JEE Main Exam 2022 - Session 1

29 June 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. Three point charges each of charge q are arranged at the vertices of an equilateral triangle of side a as shown. The electric field at the centroid of the triangle is $x\frac{kq}{a^2}$. The value of x is

A) 0

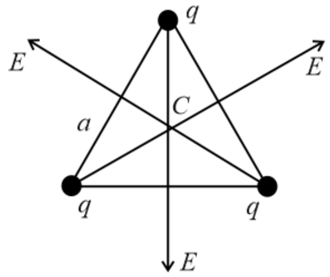
B) 3

C) 2

D) 1

Answer: 0

Solution:



Electric field is a vector quantity. Therefore, the net electric field will be vector sum of electric fields due to all three points. From the symmetry,

$E_{\text{centroid}} = 0$

$\Rightarrow x = 0$

Q.2. A ball can be thrown to a maximum horizontal distance of 100 m. With the same effort, if the ball is thrown vertically upwards. The maximum height attained by the ball is

A) 25 m

B) 50 m

C) 100 m

D) 200 m

Answer: 50 m

Solution: Horizontal range is maximum if the angle of projection is 45° and is given by $R_{\max}=\frac{u^2}{g}$

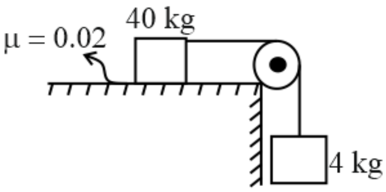
Here, $R_{max}=100\ \mathrm{m}$

Then, $100=\frac{u^2}{g}\quad\cdots(1)$

If the ball is thrown vertically upward, then the ball will attain the maximum height from the ground as $H_{\max}=\frac{u^2}{2g}$

$H_{\max}=\frac{100}{2}=50\ \mathrm{m}$

Q.3. Two blocks are connected with the help of a string as shown below:



If the coefficient of friction between $40\ \mathrm{kg}$ block and horizontal surface is 0.02, then find the acceleration of the system.

A) $\frac{8}{11}\ \mathrm{m\ s^{-2}}$

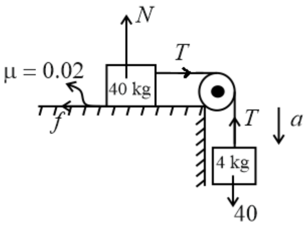
B) $8\ \mathrm{m\ s^{-2}}$

C) $\frac{11}{8}\ \mathrm{m\ s^{-2}}$

D) $10\ \mathrm{m\ s^{-2}}$

Answer: $\frac{8}{11}\ \mathrm{m\ s^{-2}}$

Solution:



For hanging block,

$40-T=4a\quad\cdots(1)$

Now, maximum value of friction force will be,

$f_{max}=\mu N=0.02\times 40\times 10=8\ \mathrm{N}$

For block on table, $T-8=40a\quad\cdots(2)$

From equation(1) and equation (2), we get $32=44a\ \Rightarrow a=\frac{32}{44}=\frac{8}{11}\ \mathrm{m\ s^{-2}}$

Q.4. Two cells of emf ε and internal resistance r are connected in series and then in parallel with external resistance $R=2\ \Omega$. If the current in both the cases is same across the external resistance, what is the value of r?

A) $6\ \Omega$

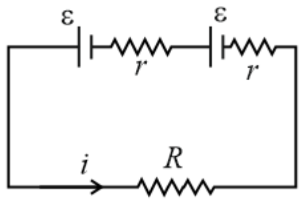
B) $2\ \Omega$

C) $8\ \Omega$

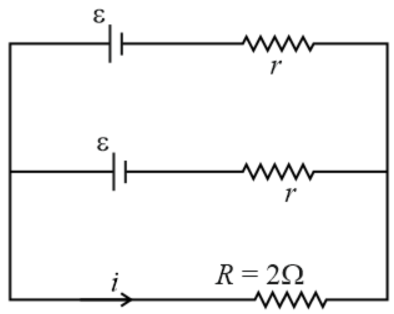
D) $10\ \Omega$

Answer: $2\ \Omega$

Solution:



When the cells are connected in series $i_s = \frac{2\varepsilon}{2r+R} = \frac{2\varepsilon}{2r+2}$



When they are connected in parallel, the emf is $\varepsilon_p = \frac{\frac{\varepsilon}{r}+\frac{\varepsilon}{r}}{\frac{1}{r}+\frac{1}{r}} = \varepsilon$ and the effective internal resistance will be $r_p = \frac{r\times r}{r+r} = \frac{r}{2}$.

Therefore, the current will be,

$i_p = \frac{\varepsilon}{R+\frac{r}{2}} = \frac{\varepsilon}{2+\frac{r}{2}} = \frac{2\varepsilon}{4+r}$

Since,
$i_s = i_p$
$\Rightarrow \frac{2\varepsilon}{2r+2} = \frac{2\varepsilon}{4+r} \Rightarrow r = 2\ \Omega$

Q.5. The initial length of tower is $125\ \mathrm{m}$. How much should be the increase in length of tower to increase the length of radio transmission by 2 times?

A) $375\ \mathrm{m}$

B) $500\ \mathrm{m}$

C) $250\ \mathrm{m}$

D) $650\ \mathrm{m}$

Answer: $375\ \mathrm{m}$

Solution: Maximum distance covered is given by $d = \sqrt{2Rh}$, here R is the radius.

Now, to increase the length of radio transmission to 2 times, h must be made 4 times. Then, $h = 4 \times 125 = 500\ \mathrm{m}$.

Thus, increase in length of tower is $\Delta h = 500 - 125 = 375\ \mathrm{m}$

Q.6. Half life of a radioactive material is 5 years. How much time will it take to reduce the number of active nuclei to 6.25% of initial value?

A) $20\ years$

B) $30\ years$

C) $40\ years$

D) $50\ years$

Answer: $20\ years$

Solution: 6.25% of the initial value means, final number of nuclei should be, $\left(\frac{N_0}{16}\right)$.

Number of nuclei remaining after n half lives is $N=\frac{N_0}{2^n}$

$6.25\% N_0=\frac{N_0}{2^n}$

$\frac{6.25}{100} N_0=\frac{N_0}{2^n} \Rightarrow \frac{625}{10^4}=\frac{1}{2^n}$

$\Rightarrow 2^n=\frac{10^4}{5^4} \Rightarrow 2^n=2^4$

$\Rightarrow n=4$

Therefore, time taken will be, $n \times t_{\frac{1}{2}}=4 \times 5=20\ years$

Q.7. A satellite has time period of $7\ \mathrm{h}$ near the surface of earth(consider distance from centre of earth to satellite as R), find its period of revolution if distance from centre of earth to satellite is increased to $3R$.

A) $36\ \mathrm{h}$

B) $45\ \mathrm{h}$

C) $55\ \mathrm{h}$

D) $25\ \mathrm{h}$

Answer: $36\ \mathrm{h}$

Solution: From Kepler's Third Law(The Law of Periods), we know

$T^2 \propto R^3 \Rightarrow \frac{T_2^2}{T_1^2}=\left(\frac{R_2}{R_1}\right)^3=\left(\frac{3R}{R}\right)^3$ Therefore, $T_2=\sqrt{27} T_1$

$=5.19 \times 7 \approx 36\ \mathrm{h}$

Q.8. Find the length of a simple pendulum if its equation is given by $y=A \sin (\pi t+\pi)$

A) $89\,\mathrm{cm}$

B) $98\,\mathrm{cm}$

C) $97.2\,\mathrm{cm}$

D) $99.4\,\mathrm{cm}$

Answer: $99.4\,\mathrm{cm}$

Solution: Comparing the given equation $y=A \sin (\pi t+\pi)$ with a standard equation $y=A \sin (\omega t+\phi)$. We have

$\omega=\pi\ \mathrm{rad\ s}^{-1}$

Since time period is $T=\frac{2\pi}{\omega}=2$ s, it is a second's pendulum.

$T=2\pi \sqrt{\frac{L}{g}} \Rightarrow L=\frac{g}{\pi^2}=99.4\ \mathrm{cm}$

Q.9. The moment of inertia of a thin uniform rod rotating about the perpendicular axis passing through one end is I_1. The same rod is bent into a ring and its moment of inertia about the diameter is I_2. The ratio $\frac{I_1}{I_2} = \frac{x\pi^2}{3}$, find x.

A) 8

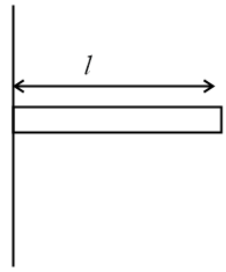
B) 4

C) 5

D) 7

Answer: 8

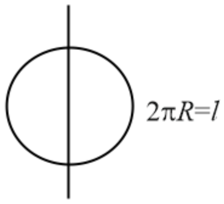
Solution:



Moment of inertia of rod rotating about an axis passing through one end is $I_1 = \frac{ml^2}{3} \quad \cdots(1)$

Moment of inertia of ring rotating about one of its diameters is $I_2 = \frac{mR^2}{2}$.

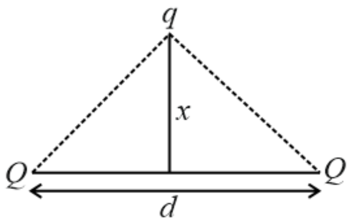
Since the rod is bent into a ring, the circumference of the ring and the length of the rod are equal, $l = 2\pi R$



Then, $I_2 = \frac{mR^2}{2} = \frac{ml^2}{8\pi^2}$

Thus, the ratio $\frac{I_1}{I_2} = \frac{8\pi^2}{3}$, so $x = 8$.

Q.10. There are two charges Q at a distance d apart. A charge q is placed at a distance x from the mid-point of the line joining Q on its perpendicular bisector. Value of x for which q experiences maximum force is



A) $\frac{d}{2}$

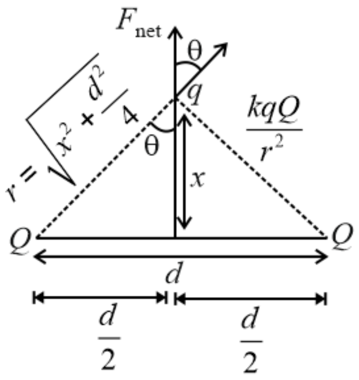
B) $\frac{d}{\sqrt{2}}$

C) d

D) $\frac{d}{2\sqrt{2}}$

Answer: $\frac{d}{2\sqrt{2}}$

Solution:



The distance between q and Q is $r = \sqrt{x^2 + \left(\frac{d}{2}\right)^2} = \sqrt{x^2 + \frac{d^2}{4}}$.

The force between q and Q will be $F = \frac{kqQ}{r^2}$.

From the given figure we can see that only vertical components of the forces will add, and the horizontal components will get cancelled out. Therefore,

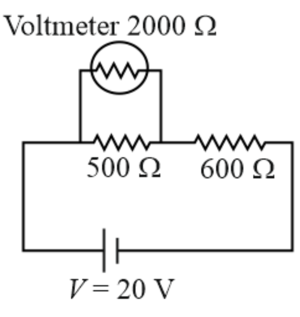
$$F_{net} = \frac{2kqQ}{r^2}\cos\theta$$
$$= \frac{2kqQ}{r^2} \times \frac{x}{r} = \frac{kqQx}{\left(x^2+\frac{d^2}{4}\right)^{\frac{3}{2}}}$$

For force to be maximum, $\frac{\mathrm{d}F_{net}}{\mathrm{d}x} = 0$.

$$\Rightarrow kqQ\left[\frac{\left(x^2+\frac{d^2}{4}\right)^{\frac{3}{2}} - x \times \frac{3}{2}\left(x^2+\frac{d^2}{4}\right)^{\frac{1}{2}} \times 2x}{\left(\left(x^2+\frac{d^2}{4}\right)^{\frac{3}{2}}\right)^2}\right] = 0$$

$$\Rightarrow \left(x^2 + \frac{d^2}{4}\right) = 3x^2 \Rightarrow 2x^2 = \frac{d^2}{4} \Rightarrow x = \frac{d}{2\sqrt{2}}$$

Q.11. In the given circuit diagram, what will be the reading of the voltmeter?



A) $12\ \mathrm{V}$

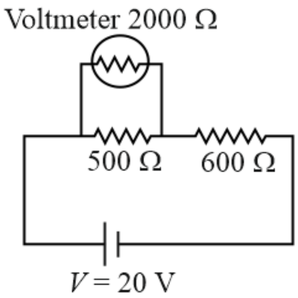
B) $2\ \mathrm{V}$

C) $8\ \mathrm{V}$

D) $20\ \mathrm{V}$

Answer: $8 \mathrm{~V}$

Solution:



The voltmeter resistance is given to be 2000Ω. The equivalent resistance in the given circuit is $R_{eq}=\frac{500 \times 2000}{2000+500}+600=1000 \Omega$.

Now, current in the circuit is $i=\frac{20}{1000} \mathrm{~A}$.

Reading of the voltmeter will be, $V=20-600 \times \frac{20}{1000}=8 \mathrm{~V}$

Q.12. The electric potential V at any point (x, y, z)in space is given by $V=3 x^2$ where, x, y, z all are in metre. The electric field at $(1 \mathrm{~m}, 0 \mathrm{~m}, 4 \mathrm{~m})$.

A) $6 \mathrm{~V} \mathrm{~m}^{-1}$ in positive x direction

B) $3 \mathrm{~V} \mathrm{~m}^{-1}$ in negative x direction

C) $6 \mathrm{~V} \mathrm{~m}^{-1}$ in negative x direction

D) $3 \mathrm{~V} \mathrm{~m}^{-1}$ in positive x direction

Answer: $6 \mathrm{~V} \mathrm{~m}^{-1}$ in negative x direction

Solution: Electric potential $V=3 x^2$

Electric potential and electric field are related as $E=-\frac{\mathrm{d} V}{\mathrm{~d} x}=-6 x$.

At the point $(1,\ 0,\ 4)$,

$\Rightarrow E=-6 \times 1=-6 \mathrm{~V} \mathrm{~m}^{-1}$

Thus, electric field is $6 \mathrm{~V} \mathrm{~m}^{-1}$ along negative x- axis.

Q.13. An object starts moving with constant acceleration a. Distance travelled by it in first t s is $10 \mathrm{~m}$. Then the distance travelled by it in next t s is

A) $10 \mathrm{~m}$

B) $20 \mathrm{~m}$

C) $30 \mathrm{~m}$

D) $40 \mathrm{~m}$

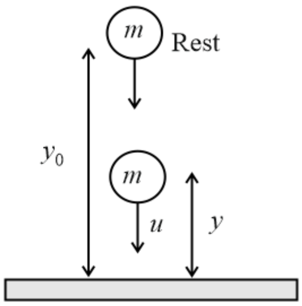
Answer: $30 \mathrm{~m}$

Solution: Initial velocity of the particle is zero. Applying equation of motion, we get $10=\frac{1}{2}\times a\times t^2$

Let distance covered in next t s is x. Then, $10+x=\frac{1}{2}\times a\times(2t)^2$

$\Rightarrow 10+x=\frac{1}{2}at^2\times 4\ \Rightarrow 10+x=10\times 4\ \Rightarrow x=30\ \mathrm{m}$

Q.14. A particle is released from rest from a height of y_0 from ground as shown. When the particle is at a height of y from ground, its kinetic energy is K then



A) $K=mg(y_0-y)$

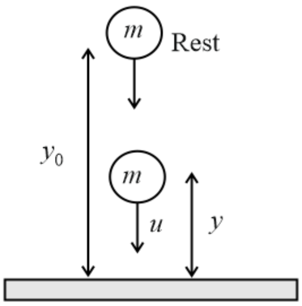
B) $K=mgy$

C) $K=-mgy$

D) $K=mg(y-y_0)$

Answer: $K=mg(y_0-y)$

Solution:



Here, gain in kinetic energy=loss in potential energy

Now loss in potential energy of particle in falling from height y_0 to y is $mg(y_0-y)$.

So, $\Delta KE=K-0=mg(y_0-y)$

Thus, kinetic energy at height y is $K=mg(y_0-y)$

Q.15. A gas expands from V_1 to V_2 by three different processes, i.e., isothermal, adiabatic and isobaric. If W_1, W_2 and W_3 are the work done respectively in the above mentioned processes, then which of the following will be the correct order?

A) $W_3>W_1>W_2$

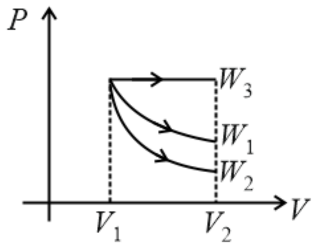
B) $W_3>W_2>W_1$

C) $W_2>W_1>W_3$

D) $W_1>W_3>W_2$

Answer: $W_3 > W_1 > W_2$

Solution: As we know, in isobaric process pressure remains constant and magnitude of slope of adiabatic curve will be more than the isothermal curve at the common point, therefore diagram for process will be as given below.



Also, $W =$ area under $P - V$ curve.

Hence,

$\therefore\ W_3 > W_1 > W_2$

Q.16. In a YDSE if the screen is shifted by 5×10^{-2} m, the fringe width changes by 3×10^{-3} cm. Find wavelength of the light if distance between the slits is 1 mm.

A) 400 nm

B) 600 nm

C) 800 nm

D) 1200 nm

Answer: 600 nm

Solution: We know that the fringe width in YDSE is $\beta = \frac{\lambda D}{d}$

$\Rightarrow \Delta\beta = \frac{\lambda}{d}\Delta D$

$\Rightarrow \lambda = \frac{(\Delta\beta)d}{\Delta D} = \frac{3\times10^{-3}\times10^{-2}\times10^{-3}}{5\times10^{-2}} = 6 \times 10^{-7}\ \mathrm{m} = 600\ \mathrm{nm}$

Q.17. Speed of light for media A and B are $2 \times 10^8\ \mathrm{m\ s^{-1}}$ and $1.5 \times 10^8\ \mathrm{m\ s^{-1}}$ respectively. If a light ray goes from B to A, then the phenomenon of total internal reflection occurs for an angle of incidence θ, such that

A) $\theta > \sin^{-1}\left(\frac{4}{3}\right)$

B) $\theta < \sin^{-1}\left(\frac{4}{3}\right)$

C) $\theta > \sin^{-1}\left(\frac{3}{4}\right)$

D) $\theta < \sin^{-1}\left(\frac{3}{4}\right)$

Answer: $\theta > \sin^{-1}\left(\frac{3}{4}\right)$

Solution: Given: $v_A = 2 \times 10^8 \text{ m s}^{-1}$ and $v_B = 1.5 \times 10^8 \text{ m s}^{-1}$

For critical angle: $\sin(\theta_c) = \frac{\mu_A}{\mu_B} = \frac{c}{v_A} \times \frac{v_B}{c} = \frac{1.5 \times 10^8}{2 \times 10^8} = \frac{3}{4}$

$\Rightarrow \theta_c = \sin^{-1}\left(\frac{3}{4}\right)$

Now for TIR, $\theta > \theta_C$

Q.18. The diameter of a sphere is measured using a vernier caliper scale with least count 0.1 mm. 5^{th} division of vernier scale was found to coincide exactly with one of the main scale and reading on main scale is 1.8 cm. If the given vernier caliper has positive zero error of 0.05 cm, then the radius of the sphere is ______ $\times 10^{-2} \text{ cm}$.

A) 90

B) 180

C) 190

D) 170

Answer: 90

Solution: Reading of the vernier caliper is taken as

$d = \text{MSR} + (\text{LC} \times \text{VSR})\text{-Zero error}$

$\Rightarrow d = 18 + (0.1 \times 5) - 0.5 = 18 \text{ mm}$

Therefore, radius $r = \frac{d}{2} = 9 \text{ mm} = 90 \times 10^{-2} \text{ cm}$

Q.19. Two long parallel wires carrying equal currents which are 8 cm apart produce a magnetic field of $200 \mu\text{T}$ midway between them. The magnitude of the current in each wire is

A) Conductor carry 6 A each in opposite direction

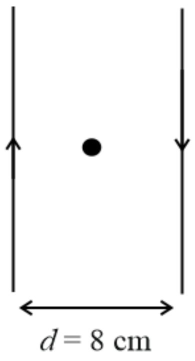
B) Conductor carry 3 A each in same direction

C) Conductor carry 20 A each in opposite direction

D) Conductor carry 6 A each in same direction

Answer: Conductor carry 20 A each in opposite direction

Solution:



Magnetic field in parallel wires is given by $B=\frac{\mu_0 I}{2\pi d}$

At midway, $\text{distance}=\frac{d}{2}$

$\because \frac{\mu_0 I}{\frac{2\pi d}{2}}\times 2=B$

$\Rightarrow \frac{\mu_0}{\pi}\times\frac{2I}{d}=B$

$\Rightarrow \frac{4\times10^{-7}\times 2I}{0.08}=200\times10^{-6}$

$\Rightarrow I=\frac{2\times10^{-4}\times0.08}{2\times4\times10^{-7}}=\frac{1000}{4}\times0.08$

$\Rightarrow 20\text{ A}$

Q.20. A block of mass m is kept inside a box. The box is falling down with acceleration of $\frac{g}{4}$. Find the force exerted by block m on the surface of the box

A) $\frac{3mg}{2}$

B) $\frac{3mg}{4}$

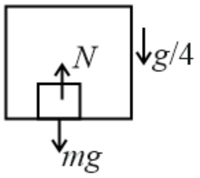
C) $\frac{4mg}{5}$

D) $\frac{3mg}{5}$

Answer: $\frac{3mg}{4}$

Solution: The forces acting on the block are shown below.

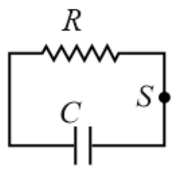
Here, N is normal force and box is falling with acceleration $\frac{g}{4}$.



From figure, $N=m\left(g-\frac{g}{4}\right)$

$=\frac{3mg}{4}$

Q.21. In the shown RC discharging circuit switch S is closed at $t=0$. The circuit takes t_1 time for charge on capacitor to reduce to $\frac{1}{8}^{\text{th}}$ of original value and takes t_2 time for charge to reduce to $\frac{1}{2}$ of the original value. Then $\frac{t_1}{t_2}$ is equal to



A) 2

B) 3

C) $\sqrt{3}$

D) $2\sqrt{2}$

Answer: 3

Solution: We know that in the discharging circuit the charge is given by,

$q=q_0 e^{\frac{-t}{RC}} \Rightarrow \frac{t}{RC}=\ln\left(\frac{q_0}{q}\right)$

$\therefore \frac{t_1}{t_2}=\frac{\ln(8)}{\text{in}(2)}=3 \Rightarrow \frac{t_1}{t_2}=3$

Q.22. If the units of length and force is increased by 4 times, the unit of energy will become n times. The value of n is

A) 16

B) 1

C) 4

D) 8

Answer: 16

Solution: Energy is equivalent to work. Therefore.

$[E]=[\text{force}]\times[\text{length}]$

$\frac{1New}{1Old}=\frac{((4F)\times(4l))}{F\times l}$

$\Rightarrow$ 1 New unit=16 times old unit.

Section B: Chemistry

Q.1. The spin-only magnetic moment of the compound $[MnCl_6]^{4-}$ is

A) 4.89

B) 5.91

C) 2.83

D) 1.73

Answer: 5.91

Solution: $[MnCl_6]^{4-}$

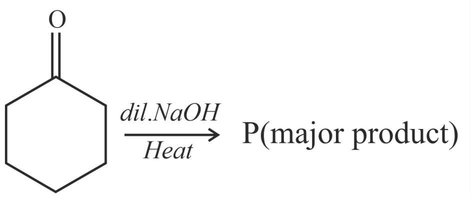
$Mn^{2+}: [Ar]3d^5$

Number of unpaired electrons $= 5$

Magnetic moment of the complex,

$\mu = \sqrt{n(n+2)} = \sqrt{35} = 5.91$ B.M.

Q.2.



Calculate the number of π bonds in product P.

A) 1

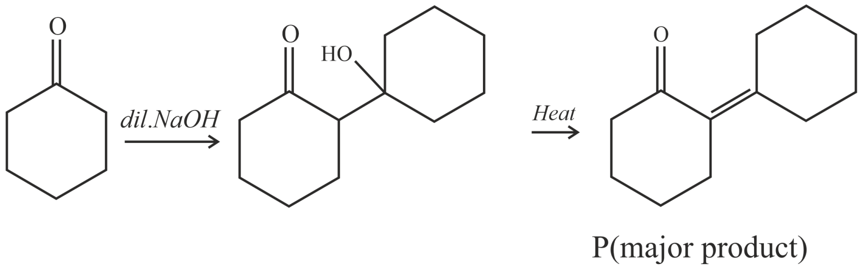
B) 2

C) 3

D) 4

Answer: 2

Solution: In an aldol condensation, an enolate ion reacts with a carbonyl compound in the presence of acid/base catalyst to form a β-hydroxy aldehyde or β-hydroxy ketone, followed by dehydration to give a conjugated enone. It is a useful carbon-carbon bond-forming reaction.



Q.3. Which of the following species have carbonate ion?

A) Washing Soda

B) Caustic Soda

C) Baking Soda

D) All of the above

Answer: Washing Soda

Solution: Washing soda $(\mathrm{Na_2CO_3.10H_2O})$ contains carbonate ion. Baking soda $(\mathrm{NaHCO_3})$ contains bicarbonate ion. Caustic soda is NaOH.

Q.4. Dichlorodiphenyltrichloroethane act as:

A) Antiseptic

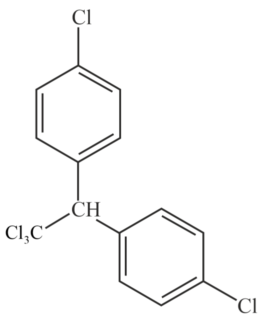
B) Disinfectant

C) Pesticide

D) Water softner

Answer: Pesticide

Solution: Dichlorodiphenyltrichloroethane, commonly known as DDT, is a colorless, tasteless, and almost odorless crystalline chemical compound, an organochloride. Originally developed as an insecticide or pesticide, it became infamous for its environmental impacts.



Q.5. Consider the following calculation:

$\frac{0.002858 \times 0.112}{0.5702} = \mathrm{X}$

What is the value of X?

A) 0.00056

B) 0.000561

C) 0.000563

D) 0.0005

Answer: 0.000561

Solution: In multiplication and division, the result must be reported with no more significant figures as in the measurement with few significant figures

$\frac{0.002858 \times 0.112}{0.5702} = 0.000561$

Q.6. How many chiral alcohols have molecular formula $C_4H_{10}O$ (including stereoisomers)?

A) 1

B) 2

C) 3

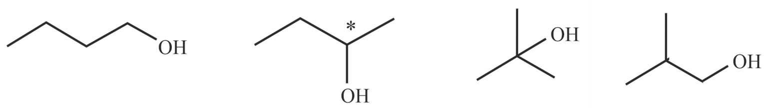
D) 4

Answer: 2

Solution: Molecular formula $= C_4H_{10}O$

Degree of unsaturation $= (C+1) - \frac{H+X-N}{2} = 5 - \frac{10}{2} = 0$

Possible alcohols are:



2-butanol exhibits optical isomerism, hence, two stereoisomers need to be considered.

Q.7. What is formed by the mixture of chloroxylenol and terpineol?

A) Disinfectant

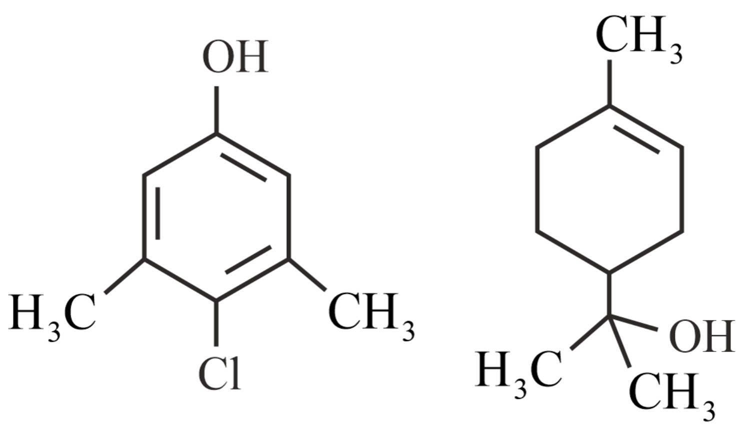
B) Antibiotic

C) Antiseptic

D) Antacid

Answer: Antiseptic

Solution: Commonly used antiseptic, dettol is a mixture of chloroxylenol and terpineol.



Chloroxylenol Terpineol

Q.8. Which of the following is correct when aniline undergoes Friedel Crafts reaction?

A) Product is aniline

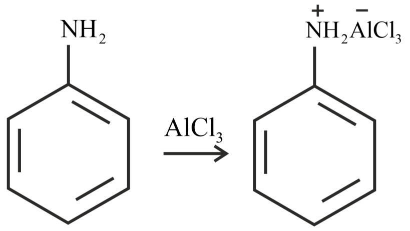
B) Product is substituted aniline

C) Product is amide

D) Friedel craft reaction does not occur

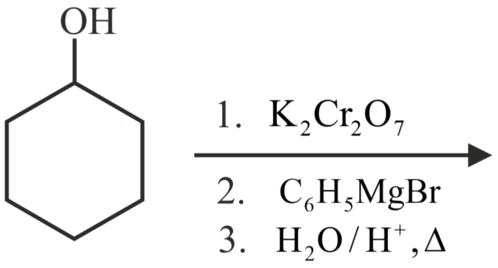
Answer: Friedel craft reaction does not occur

Solution: Aniline does not undergo Friedel-crafts reaction due to salt formation with aluminium chloride, the Lewis acid, which is used as a catalyst, whereas aniline is a strong base. So, an acid-base reaction will take place. Thus, aniline reacts with $AlCl_3$ to form a salt.



Due to the positive charge on the nitrogen atom (shows $-I$ effect and it is meta directing), electrophilic substitution in the benzene ring is deactivated. Hence, the Friedel-Crafts reaction will not take place in aniline.

Q.9.



Number of sp^2 hybridized carbons present in the product of the above reaction is

A) 6

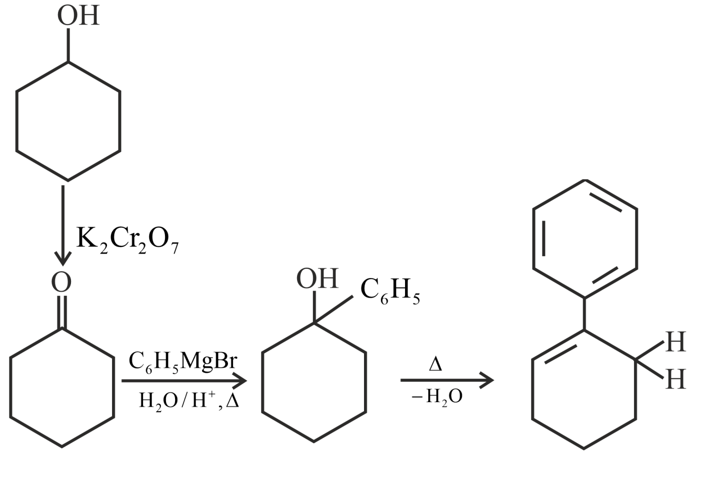
B) 7

C) 8

D) 5

Answer: 8

Solution:



Q.10. The shape and number of lone pairs present in one molecule of BrF_3 are respectively:

A) Tetrahedral, five

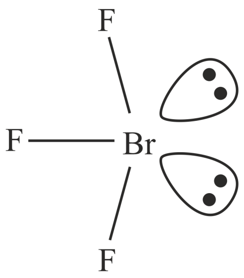
B) T-shape, two

C) Trigonal planar, zero

D) T-shape, eleven

Answer: T-shape, eleven

Solution:



Bromine undergo sp^3d hybridization and the shape of the molecule is T-shape. Each fluorine is having 3 lone pairs and bromine is having 2 lone pairs. Hence, total 11 lone pairs are there.

Q.11. Statement I: Dacron is a polymer.

Statement II: It is a condensation polymer of terephthalic acid and ethylene glycol.

A) Both statements are correct and statement II is correct explanation

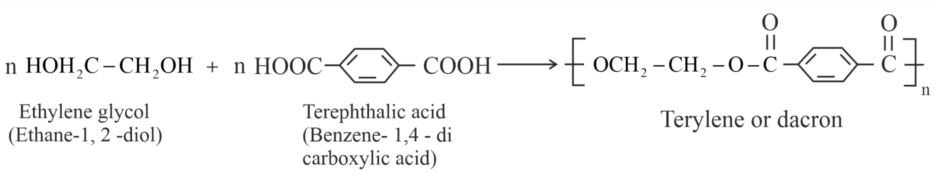
B) Both statements are correct and statement II is not the correct explanation

C) Statement I is true and statement II is false

D) Statement I is false and statement II is true

Answer: Both statements are correct and statement II is correct explanation

Solution: The formation of terylene or Dacron by the interaction of ethylene glycol and terephthalic acid is an example of this type of polymerisation.



Q.12. CH_4, NH_4^+, BH_4^- which statement is true about them

A) they are not isoelectronic species

B) 2 of them are isoelectronic and all tetrahedral structure

C) all are isoelectronic and tetrahedral structure

D) All are isoelectronic and 2 are tetrahedral

Answer: all are isoelectronic and tetrahedral structure

Solution: All the three species given are isoelectronic, and the central atom shows sp^3 hybridization. Hence, they have tetrahedral structure.

Q.13. Which of the following reaction is most likely responsible for acid rain?

A) $H_2S + O_2 \rightarrow S + H_2O$

B) $S + NaOH \rightarrow Na_2S + Na_2S_2O_3 + H_2O$

C) $I_2 + Na_2S_2O_3 \rightarrow Na_2S_4O_6 + NaI$

D) $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$

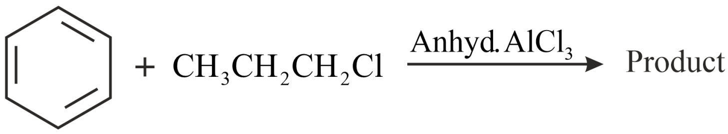
Answer: $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$

Solution: Acid rain is a by product of a variety of human activities that emit the oxides of sulphur and nitrogen in the atmosphere. As mentioned earlier, burning of fossil fuels (which contain sulphur and nitrogenous matter) such as coal and oil in power stations and furnaces or petrol and diesel in motor engines produce sulphur dioxide and nitrogen oxides. SO_2 and NO_2 after oxidation and reaction with water are major contributors to acid rain, because polluted air usually contains particulate matter that catalyse the oxidation.

$2SO_2(g) + O_2(g) + 2H_2O(l) \rightarrow 2H_2SO_4(aq)$

$4NO_2(g) + O_2(g) + 2H_2O(l) \rightarrow 4HNO_3(aq)$

Q.14.



The stable intermediate formed during the reaction is:

A) $CH_3CH_2\overset{+}{C}H_2$

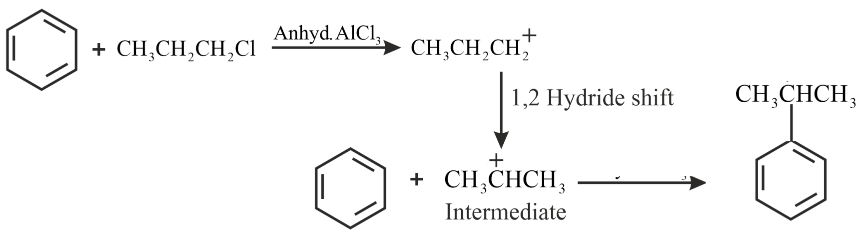
B) $CH_3\overset{+}{C}HCH_3$

C) $CH_3CH_2\overset{-}{C}H_2$

D) $CH_3\overset{-}{C}HCH_3$

Answer: $CH_3\overset{+}{C}HCH_3$

Solution:



Q.15. Match the following

P	Siderite	I	$ZnCO_3$
Q	Malachite	II	ZnS
R	Sphalerite	III	$Cu_2(OH)_2CO_3$
S	Calamine	IV	$FeCO_3$

A) P-IV, Q-III, R-I, S-II

B) P-IV, Q-III, R-II, S-I

C) P-I, Q-II, R-III, S-IV

D) P-III, Q-IV, R-I, S-II

Answer: P-IV, Q-III, R-II, S-I

Solution: Siderite - $FeCO_3$

Malachite is a copper carbonate hydroxide mineral with the formula $Cu_2CO_3(OH)_2$

Sphalerite is a sulphide mineral with the chemical formula ZnS

Calamine is $ZnCO_3$

Q.16. A Vessel contain 16 gm of hydrogen and 128 gm of Oxygen at standard temperature and pressure then the volume of vessel in cm^3 is

A) $2.72 \times 10^3\ cm^3$

B) $2.72 \times 10^5\ cm^3$

C) $2.72 \times 10^4\ cm^3$

D) $2.72 \times 10^2\ cm^3$

Answer: $2.72 \times 10^5\ cm^3$

Solution: One mole of a gas occupies a volume of 22.7 litres at STP

Number of moles of H_2, $n_{H_2} = \frac{16}{2} = 8$

Number of moles of O_2, $n_{O_2} = \frac{128}{32} = 4$

Total number of moles of the gas $= 12$

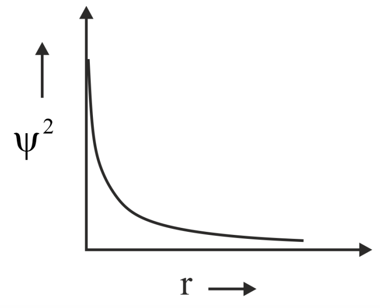
Total volume $= 22.7 \times 12$ in litres

Total volume in $cm^3 = 22.7 \times 12 \times 1000$

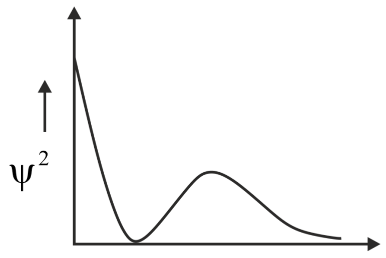
$= 2.72 \times 10^5\ cm^3$

Q.17. Ψ^2 vs r plot for $2s$ orbital of single-electron species.

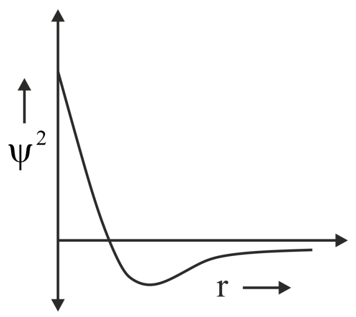
A)



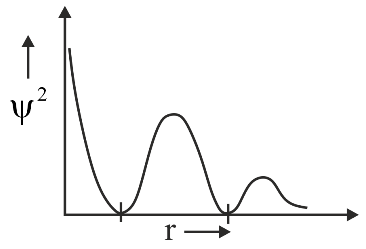
B)



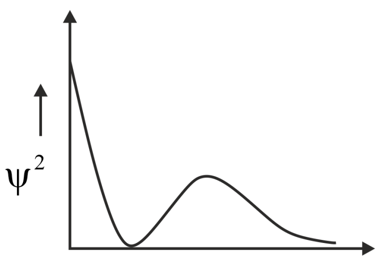
C)



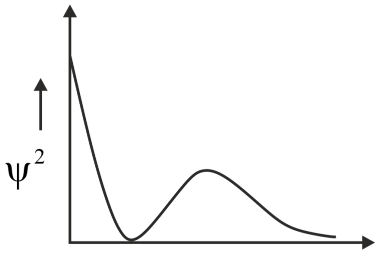
D)



Answer:



Solution: Ψ^2 vs r plot for 2s orbital of single-electron species will be



Here Ψ^2 represent probability density of finding an electron wave at a position around nucleus. The 2s orbital will also have a nodal plane.

$n - l - 1 = 2 - 0 - 1 = 1$

Q.18. Statement I: Ionization energy of oxygen is less than that of nitrogen.

Statement II: $2p$ subshell of nitrogen is half filled.

A) Both statements are correct and statement II is correct explanation of statement I

B) Both statements are correct and statement II is not the correct explanation for statement I

C) Statement I is incorrect and statement II is correct

D) Statement I is correct and statement II is correct

Answer: Both statements are correct and statement II is correct explanation of statement I

Solution: Ionization energy of oxygen is less than that of nitrogen because $2p$ subshell of nitrogen is half filled, and half filled orbitals have some extra stability, so removing electrons from them will be little more difficult.

$N = 1s^2 2s^2 2p^3$

$O = 1s^2 2s^2 2p^4$

Q.19. The temperature dependence of rate constant in a reaction is given by $k = Ae^{\frac{-2600}{T}}$. The activation energy is: [Symbols have their usual meanings]

A) $15.48\ kJ/mol$

B) $21.62\ kJ/mol$

C) $35.82\ kJ/mol$

D) $42.63\ kJ/mol$

Answer: $21.62\ kJ/mol$

Solution: From Arrhenius equation,

by $k = Ae^{\frac{-2600}{T}}$

Comparing,

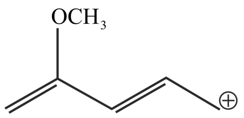
$\frac{Ea}{R} = 2600$

$Ea = 2600 \times 8.314 = 21616.4 \text{ J/mol}$

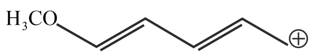
The activation energy is 21.62 kJ/mol.

Q.20. Most stable carbocation is:

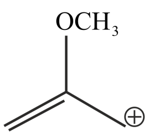
A)



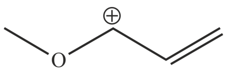
B)



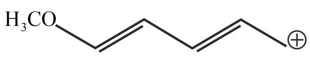
C)



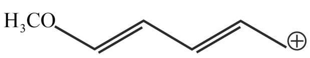
D)



Answer:



Solution:



The above carbocation is a more stable carbocation due to the presence of continuous conjugation and due to +M effect of methoxy group.

Q.21. Elevation in boiling point of 1.5 molal solution is $4K$ and depression in freezing point of 4.5 molal solution of same salt is $4K$. Find the ratio $\frac{K_b}{K_f}$ of the salt.

A) $2:1$

B) $4:1$

C) $3:1$

D) $3:2$

Answer: $3:1$

Solution: Elevation in boiling point $\Delta T_b = ik_b m_1$

Depression in freezing point $\Delta T_f = ik_f m_2$

$4 = ik_b(1.5)$

$4 = ik_f(4.5)$

Dividing both:

$\frac{k_b}{k_f} = \frac{3}{1}$

Q.22. Which structure of protein does not get affected on heating?

A) Primary structure

B) Secondary structure

C) Tertiary structure

D) Quaternary structure

Answer: Primary structure

Solution: Primary structure of proteins: Proteins may have one or more polypeptide chains. Each polypeptide in a protein has amino acids linked with each other in a specific sequence and it is this sequence of amino acids that is said to be the primary structure of that protein.

Secondary structure of proteins: The secondary structure of protein refers to the shape in which a long polypeptide chain can exist.

Tertiary structure of proteins: The tertiary structure of proteins represents overall folding of the polypeptide chains i.e., further folding of the secondary structure.

Quaternary structure of proteins: Some proteins are composed of two or more polypeptide chains referred to as subunits. The spatial arrangement of these subunits with respect to each other is known as quaternary structure.

During denaturation Quaternary, secondary and tertiary structures are destroyed, but primary structure remains intact.

Section C: Mathematics

Q.1. $\lim_{x \to 1} \frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to

A) $2\pi^2$

B) π^2

C) $3\pi^2$

D) $\frac{\pi^2}{2}$

Answer: π^2

Solution: $\lim_{x \to 1} \frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$

$= \lim_{x \to 1} \frac{(x^2-1)\sin^2(\pi x)}{(x^2-1)(x^2-2x+1)}$

$= \lim_{h \to 0} \frac{\sin^2(\pi(1+h))}{(1+h-1)^2}$

$= \lim_{h \to 0} \frac{\sin^2 \pi h}{h^2} = \lim_{h \to 0} \left(\frac{\sin \pi h}{\pi h}\right)^2 \pi^2 = \pi^2$

Q.2. If the line $\frac{x-2}{3} = \frac{y-2}{4} = \frac{z+6}{2}$ intersects the plane $2x + 4y + 3z = 0$ at point P, then the distance OP (where O is origin) is

A) $\frac{\sqrt{8069}}{7}$

B) $\frac{\sqrt{9053}}{14}$

C) $\frac{\sqrt{7084}}{7}$

D) $\frac{\sqrt{9017}}{14}$

Answer: $\frac{\sqrt{9053}}{14}$

Solution: $\frac{x-2}{3} = \frac{y-2}{4} = \frac{z+6}{2} = \lambda$

Let a point on the given line be $(3\lambda + 2, 4\lambda + 2, 2\lambda - 6)$

This point lies on the plane, i.e. $2(3\lambda + 2) + 4(4\lambda + 2) + 3(2\lambda - 6) = 0$

$\Rightarrow 28\lambda = 6 \Rightarrow \lambda = \frac{3}{14}$

So, the point is $\left(\frac{37}{14}, \frac{40}{14}, -\frac{78}{14}\right)$

Hence, $OP = \sqrt{\left(\frac{37}{14}\right)^2 + \left(\frac{40}{14}\right)^2 + \left(-\frac{78}{14}\right)^2} = \frac{\sqrt{9053}}{14}$

Q.3. If $\sin x = \cos^2 x$, then the number of solutions satisfying the equation in $x \in (0, 10)$ are

A) 4

B) 2

C) 0

D) 3

Answer: 4

Solution: Given, $\sin x=\cos^2 x$

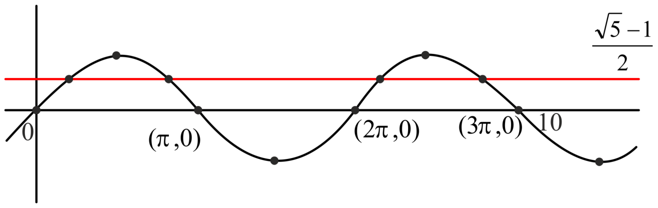
$\Rightarrow \sin x=1-\sin^2 x$

$\Rightarrow \quad \sin^2 x+\sin x-1=0$

$\Rightarrow \quad \sin x=\frac{\left(-1 \pm \sqrt{5}\right)}{2}$

$\Rightarrow \quad \sin x=\frac{\left(\sqrt{5}-1\right)}{2}$

Now plotting the graph we get,



$\Rightarrow$ Now by graph number of solutions in $(0,10)=4$

Q.4. If matrix $A=\begin{bmatrix}2 & -1\\ 0 & 2\end{bmatrix}$, then the modulus of sum of all elements of matrix B which is satisfying $B=I-{}^5C_1 \operatorname{adj}(A)+{}^5C_2(\operatorname{adj}(A))^2-{}^5C_3(\operatorname{adj}(A))^3+{}^5C_4(\operatorname{adj}(A))^4-{}^5C_5(\operatorname{adj}(A))^5$ is

A) 7

B) 2

C) 14

D) 8

Answer: 7

Solution: Simplifying the given expression we get,

$B=I-{}^5C_1\operatorname{adj}(A)+{}^5C_2(\operatorname{adj}(A))^2-{}^5C_3(\operatorname{adj}(A))^3+{}^5C_4(\operatorname{adj}(A))^4-{}^5C_5(\operatorname{adj}(A))^5$

$\Rightarrow\ B=(I-\operatorname{adj}(A))^5$

Now, $A=\begin{bmatrix}2 & -1\\0 & 2\end{bmatrix}\Rightarrow \operatorname{adj}(A)=\begin{bmatrix}2 & 1\\0 & 2\end{bmatrix}$

So, $B=\begin{bmatrix}-1 & -1\\0 & -1\end{bmatrix}^5$

$\Rightarrow\ -B=\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}^5=\begin{bmatrix}1 & 5\\0 & 1\end{bmatrix}$

$\Rightarrow B=\begin{bmatrix}-1 & -5\\0 & -1\end{bmatrix}$

Now, modulus of sum of elements of matrix $B=|-1-5-1|=|-7|=7$

Q.5. In the expansion of $\left(2x^{\frac{1}{5}}-\frac{1}{x^{\frac{1}{5}}}\right)^{15}$, coefficients of x^{-1} and x^{-3} are m and n respectively. If $m\cdot n^2={}^{15}C_r\cdot 2^r$, then the value of r is

A) 5

B) 10

C) 4

D) 6

Answer: 5

Solution: The r^{th} term of $\left(2x^{\frac{1}{5}}-\frac{1}{x^{\frac{1}{5}}}\right)^{15}$

$\Rightarrow T_{r+1}={}^{15}C_r\left(2x^{\frac{1}{5}}\right)^{15-r}\left(\frac{-1}{x^{\frac{1}{5}}}\right)^r$

For term having x^{-1}

$\frac{15-r}{5}-\frac{r}{5}=-1\Rightarrow r=10\Rightarrow m={}^{15}C_{10}\cdot 2^5\cdot(-1)^{10}$

For term having x^{-3}

$\frac{15-2r}{5}=-3\Rightarrow r=15\Rightarrow n={}^{15}C_{15}\cdot 2^0\cdot(-1)^{15}$

$\therefore mn^2={}^{15}C_{10}\cdot 2^5={}^{15}C_5\cdot 2^5$

On comparing with $m\cdot n^2={}^{15}C_r\cdot 2^r$, we get $r=5$

Q.6. If two $\mathrm{A.P.}$'s are given as $3,6,9,\cdots$ upto 78 terms and $5,9,13,\cdots$ upto 59 terms, then the sum of common terms between them will be

A) 2223

B) 2322

C) 2232

D) 2323

Answer: 2223

Solution: Given,

$3, 6, 9, \cdots$ upto 78 terms

$\Rightarrow t_{78} = 3 + 77 \times 3 = 234$

$5, 9, 13, \cdots$ upto 59 terms

$\Rightarrow t_{59} = 5 + 58 \times 4 = 237$

Common difference of common terms $= \text{LCM}\{3, 4\} = 12$

First common term is 9 and last common term is 225

So series will be $9, 21, 33, \cdots, 225 \ \Rightarrow n = 19$

$S = \frac{n}{2}[a + l] = \frac{19}{2}[9 + 225] = 2223$

Q.7. If sum of series is given by $S = 1 + \frac{5}{6} + \frac{10}{6^2} + \frac{16}{6^3} + \cdots$, then the value of S is

A) $\frac{16}{216}$

B) $\frac{301}{125}$

C) $\frac{25}{216}$

D) $\frac{276}{125}$

Answer: $\frac{276}{125}$

Solution: Solving the given expression we get,

$S = 1 + \frac{5}{6} + \frac{10}{6^2} + \frac{16}{6^3} + \cdots$ ....(i)

$\Rightarrow \frac{S}{6} = \frac{1}{6} + \frac{5}{6^2} + \frac{10}{6^3} + \cdots$ ....(ii)

Now subtracting equation (ii) from (i) we get,

$\Rightarrow \frac{5S}{6} = 1 + \frac{4}{6} + \frac{5}{6^2} + \frac{6}{6^3} + \cdots$

$\Rightarrow \left(\frac{5S}{6} - 1\right) = \frac{4}{6} + \frac{5}{6^2} + \frac{6}{6^3} + \cdots$ .....(iii)

$\Rightarrow \frac{1}{6}\left(\frac{5S}{6} - 1\right) = \frac{4}{6^2} + \frac{5}{6^3} + \cdots$ ....(iv)

Now subtracting equation (iv) from (iii) we get,

$\Rightarrow \frac{5}{6}\left(\frac{5S}{6} - 1\right) = \frac{4}{6} + \frac{1}{6^2} + \frac{1}{6^3} + \cdots$

$\Rightarrow \frac{5}{6}\left(\frac{5S}{6} - 1\right) = \frac{4}{6} + \frac{\frac{1}{36}}{1 - \frac{1}{6}}$

$\Rightarrow \frac{5}{6}\left(\frac{5S}{6} - 1\right) = \frac{4}{6} + \frac{1}{30} = \frac{21}{30}$

$\Rightarrow \left(\frac{5S}{6} - 1\right) = \frac{21 \times 6}{30 \times 5} = \frac{21}{25}$

$\Rightarrow \frac{5S}{6} = \frac{21}{25} + 1 = \frac{46}{25}$

$\Rightarrow S = \frac{46 \times 6}{25 \times 5} = \frac{276}{125}$

Q.8. The number of complex numbers z satisfying $|z| = 3$ and $\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$ are

A) 0

B) 1

C) 2

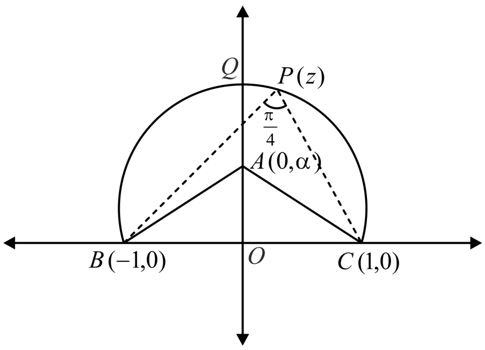
D) 3

Answer: 0

Solution: Given $|z| = 3$

$\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$

$\Rightarrow \arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$



$\Rightarrow$ z is on major arc of a circle having BC as chord and $A\ (0, \alpha)$ as centre of the circle

So $\angle OAC = \angle BPC = \frac{\pi}{4} \Rightarrow OA = OC = 1 = \alpha$

Radius $= AC = \sqrt{2}$

$OQ = \alpha +$ Radius $= 1 + \sqrt{2}$

$\Rightarrow Q = \left(0, 1 + \sqrt{2}\right)$

$|z| = 3$ represents a circle of radius 3 and centre at $(0, 0)$

i.e. both these circles do not intersect.

Hence, no z satisfies both the equation.

Q.9. Number of four-digit numbers in which first three digit number is divisible by last digit i.e. the fourth digit is

A) 2545

B) 2430

C) 2445

D) 2145

Answer: 2545

Solution: Let the four-digit number be $abcd$

Total possible numbers abc will be $9 \times 10 \times 10 = 900$

The last digit is d so there are $\left[\frac{900}{d}\right]$ possibilities for first three digits.

Total number of 4 digit numbers $= \left[\frac{900}{1}\right] + \left[\frac{900}{2}\right] + \left[\frac{900}{3}\right] + \left[\frac{900}{4}\right] + \left[\frac{900}{5}\right] + \left[\frac{900}{6}\right] + \left[\frac{900}{7}\right] + \left[\frac{900}{8}\right] + \left[\frac{900}{9}\right]$

$= 900 + 450 + 300 + 225 + 180 + 150 + 128 + 112 + 100$

$= 2545$

Q.10. ΔABC is inscribed in a circle $x^2 - \sqrt{2}x + y^2 = 0$ where $\angle ABC = \frac{\pi}{2}$, then the maximum area of triangle ABC is

A) $\frac{1}{2}$

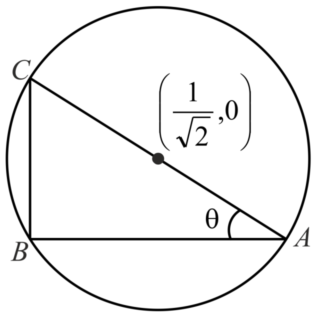
B) 2

C) $\frac{\sqrt{3}}{2}$

D) 4

Answer: $\frac{1}{2}$

Solution: $x^2 - \sqrt{2}x + y^2 = 0 \Rightarrow \left(x - \frac{1}{\sqrt{2}}\right)^2 + y^2 = \frac{1}{2}$



Centre $\equiv \left(\frac{1}{\sqrt{2}}, 0\right), \ r = \frac{1}{\sqrt{2}}$

$AC = \sqrt{2}$

$\operatorname{ar}(\Delta ABC) = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times \sqrt{2}\cos\theta \times \left(\sqrt{2}\sin\theta\right)$

$= \frac{1}{2} \times \sin 2\theta$

$\max\{\operatorname{ar}(\Delta ABC)\} = \frac{1}{2}$

{As maximum value of $\sin 2\theta = 1$ }

Q.11. If the angle of elevation of the top of a tower from the top and bottom of a pole are 30° and 60° respectively and the height of pole is 20 m, then the height of the tower is (in meters)

A) 35

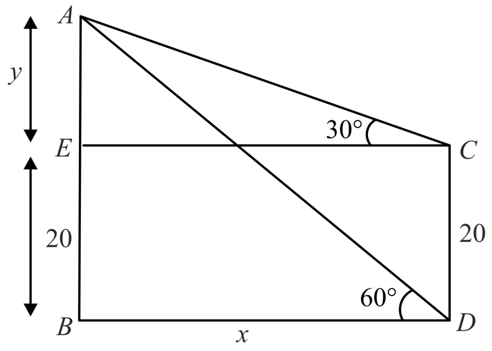
B) 30

C) 40

D) 45

Answer: 30

Solution: Plotting the diagram we get,



In ΔABD

$\tan 60^{\circ}=\frac{20+y}{x} \quad \ldots(\mathrm{i})$

In ΔAEC

$\tan 30^{\circ}=\frac{y}{x} \quad \ldots(\mathrm{ii})$

From equation (i) & $(\mathrm{ii}), \frac{(\mathrm{i})}{(\mathrm{ii})}$ we get, $\frac{\tan 60^{\circ}}{\tan 30^{\circ}}=\frac{20+y}{y} \Rightarrow \frac{\sqrt{3}}{\sqrt{1/3}}=\frac{20+y}{y}$

$\Rightarrow \frac{20+y}{y}=3 \Rightarrow y=10$

So, height of tower $=20+y=30 \mathrm{~m}$

Q.12. If α is the root of equation $x^4+x^2+1=0$, then the value of $\alpha^{1011}+\alpha^{2022}-\alpha^{3033}$ is

A) $1+2\alpha$

B) α

C) 1

D) $1+\alpha$

Answer: 1

Solution: Given, $x^4+x^2+1=0$

α is a root of $\alpha^4+\alpha^2+1=0$

$\Rightarrow \alpha^2=\omega$ or ω^2

Now simplifying $\alpha^{1011}+\alpha^{2022}-\alpha^{3033}$ we get,

$=\alpha \cdot\left(\alpha^2\right)^{505}+\left(\alpha^2\right)^{1011}-\alpha \cdot\left(\alpha^2\right)^{1516}$

$=\alpha \cdot \omega^{505}+\omega^{1011}-\alpha \cdot \omega^{1516}$

$=\alpha \omega+1-\alpha \omega=1$

Q.13. The probability that the relation $\{x, y\} \rightarrow\{x, y\}$ is symmetric and transitive both, is

A) $\frac{1}{4}$

B) $\frac{3}{8}$

C) $\frac{5}{16}$

D) $\frac{1}{8}$

Answer: $\frac{5}{16}$

Solution: Total number of elements in the cartesian product will be 4, so total possible relations will be $2^4 = 16$

The relations which will be symmetric and transitive both will be
$\{\},\ \{x,x\},\ \{y,y\},\ \{(x,x),(y,y)\},\ \{(x,x),(x,y),(y,x)\,(y,y)\}$

i.e. 5 relations

Hence, required probability will be $\frac{5}{16}$