UNITS AND MEASUREMENTS

- The **SI system :** It is the international system of units. At present internationally accepted for measurement. In this system there are seven fundamental and two supplementary quantities and their corresponding units are:

Quantity	Unit	Symbol
1. Length	metre	m
2. Mass	kilogram	kg
3. Time	second	s
4. Electric current	ampere	A
5. Temperature	kelvin	K
6. Luminous intensity	candela	cd
7. Amount of substance	mole	mol
Supplementary		
1. Plane angle	radian	rad
2. Solid angle	steradian	sr

- Dimensions : These are the powers to which the fundamental units are raised to get the unit of a physical quantity.
- Uses of dimensions
  (i) To check the correctness of a physical relation.
  (ii) To derive relationship between different physical quantities.
  (iii) To convert one system of unit into another.

$$n_1u_1 = n_2u_2$$

$$n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]$$

- Significant figures : In any measurement, the reliable digits plus the first uncertain digit are known as significant figures.
- Error : It is the difference between the measured value and true value of a physical quantity or the uncertainty in the measurements.
- Absolute error : The magnitude of the difference between the true value and the measured value is called absolute error.

$$\Delta a_1 = \bar{a} - a_1,\ \Delta a_2 = \bar{a} - a_2,\ \Delta a_n = \bar{a} - a_n$$

Mean absolute error

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n}|\Delta a_i|$$

- Relative error : It is the ratio of the mean absolute error to its true value

or relative error $= \dfrac{\overline{\Delta a}}{\bar{a}}$

- Percentage error : It is the relative error in per cent.

$$\text{Percentage error} = \left(\frac{\Delta \bar{a}}{a_{mean}}\right) \times 100\%$$

MOTION IN A STRAIGHT LINE

- Average speed, $V_{av} = \dfrac{s_1 + s_2 + s_3}{t_1 + t_2 + t_3}$
- Average acceleration, $a_{av} = \dfrac{a_1t_1 + a_2t_2}{t_1 + t_2}$
- The area under the velocity-time curve is equal to the displacement and slope gives acceleration.
- If a body falls freely, the distance covered by it in each subsequent second starting from first second will be in the ratio 1 : 3 : 5 : 7 etc.
- If a body is thrown vertically up with an initial velocity u, it takes u/g second to reach maximum height and u/g second to return, if air resistance is negligible.
- If air resistance acting on a body is considered, the time taken by the body to reach maximum height is less than the time to fall back the same height.
- For a particle having zero initial velocity if $s \propto t^{\alpha}$, where $\alpha > 2$, then particle's acceleration increases with time.
- For a particle having zero initial velocity if $s \propto t^{\alpha}$, where $\alpha < 0$, then particle's acceleration decreases with time.
- Kinematic equations :
  $v = u + a_t(t)$; $v^2 = u^2 + 2a_t(s)$

  $S = ut + \frac{1}{2}a_t(t)^2$; $S_n = u + \frac{a}{2}(2n-1)$

  applicable only when $|\vec{a}_t| = a_t$ is constant.
  a_t = magnitude of tangential acceleration, S = distance
- If acceleration is variable use calculus approach.
- Relative velocity : $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

MOTION IN A PLANE

- If T is the time of flight, h maximum height, R horizontal range of a projectile, α its angle of projection, then the relations among these quantities.

$h = \dfrac{gT^2}{8}$ ...... (1);

$gT^2 = 2R\tan\alpha$ .... (2);

$R\tan\alpha = 4h$ .... (3)

- $T = \frac{2u\sin\theta}{g}; h = \frac{u^2\sin^2\theta}{2g}$

  $R = \frac{u^2 \sin 2\theta}{g}$; $R_{max} = \frac{u^2}{g}$ when $\theta = 45°$
- For a given initial velocity, to get the same horizontal range, there are two angles of projection α and $90° - \alpha$.
- The equation to the parabola traced by a body projected horizontally from the top of a tower of height y, with a velocity u is $y = gx^2/2u^2$, where x is the horizontal distance covered by it from the foot of the tower.
- Equation of trajectory is $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$, which is parabola.
- Equation of trajectory of an oblique projectile in terms of range (R) is $y = x\tan\theta\left(1 - \frac{x}{R}\right)$
- Maximum height is equal to n times the range when the projectile is launched at an angle $\theta = \tan^{-1}(4n)$.
- In a uniform circular motion, velocity and acceleration are constants only in magnitude. Their directions change.
- In a uniform circular motion, the kinetic energy of the body is a constant. $W = 0$, $\vec{a} \neq 0$, $\vec{P} \neq$ constant, $\vec{L}$=constant
- Centripetal acceleration, $a_r = \omega^2 r = \frac{v^2}{r} = \omega v$ (always applicable)

  $\vec{a}_r = \vec{\omega} \times \vec{v}$

LAWS OF MOTION

- Newton's first law of motion or law of inertia : It is resistance to change.
- Newton's second law : $\vec{F} = m\vec{a}$, $\vec{F} = d\vec{p}/dt$
- Impulse: $\Delta\vec{p} = \vec{F}\Delta t$, $p_2 - p_1 = \int_1^2 F\,dt$
- Newton's third law: $\vec{F}_{12} = -\vec{F}_{21}$
- Frictional force $f_s \le (f_s)_{max} = \mu_s R$; $f_k = \mu_k R$
- Circular motion with variable speed. For complete circles, the string must be taut in the highest position, $u^2 \ge 5ga$. Circular motion ceases at the instant when the string becomes slack, i.e., when T = 0, range of values of u for which the string does go slack is $\sqrt{2ga} < u < \sqrt{5ga}$.
- Conical pendulum : $\omega = \sqrt{g/h}$ where h is height of a point of suspension from the centre of circular motion.
- The acceleration of a lift

  $a = \frac{\text{actual weight} - \text{apparent weight}}{\text{mass}}$

  If 'a' is positive lift is moving down, and if it is negative the lift is moving up.
- On a banked road, the maximum permissible speed V_{max}

  $= \left(R_g \frac{u_s + \tan\theta}{1 - u_s \tan\theta}\right)^{1/2}$

WORK, ENERGY AND POWER

- Work done $W = FS\cos\theta$
- Relation between kinetic energy E and momentum, $P = \sqrt{2mE}$

  K.E. $= \frac{1}{2}mV^2$; P.E. = mgh
- If a body moves with constant power, its velocity (v) is related to distance travelled (x) by the formula $v \propto x^{3/2}$.
- Power $P = \frac{W}{t} = F.V$
- Work due to kinetic force of friction between two contact surfaces is always negative. It depends on relative displacement between contact surfaces. $W_{FK} = -F_K(S_{rel})$.
- $\Sigma W = \Sigma \Delta K$, $\Sigma W \Rightarrow$ total work due to all kinds of forces, $\Sigma \Delta K \Rightarrow$ total change in kinetic energy.
- $\Sigma W_{conservative} = -\Sigma \Delta U$; $\Sigma W_{conservative} \Rightarrow$ Total work due to all kinds of conservative forces.

  $\Sigma \Delta u \Rightarrow$ Total change in all kinds of potential energy.
- Coefficient of restitution $e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$
- The total momentum of a system of particles is a constant in the absence of external forces.

SYSTEMS OF PARTICLES AND ROTATIONAL MOTION

- The centre of mass of a system of particles is defined as the point whose position vector is $R = \frac{\sum m_i r_i}{M}$
- The angular momentum of a system of n particles about the origin is $L = \sum_{i=1}^{n} r_i \times p_i$;

  $L = mvr = I\omega$
- The torque or moment of force on a system of n particles about the origin is $\tau = \sum_i r_i \times F_i$
- The moment of inertia of a rigid body about an axis is defined by the formula $I = \sum m_i r_i^2$
- The kinetic energy of rotation is $K = \frac{1}{2}I\omega^2$
- The theorem of parallel axes : $I_z' = I_z + Ma^2$

  Theorem of perpendicular axes : $I_z = I_x + I_y$

- For rolling motion without slipping $v_{cm} = R\omega$. The kinetic energy of such a rolling body is the sum of kinetic energies of translation and rotation : $K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$
- A rigid body is in mechanical equilibrium if
  (a) It is translational equilibrium i.e., the total external force on it is zero : $\Sigma F_i = 0$.
  (b) It is rotational equilibrium i.e., the total external torque on it is zero : $\Sigma \tau_i = \Sigma r_i \times F_i = 0$.
- If a body is released from rest on rough inclined plane, then for pure rolling $\mu_r \geq \frac{n}{n+1}\tan\theta \ (I_c = nmr^2)$

  Rolling with sliding $0 < \mu_s < \left(\frac{n}{n+1}\right)\tan\theta$;

  $\frac{g\sin\theta}{n+1} < a < g\sin\theta$

GRAVITATION

- Newton's universal law of gravitation

  Gravitational force $F = \frac{Gm_1m_2}{r^2}$

  $G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$
- The acceleration due to gravity.
  (a) at a height h above the Earth's surface

  $g(h) = \frac{GM_E}{(R_E+h)^2} = g\left(1 - \frac{2h}{R_E}\right)$ for $h << R_E$

  $g(h) = g(0)\left(1 - \frac{2h}{R_E}\right)$ where $g(0) = \frac{GM_E}{R_E^2}$

  (b) at depth d below the Earth's surface is

  $g(d) = \frac{GM_E}{R_E^2}\left(1 - \frac{d}{R_E}\right) = g(0)\left(1 - \frac{d}{R_E}\right)$

  (c) with latitude λ $g^1 = g - R\omega^2\cos^2\lambda$
- Gravitational potential $V_g = -\frac{GM}{r}$
- Intensity of gravitational field $I = \frac{GM}{r^2}$
- The gravitational potential energy

  $V = -\frac{Gm_1m_2}{r} + \text{constant}$
- The escape speed from the surface of the Earth is $v_e = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E}$ and has a value of $11.2\ km\ s^{-1}$.
- Orbital velocity, $v_{orbi} = \sqrt{\frac{GM_E}{R_E}} = \sqrt{gR_E}$
- A geostationary (geosynchronous communication) satellite moves in a circular orbit in the equatorial plane at a approximate distance of 4.22×10^4 km from the Earth's centre.
- Kepler's 3rd law of planetary motion.

  $T^2 \propto a^3$; $\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$

MECHANICAL PROPERTIES OF SOLIDS

- Hooke's law : stress $\propto$ strain
- Young's modulus of elasticity $Y = \frac{F\Delta\ell}{A\ell}$
- Compressibility $= \frac{1}{\text{Bulk modulus}}$
- $Y = 3k(1 - 2\sigma)$
- $Y = 2n(1 + \sigma)$
- If S is the stress and Y is Young's modulus, the energy density of the wire E is equal to $S^2/2Y$.
- If α is the longitudinal strain and E is the energy density of a stretched wire, Y Young's modulus of wire, then E is equal to $\frac{1}{2}Y\alpha^2$
- Thermal stress $= \frac{F}{A} = Y\alpha\Delta\theta$

MECHANICAL PROPERTIES OF FLUIDS

- Pascal's law : A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.
  Pressure exerted by a liquid column $P = h\rho g$
- Bernoulli's principle
  $P + \rho v^2/2 + \rho gh = \text{constant}$
- Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of interface.
- Stokes' law states that the viscous drag force F on a sphere of radius a moving with velocity v through a fluid of viscosity η $F = -6\pi\eta av$.
- Terminal velocity $V_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$
- The surface tension of a liquid is zero at boiling point. The surface tension is zero at critical temperature.
- If a drop of water of radius R is broken into n identical drops, the work done in the process is $4\pi R^2 S(n^{1/3} - 1)$ and fall in temperature $\Delta q = \frac{3T}{J}\sqrt{\frac{1}{r} - \frac{1}{R}}$
- Two capillary tubes each of radius r are joined in parallel. The rate of flow is Q. If they are replaced by single capillary tube of radius R for the same rate of flow, then $R = 2^{1/4}r$.
- Ascent of a liquid column in a capillary tube $h = \frac{2s\cos\phi}{r\rho g}$
- Coefficient of viscosity, $n = -\frac{F}{A\left(\frac{dv}{dx}\right)}$
- Velocity of efflux $V = \sqrt{2gh}$

THERMAL PROPERTIES OF MATTER

- Relation between different temperature scales :

$$\frac{C}{100}=\frac{F-32}{100}=\frac{K-273}{100}$$

- The coefficient of linear expansion (α_ℓ), superficial (β) and volume expansion (α_v) are defined by the relations :

$$\frac{\Delta\ell}{\ell}=\alpha_\ell\Delta T\ ;\ \frac{\Delta A}{A}=\beta\Delta T\ ;\ \frac{\Delta V}{V}=\alpha_V\Delta T$$

$\alpha_v=3\alpha_\ell$; $\beta=2\alpha_\ell$

- In conduction, heat is transferred between neighbouring parts of a body through molecular collisions, without any flow of matter. The rate of flow of heat $H=KA\frac{T_C-T_D}{L}$, where K is the thermal conductivity of the material of the bar.
- Convection involves flow of matter within a fluid due to unequal temperatures of its parts.
- Radiation is the transmission of heat as electromagnetic waves.
- Stefan's law of radiation : $E=\sigma T^4$, where the constant σ is known as Stefan's constant $=5.67\times10^{-8}\ wm^{-2}k^{-4}$.
- Wein's displacement law : $\lambda_m T$ = constant, where constant is known as Wein's constant $=2.898\times10^{-3}$ mk.
- Newton's law of cooling: $\frac{dQ}{dt}=-k\,(T_2-T_1)$; where T_1 is the temperature of the surrounding medium and T_2 is the temperature of the body.
- Heat required to change the temperature of the substance, $Q=mc\Delta\theta$

  c = specific heat of the substance
- Heat absorbed or released during state change $Q=mL$

  L = latent heat of the substance
- Mayer's formula $c_p-c_v=R$

THERMODYNAMICS

- First law of thermodynamics: $\Delta Q=\Delta U+\Delta W$, where ΔQ is the heat supplied to the system, ΔW is the work done by the system and ΔU is the change in internal energy of the system.
- In an isothermal expansion of an ideal gas from volume V_1 to V_2 at temperature T the heat absorbed (Q) equals the work done (W) by the gas, each given by

$$Q=W=nRT\ln\left(\frac{V_2}{V_1}\right)$$

- In an adiabatic process of an ideal gas $PV^\gamma=TV^{\gamma-1}=\frac{T^\gamma}{P^{\gamma-1}}$ = constant, where $\gamma=\frac{C_p}{C_v}$.
- Work done by an ideal gas in an adiabatic change of state from (P_1,V_1,T_1) to (P_2,V_2,T_2) is $W=\frac{nR\,(T_1-T_2)}{\gamma-1}$
- The efficiency of a Carnot engine is given by

$$\eta=1-\frac{T_2}{T_1}$$

- **Second law of thermodynamics:** No engine operating between two temperatures can have efficiency greater than that of the Carnot engine.
- Entropy or disorder $S=\frac{\delta Q}{T}$

KINETIC THEORY

- Ideal gas equation $PV=nRT$
- Kinetic theory of an ideal gas gives the relation $P=\frac{1}{3}nm\overline{v}^2$, Combined with the ideal gas equation it yields a kinetic interpretation of temperature.

$$\frac{1}{2}nm\overline{v}^2=\frac{3}{2}k_BT\ ,\ v_{rms}=(\overline{v}^2)^{1/2}=\sqrt{\frac{3k_BT}{m}}$$

- The law of equipartition of energy is stated thus: the energy for each degree of freedom in thermal equilibrium is $1/2\,(k_BT)$
- The translational kinetic energy $E=\frac{3}{2}k_BNT$. This leads to a relation $PV=\frac{2}{3}E$.
- Degree of freedom : Number of directions in which it can move freely.
- Root mean square (rms) velocity of the gas

$$C=\sqrt{\frac{3RT}{M}}=\sqrt{\frac{3P}{\rho}}$$

- Most probable speed $V_{mp}=\sqrt{\frac{2RT}{M}}=\sqrt{\frac{2KT}{m}}$
- Mean free path $\lambda=\frac{KT}{\sqrt{2}\pi d^2P}$

OSCILLATIONS

- Displacement in SHM : $Y=a\sin\omega t$ or, $y=a\cos\omega t$
- The particle velocity and acceleration during SHM as functions of time are given by,

  $v(t)=-\omega A\sin(\omega+\phi)$ (velocity),

  $a(t)=-\omega^2A\cos(\omega t+\phi)=-\omega^2x(t)$ (acceleration)

Velocity amplitude $v_m=\omega A$ and acceleration amplitude $a_m=\omega^2A$.

- A particle of mass m oscillating under the influence of a Hooke's law restoring force given by $F=-k\,x$ exhibits simple harmonic motion with $\omega=\sqrt{\frac{k}{m}}$ (angular frequency),

$$T=2\pi\sqrt{\frac{m}{k}}\ \text{(period)}$$

Such a system is also called a linear oscillator.

- Time period for conical pendulum $T=2\pi\sqrt{\left(\frac{\ell\cos\theta}{g}\right)}$ where θ angle between string & vertical.
- Energy of the particle $E=k+u=\frac{1}{2}m\omega^2A^2$

WAVES

- The displacement in a sinusoidal wave y (x, t) = a sin (kx – ωt + ϕ) where ϕ is the phase constant or phase angle.
- Equation of plane progressive wave :
$$= a \sin 2\pi\left(\frac{t}{T}-\frac{x}{V}\right)$$
- Equation of stationary wave :
$$Y = 2a \sin \frac{2\pi t}{T} \cos \frac{2\pi x}{\lambda}$$
- The speed of a transverse wave on a stretched string $v = \sqrt{T/\mu}$.
- Sound waves are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed v of sound wave in a fluid having bulk modulus B and density μ is $v = \sqrt{B/\rho}$.
- The speed of longitudinal waves in a metallic bar is $v = \sqrt{Y/\rho}$

  For gases, since B = γ P, the speed of sound is $v = \sqrt{\gamma P/\rho}$
- The interference of two identical waves moving in opposite directions produces standing waves. For a string with fixed ends, standing wave y (x, t) = [2a sin kx] cos ωt
- The separation between two consecutive nodes or antinodes is λ/2.
- A stretched string of length L fixed at both the ends vibrates with frequencies $f = \frac{1}{2}\frac{v}{2L}$.

  The oscillation mode with lowest frequency is called the fundamental mode or the first harmonic.
- A pipe of length L with one end closed and other end open (such as air columns) vibrates with frequencies given by
$$f = \left(n+\frac{1}{2}\right)\frac{v}{2L}, n = 0, 1, 2, 3, ...$$
  The lowest frequency given by v/4L is the fundamental mode or the first harmonic.

  Open organ pipe $n_1 : n_2 : n_3$ .... 1, 2, 3......., $n = \frac{V}{2l}$
- Beats arise when two waves having slightly different frequencies, f_1 and f_2 and comparable amplitudes, are superposed. The beat frequency $f_{beat} = f_1 - f_2$
- The Doppler effect is a change in the observed frequency of a wave when the source S and the observer O moves relative to the medium. $f = f_0\left(\frac{v \pm v_0}{v \pm v_s}\right)$

ELECTRO-STATICS

- Coulomb's Law : $\vec{F}_{21}$ = force on q_2 due to $q_1 = \frac{k\,(q_1 q_2)}{r_{21}^2}\hat{r}_{21}$ where $k = \frac{1}{4\pi\varepsilon_0}$ $= 9 \times 10^9\ Nm^2\,C^{-2}$
- Electric field due to a point charge q has a magnitude $|q|/4\pi\varepsilon_0 r^2$
- Field of an electric dipole in its equatorial plane
$$E = \frac{-\vec{p}}{4\pi\varepsilon_0}\frac{1}{(a^2+r^2)^{3/2}} \cong \frac{-\vec{p}}{4\pi\varepsilon_0 r^3}, \quad \text{for } r >> a$$
  Dipole electric field on the axis at a distance r from the centre:
$$\vec{E} = \frac{2\vec{p}r}{4\pi\varepsilon_0(r^2-a^2)^2} \cong \frac{2\vec{p}}{4\pi\varepsilon_0 r^3} \text{ for } r >> a$$
  Dipole moment $\vec{p} = q2a$
- In a uniform electric field $\vec{E}$, a dipole experiences a torque $\vec{\tau}$ given by $\vec{\tau} = \vec{p} \times \vec{E}$ but experiences no net force.

  The flux Δϕ of electric field $\vec{E}$ through a small area element $\Delta\vec{S}$ is given by $\Delta\phi = \vec{E}.\Delta\vec{S}$
- Gauss's law: The flux of electric field through any closed surface S is $1/\varepsilon_0$ times the total charge enclosed i.e., Q
- Thin infinitely long straight wire of uniform linear charge density λ : $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{n}$
- Infinite thin plane sheet of uniform surface charge density σ
$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$
- Thin spherical shell of uniform surface charge density σ :
$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0 r^2}\hat{r} \quad (r \geq R)\ ;\ \vec{E} = 0 \quad (r < R)$$
- Electric Potential : $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$.
- An equipotential surface is a surface over which potential has a constant value.
- Potential energy of two charges q_1, q_2 at $\vec{r}_1, \vec{r}_2$ is given by $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$, where r_{12} is distance between q_1 and q_2.
- Capacitance C = Q/V, where Q = charge and V = potential difference
- For a parallel plate capacitor (with vacuum between the plates), $C = \varepsilon_0\frac{A}{d}$.
- The energy U stored in a capacitor of capacitance C, with charge Q and voltage V is
$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$
- For capacitors in the series combination,
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$
  In the parallel combination, $C_{eq} = C_1 + C_2 + C_3 + ...$ where C_1, C_2, C_3... are individual capacitances.

CURRENT ELECTRICITY

- Electric current, $I = \frac{q}{t}$
- Current density j gives the amount of charge flowing per second per unit area normal to the flow, $\vec{J} = nqv_d$
- Mobility, $\mu = \frac{V_d}{E}$ and $V_d = \frac{I}{Ane}$
- Resistance $R = \rho\frac{\ell}{A}$, ρ = resistivity of the material
- Equation $\vec{E} = \rho\vec{J}$ another statement of Ohm's law, ρ = resistivity of the material.
- Ohm's law $I \propto V$ or $V = RI$
- (a) Total resistance R of n resistors connected in series $R = R_1 + R_2 + ... + R_n$
  (b) Total resistance R of n resistors connected in parallel $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$.
- Kirchhoff's Rules – (a) Junction rule: At any junction of circuit elements, the sum of currents entering the junction must equal the sum of currents leaving it.
  (b) Loop rule: The algebraic sum of changes in potential around any closed loop must be zero.
- The Wheatstone bridge is an arrangement of four resistances R_1, R_2, R_3, R_4. The null-point condition is given by $\frac{R_1}{R_2} = \frac{R_3}{R_4}$
- The potentiometer is a device to compare potential differences. The device can be used to measure potential difference; internal resistance of a cell and compare emf's of two sources. Internal resistance $r = R\left(\frac{\ell_1}{\ell_2} - 1\right)$
- RC circuit : During charging : $q = CE(1 - e^{-t/RC})$
  During discharging : $q = q_0 e^{-t/RC}$
- According 'to Joule's Heating law, $H = I^2Rt$

MAGNETISM

- The total force on a charge q moving with velocity v i.e., Lorentz force. $\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$.
- A straight conductor of length ℓ and carrying a steady current I experiences a force $\vec{F}$ in a uniform external magnetic field $\vec{B}$, $\vec{F} = I\vec{\ell} \times \vec{B}$, the direction of ℓ is given by the direction of the current.
- Biot-Savart law $d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \vec{r}}{r^3}$.
- The magnitude of the magnetic field due to a circular coil of radius R carrying a current I at an axial distance x from the centre is $B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$.
- The magnitude of the field B inside a long solenoid carrying a current I is : $B = \mu_0 nI$. For a toroid one obtains, $B = \frac{\mu_0 NI}{2\pi r}$.
- Ampere's Circuital Law : $\oint_C \vec{B}.d\vec{\ell} = \mu_0 I$, where I refers to the current passing through S.
- Force between two long parallel wires $F = \frac{\mu_0 I_1 I_2}{2\pi a} Nm^{-1}$.
  The force is attractive if currents are in the same direction and repulsive currents are in the opposite direction.
- For current carrying coil $\vec{M} = NI\vec{A}$; torque = $\vec{\tau} = \vec{M} \times \vec{B}$
- Conversion of (i) galvanometer into ammeter, $S = \left(\frac{I_g}{I - I_g}\right)G$
  (ii) galvanometer into voltmeter, $S = \frac{V}{I_g} - G$
- The magnetic intensity, $\vec{H} = \frac{\vec{B}_0}{\mu_0}$.
- The magnetisation $\vec{M}$ of the material is its dipole moment per unit volume. The magnetic field B in the material is, $\vec{B} = \mu_0(\vec{H} + \vec{M})$
- For a linear material $\vec{M} = \chi\vec{H}$. So that $\vec{B} = \mu\vec{H}$ and χ is called the magnetic susceptibility of the material.
  $\mu = \mu_0\mu_r$; $\mu_r = 1 + \chi$.

ELECTRO-MAGNETIC INDUCTION

- The magnetic flux $\phi_B = \vec{B}.\vec{A} = BA\cos\theta$, where θ is the angle between $\vec{B}$ & $\vec{A}$.
- Faraday's laws of induction : $\varepsilon = -N\frac{d\phi_B}{dt}$
- Lenz's law states that the polarity of the induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produces it.
- The induced emf (motional emf) across ends of a rod $\varepsilon = B\ell v$
- The self-induced emf is given by, $\varepsilon = -L\frac{dI}{dt}$
  L is the self-inductance of the coil.
  $L = \frac{\mu_0 N^2 A}{\ell}$
- A changing current in a coil (coil 2) can induce an emf in a nearby coil (coil 1).
  $\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$, M_{12} = mutual inductance of coil 1 w.r.t coil 2.
  $M = \frac{\mu_0 N_1 N_2 A}{\ell}$
- Growth of current in an inductor, $i = i_0[1 - e^{-Rt/L}]$
  For decay of current, $i = i_0 e^{-Rt/L}$

ALTERNATING CURRENT

- For an alternating current $i = i_m \sin \omega t$ passing through a resistor R, the average power loss P (averaged over a cycle) due to joule heating is $(1/2)i^2_m R$.

  E.m.f, $E = E_0 \sin \omega t$
- Root mean square (rms) current $I = \frac{i_m}{\sqrt{2}} = 0.707\, i_m$. $E_{rms} = \frac{E_0}{\sqrt{2}}$
- The average power loss over a complete cycle $P = VI\cos\phi$. The term $\cos\phi$ is called the power factor.
- An ac voltage $v = v_m \sin\omega t$ applied to a pure inductor L, drives a current in the inductor $i = i_m \sin(\omega t - \pi/2)$, where $i_m = v_m/X_L$. $X_L = \omega L$ is called inductive reactance.
- An ac voltage $v = v_m \sin\omega t$ applied to a capacitor drives a current in the capacitor: $i = i_m \sin(\omega t + \pi/2)$. Here,

  $i_m = \frac{v_m}{X_C}$, $X_C = \frac{1}{\omega C}$ is called capacitive reactance.
- An interesting characteristic of a series RLC circuit is the phenomenon of resonance. The circuit exhibits resonance, i.e., the amplitude of the current is maximum at the resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}} (X_L = X_C)$.
- Impedance $z = \sqrt{R^2 + (x_L - x_C)^2}$
- Transformation ratio, $K = \frac{N_S}{N_P} = \frac{E_S}{E_P} = \frac{I_P}{I_S}$
- Step up transformer : $N_S > N_P$; $E_S > E_P$; $I_P > I_S$
- Step down transformer $N_P > N_S$; $E_P > E_S$ and $I_P < I_S$
- The quality factor Q defined by $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$ is an indicator of the sharpness of the resonance, the higher value of Q indicating sharper peak in the current.

RAY OPTICS

- Reflection is governed by the equation $\angle i = \angle r'$ and refraction by the Snell's law, $\sin i/\sin r = n$, where the incident ray, reflected ray, refracted ray and normal lie in the same plane.
- Mirror equation: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

  Magnification $M = \frac{V}{u} = \frac{I}{O}$
- Prism Formula $n_{21} = \frac{n_2}{n_1} = \frac{\sin[(A + D_m)/2)]}{\sin(A/2)}$, where D_m is the angle of minimum deviation.
- Dispersion is the splitting of light into its constituent colours. The deviation is maximum for violet and minimum for red.
- Scattering $\propto \frac{1}{\lambda^4}$
- Dispersive power $\omega = \frac{\delta_v - \delta_r}{\delta}$, where δ_v, δ_r are deviation of violet and red and δ the deviation of mean ray (usually yellow).
- For refraction through a spherical interface (from medium 1 to 2 of refractive index n_1 and n_2, respectively)

  $\frac{n_2}{v} - \frac{n_1}{v} = \frac{n_2 - n_1}{R}$
- Refractive index of a medium $\mu = \frac{C}{V}$ $(C = 3 \times 10^8$ m/s)

  $r = \frac{1}{\sin C}$ (C = Critical angle)
- Condition for TIR : 1. Ray of light must travel from denser to rarer medium 2. Angle of incidence in denser medium > critical angle.
- Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$
- Lens maker's formula : $\frac{1}{f} = \frac{(n_2 - n_1)}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
- The power of a lens $P = 1/f$. The SI unit for power of a lens is dioptre (D): $1\,D = 1\,m^{-1}$.
- If several thin lenses of focal length f_1, f_2, f_3,.. are in contact, the effective focal $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...$
- The total power of a combination of several lenses $P = P_1 + P_2 + P_3 + ...$
- Chromatic aberration if satisfying the equation

  $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ or in terms of powers $\omega_1 P_1 + \omega_2 P_2 = 0$.
- For compound microscope $M = \frac{V_0}{u_0}\left(1 + \frac{D}{f_e}\right)$ when final image at D

  $M = \frac{V_0}{u_0} \cdot \frac{D}{f_e}$ when final image at infinity.

WAVE OPTICS

- Wavefront : It is the locus of all the particles vibrating in the same phase.
- The resultant intensity of two waves of intensity $I_0/4$ of phase difference ϕ at any points $I = I_0 \cos^2\left[\frac{\phi}{2}\right]$, where I_0 is the maximum density.
- Intensity $I \propto (\text{amplitude})^2$
- Condition for dark band : $\delta = (2n - 1)\frac{\lambda}{2}$, for bright band : $\delta = m\lambda$

- Fringe width $\beta = \frac{D\lambda}{d}$
- A thin film of thickness t and refractive index μ appears dark by reflection when viewed at an angle of refraction r if
  $2\mu t \cos r = n\lambda$ (n = 1, 2, 3, etc.)
- A single slit of width a gives a diffraction pattern with a central maximum. The intensity falls to zero at angles of $\pm\frac{\lambda}{a}, \pm\frac{2\lambda}{a}$, etc.
- Amplitude of resultant wave $R = \sqrt{a^2 + b^2 + 2ab\cos\phi}$
- Intensity of wave $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$
- Brewster law : $\mu = \tan i_p$

MODERN PHYSICS

- Energy of a photon $E = h\nu = \frac{hc}{\lambda}$
- Momentum of a photon $P = \frac{h}{\lambda}$
- Einstein's photoelectric equation

$$\frac{1}{2}mv_{max}^2 = V_0 e = h\nu - \phi_0 = h(\nu - \nu_0)$$

- Mass defect,
  $\Delta M = (Z m_p + (A - Z)m_n) - M; \quad \Delta E_b = \Delta M c^2$.
  1 amu = 931 MeV
- $E_n = -\frac{Z^2}{n^2} \times 13.6 eV$ (For hydrogen like atom)
- According to Bohr's atomic model, angular momentum for the electron revolving in stationary orbit, $mvr = nh/2\pi$
- Radius of the orbit of electron $r = \frac{n^2h^2}{4\pi^2 mkze^2}$
- Bragg's law : $2d \sin\theta = n\lambda$.
- Radius of the nucleus $R = R_o A^{1/3}$
- Law of radioactive decay : $N = N_0 e^{-\lambda t}$.
  Activity $= \frac{dN}{N} = -\lambda N$ (unit is Becquerel)
- Half life period, $T_{1/2} = \frac{0.693}{\lambda}$
- Characteristics X-rays : $\lambda_{K_\alpha} < \lambda_{L_\alpha}$
  Moseley law : $\nu = a(Z - b)^2$
- Pure semiconductors are called 'intrinsic semiconductors'. The presence of charge carriers (electrons and holes) number of electrons (n_e) is equal to the number of holes (n_h).
- The number of charge carriers can be changed by 'doping' of a suitable impurity in pure semiconductors known as extriasic semiconductors (n-type and p-type).
- In n-type semiconductors, $n_e >> n_h$ while in p-type semiconductors $n_h >> n_e$.
- n-type semiconducting Si or Ge is obtained by doping with pentavalent atoms (donors) like As, Sb, P, etc., while p-type Si or Ge can be obtained by doping with trivalent atom (acceptors) like B, Al, In etc.
- In forward bias (n-side is connected to negative terminal of the battery and p-side is connected to the positive), the barrier is decreased while the barrier increases in reverse bias.
- Diodes can be used for rectifying an ac voltage (restricting the ac voltage to one direction).
- Zener diode is one such special purpose diode. In reverse bias, after a certain voltage, the current suddenly increases (breakdown voltage) in a Zener diode. This property has been used to obtain voltage regulation.
- The important transistor parameters for CE-configuration are:

Input resistance $r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$

Output resistance $r_0 = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B}$

Current amplification factor, $\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}}$

The voltage gain of a transistor amplifier in common emitter configuration is:

$A_v = \left(\frac{v_0}{v_i}\right) = \beta\frac{R_C}{R_B}$, where R_C and R_B are respectively the resistances in collector and base sides of the circuit.

- The important digital circuits performing special logic operations are called logic gates. These are: OR, AND, NOT, NAND, and NOR gates. NAND gate is the combination of NOT and AND gate. NOR gate is the combination of NOT and OR gate.

COMMUNICATION SYSTEMS

- Transmitter, transmission channel and receiver are three basic units of a communication system.
- Two important forms of communication system are: Analog and Digital. The information to be transmitted is generally in continuous waveform for the former while for the latter it has only discrete or quantised levels.
- Low frequencies cannot be transmitted to long distances. Therefore, they are superimposed on a high frequency carrier signal by a process known as modulation.
  In the process of modulation, new frequencies called sidebands are generated on either side.
- If an antenna radiates electromagnetic waves from a height h_T, then the range $d_T \sqrt{2Rh_T}$ R = radius of earth.
- Effective range, $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$
  h_T = height of transmitting antenna; h_R = height of receiving antenna
- Critical frequency $V_c = 9(N_{max})^{1/2}$
  where N_{max} = no. density of electrons/m^3
- Skip distance, $D_{skip} = 2h\left(\frac{V_{max}}{V_c}\right)^2 - 1$
  h = height of reflecting layer of atmosphere.
- Power radiated by an antenna $\propto \frac{1}{\lambda^2}$