JEE Main Exam 2022 - Session 1

29 June 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. At a certain place, the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.50 oersted. The earth's total magnetic field is,

A) $\sqrt{3}$ oersted

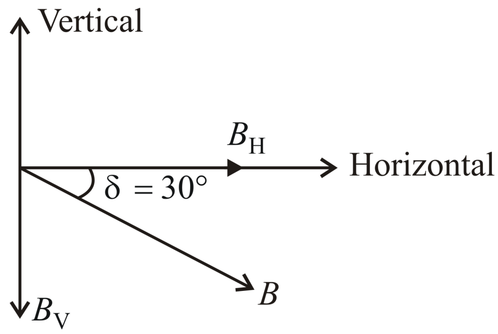
B) 1 oersted

C) $\frac{1}{\sqrt{3}}$ oersted

D) $\frac{1}{2}$ oersted

Answer: $\frac{1}{\sqrt{3}}$ oersted

Solution:



Angle of dip, $\delta = 30^{\circ}$

Angle of dip or magnetic dip is the angle the net earth's magnetic field and it's horizontal component.

$B_H = B\cos\delta$

$B = \frac{B_H}{\cos\delta}$

Magnetic field, $B = \frac{0.5}{\sqrt{3}/2} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$ oersted.

Q.2. Fringe width in $YDSE$ for wavelength $\lambda = 5000$ Å is 1.2 mm. Then the fringe width for a light wave when the distance between the slits becomes twice and the wavelength changes to 6000 Å, is

A) 0.52 mm

B) 0.72 mm

C) 8.5 mm

D) 1.32 mm

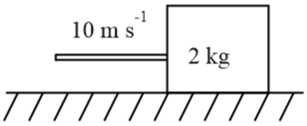
Answer: $0.72\ \mathrm{mm}$

Solution: We know that fringe width is given by, $\beta = \frac{\lambda D}{d}$

$\Rightarrow \frac{\beta'}{\beta} = \frac{\lambda'}{\lambda} \times \frac{d}{d'} = \frac{6000}{5000} \times \frac{d}{2d}$

$\Rightarrow \beta' = \frac{6}{5\times 2} \times 1.2 = 0.6 \times 1.2 = 0.72\ \mathrm{mm}$

Q.3. Water flowing at velocity $10\ \mathrm{m\ s^{-1}}$ through a nozzle of cross-sectional area $1\ \mathrm{cm}^2$, strikes a $2\ \mathrm{kg}$ block at rest as shown. If water flow comes to rest immediately after striking the block, then initial acceleration of the block is:



A) $3\ \mathrm{m\ s^{-2}}$

B) $5\ \mathrm{m\ s^{-2}}$

C) $4\ \mathrm{m\ s^{-2}}$

D) $2\ \mathrm{m\ s^{-2}}$

Answer: $5\ \mathrm{m\ s^{-2}}$

Solution: Let velocity of water jet be v.

Mass of water jet striking the block per second, $m = \rho A v$, where, ρ is density and A is cross-sectional area.

Initial momentum, $p_i = (\rho A v)v$ and final momentum $p_f = 0$

So, force applied is $F = \frac{\Delta p}{\Delta t} = \frac{(\rho A v)\times v}{1}$

$\Rightarrow ma = \rho A v^2$

Acceleration of block, $a = \frac{10^3 \times 10^{-4} \times (100)}{2}$

$a = 5\ \mathrm{m\ s^{-2}}$

Q.4. **Assertion** : If energy of incident photon is less than work function of a metal, electrons are not ejected from it.

Reason : If maximum kinetic energy of ejected electrons is zero, then work function of the metal is equal to energy of incident photon.

A) Both assertion and reason are true and reason is correct explanation of assertion.

B) Both assertion and reason are true but reason is not correct explanation.

C) Assertion is true and reason is false

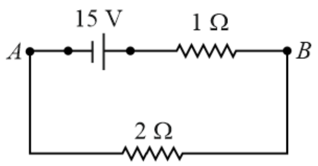
D) Assertion is false and reason is true

Answer: Both assertion and reason are true and reason is correct explanation of assertion.

Solution: According to the reason statement, the work function is defined as the energy of the incident photon when the maximum kinetic energy of the emitted photon is zero. Therefore, no electron will be ejected from a metal if the energy of incident photon is less than work function of the metal.

Therefore, option A is the correct answer.

Q.5. In the following circuit, the value of $V_B - V_A$ is



A) $5 \mathrm{~V}$

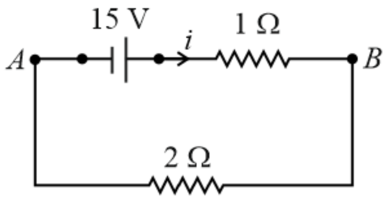
B) $15 \mathrm{~V}$

C) $10 \mathrm{~V}$

D) $7.5 \mathrm{~V}$

Answer: $10 \mathrm{~V}$

Solution:



Combined resistance value in the circuit is, 3Ω.

$i = \frac{15}{3} = 5 \mathrm{~A}$

Therefore, $V_B - V_A = i \times 2 = 10 \mathrm{~V}$

Q.6. A ball is dropped from a height of $180 \mathrm{~m}$. After $2 \mathrm{~s}$, another ball is thrown downwards with speed u so that both balls collide at a height $100 \mathrm{~m}$. The value of u is(take $g = 10 \mathrm{~m~s^{-2}}$)

A) $10 \mathrm{~m~s^{-1}}$

B) $20 \mathrm{~m~s^{-1}}$

C) $40 \mathrm{~m~s^{-1}}$

D) $30 \mathrm{~m~s^{-1}}$

Answer: $30 \mathrm{~m~s^{-1}}$

Solution: Displacement covered by first ball, $(180 - 100) = 0 + \frac{1}{2} \times 10 \times t^2 \quad \Rightarrow t = 4 \mathrm{~s}$

Now, the second body gets only, $t_2 = 4 - 2 = 2 \mathrm{~s}$.

Displacement covered by second will be same,

$80 = u \times 2 + \frac{1}{2} \times 10 \times (2)^2$

$\Rightarrow u = \frac{80-20}{2} = 30 \mathrm{~m~s^{-1}}$

Q.7. $\vec{A}$ and $\vec{B}$ are two vectors having equal magnitude. If magnitude of $\vec{A} + \vec{B}$ is two times magnitude of $\vec{A} - \vec{B}$, then angle between A and B is ?

A) $\cos^{-1}\left(\frac{3}{5}\right)$

B) $\cos^{-1}\left(\frac{1}{2}\right)$

C) $\cos^{-1}\left(\frac{1}{3}\right)$

D) $\cos^{-1}\left(\frac{2}{5}\right)$

Answer: $\cos^{-1}\left(\frac{3}{5}\right)$

Solution:

Given: $\left|\overrightarrow{A}+\overrightarrow{B}\right|=2\left|\overrightarrow{A}-\overrightarrow{B}\right|$

We have,

$A^2+A^2+2A^2\cos\theta=4\left(A^2+A^2-2A^2\cos\theta\right)$

Simplifying we get

$2A^2+2A^2\cos\theta=8A^2-8A^2\cos\theta$

$\Rightarrow 1+\cos\theta=4-4\cos\theta$

$\Rightarrow 5\cos\theta=3$
$\Rightarrow \cos\theta=\frac{3}{5}$
$\Rightarrow \theta=\cos^{-1}\frac{3}{5}$

Q.8. A rod of original length L hangs with one end free. It expands to L_1 & L_2 on hanging a mass of $1\,\mathrm{kg}$ & $2\,\mathrm{kg}$ from its free end respectively. Find the length L (in terms of L_1 and L_2)

A) $2L_1-L_2$

B) $3L_1-L_2$

C) $(L_1L_2)^{0.5}$

D) $(L_1L_2)^{0.3}$

Answer: $2L_1-L_2$

Solution:

From Hooke's law, $\Delta L=\frac{FL}{AY}\Rightarrow\Delta L\propto F$. Therefore,

$\frac{L_2-L}{L_1-L}=\frac{m_2g}{m_1g}=\frac{2}{1}\Rightarrow L=2L_1-L_2$

Q.9. $\left(P+\frac{a}{V^2}\right)(V-b)=RT$ (For $1\,\mathrm{mol}$ of gas), $\frac{a}{b}$ will be dimensionally equal to:

A) $\frac{P}{V}$

B) PV

C) PV^3

D) $\frac{P}{V^2}$

Answer: PV

Solution: $\left(P+\frac{a}{V^2}\right)(V-b)=nRT$

Dimensions of b should be equal to dimensions of V. Similarly, dimensions of a will be equal to dimensions of PV^2.

Therefore, dimensions of $\left[\frac{a}{b}\right]=\left[\frac{PV^2}{V}\right]=[PV]$.

Q.10. A Carnot engine takes 300 calories of heat from a source at $227^{\circ}\mathrm{C}$ and rejects 225 calories of heat to the sink. The temperature of the sink is?

A) $102^{\circ}\mathrm{C}$

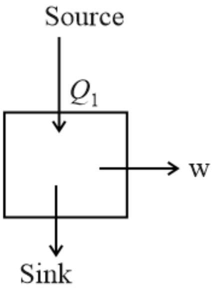
B) $109^{\circ}\mathrm{C}$

C) $135^{\circ}\mathrm{C}$

D) $127^{\circ}\mathrm{C}$

Answer: $102^{\circ}\mathrm{C}$

Solution:



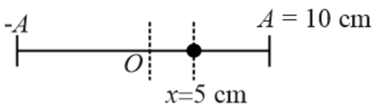
Efficiency of Carnot engine is given by $\eta=1-\frac{Q_2}{Q_1}=1-\frac{T_2}{T_1}$.

$\Rightarrow 1-\frac{225}{300}=1-\frac{T_2}{500}$

$T_2=\frac{5}{3}\times 225=375\ \mathrm{K}$

So, temperature of sink is $T_2=375-273=102^{\circ}\mathrm{C}$

Q.11. A particle performing SHM with $A=10\ \mathrm{cm}$, is given impulse at $x=5\ \mathrm{cm}$, such that velocity becomes three times. Find the new amplitude.



A) $15\,\mathrm{cm}$

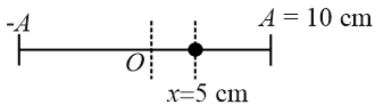
B) $10\sqrt{7}\ \mathrm{cm}$

C) $12\,\mathrm{cm}$

D) $18\sqrt{3}\,\mathrm{cm}$

Answer: $10\sqrt{7}\ \mathrm{cm}$

Solution:



In SHM, velocity of the particle at any displacement, x is given by, $v = \sqrt{A^2 - x^2}$.

Therefore,

$$\Rightarrow \frac{v}{3v} = \frac{\left(\sqrt{A_1^2 - 25}\right)}{\sqrt{A_2^2 - 25}} \Rightarrow \sqrt{A_2^2 - 25} = 3 \times \sqrt{75}$$

$$\Rightarrow A_2^2 = 700 \ \Rightarrow A_2 = 10\sqrt{7} \text{ cm}$$

Q.12. Escape velocity on planet A is given as 12 m s^{-1}. If density of another planet B is 4 times that of planet A, and its radius is half of that of planet A, then the escape velocity from planet B is

A) 24 m s^{-1}

B) 12 m s^{-1}

C) 36 m s^{-1}

D) 6 m s^{-1}

Answer: 12 m s^{-1}

Solution: We know that escape velocity is given by ,

$$v = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{M_A}{M_B} \times \frac{R_B}{R_A}} = \sqrt{\frac{\rho_A {R_A}^3}{\rho_B {R_B}^3} \times \frac{R_B}{R_A}} = \sqrt{\frac{\rho_A {R_A}^2}{\rho_B {R_B}^2}} = \sqrt{\frac{1}{4} \times \frac{4}{1}}$$

$$\Rightarrow v_B = v_A = 12 \text{ m s}^{-1}$$

Q.13. Average intensity of an electromagnetic wave is $I = 0.22 \text{ W m}^{-2}$ then magnetic field amplitude of this wave is

A) $2.8 \times 10^{-6} \text{ T}$

B) $2.3 \times 10^{-7} \text{ T}$

C) $5.8 \times 10^{-6} \text{ T}$

D) $4.3 \times 10^{-8} \text{ T}$

Answer: $4.3 \times 10^{-8} \text{ T}$

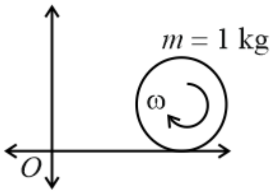
Solution: Using the relation between $I = \frac{B_0 E_0}{2\mu}$.

Since, $E_0 = cB_0$, then $I = \frac{B_0^2}{2\mu_0} \times c$

So, amplitude of magnetic field is $B_0 = \sqrt{\frac{0.22 \times 2 \times 4\pi \times 10^{-7}}{3 \times 10^8}}$

$B_0 = 4.3 \times 10^{-8} \text{T}$

Q.14. A spherical shell of radius R is rolling on a level horizontal surface as shown. Then the angular momentum about the origin is



A) $\frac{2}{3}mR^2\omega$

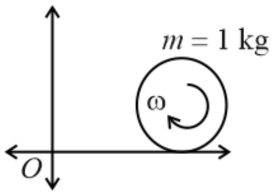
B) $\frac{7}{3}mR^2\omega$

C) $\frac{5}{3}mR^2\omega$

D) $\frac{1}{3}mR^2\omega$

Answer: $\frac{5}{3}mR^2\omega$

Solution:



Angular momentum of a rotating and translating body is given by,

$L_O = mvR + I\omega = mvR + \frac{2}{3}MR^2\omega$

For rolling, $v = \omega R$

$\Rightarrow L_O = m\omega R^2\left(1 + \frac{2}{3}\right) = \frac{5}{3}m\omega R^2$

Q.15. Velocity of a particle in terms of displacement is, $v = b(x)^{\frac{5}{2}}$. Find work done in displacing particle from $x = 0 \text{ m}$ to $x = 4 \text{ m}$. (Given, $b = 0.25$ & $m = 0.5 \text{ kg}$)

A) 2 J

B) 4 J

C) 8 J

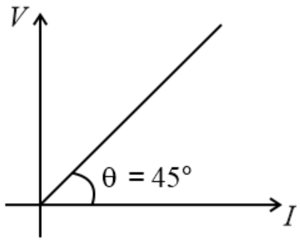
D) 16 J

Answer: 16 J

Solution: As we know, work done by all forces will be, $W_{\text{all}} = \Delta K$

$= \frac{1}{2}mb^2\left(x_2^5 - x_1^5\right) = \frac{1}{2} \times 0.5 \times \frac{1}{16} \times 4^5 = 16 \text{ J}$

Q.16. Voltage vs current graph is shown as below. If the diameter of the wire is $2.4\ \mathrm{cm}$ and length of wire is $31.4\ \mathrm{cm}$, then find its resistivity.



A) $0.144\ \mathrm{cm}$

B) $0.164\ \mathrm{cm}$

C) $0.182\ \mathrm{cm}$

D) $0.204\ \mathrm{cm}$

Answer: $0.144\ \mathrm{cm}$

Solution: Since, slope of V–I graph represents the resistance, so $R = \tan 45^\circ = 1\ \Omega$

Now, resistance of wire is given by $R = \frac{\rho L}{A}$

So, resistivity of the wire is $\rho = \frac{A}{RL} = \frac{\pi r^2}{RL} = \frac{\pi\left(1.2^2 \times 10^{-4}\right)}{31.4 \times 10^{-2}} = 1.44 \times 10^{-3}\ \mathrm{m} = 0.144\ \mathrm{cm}$

Q.17. Two rods of thermal conductivity K and $2K$ and length $L_1 = 0.4\ \mathrm{m}$ and $L_2 = 0.2\ \mathrm{m}$ with equal cross-sectional area are connected in series. What is the equivalent thermal conductivity of the rod?

A) $\frac{K}{2}$

B) $2K$

C) $\frac{3}{2}K$

D) $\frac{6}{5}K$

Answer: $\frac{6}{5}K$

Solution: The thermal resistance of rod 1 will be $R_1 = \frac{l_1}{KA} = \frac{0.4}{KA}$ and rod 2 will be $R_2 = \frac{l_2}{2KA} = \frac{0.2}{2KA} = \frac{0.1}{KA}$.

Since they are connected in series, $R_{eq} = R_1 + R_2 = \frac{0.5}{KA}$.

If equivalent thermal conductivity is K_{eq}, then $R_{eq} = \frac{l_1 + l_2}{K_{eq}A} = \frac{0.6}{K_{eq}A}$. Therefore,

$\frac{0.5}{KA} = \frac{0.6}{K_{eq}A} \Rightarrow K_{eq} = \frac{6}{5}K$

Q.18. A particle of charge $40\ \mu\mathrm{C}$ and $100\ \mathrm{mg}$ mass is thrown with velocity $200\ \mathrm{m\ s^{-1}}$ against a uniform electric field of $10^5\ \mathrm{N\ C^{-1}}$. How much distance will it travel before coming to momentary rest?

A) $0.5\ \mathrm{m}$

B) $0.4\ \mathrm{m}$

C) $0.6\ \mathrm{m}$

D) $0.3 \mathrm{~m}$

Answer: $0.5 \mathrm{~m}$

Solution: Force experienced by a particle with charge q in an electric field $F=qE$.

Now, the acceleration produced is given as $a=\frac{F}{m}=\frac{qE}{m}=\frac{40 \times 10^{-6} \times 10^{5}}{1 \times 10^{-4}}$

$=4 \times 10^{4} \mathrm{~m} \mathrm{~s}^{-2}$ (As the particle is projected against the electric field, hence it is decelerated)

Using, $v^{2}=u^{2}-2as$

We have, $0^{2}=u^{2}-2as$

$s=\frac{v^{2}}{2a}=\frac{(200)^{2}}{2 \times 4 \times 10^{4}}=0.5 \mathrm{~m}$

Q.19. In optical communication system operating at $1000 \mathrm{~nm}$, only 2% of source frequency is used for TV transmission having a bandwidth of $8 \mathrm{~kHz}$. Find the number of channels that can be transmitted.

A) 7.5×10^{8}

B) 3×10^{8}

C) 1.5×10^{12}

D) 0.75×10^{12}

Answer: 7.5×10^{8}

Solution: Given, $\lambda=1000 \mathrm{~nm}=10^{-6} \mathrm{~m}$

$\Rightarrow f=\frac{c}{\lambda}=\frac{3 \times 10^{8}}{10^{-6}}=3 \times 10^{14} \mathrm{~Hz} \Delta f=\left(\frac{2}{100}\right) \times 3 \times 10^{14}=6 \times 10^{12} \mathrm{~Hz} \ N=\frac{\text{Total bandwidth of channel}}{\text{Bandwidth needed per channel}}=\frac{6 \times 10^{12}}{8000}$
$=7.5 \times 10^{8}$

Q.20. Equation of a wave is given as, $y=10 sin 2\pi\left(nt-\frac{x}{\lambda}\right)$. If maximum velocity of the particle is four times the wave velocity, find the wavelength of the wave.

A) $5\pi \mathrm{~m}$

B) $10\pi \mathrm{~m}$

C) $15\pi \mathrm{~m}$

D) $20\pi \mathrm{~m}$

Answer: $5\pi \mathrm{~m}$

Solution: As we know, $v_{p}=\frac{\mathrm{d}y}{\mathrm{d}t}=(10 \times 2\pi n) \times \cos 2\pi\left(nt-\frac{x}{\lambda}\right)$

Therefore, $\left(v_{p}\right)_{max}=20\pi n$

Now as we know, $v=\lambda n$

As given in the question,

$\left(v_{p}\right)_{max}=4v$
$\Rightarrow 20\pi n=4\lambda n$
$\Rightarrow \lambda=5\pi \mathrm{~m}$

Q.21. A particle of mass m initially at rest, breaks into three parts in mass ratio $1:1:2$. The velocities of the first two parts directed perpendicular to each other are $30 \mathrm{~m~s}^{-1}$ and $40 \mathrm{~m~s}^{-1}$. Find the velocity of the third part.

A) $35 \mathrm{~m~s}^{-1}$

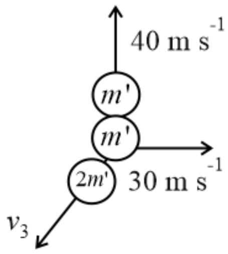
B) $40 \mathrm{~m~s}^{-1}$

C) $25 \mathrm{~m~s}^{-1}$

D) $15 \mathrm{~m~s}^{-1}$

Answer: $25 \mathrm{~m~s}^{-1}$

Solution:



Let the masses of the three parts be m', m' and $2m'$.

Momentum of first part $p_1 = m' \times 30 = 30m'$ and momentum of second part $p_2 = m' \times 40 = 40m'$.

Third part will fly in a direction opposite to the direction of resultant of p_1 and p_2, so that total momentum of the system after explosion remains zero.

Momentum of the third part is $p_3 = \sqrt{p_1^2 + p_2^2} = \sqrt{(30m')^2 + (40m')^2} = 50m'$

Velocity of third part, $v = \frac{50m'}{2m'}$ $=25 \mathrm{~m~s}^{-1}$

Q.22. Two coils heat same amount of water separately in $60 \mathrm{~min}$ and $20 \mathrm{~min}$. Then time taken to heat same amount of water, if they are connected in parallel with same source is

A) $10 \mathrm{~min}$

B) $5 \mathrm{~min}$

C) $7.5 \mathrm{~min}$

D) $15 \mathrm{~min}$

Answer: $15 \mathrm{~min}$

Solution: If H is the heat required to heat the water, then

$P_1 \times 60 = H \Rightarrow P_1 = \frac{H}{60}$ and $P_2 \times 20 = H \Rightarrow P_2 = \frac{H}{20}$

When the coils are connected in parallel their powers will get added. If the required time now is t, then

$(P_1 + P_2)t = H$

$\Rightarrow t = \frac{H}{\frac{H}{60} + \frac{H}{20}} = 15 \mathrm{~min}$

Section B: Chemistry

Q.1. 0.5 g of an organic compound on Kjeldahl's analysis gave enough ammonia to just neutralize $10\ \mathrm{cm}^3$ of $1\ \mathrm{M}\ H_2SO_4$.The percentage of nitrogen in the compound is:

(Atomic mass: $\mathrm{N}=14,\ \mathrm{H}=1$)

A) 56

B) 28

C) 42

D) 14

Answer: 56

Solution: Kjeldahl is a method to determine percentage composition of nitrogen in a nitrogenous organic compound by suitable titration technique.

Now, as per the reaction:

$2NH_3+H_2SO_4\rightarrow (NH_4)_2SO_4$

Given that:

Volume of $H_2SO_4=\ 10\ \mathrm{mL}$

Molarity of H_2SO_4 solution$=\ 1\ \mathrm{M}$

We know that,

$\mathrm{Molarity}=\frac{\text{number of moles of solute}}{\text{volume of solution(in Litre)}}$

$\mathrm{M}=\frac{\text{moles}\times 1000}{\text{volume of solution (mL)}}$

$1=\frac{\text{moles}\times 1000}{10(\mathrm{mL})}$

$\text{moles}\ \left(H_2SO_4\right)=0.01\,\mathrm{mol}$

According to above reaction, 1 mole of H_2SO_4 reacts with 2 mole of NH_3.

Hence, moles $\left(NH_3\right)=0.02$ mol

We know that,

$\text{weight}=\text{moles}\times\text{molecular mass}$

weight of $NH_3=0.02\times 17=0.34$ g

Since, 17 g of NH_3 contains $=\ 14$ g of N

Therefore, 0.34 g of NH_3 will contain $=\frac{14}{17}\times 0.34=0.28$ g

$\text{Percentage composition}=\frac{\text{mass of nitrogen}}{\text{mass of compound}}\times 100$

$\%\mathrm{N}=\frac{0.28}{0.5}\times 100=56\%$

Q.2. The sugar units present in natural DNA and RNA, respectively are

A) D-2-deoxyribose and L-ribose

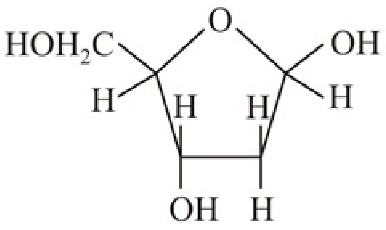
B) L-2-deoxyribose and D-ribose

C) D-2-deoxyribose and D-ribose

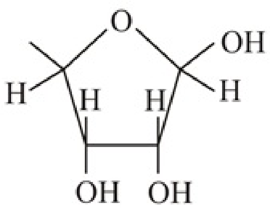
D) L-2-deoxyribose and L-ribose

Answer: D-2-deoxyribose and D-ribose

Solution: Deoxyribo nucleic acid or DNA contains 2-deoxy-D-ribose sugar unit, whereas ribo nucleic acid or RNA contains D-ribose sugar unit.



D-2 deoxyribose in DNA



D-ribose in RNA

Q.3. Arrange the following in the increasing order of their covalent character.

A) $CsCl < RbCl < NaCl < LiCl$

B) $LiCl < NaCl < RbCl < CsCl$

C) $CsCl < NaCl < LiCl < RbCl$

D) $LiCl < RbCl < NaCl < CsCl$

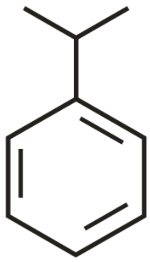
Answer: $CsCl < RbCl < NaCl < LiCl$

Solution: Lithium chloride is the alkali metal chloride which is most covalent in nature. This is due to the small size and high polarising power of lithium. That is, smaller the cation, greater is the polarising power and hence, greater is the covalent character.

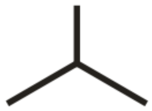
So order of covalent character will be $CsCl < RbCl < NaCl < LiCl$

Q.4. Which of the following has asymmetric carbon?

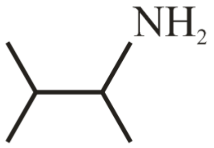
A)



B)



C)



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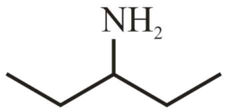
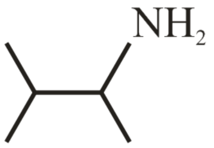
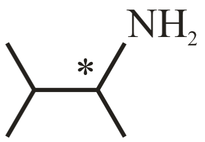


Answer:



Solution: An asymmetric carbon atom (chiral carbon) is a carbon atom that is attached to four different types of atoms or groups of atoms.



The carbon marked with *, contains four different groups attached to it. Hence, it is an asymmetric carbon.

Q.5. Which of the following amino acids contain sulphur?

A) Histamine

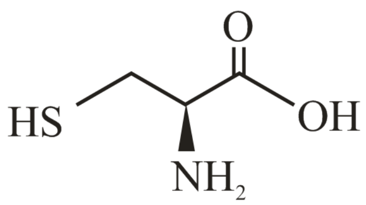
B) Cimetidine

C) Cysteine

D) Ranitidine

Answer: Cysteine

Solution: Amino acids that contains sulphur is cysteine. The structure is shown below



Cysteine

Q.6. The electronic configuration of element of atomic number 78 is:

A) $[Xe]3f^{14}4d^{10}5s^0$

B) $[Kr]3f^{14}4d^8$

C) $[Xe]3f^{14}4d^{10}5s^2$

D) $[Xe]4f^{14}5d^9 6s^1$

Answer: $[Xe]4f^{14}5d^9 6s^1$

Solution: The atomic number of Platinum is 78. Platinum belongs Nickel group, and it is 5d series element. Electronic configuration of Platinum is$1s^2 2s^2 p^6 3s^2 p^6 d^{10} 4s^2 p^6 d^{10} f^{14} 5s^2 p^6 d^9 6s^1$

Q.7. The element in which during the extraction process CN compound is not formed

A) Zn

B) Ag

C) Gold

D) Cu

Answer: Cu

Solution: In the metallurgy of silver and gold, the respective metal is leached with a dilute solution of $NaCN$ or KCN in the presence of air, which supplies O_2. The metal is obtained later by replacement reaction.

$$4M(s) + 8CN^-(aq) + 2H_2O(aq) + O_2(g) \rightarrow 4[M(CN)_2]^-(aq) + 4OH^-(aq)\ [M = Ag \text{ or } Au]$$

$$2[M(CN)_2]^-(aq) + Zn(s) \rightarrow [Zn(CN)_4]^{2-}(aq) + 2M(s)$$

Copper metal is not involved in this process.

Q.8. For which of the following metal, lowest wavelength of incident radiation required for photoelectric effect.

A) Li

B) Cs

C) Rb

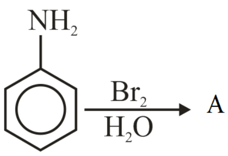
D) Na

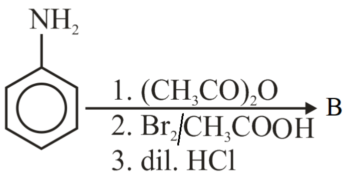
Answer: Li

Solution: The total energy of the incoming photon, E_{photon}, must be equal to the kinetic energy of the ejected electron, $KE_{electron}$, plus the energy required to eject the electron from the metal. The energy required to free the electron from a particular metal is also called the metal's work function, which is represented by the symbol Φ (in units of J):

The lowest wavelength means the highest energy. Li metal requires the highest energy required to eject the electron as its ionisation energy is high among the given metals.

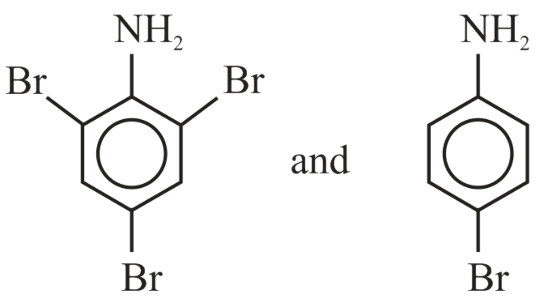
Q.9.



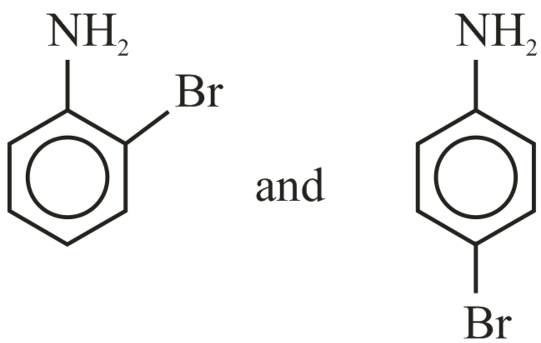


A and B respectively are:

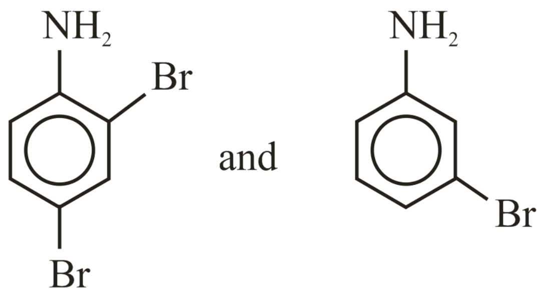
A)



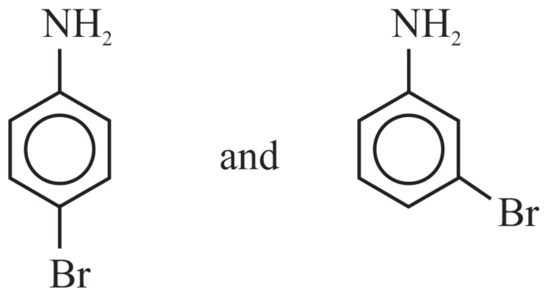
B)



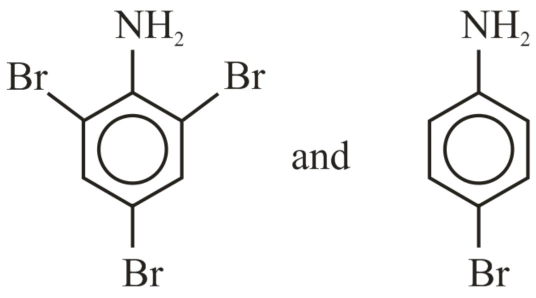
C)



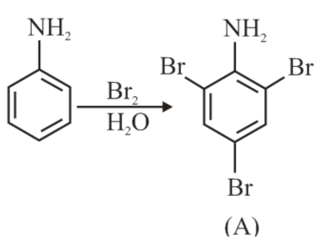
D)

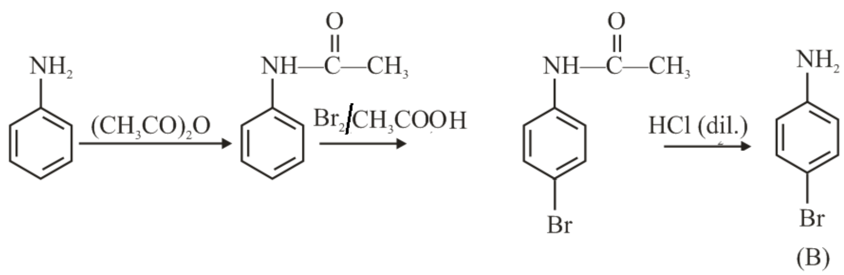


Answer:



Solution: In aniline $-NH_2$ is a strong activating group. So, on direct bromination, substitution will occur at all ortho and para positions. And also we can not use lewis acid since it will react with lewis acid. But if we first convert this into amide then only para substitution will be major product.





Q.10. Which of the following is present in rainwater that affects the Taj Mahal and causes damage to the monument?

A) H_2SO_4

B) H_3PO_4

C) Phenol

D) Lactic acid

Answer: H_2SO_4

Solution: The air around the city of Agra, where the Taj Mahal is located, contains fairly high levels of sulphur and nitrogen oxides. It is mainly due to a large number of industries and power plants around the area. Use of poor quality of coal, kerosene and firewood as fuel for domestic purposes add up to this problem. The resulting acid rain reacts with marble, $CaCO_3$ of Taj Mahal $(CaCO_3 + H_2SO_4 \rightarrow CaSO_4 + H_2O + CO_2)$ causing damage to this wonderful monument.

Q.11. Statement I: Phenol is a weaker acid as compared to acetic acid.

Statement II: Phenol is a weaker acid than alcohol and water.

A) Statement I and II both are correct

B) Statement I is correct but statement II is incorrect

C) Statement I is incorrect but statement II is correct

D) Statement I and II both are incorrect

Answer: Statement I is correct but statement II is incorrect

Solution: Acetate ion is better stabilized by resonance than phenoxide ion.

Phenoxide ion -ve charge on oxygen to carbon in resinating structures

Acetate ion -ve charge on oxygen to oxygen in resinating structures

The both phenoxide and acetate ions are resonance stablized. In acetate ion, all the canonical structures are equally contributing. More contributing the resonance structures more will be stability. Hence, acetic acid is a better acid than phenol.

Phenols are much more acidic than alcohols because the negative charge in the phenoxide ion is not localized on the oxygen atom as it is in an alkoxide ion but is delocalised as it is shared by a number of carbon atoms in the benzene ring.

Q.12. Match elements given in column I with their uses given in column II.

	Column I (Element)		Column II (Uses)
i	Cs	a	High temperature thermometer
ii	Ga	b	Water proofing
iii	B	c	Photoelectric cell
iv	Si	d	Bullet-proof vest

A) i-c, ii-a, iii-d, iv-b

B) i-d, ii-c, iii-a, iv-b

C) i-b, ii-c, iii-d, iv-a

D) i-d, ii-a, iii-b, iv-c

Answer: i-c, ii-a, iii-d, iv-b

Solution: i-c, ii-a, iii-d, iv-b

	Column I (Element)		Column II (Uses)
i	Cs	c	Photoelectric cell
ii	Ga	b	High temperature thermometer
iii	B	d	Bullet-proof vest
iv	Si	b	Water proofing

Q.13. Solubility of $AgCl$ is maximum in

A) $0.1\ M\ HCl$

B) $0.1\ M\ KCl$

C) Deionised water

D) $0.01\ M\ NH_3$

Answer: $0.01\ M\ NH_3$

Solution: Due to the common ion effect of chloride, the solubility of silver chloride is less in HCl and KCl, compared to solubility in water. But in ammonia the solubility is maximum due to the following reaction.

$$AgCl(s) \rightleftharpoons Ag^{+}(aq) + Cl^{-}(aq)$$

$$Ag^{+}\left(aq\right) + 2NH_3\left(aq\right) \leftrightharpoons \left[Ag\left(NH_3\right)_2\right]^{+}$$

Q.14. Manganate ion disproportionate in neutral medium. Find spin-only magnetic moment of the species formed by oxidation of manganate ion (in B.M.)

A) 0

B) 1.73

C) 2.89

D) 3.8

Answer: 0

Solution:

$$MnO_4^{2-} \longrightarrow \underset{\substack{\text{oxidised form} \\ d^0(Mn^{+7})}}{MnO_4^{-}} + \underset{\text{reduced form}}{MnO_2}$$

As we can see in this reaction oxidised product of reaction is MnO_4^- which has no unpaired electrons. So, magnetic moment will be zero.

Q.15. Consider the following reaction

$$C_6H_{12}O_6 \xrightarrow{\text{Zymase}} A \xrightarrow{\text{NaOI}} B + CHI_3$$

The number of carbon atoms in B is

A) 1

B) 2

C) 3

D) 4

Answer: 1

Solution:

$$C_6H_{12}O_6 \xrightarrow{\text{zymase}} \underset{(A)}{2C_2H_5OH(aq)} + 2CO_2\left(g\right)$$

$$C_2H_5OH\left(aq\right) \xrightarrow{\text{NaOI}} \underset{(B)}{HCOO^-Na^+} + CHI_3$$

Q.16. Given, activation energy (E_a) for a chemical reaction is 23566 J. Initial temperature is 310 K and final temperature is 300 K. If the rate constant of the reaction in initial and final condition is K_1 and K_2 respectively and the value of $\frac{K_2}{K_1}$ is $x \times 10^{-3}$. Find the value of x.

[Given, $\log(0.75) = 0.13$]

A) 250

B) 750

C) 500

D) 100

Answer: 750

Solution:

$$\ln \frac{K_2}{K_1} = \frac{Ea}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log \frac{K_2}{K_1} = \frac{23566}{8.314 \times 2.303}\left[\frac{1}{310} - \frac{1}{300}\right]$$

$$\log \frac{K_2}{K_1} = -0.13$$

$$\frac{K_2}{K_1} = 10^{-0.13}$$

$$\frac{K_2}{K_1} = 7.5 \times 10^{-1}$$

$$\frac{K_2}{K_1} = 750 \times 10^{-3}$$

Q.17. Consider the following reaction:

$$Fe_2O_3 + 3CO \rightarrow 2Fe + 3CO_2$$

If 4640 g of Fe_2O_3 is allowed to react with 90 moles of CO. What is the weight (in g) of Fe produced?

A) 4438

B) 3248

C) 4640

D) 9280

Answer: 3248

Solution: $Fe_2O_3 + 3CO \rightarrow 2Fe + 3CO_2$

$n_{Fe_2O_3} = \frac{4640}{160} = 29 \text{ mol}$

$n_{CO} = 90 \text{ mol}$

1 mol of Fe_2O_3 reacts with 3 mol of CO

29 mol of Fe_2O_3 reacts with $3 \times 29 = 87 \text{ mol}$ of CO

Fe_2O_3 is the limiting reagent.

Moles of Fe produced $= 29 \times 2 = 58 \text{ mol}$

Weight of Fe produced $= 58 \times 56 = 3248 \text{ g}$

Q.18. Which of the following polymer can regain its shape?

A) Nylon- $6, 6$

B) Buna-S

C) Terylene

D) Bakelite

Answer: Buna-S

Solution: Synthetic rubber is any vulcanisable rubber like polymer, which is capable of getting stretched to twice its length. However, it returns to its original shape and size as soon as the external stretching force is released. Thus, synthetic rubbers are either homopolymers of $1, 3$ - butadiene derivatives or copolymers of $1, 3$ - butadiene or its derivatives with another unsaturated monomer.

Buna-S is an example of synthetic rubber. The other unsaturated monomer is styrene.

Q.19. On reaction of white phosphorous with alkali in inert atmosphere, salt of which of the following acid is obtained?

A) Phosphoric acid

B) Phosphinic acid

C) Hypophosphoric acid

D) Orthophosphoric acid

Answer: Phosphinic acid

Solution: White phosphorus is a translucent white waxy solid. It is poisonous, insoluble in water but soluble in carbon disulphide and glows in dark (chemiluminescence). It dissolves in boiling $NaOH$ solution in an inert atmosphere giving PH_3.

$$P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + \underset{\text{(sodium hypophosphite)}}{3NaH_2PO_2}$$

Sodium hypophosphite is a salt of Phosphinic acid (Hypophosphorous acid).

Q.20. Find the molar mass of an ideal gas at 100 mm Hg pressure at 235^{o} C temperature having a density of 0.46 g/mL.

(Round off to nearest integer)

Use: $R = 0.0821 \text{ L atm mol}^{-1} \text{ K}^{-1}$

A) 146 g/mol

B) 235 g/mol

C) 292 g/mol

D) $270 \mathrm{~g} / \mathrm{mol}$

Answer: $146 \mathrm{~g} / \mathrm{mol}$

Solution: $\mathrm{M}=\frac{\rho \mathrm{RT}}{\mathrm{P}}$

$\rho=0.46 \frac{\mathrm{g}}{\mathrm{L}} ; \mathrm{P}=\frac{100}{760} \mathrm{atm} ; \mathrm{T}=235^{\circ} \mathrm{C}=508 \mathrm{~K}$

$\mathrm{M}=\frac{0.46 \times 0.0821 \times 508 \times 760}{100}$

$\mathrm{M}=145.81 \frac{\mathrm{g}}{\mathrm{mol}} \approx 146 \frac{\mathrm{g}}{\mathrm{mol}}$

Q.21. Statement 1 : formation of ester from carboxylic acid and alcohol is an example of nucleophilic substitution reaction.

Statement 2 : EWG increase rate of reaction.

A) $\mathrm{S}-1$ is true and $\mathrm{S}-2$ false

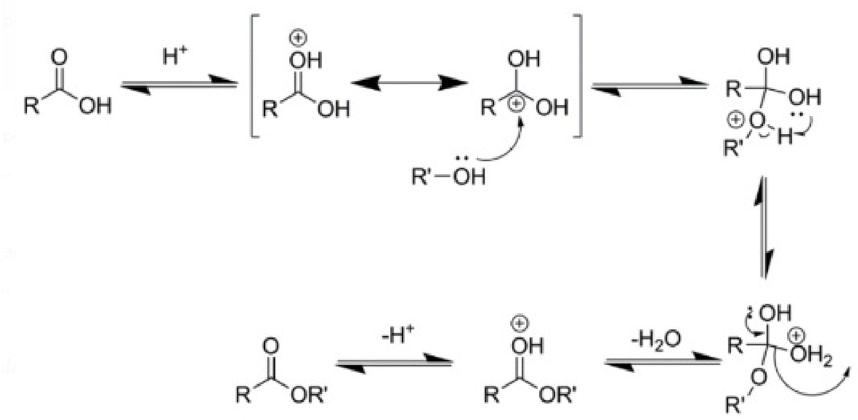
B) $\mathrm{S}-1$ is false and $\mathrm{S}-2$ false

C) $\mathrm{S}-1$ is true and $\mathrm{S}-2$ true

D) $\mathrm{S}-1$ is false and $\mathrm{S}-2$ true

Answer: $\mathrm{S}-1$ is true and $\mathrm{S}-2$ true

Solution:



In the above reaction OH group is replaced by OR group. Hence, nucleophilic substitution reaction is involved. Electron withdrawing groups increases the stability of $\mathrm{R}\overset{\oplus}{\mathrm{C}}(\mathrm{OH})_2$, hence, rate of reaction increases.

Section C: Mathematics

Q.1. The mirror image of the point $P(2,4,7)$ with respect to the plane $3x-y+4z=2$ is

A) $\left(-5,\frac{19}{3},-\frac{7}{3}\right)$

B) $\left(-\frac{58}{13},\frac{80}{13},-\frac{21}{13}\right)$

C) $\left(-5,-\frac{19}{3},7\right)$

D) $\left(-5,\frac{19}{3},\frac{7}{2}\right)$

Answer: $\left(-\frac{58}{13},\frac{80}{13},-\frac{21}{13}\right)$

Solution: Given, equation of $3x-y+4z=2$ and point $P(2,4,7)$

Now using the formula of image

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=-2\times\frac{(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$

We get,

$\frac{x-2}{3}=\frac{y-4}{-1}=\frac{z-7}{4}=-\frac{2(6-4+28-2)}{26}$

$\Rightarrow\ \frac{x-2}{3}=\frac{y-4}{-1}=\frac{z-7}{4}=-\frac{28}{13}$

$\Rightarrow\ x=-\frac{58}{13},y=\frac{80}{13},z=-\frac{21}{13}$

Q.2. For a complex number z if $\left(|z-2|\le 1,\ z(i+1)+\bar{z}\,(1-i)\le 2\right)$ and $|z-2i|$ attains maximum value and minimum value at z_1 & z_2 respectively, then the value of $|z_1|^2+|z_2|^2$ is

A) $7-\sqrt{2}$

B) $10-\sqrt{2}$

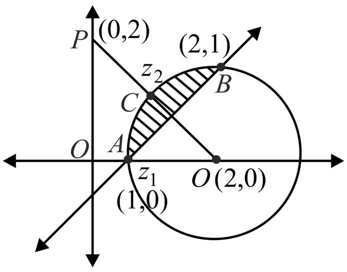
C) $14-4\sqrt{2}$

D) $14+\sqrt{2}$

Answer: $14-4\sqrt{2}$

Solution: $z(i+1)+\bar{z}(1-i)\leq 2 \ \Rightarrow x-y\leq 1 \ \ \ldots(\text{i})$

and $|z-2|\leq 1\Rightarrow (x-2)^2+y^2\leq 1$



Maximum value $=|z_1|=PA=PB=\sqrt{5}$ and

minimum value $=|z_2|=PO-OC=2\sqrt{2}-1=PC$

$\Rightarrow |z_1|^2+|z_2|^2=5+\left(2\sqrt{2}-1\right)^2=14-4\sqrt{2}$

Q.3. If $\left|\vec{a}\right|=\left|\vec{b}\right|=2$ & $\left|\vec{a}+\vec{b}\right|=2\left|\vec{a}-\vec{b}\right|$, then the angle between $\vec{a}$ and $\vec{b}$ is

A) $\sin^{-1}\frac{3}{5}$

B) $\sin^{-1}\frac{4}{5}$

C) $\tan^{-1}\frac{1}{2}$

D) $\frac{\pi}{3}$

Answer: $\sin^{-1}\frac{4}{5}$

Solution: Given $\left|\vec{a}+\vec{b}\right|=2\left|\vec{a}-\vec{b}\right|$

$\Rightarrow \left|\vec{a}+\vec{b}\right|^2=4\left|\vec{a}-\vec{b}\right|^2$

$\Rightarrow 8+2\vec{a}\cdot\vec{b}=32-8\vec{a}\cdot\vec{b}$

$\Rightarrow 10\vec{a}\cdot\vec{b}=24$

$\Rightarrow \vec{a}\cdot\vec{b}=\frac{12}{5}\Rightarrow \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta=\frac{12}{5}\Rightarrow \theta=\cos^{-1}\frac{3}{5}=\sin^{-1}\frac{4}{5}$

Q.4. If the solution of differential equation $x\frac{dy}{dx}-y=\sqrt{y^2+16x^2}$ is given by $y=y(x)$ and $y(1)=3$, then the value of $y(2)$ is

A) 15

B) 5

C) 14

D) 17

Answer: 15

Solution: Given,

$x\frac{dy}{dx}-y=\sqrt{y^2+16x^2}$

$\Rightarrow\ \frac{dy}{dx}=\frac{\sqrt{y^2+16x^2}+y}{x}$

Let $y=xt\Rightarrow\frac{dy}{dx}=x\frac{dt}{dx}+t$

$\Rightarrow x\frac{dt}{dx}+t=t+\sqrt{t^2+16}$

$\Rightarrow\ \frac{dt}{\sqrt{t^2+16}}=\frac{dx}{x}$

Now integrating both side we get,

$\Rightarrow\int\frac{dt}{\sqrt{t^2+16}}=\int\frac{dx}{x}$

$\Rightarrow\ \ln\left|t+\sqrt{t^2+16}\right|=\ln x+\ln c$

$\Rightarrow\ \frac{y}{x}+\frac{\sqrt{y^2+16x^2}}{x}=cx$

$\because\ \ y(1)=3$ so $\Rightarrow\ \frac{3}{1}+\frac{\sqrt{3^2+16\times1^2}}{1}=c\times1\Rightarrow c=8$

So solution becomes,

$\frac{y}{x}+\frac{\sqrt{y^2+16x^2}}{x}=8x$

Now $y(2)=\frac{y}{2}+\frac{\sqrt{y^2+16\times2^2}}{2}=8\times2$

$\Rightarrow y+\sqrt{y^2+64}=32$

$\Rightarrow y=15$

Q.5. $\int_0^5\cos\pi\left(x-\left[\frac{x}{2}\right]\right)dx=$ (where $[\cdot]$ is the greatest integer function)

A) 0

B) 2

C) 3

D) 4

Answer: 0

Solution: $I=\int_0^5\cos\left(\left[\frac{x}{2}\right]\pi-\pi x\right)dx$

$\because x\in(0,5)\ \therefore x\in\left(0,\frac{5}{2}\right)\Rightarrow\left[\frac{x}{2}\right]\in\{0,1,2\}$

$I=\int_0^2\cos(-\pi x)dx+\int_2^4\cos(\pi-\pi x)dx+\int_4^5\cos(2\pi-\pi x)dx$

$\Rightarrow I=\left[\frac{\sin\pi x}{\pi}\right]_0^2+\left[-\frac{\sin\pi x}{\pi}\right]_2^4+\left[\frac{\sin\pi x}{\pi}\right]_4^5=0$

Q.6. If length of wire is $22\ \text{cm}$ and it is cut into 2 pieces and made into a square and an equilateral triangle, then what will be side of triangle such that combined area of square and triangle is minimum.

A) $\frac{22\sqrt{3}}{3+3\sqrt{3}}$

B) $\frac{22\sqrt{3}}{4+3\sqrt{3}}$

C) $\frac{22}{4+3\sqrt{3}}$

D) $\frac{22\sqrt{3}}{3+4\sqrt{3}}$

Answer: $\frac{22\sqrt{3}}{4+3\sqrt{3}}$

Solution: Let the side of square be $\frac{22-l}{4}$ and side of triangle be $\frac{l}{3}$,

So, total area is given by $\Delta = \frac{\sqrt{3}}{4}\left(\frac{l}{3}\right)^2 + \left(\frac{22-l}{4}\right)^2$

$\Rightarrow \Delta = \frac{1}{12\sqrt{3}} \cdot l^2 + \frac{1}{16}(22-l)^2$

Differentiating with respect to l to get maxima and minima points,

$\Rightarrow \frac{d\Delta}{dl} = \frac{1}{6\sqrt{3}} \cdot l - \frac{1}{8}(22-l) = 0 \Rightarrow l = \frac{66\sqrt{3}}{4+3\sqrt{3}}$

Also $\frac{d^2\Delta}{dl^2} = \frac{1}{6\sqrt{3}} + \frac{1}{8}$ ($+$ve. which is case of minima)

So, side of triangle will be $= \frac{l}{3} = \frac{22\sqrt{3}}{4+3\sqrt{3}}$

Q.7. The domain of $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is

A) $\left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

B) $\left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right)$

C) $\left(-\infty, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$

D) $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

Answer: $\left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

Solution:

$$f(x)=\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$$

Case I: $-1\leq\frac{1}{4x^2-1}\leq 1$

$\Rightarrow 4x^2-1\geq 1$ or $4x^2-1\leq -1$ and $4x^2-1\neq 0$

$\therefore x^2\geq\frac{1}{2}$ or $x^2\leq 0\Rightarrow x=0$ and $x\neq\pm\frac{1}{2}$

$\therefore x\in\left(-\infty,-\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},\infty\right)\cup\{0\}$

Case II: $-1\leq\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\leq 1$

$\Rightarrow -\frac{\pi}{2}\leq\sin^{-1}\left(\frac{1}{4x^2-1}\right)\leq\frac{\pi}{2}$ which is always true.

Hence, $x\in\left(-\infty,-\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},\infty\right)\cup\{0\}$

Q.8. If $A=\left[a_{ij}\right]_{3\times 3}$ where $a_{ij}=2^{j-i}$, where, $i,j\in\{1,2,3\}$, then the value of $A^2+A^3+A^4+\cdots A^{10}$ is

A) $A\left(\frac{3^{10}-1}{2}\right)$

B) $A\left(\frac{3^{10}+1}{2}\right)$

C) $A\left(\frac{3^{10}+3}{2}\right)$

D) $A\left(\frac{3^{10}-3}{2}\right)$

Answer: $A\left(\frac{3^{10}-3}{2}\right)$

Solution: Given, $A=\left[a_{ij}\right]_{3\times 3}$ where $a_{ij}=2^{j-i}$ we get,

$$A=\begin{bmatrix}1 & 2 & 4\\ \frac{1}{2} & 1 & 2\\ \frac{1}{4} & \frac{1}{2} & 1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 2 & 4\\ \frac{1}{2} & 1 & 2\\ \frac{1}{4} & \frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 4\\ \frac{1}{2} & 1 & 2\\ \frac{1}{4} & \frac{1}{2} & 1\end{bmatrix}$$

$$\Rightarrow A^2=\begin{bmatrix}3 & 6 & 12\\ \frac{3}{2} & 3 & 6\\ \frac{3}{4} & \frac{3}{2} & 3\end{bmatrix}=3A\ \Rightarrow A^3=3A^2=9A$$

Now, $A^2+A^3+A^4+\cdots A^{10}$

$=3A+3^2A+3^3A+\cdots 3^9A$

$=3A\left[\frac{3^9-1}{3-1}\right]=A\left[\frac{3^{10}-3}{2}\right]$

Q.9. The area bounded by the curves $y^2=8x, y=2\sqrt{2}x$ outside the triangle by $y=2\sqrt{2}x, y=0, x=1$ is

A) $\frac{\sqrt{3}}{3}$

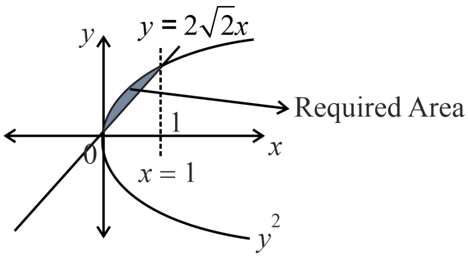
B) $\frac{\sqrt{2}}{3}$

C) $\frac{2}{\sqrt{3}}$

D) $\frac{2}{3}$

Answer: $\frac{\sqrt{2}}{3}$

Solution: Plotting the with given data $y^2 = 8x, y = 2\sqrt{2}x$ and $y = 2\sqrt{2}x, y = 0, x = 1$ we get,



Required Area $= \int_0^1 \left(\sqrt{8x} - 2\sqrt{2}x\right) dx$

$= \left[2\sqrt{2}\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2\sqrt{2}\frac{x^2}{2}\right]_0^1$

$= \frac{4}{3}\sqrt{2} - \sqrt{2}$

$= \left(\frac{\sqrt{2}}{3}\right)$ sq.unit

Q.10. If $(p \vee q) O (p \wedge q \Rightarrow q)$ is a tautology, then which operator should replace O in the given statement?

A) $\vee$

B) $\wedge$

C) $\Leftrightarrow$

D) None of these

Answer: $\vee$

Solution: $\because (p \wedge q) \Rightarrow q \equiv$ ~ $(p \wedge q) \vee q \equiv$ ~$p \vee$ ~$q \vee q \equiv$ ~$p \vee t \equiv t$

So, O can be $\vee$ as $(p \vee q) \vee t \equiv t$

Q.11. The number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$ is equal to

A) 1

B) 2

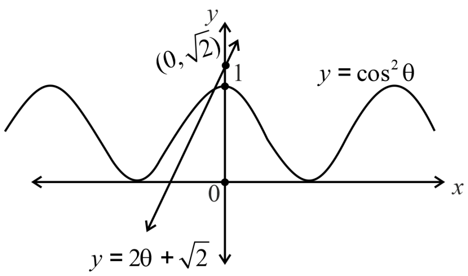
C) 3

D) 4

Answer: 1

Solution: $2\theta-\cos^2\theta+\sqrt{2}=0$

$\Rightarrow\ 2\theta+\sqrt{2}=\cos^2\theta$



As per the graph drawn there is only one point of intersection, so only one solution of the given equation.

Q.12. If $a_0=a_1=0,\ a_{n+2}=2a_{n+1}-a_n+1,\ n\geq 0$, then $\sum_{n=2}^{\infty}\frac{a_n}{7^n}$ is equal to

A) $\frac{21}{16}$

B) $\frac{7}{216}$

C) $\frac{49}{216}$

D) $\frac{16}{49}$

Answer: $\frac{7}{216}$

Solution: Given $a_{n+2}=2a_{n+1}-a_n+1$

Dividing by 7^{n+2}, we get

$\frac{a_{n+2}}{7^{n+2}}=\frac{2}{7}\cdot\frac{a_{n+1}}{7^{n+1}}-\frac{1}{49}\cdot\frac{a_n}{7^n}+\frac{1}{7^{n+2}}$

So, $\sum_{n=2}^{\infty}\frac{a_{n+2}}{7^{n+2}}=\frac{2}{7}\sum_{n=2}^{\infty}\frac{a_{n+1}}{7^{n+1}}-\frac{1}{49}\sum_{n=2}^{\infty}\frac{a_n}{7^n}+\sum_{n=2}^{\infty}\frac{1}{7^{n+2}}$

Let
$\sum_{n=2}^{\infty}\frac{a_n}{7^n}=P$

$\Rightarrow\left(P-\frac{a_3}{7^3}-\frac{a_2}{7^2}\right)=\frac{2}{7}\left(P-\frac{a_2}{7^2}\right)-\frac{1}{49}P+\frac{1}{7^4}\left(\frac{1}{1-\frac{1}{7}}\right)$

$(\because a_2=1\ \&\ a_3=3)$

$\Rightarrow P\left(1-\frac{2}{7}+\frac{1}{49}\right)=\frac{3}{7^3}+\frac{1}{7^2}-\frac{2}{7^3}+\frac{1}{6\cdot 7^3}$

$\Rightarrow\frac{36P}{49}=\frac{1}{42}\Rightarrow P=\frac{7}{216}$

Q.13. A circle C_1 is $x^2+y^2=2$ and another circle C_2 is $(x-3)^2+(y-2)^2=5$. If the tangent at $(1,-1)$ on C_1 intersect at two points on C_2 at A and B and tangents to C_2 at A and B meet at M, then the area of triangle ΔABM is

A) 13.5

B) 13.2

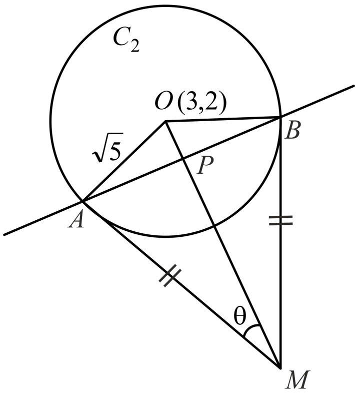
C) 10.5

D) 10.2

Answer: 13.5

Solution: Equation of tangent at $(1,-1)$ on C_1 is given by $x-y=2$ $\quad\ldots(1)$

Plotting graph of intersection of tangent to other circle at points A and B,



Now by perpendicular distance formula we get, $OP=\frac{1}{\sqrt{2}}$

Now in ΔOAP, the length of $AP=\frac{3}{\sqrt{2}}$

Again in ΔOAP, $\tan(90^\circ-\theta)=\frac{AP}{OP}=3\Rightarrow\cot\theta=3$

So, $\sin\theta=\frac{AP}{AM}\Rightarrow AM=3\sqrt{5}$

Now area of $\Delta ABM=\frac{1}{2}AM^2\cdot\sin 2\theta$

$=\left(3\sqrt{5}\right)^2\cdot\sin\theta\cdot\cos\theta$

$=45\left(\frac{3}{10}\right)\ =13.5$

Q.14. The probability of making a singular matrix 2×2 taking elements from the set $\{2,3,5,7,11,13,17,19,23,29\}$ is

A) $\frac{1}{1000}$

B) $\frac{19}{1000}$

C) $\frac{41}{1000}$

D) $\frac{9}{500}$

Answer: $\frac{19}{1000}$

Solution: For a matrix to be singular $\Delta=0$, then

Case I: All entries are same.

Number of matrices $=10$

Case II: When two same prime numbers are used in matrix.

Number of such matrices $={}^{10}C_2 \cdot 2! \cdot 2! = 180$

Total cases will be $10 \times 10 \times 10 \times 10 = 10^4$

So,required probability $=\frac{190}{10^4}=\frac{19}{1000}$

Q.15. If α and β are the roots of the equation $x^2+2i-1=0$, then the value of $\left|\alpha^8+\beta^8\right|$ is equal to

A) 50

B) 14

C) 48

D) 36

Answer: 50

Solution: Since α and β are the roots of the equation $x^2+2i-1=0$

so $\alpha^2=-2i+1=\beta^2$

$\Rightarrow \alpha^4=(-2i+1)^2=4i^2+1-4i=-3-4i$

$\Rightarrow \alpha^8=(-3-4i)^2=-7+24i$

Similarly, $\beta^8=-7+24i$

$\therefore \left|\alpha^8+\beta^8\right|=|-14+48i|=50$