1, then the sum of the roots of the quadratic equation $f(x) = 0$ is

A) $\frac{11}{3}$

B) $\frac{8}{3}$

C) $-\frac{11}{3}$

D) $\frac{3}{11}$

Answer: $\frac{11}{3}$

Solution: Let $f(x) = ax^2 + bx + c$

Given, $f(-2) + f(3) = 0$

$\Rightarrow 4a - 2b + c + 9a + 3b + c = 0$

$\Rightarrow \quad 13a + b + 2c = 0 \qquad \ldots(\text{i})$

Since -1 is a root of $f(x) = 0$. So, $f(-1) = 0$

$\Rightarrow a - b + c = 0 \qquad \ldots(\text{ii})$

On solving equations (i) & (ii), we get

$11a + 3b = 0\ \Rightarrow -\frac{b}{a} = \frac{11}{3}$

Now, we know that sum of the roots of $ax^2 + bx + c = 0$ is $\frac{-b}{a}$.

$\therefore$ The sum of the roots of $f(x) = 0$ is $\frac{11}{3}$.

Q.12. Let $f(x) + f(x+k) = n$. If $I_1 = \int_0^{4k} f(x)dx$ and $I_2 = \int_{-k}^{3k} f(x)dx$, then $I_1 + I_2$ equals

A) nk

B) $2nk$

C) $3nk$

D) $4nk$

Answer: $4nk$

Solution: Given $f(x)+f(x+k)=n \quad \ldots$(i)

$I_1=\int_0^{4k} f(x)dx$

Now $I_2=\int_{-k}^{3k} f(x)dx$

Let $x=t-k \Rightarrow dx=dt$

$\Rightarrow \quad I_2=\int_0^{4k} f(t-k)dt$

$=\int_0^{4k}(n-f(t))dt \quad \{$as $f(t-k)=n-f(t)\}$

$=\int_0^{4k} ndt-\int_0^{4k} f(t)dt$

$=4nk-I_1$

Hence, $I_1+I_2=4nk$

Q.13. The image of point $P(3,2,3)$ with respect to the line $\frac{x-3}{3}=\frac{y-2}{4}=\frac{z-1}{5}$ is S. If a point $Q(\alpha,\beta,\gamma)$ divides PS internally in the ratio $1:3$, then (α,β,γ) is

A) $\left(\frac{33}{10},\frac{12}{5},\frac{5}{2}\right)$

B) $\left(\frac{33}{10},\frac{5}{2},\frac{12}{5}\right)$

C) $\left(\frac{12}{5},\frac{33}{10},\frac{5}{2}\right)$

D) $(1,0,1)$

Answer: $\left(\frac{33}{10},\frac{12}{5},\frac{5}{2}\right)$

Solution: Given that the image of point $P(3,2,3)$ with respect to the line $\frac{x-3}{3}=\frac{y-2}{4}=\frac{z-1}{5}$ is S.

Let a point on the line be $(3\lambda+3,4\lambda+2,5\lambda+1)$

D.R.'s of perpendicular from P to the line be $(3\lambda,4\lambda,5\lambda-2)$

Now $9\lambda+16\lambda+25\lambda-10=0 \Rightarrow \lambda=\frac{1}{5}$

$\therefore$ Foot of the perpendicular $M \equiv \left(\frac{18}{5},\frac{14}{5},2\right)$

Point dividing P and S in ratio $1:3$ will be mid-point of P & foot of perpendicular (M)

i.e., $\left(\frac{\frac{18}{5}+3}{2},\frac{\frac{14}{5}+2}{2},\frac{2+3}{2}\right)=\left(\frac{33}{10},\frac{12}{5},\frac{5}{2}\right)$

Q.14. If $2xye^{\frac{x^2}{y}}dx+\left(y-x^2e^{\frac{x^2}{y}}\right)dy=0$, then

A) $e^{\frac{x^2}{y}}+\ln y=c$

B) $e^{\frac{x^2}{y}} + \frac{y^2}{2} = c$

C) $x \cdot e^{\frac{x^2}{y}} + y = c$

D) $y \cdot e^{\frac{x^2}{y}} - \ln y = c$

Answer: $e^{\frac{x^2}{y}} + \ln y = c$

Solution: We have, $2xye^{\frac{x^2}{y}}dx + \left(y - x^2 e^{\frac{x^2}{y}}\right)dy = 0$

$$\Rightarrow e^{\frac{x^2}{y}}\left[2xydx - x^2dy\right] = -ydy$$

$$\Rightarrow e^{\frac{x^2}{y}}\left[\frac{yd(x^2) - x^2dy}{y^2}\right] = -\frac{dy}{y}$$

$$\Rightarrow e^{\frac{x^2}{y}}d\left(\frac{x^2}{y}\right) = -\frac{dy}{y}$$

$$\Rightarrow e^{\frac{x^2}{y}}d\left(\frac{x^2}{y}\right) + \frac{dy}{y} = 0$$

$$\Rightarrow \int e^{\frac{x^2}{y}}d\left(\frac{x^2}{y}\right) + \int \frac{1}{y}dy = \int 0dy$$

$$\Rightarrow e^{\frac{x^2}{y}} + \ln y = c$$

Q.15. If one of the diameter of the circle $\left(x - \sqrt{2}\right)^2 + \left(y - 3\sqrt{2}\right)^2 = 6$ is a chord of another circle $\left(x - 2\sqrt{2}\right)^2 + \left(y - 2\sqrt{2}\right)^2 = r^2$, then the value of r is equal to

A) $\sqrt{10}$

B) $\sqrt{5}$

C) 2

D) $\sqrt{8}$

Answer: $\sqrt{10}$

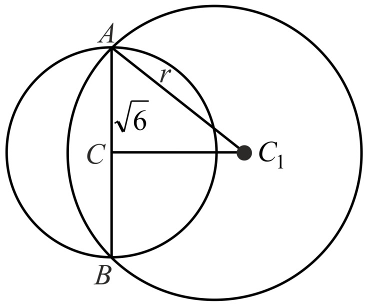
Solution:

Given that, one of the diameter of the circle $\left(x-\sqrt{2}\right)^2+\left(y-3\sqrt{2}\right)^2=6$ is a chord of another circle $\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$.

Let, C and C_1 be the centres of the circles $\left(x-\sqrt{2}\right)^2+\left(y-3\sqrt{2}\right)^2=6$ and $\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$ respectively.

So, $C\left(\sqrt{2},\ 3\sqrt{2}\right)$ & $C_1\left(2\sqrt{2},\ 2\sqrt{2}\right)$

The position of the two circles as shown in figure.



Now, $CC_1=\sqrt{\left(2\sqrt{2}-\sqrt{2}\right)^2+\left(2\sqrt{2}-3\sqrt{2}\right)^2}=\sqrt{2+2}=2$

It is clear from the above diagram, ΔACC_1 is a right angled triangle.$\Rightarrow r^2=(AC)^2+(CC_1)^2=\left(\sqrt{6}\right)^2+2^2\Rightarrow r^2=10$
$\Rightarrow r=\sqrt{10}$

Q.16. There are seven students in a class with average score of 62 . A student fails if he gets less than 50. The worst case that maximum number of students fail in the class if the given variance is 30, will be

A) 1

B) 2

C) 3

D) 4

Answer: 1

Solution:

Given, mean $\bar{x}=\frac{\sum_{i=1}^{7}x_i}{7}=62$

and variance $=\frac{1}{7}\sum_{i=1}^{7}(\bar{x}-x_i)^2=30$

$\Rightarrow\sum_{i=1}^{7}(62-x_i)^2=210$

When a student scores 49 then $(62-49)^2=169$

Since a student fails if he gets less than 50, so at most one student can score less than 50.

Q.17. For the two relations $R_1=\{(a,b);a,b\in N:|a-b|\le 13\}$ and $R_2=\{(a,b);a,b\in N:|a-b|\ne 13\}$, the transitive relation is(are)

A) only R_1

B) only R_2

C) both R_1 and R_2

D) neither R_1 nor R_2

Answer: neither R_1 nor R_2

Solution: To check if R_1 is transitive -

Let $a=16,\ b=4,\ c=1$

Here $|a-b|=|16-4|\leq 13$ holds true

now $|b-c|=|4-1|\leq 13 \Rightarrow |3| \leqslant 13$ also holds true

but $|a-c|=|16-1| \nleq 13$

Hence R_1 is not transitive

Similarly, to check transitivity of R_2

Let $a=16,\ b=4,\ c=3$

Here $|a-b|=|16-4| \neq 13 \text{ and } |b-c|=|4-3| \neq 13$

but $|a-c|=|16-3|=13$

Hence, R_2 is also not transitive.

Q.18. Let, $A=\begin{bmatrix} 1+i & 1 \\ -1 & 0 \end{bmatrix}$ then the total number of elements in the set $\{n \in \{1,2,3,....,100\} : A^n = A\}$ is

A) 25

B) 28

C) 32

D) 35

Answer: 25

Solution: We have $A=\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}$

$A^2=A\cdot A=\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}=\begin{bmatrix}i & 1+i\\ -i+1 & -i\end{bmatrix}$

$A^3=A^2\cdot A=\begin{bmatrix}i & 1+i\\ 1-i & -i\end{bmatrix}\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}=\begin{bmatrix}0 & i\\ 1 & 1-i\end{bmatrix}$

$A^4=A^3\cdot A=\begin{bmatrix}0 & i\\ 1 & 1-i\end{bmatrix}\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$

$\therefore \quad A^4=I$

So, $A^5=A^4\cdot A=I\cdot A=A$

$A^6=A^4\cdot A^2=I\cdot A^2=A^2$ and so on

$\because A^1=A^5=A^9=.....=A^{97}=A$

Hence, possible values of n, such that $A^n=A$

$=\{1,5,9,\ldots,97\}$

Clearly, above sequence is in A.P. where

$a=1,\ d=4\ \&\ t_n=97\ \Rightarrow a+(n-1)d=97$

$\Rightarrow\ \ 1+(n-1)4=97\ \Rightarrow\ \ n=25$

$\therefore$ The number of elements in the given set $=25$.

Q.19. If S_n represents the sum of infinite terms of a G.P. whose first term and common ratio are n^2 and $\frac{1}{(n+1)^2}$ respectively, then the value of $\frac{1}{26}+\sum_{n=0}^{50}\left(S_n+\frac{2}{n+1}-n-1\right)=$

A) 41652

B) 41650

C) 46150

D) 46152

Answer: 41652

Solution: We have, $a=n^2$, $r=\frac{1}{(n+1)^2}$

$S_n=\frac{n^2}{1-\frac{1}{(n+1)^2}}=\frac{n^2(n+1)^2}{n(n+2)}$

$=\frac{n[n(n+2)+1]}{n+2}=n^2+\frac{n}{n+2}$

$=n^2+\frac{n+2-2}{n+2}=n^2+1-\frac{2}{n+2} \quad \ldots(\mathrm{i})$

Now, $\sum_{n=0}^{50}\left(S_n+\frac{2}{n+1}-n-1\right)$

From equation (i),

$=\sum_{n=0}^{50}\left(n^2+1-\frac{2}{n+2}+\frac{2}{n+1}-n-1\right)$

$=\sum_{n=0}^{50}\left(n^2-n\right)+2\sum_{n=0}^{50}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

$=\sum_{n=0}^{50}n^2-\sum_{n=0}^{50}n+2\left[\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{51}-\frac{1}{52}\right)\right]$

$=\frac{50(51)(101)}{6}-\frac{50(51)}{2}+2\left[1-\frac{1}{52}\right]$

$=(25)(17)(101)-25(51)+2\left(\frac{51}{52}\right)$

$=25(1717-51)+\frac{51}{26}=25(1666)+\frac{51}{26}$

$=41650+\frac{51}{26}$

$\therefore \frac{1}{26}+\sum_{n=0}^{50}\left(S_n+\frac{2}{n+1}-n-1\right)=41650+2=41652$