

JEE Main Exam 2022 - Session 2

25 Jul 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. A projectile is thrown at some angle θ . If the range of the projectile is equal to the maximum height attained by it, find the value of $\tan(\theta)$

- A) 4
- B) $\frac{1}{4}$
- C) 2
- D) $\frac{1}{2}$

Answer: 4

Solution: The range and the maximum height of the projectile are given as below,

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin \theta \cos \theta}{g}$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

Therefore,

$$\begin{aligned} \text{As } H &= R \\ \Rightarrow \tan \theta &= 4 \end{aligned}$$

Q.2. The length of a seconds pendulum when it is at height $2R$ from the Earth's surface is

- A) $\frac{2}{9}$ m
- B) $\frac{1}{9}$ m
- C) $\frac{4}{9}$ m
- D) $\frac{5}{9}$ m

Answer: $\frac{1}{9}$ m



Solution: The acceleration due to gravity at height $2R$ from the surface of the Earth will be,

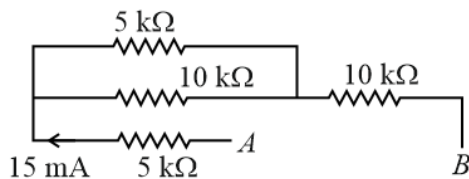
$$\begin{aligned} g' &= \frac{gR^2}{(R+h)^2} \\ &= \frac{gR^2}{(R+2R)^2} \\ &= \frac{g}{9} \end{aligned}$$

Length of the second's pendulum at the surface of the Earth is 1 m.

Since, time period of a pendulum is given by,

$$\begin{aligned} T &= 2\pi\sqrt{\frac{l}{g}} \\ \Rightarrow \frac{T'}{T} &= \sqrt{\frac{l'}{l} \times \frac{g}{g'}} \\ \Rightarrow 1 &= \sqrt{\frac{l'}{l} \times \frac{g}{g'}} \\ \Rightarrow l' &= \frac{l}{9} = \frac{1}{9} \text{ m} \end{aligned}$$

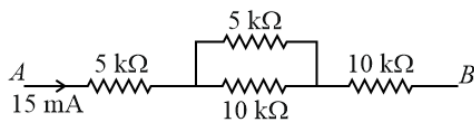
Q.3. In the circuit shown, find potential difference across point A and B .



- A) 275 V
- B) 27.5 V
- C) 40 V
- D) 30 V

Answer: 275 V

Solution: The circuit can be redrawn as

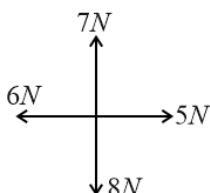


Therefore, equivalent resistance between point A and B is $R_{AB} = 5 + \frac{10 \times 5}{10 + 5} + 10 = \frac{55}{3} \text{ k}\Omega$

Therefore, the potential difference between A and B

$$V_{AB} = (15 \times 10^{-3}) \times \left(\frac{55}{3} \times 10^3\right) \text{ V} = 275 \text{ V}$$

Q.4. Find the force required so as to make net force zero.



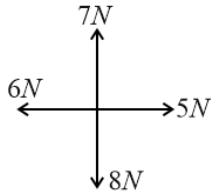
- A) $\sqrt{2} \text{ N}$, 45° with x -axis



- B) $\sqrt{3}$ N, 45° with x -axis
 C) 2 N, 60° with x -axis
 D) 4 N, 30° with x -axis

Answer: $\sqrt{2}$ N, 45° with x -axis

Solution:



Vector Sum of the force shown $= (5 - 6)\hat{i} + (7 - 8)\hat{j} = (-\hat{i} - \hat{j})$ N

Force required to make net force zero $\vec{F} = -(-\hat{i} - \hat{j}) = (\hat{i} + \hat{j})$ N.

Therefore, the required force is $F = \sqrt{1^2 + 1^2} = \sqrt{2}$ N at $\tan \theta = \frac{1}{1} \Rightarrow \theta = 45^\circ$ with x -axis

Q.5. An electron moving with kinetic energy 0.1 keV enters perpendicularly to the field 10^{-4} T. Find frequency of revolution.

- A) 2.8×10^6 Hz
 B) 4.1×10^6 Hz
 C) 6.0×10^6 Hz
 D) 7.0×10^6 Hz

Answer: 2.8×10^6 Hz

Solution: Frequency of revolution is given by, $f = \frac{qB}{2\pi m}$

$$= \frac{(1.6 \times 10^{-19}) \times 10^{-4}}{2 \times 3.14 \times (9.1 \times 10^{-31})}$$

$$\Rightarrow f = 2.799 \times 10^6 \text{ Hz}$$

$$\Rightarrow f \approx 2.8 \times 10^6 \text{ Hz}$$

Q.6. Find the amplitude of magnetic field B_0 , if amplitude of electric field is $E_0 = 540 \text{ N C}^{-1}$.

(speed of light, $c = 3 \times 10^8 \text{ m s}^{-1}$)

- A) 18×10^{-7} T
 B) 36×10^{-8} T
 C) 18×10^{-8} T
 D) 36×10^{-7} T

Answer: 18×10^{-7} T

Solution: Speed of light in terms of amplitude of electric and magnetic field can be written as,

$$\begin{aligned} B_0 &= \frac{E_0}{c} \\ &= \frac{540}{3 \times 10^8} \\ &= 18 \times 10^{-7} \text{ T} \end{aligned}$$

Q.7. Find percentage error in heat generated, if percentage error in resistance is 1%, time is 3% and current is 2%.

- A) 4
 B) 6



C) 2

D) 8

Answer: 8

Solution: Heat generated through resistance is given by, $H = i^2 R t$

Therefore, for very small changes we can write,

$$\Rightarrow \frac{\Delta H}{H} \times 100 = 2 \times \frac{\Delta i}{i} \times 100 + \frac{\Delta R}{R} \times 100 + \frac{\Delta t}{t} \times 100$$

$$\Rightarrow \frac{\Delta H}{H} \times 100 = 4\% + 1\% + 3\%$$

$$\Rightarrow \frac{\Delta H}{H} \times 100 = 8\%$$

Q.8. Find the percentage change in weight of an object, when it is taken from surface of earth to height $\frac{R}{4}$, above the surface of earth.

A) 16%

B) 36%

C) 25%

D) 20%

Answer: 36%

Solution: Acceleration due to gravity at height h is given by, $g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$

At height $\frac{R}{4}$,

$$g' = \frac{g}{\left(1 + \frac{1}{4}\right)^2} = \frac{16}{25}g$$

Change in weight

$$= mg - mg'$$

$$= mg \left(1 - \frac{16}{25}\right)$$

$$= \frac{9}{25}mg$$

$$\text{Percentage change} = \frac{9}{25} \times 100\%$$

$$= 36\%$$

Q.9. Two coils having 5 turns and 2 turns respectively, carries equal current. Magnetic field at their respective centres is B_1 and B_2 respectively. Value of $\frac{B_1}{B_2}$ is
(Both coils have equal radii)

A) $\frac{5}{2}$

B) $\frac{2}{5}$

C) $\frac{1}{6}$

D) $\frac{1}{5}$

Answer: $\frac{5}{2}$

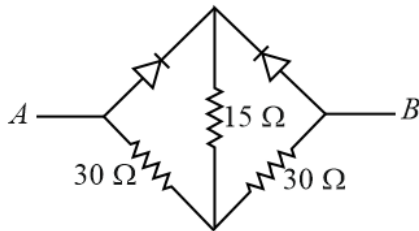


Solution: Magnetic field due to coils having N turns is given by, $B = N \left(\frac{\mu_0 i}{2r} \right)$

Therefore,

$$\frac{B_1}{B_2} = \frac{N_1}{N_2} = \frac{5}{2}$$

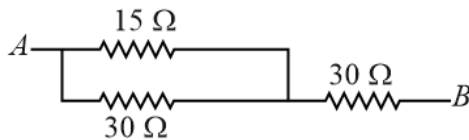
Q.10. If A is at higher potential than B , then equivalent resistance (in Ω) across AB is _____.



- A) 40 Ω
- B) 50 Ω
- C) 60 Ω
- D) 70 Ω

Answer: 40 Ω

Solution: If A is connected to a higher potential compared to B , then left diode will be in forward bias and hence will not offer any resistance & the modified circuit will be as given below.



$$R_{eq} = \left(\frac{15 \times 30}{15 + 30} \right) + 30$$

$$\Rightarrow R_{eq} = 40 \Omega$$

Q.11. One cell (1.2 V) is balanced in a potentiometer against a length of 72 cm. Another cell (1.8 V) is balanced at some other length. The difference in length (in cm) is

- A) 36 cm
- B) 54 cm
- C) 72 cm
- D) 18 cm

Answer: 36 cm

Solution: EMF of the cell will be proportional to the balancing length. Therefore,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} \text{ and } l_1 = 72 \text{ cm}$$

$$\Rightarrow l_2 = \frac{1.8}{1.2} \times 72 = 108 \text{ cm}$$

$$\therefore \Delta l = l_2 - l_1 = 108 - 72 = 36 \text{ cm}$$

Q.12. De Broglie wavelength of proton and deuteron have ratio 1 : $\sqrt{2}$. Find the ratio of their potential difference through which they were accelerated.

- A) 4 : 1
- B) 4 : 3



C) 3 : 4

D) 1 : 1

Answer: 4 : 1

Solution: De Broglie wavelength of a particle having linear momentum mv is given by,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m(KE)}}$$

Kinetic energy gained through potential difference will be equal to qV .

Therefore,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$$

Now,

$$\begin{aligned} \Rightarrow \frac{\lambda_1}{\lambda_2} &= \frac{1}{\sqrt{2}} \\ \Rightarrow \sqrt{\frac{(2m)(e)V_2}{m(e)V_1}} &= \frac{1}{\sqrt{2}} \\ \Rightarrow \frac{V_1}{V_2} &= \frac{4}{1} \end{aligned}$$

Q.13. Find the total number of different wavelengths that may be observed when electron de-excites from $n = 5$ in a hydrogen atom

A) 10

B) 5

C) 8

D) 15

Answer: 10

Solution: Total number of wavelengths that may be observed are given by,

$${}^nC_2 = {}^5C_2 = \frac{5 \times 4}{2} = 10$$

Q.14. A bullet of mass 200 g travelling with a speed of 10 m s^{-1} gets embedded in a block of mass 9.8 kg, kept on a smooth surface. Find the loss in kinetic energy of the system in J.

A) 9.8

B) 4.9

C) 10

D) 5

Answer: 9.8

Solution: If the final velocity of the bullet and the block is v , applying conservation of momentum,

$$\begin{aligned} mu &= (m + M)v \\ \Rightarrow 0.2 \times 10 &= (0.2 + 9.8)v \\ \Rightarrow v &= 0.2 \text{ m s}^{-1} \end{aligned}$$

Loss in kinetic energy will be,

$$K_i - K_f = \left(\frac{1}{2} \times 0.2 \times 10^2 \right) - \left(\frac{1}{2} \times 10 \times 0.2^2 \right) = 9.8 \text{ J}$$

Q.15. If maximum amplitude of a modulated wave is 6 V and minimum amplitude is 2 V. Find the value of modulation index.

A) 50%

B) 60%

C) 70%



D) 80%

Answer: 50%

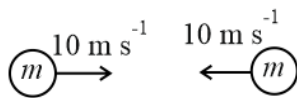
Solution: If amplitude of carrier wave is A_c and amplitude of message wave is A_m ,

$$A_{max} = A_c + A_m \text{ and } A_{min} = A_c - A_m$$

Modulation index,

$$M_i = \frac{A_m}{A_c} = \frac{\frac{A_{max} - A_{min}}{2}}{\frac{A_{max} + A_{min}}{2}} = \frac{6-2}{6+2} = \frac{1}{2} = 50\%$$

Q.16. Two identical particles each of mass $m = 1$ kg, collide with each other elastically. If collision lasts for 0.05 s then average force of collision is



A) 200 N

B) 400 N

C) 100 N

D) 300 N

Answer: 400 N

Solution: Since the masses are identical and collision is elastic, velocities will be interchanged after collision.

$$\text{Impulse on one ball } J = m(10 - (-10)) = 20 \text{ N s}$$

(As the ball will bounce back with same speed)

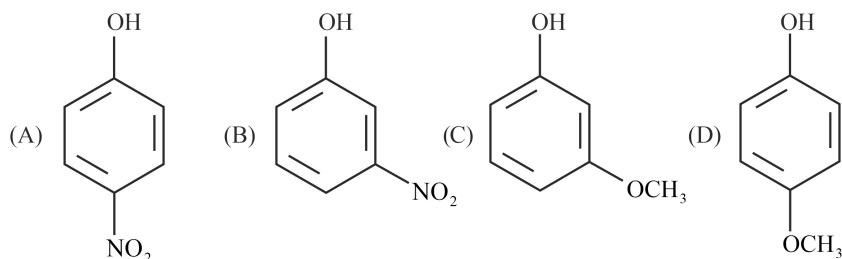
$$F_{avg} = \frac{J}{\Delta t} = \frac{20}{0.05} \text{ N}$$

$$F_{avg} = 400 \text{ N}$$



Section B: Chemistry

Q.17. Which of the following is correct decreasing order of acidity?



A) $A > B > C > D$

B) $B > C > A > D$

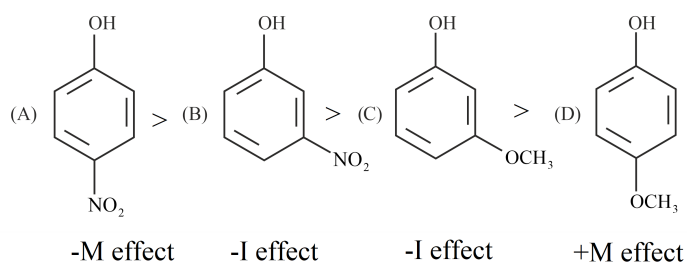
C) $C > A > B > D$

D) $D > A > B > C$

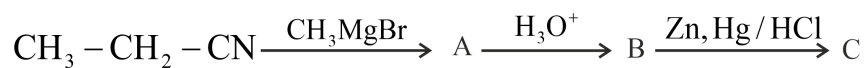
Answer: $A > B > C > D$

Solution: Acidic nature is more, if the stability of the conjugate base is more. Electron withdrawing groups increases the stability. Nitro group at para position shows $-M$ effect and meta position it shows $-I$ effect only.

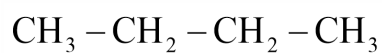
Methoxy group at meta position shows $-I$ effect, but effect is less than nitro group. Methoxy group at para position shows $+M$ effect that decreases the acidic nature.



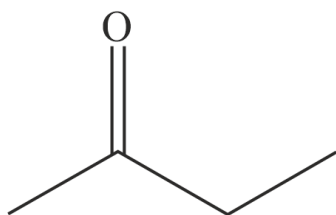
Q.18. From the given scheme, identify C.



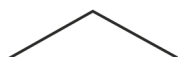
A)



B)

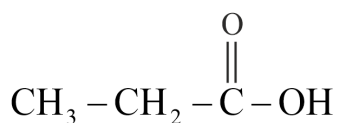


C)

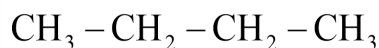




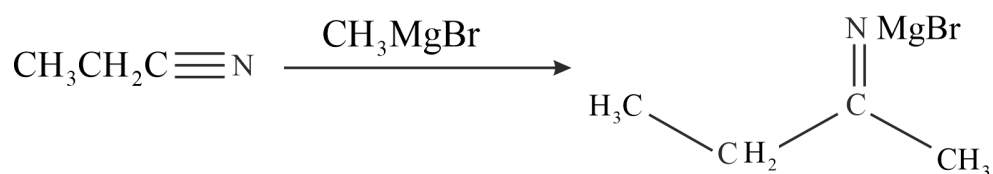
D)



Answer:

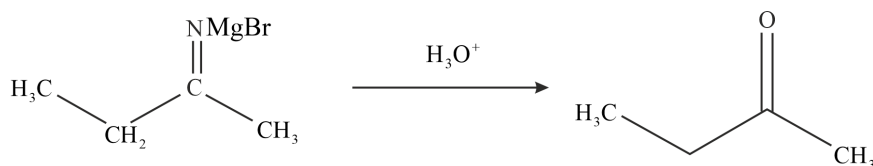


Solution: First, in this reaction the nitrile will react with the Grignard reagent. The reaction can be written as:

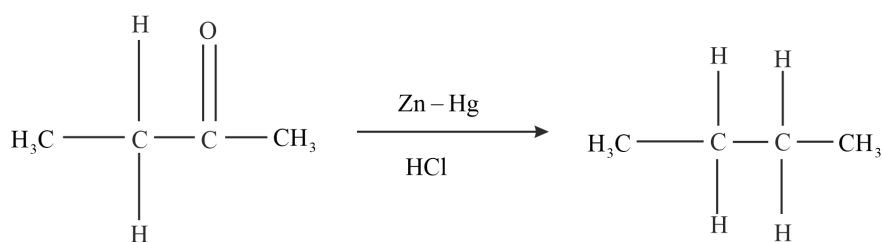


Now, in this step we can see that the nitrogen bond (C – N) is broken, and it leads to the formation of intermediate i.e. imine. In this step, Mg Br gets attached to the nitrogen atom.

Now, this reaction will further take place accordingly in the presence of aqueous acid. The reaction can be written as:



Clemmenson reagent acts only on aldehydes and ketones. Therefore, butan-2-one will get reduced to butane when we treat it with zinc and hydrochloric acid.



Q.19. The correct order of first ionisation enthalpy among following elements Be, B, C, N, O is

- A) $\text{B} < \text{Be} < \text{C} < \text{O} < \text{N}$
B) $\text{B} < \text{Be} < \text{C} < \text{N} < \text{O}$
C) $\text{Be} < \text{B} < \text{C} < \text{N} < \text{O}$
D) $\text{Be} < \text{B} < \text{C} < \text{O} < \text{N}$

Answer: $\text{B} < \text{Be} < \text{C} < \text{O} < \text{N}$

Solution: The ionisation enthalpy increases as we go from left to right in a period, while it decreases as we come down a group. So according to this, the order will be $\text{Be} < \text{B} < \text{C} < \text{N} < \text{O}$. But there are two exceptions, in the case of Be and N because of the highly stable $2s^2$ (completely filled orbital) and $2s^2 2p^3$ (half-filled orbital $2p^3$) valence configuration. Due to this, the correct order is:
 $\text{B} < \text{Be} < \text{C} < \text{O} < \text{N}$.

Q.20. Which of the following biomolecule have 1, 2-Glycosidic linkage?

- A) Sucrose



- B) Mannose
- C) Fructose
- D) Lactose

Answer: Sucrose

Solution: Sucrose is composed of a molecule of glucose joined to a molecule of fructose by an $\alpha - 1, \beta - 2$ -glycosidic linkage.

Mannose is a sugar monomer of the aldohexose series of carbohydrates. It is a C - 2 epimer of glucose.

Fructose, or fruit sugar, is a ketonic simple sugar found in many plants, where it is often bonded to glucose to form the disaccharide sucrose.

Lactose is composed of a molecule of galactose joined to a molecule of glucose by a $\beta - 1, 4$ -glycosidic linkage. It is a reducing sugar that is found in milk.

Q.21. The site which is occupied by inhibitors which changes the shape of active site of the catalyst:

- A) Inhibition site
- B) Competitive site
- C) Allosteric site
- D) Active site

Answer: Allosteric site

Solution: Some drugs do not bind to the enzyme's active site. These bind to a different site of enzyme which is called allosteric site. This binding of inhibitor at allosteric site changes the shape of the active site in such a way that substrate cannot recognise it.

Non-competitive inhibitor changes the active site of enzyme after binding at allosteric site

Q.22. The correct order of density of following elements is: (Be, Mg, Ca, Sr)

- A) $\text{Be} > \text{Mg} > \text{Ca} > \text{Sr}$
- B) $\text{Ca} > \text{Mg} > \text{Be} > \text{Sr}$
- C) $\text{Ca} < \text{Mg} < \text{Be} < \text{Sr}$
- D) $\text{Mg} < \text{Ca} < \text{Sr} < \text{Be}$

Answer: $\text{Ca} < \text{Mg} < \text{Be} < \text{Sr}$

Solution: The size of alkali metals increases down the group, so volume also shows increment, and since volume is inversely proportional to the density but in the case Sr and Ba, increase in the volume is less compared to increasing mass.

So, down the group density decreases first and then increases.

Therefore, the correct order of density is as follows:

$\text{Ca} < \text{Mg} < \text{Be} < \text{Sr}$

Q.23. Which of the following is correct order of acidic strength?

- A) $\text{BrO}_4^- > \text{CO}_2 > \text{NO} > \text{N}_2\text{O}$
- B) $\text{BrO}_4^- > \text{NO} > \text{CO}_2 > \text{N}_2\text{O}$
- C) $\text{BrO}_4^- < \text{CO}_2 < \text{NO} < \text{N}_2\text{O}$
- D) $\text{BrO}_4^- < \text{N}_2\text{O} < \text{NO} < \text{CO}_2$

Answer: $\text{BrO}_4^- < \text{N}_2\text{O} < \text{NO} < \text{CO}_2$

Solution: Perbromate ion is formed from the acid perbromic acid. Hence, among the given species it is less acidic. Carbon dioxide is anhydride of carbonic acid, it more acidic compound among the given species. Nitric oxide and nitrous oxide are neutral oxides. Acidic strength of these oxides can be compared based on oxidation states. More the oxidation state, more is the acidic nature. Hence, nitric oxide is more acidic than nitrous oxide.

Q.24. Find the sum of the total lone pairs in the following compounds:

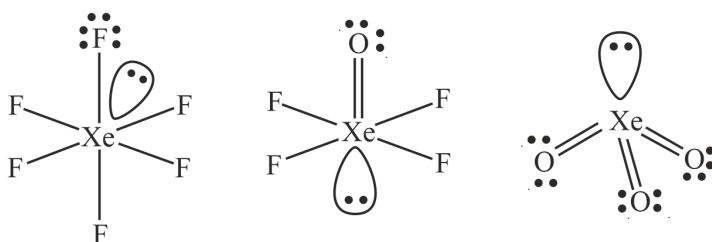
$\text{XeF}_6, \text{XeOF}_4, \text{XeO}_3$



- A) 21
B) 31
C) 41
D) 51

Answer: 41

Solution:



Each xenon is having one lone pair each. Each fluorine is having three lone pairs each and each oxygen is having two lone pairs each. Hence, the total number of lone pairs = $3(\text{Xe}) + 30(\text{F}) + 8(\text{O}) = 41$.

Q.25. How many of the following oxides are acidic?

NO, N₂O, P₄O₁₀, B₂O₃, N₂O₅, CaO

- A) 5
B) 3
C) 2
D) 1

Answer: 3

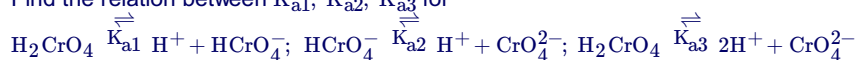
Solution: Based on their acid-base characteristics oxides are classified as acidic or basic. An oxide that combines with water to give an acid is termed as an acidic oxide. The oxide that gives a base in water is known as a basic oxide.

P₄O₁₀, B₂O₃, N₂O₅ are acidic oxides.

NO, N₂O are neutral oxides.

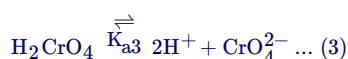
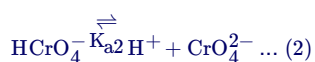
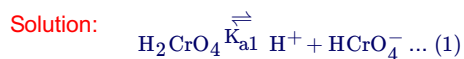
CaO is basic oxide.

Q.26. Find the relation between K_{a1}, K_{a2}, K_{a3} for



- A) $K_{a3} = K_{a1} + K_{a2}$
B) $K_{a3} = \frac{K_{a1}}{K_{a2}}$
C) $K_{a3} = K_{a1} - K_{a2}$
D) $K_{a3} = K_{a1} \times K_{a2}$

Answer: $K_{a3} = K_{a1} \times K_{a2}$



On adding equation (1) and equation (2) we get equation (3). So, $K_{a3} = K_{a1} \times K_{a2}$.



Q.27. Match Column I with Column II.

Column I		Column II	
A.	Nylon-6, 6	P.	Buckets
B.	Low density polythene	Q.	Toys
C.	High density polythene	R.	Bristles of brush
D.	Teflon	S.	Non-stick utensils

A) A-R, B-Q, C-P, D-S

B) A-P, B-Q, C-R, D-S

C) A-P, B-S, C-Q, D-R

D) A-S, B-Q, C-P, D-R

Answer: A-R, B-Q, C-P, D-S

Solution:

Column I		Column II	
A.	Nylon-6, 6	R.	Bristles of brush
B.	Low density polythene	Q.	Toys
C.	High density polythene	P.	Buckets
D.	Teflon	S.	Non-stick utensils

Q.28. $A = [Ni(en)_3]^{2+}$, $B = [Ni(NH_3)_6]^{2+}$, $C = [Ni(H_2O)_6]^{2+}$, CFSE energy order is:

A) $B > A > C$

B) $C > A > B$

C) $B > C > A$

D) $A > B > C$

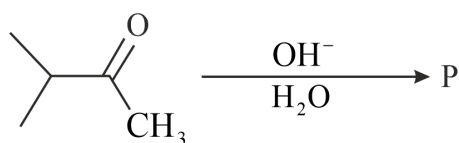
Answer: $A > B > C$

Solution: CFSE increases on increasing the splitting power of ligands and decreases on decreasing the splitting power of ligands. splitting power of ligand is decided according to spectrochemical series

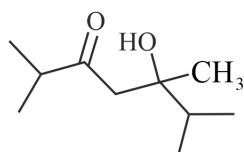


So, CFST energy order will be $A > B > C$

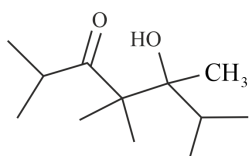
Q.29. Find product P.



A)

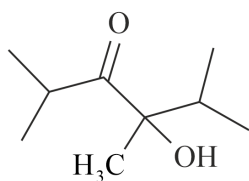


B)

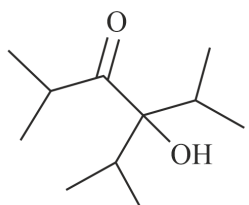




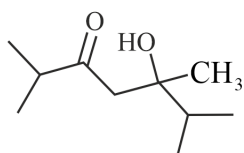
C)



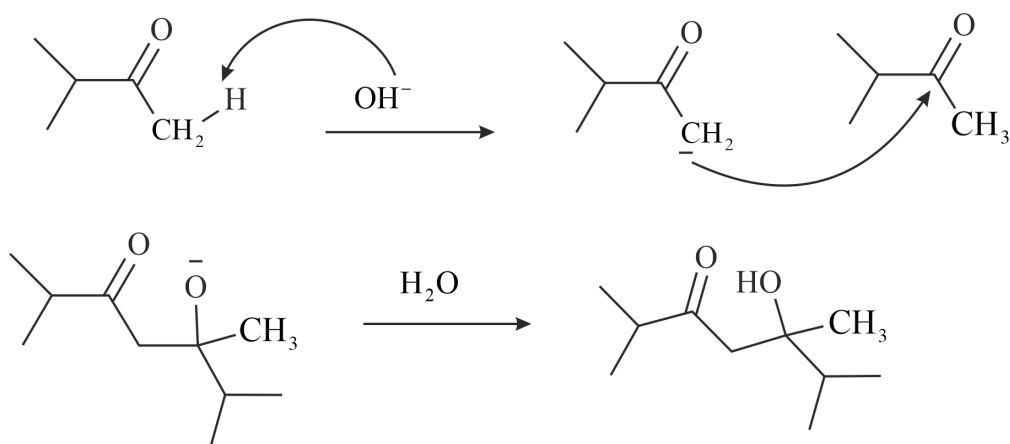
D)



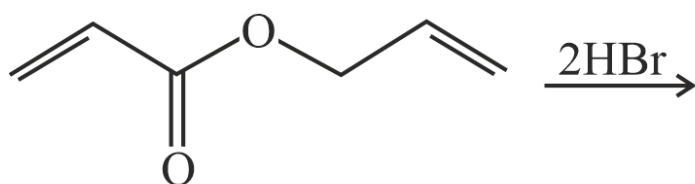
Answer:



Solution: This reaction is an example of aldol reaction. The product formation is shown below.

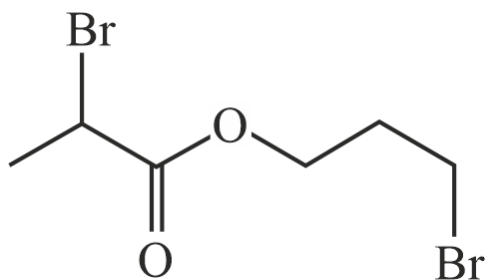


Q.30. Which of the following options represent the correct product of the following reaction.

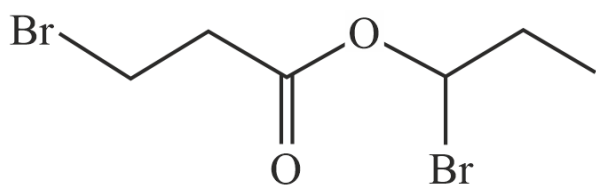




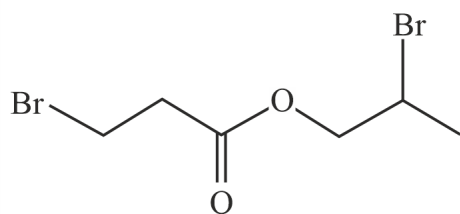
A)



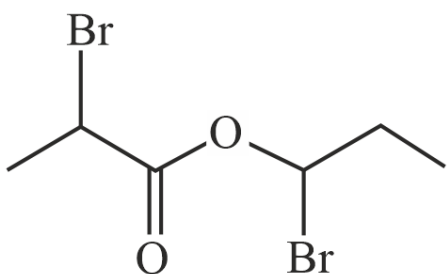
B)



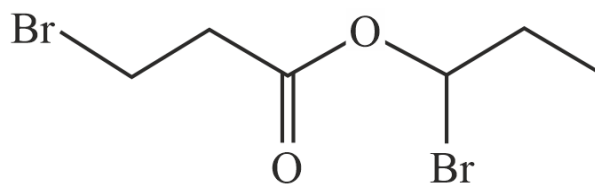
C)



D)

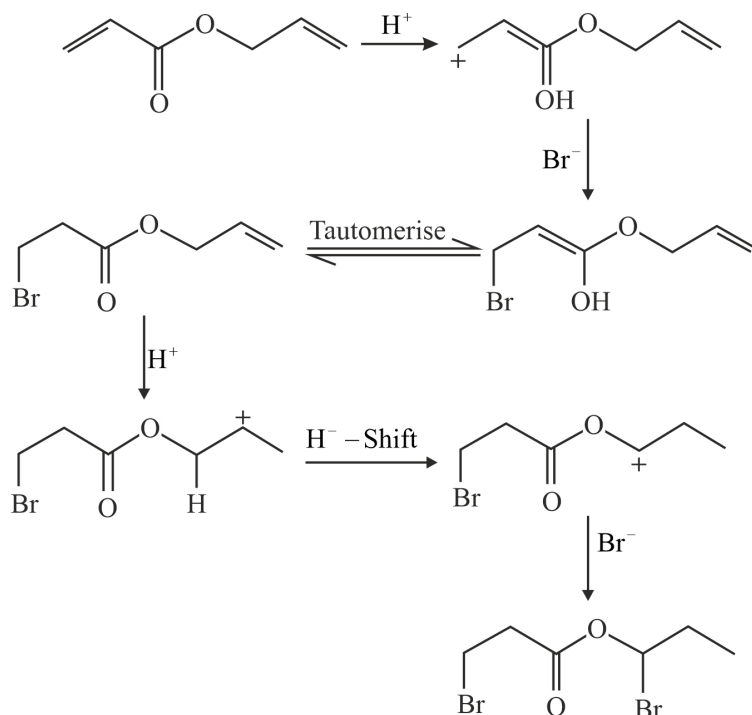


Answer:





Solution:



Q.31. Match the following:

1. Nicotine	A. Harmful for bones
2. Sulphates	B. Laxative effect
3. Fluoride	C. Pesticide
4. Sodium arsenite	D. Herbicide

- A) 1-C, 2-A, 3-B, 4-D
 B) 1-C, 2-B, 3-A, 4-D
 C) 1-A, 2-C, 3-B, 4-D
 D) 1-A, 2-D, 3-B, 4-C

Answer: 1-C, 2-B, 3-A, 4-D

Solution:

1. Nicotine	C. Pesticide
2. Sulphates	B. Laxative effect
3. Fluoride	A. Harmful for bones
4. Sodium arsenite	D. Herbicide

Q.32. An electron is present in the 4th excited state in H-atom. When it jumps to ground state then find the maximum number of wavelengths emitted?

- A) 6
 B) 10
 C) 15
 D) 20

Answer: 10

Solution: Maximum number of spectral lines = $\frac{\Delta n(\Delta n + 1)}{2}$

$$\frac{4(4+1)}{2} = \frac{20}{2} = 10$$

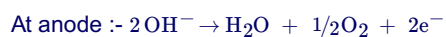
Q.33. High purity H₂ can be produced by:



- A) Natural gas reforming
- B) Electrolysis of warm solution of barium hydroxide
- C) Photoelectro chemical water splitting
- D) Treatment of alkali with Zn

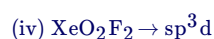
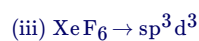
Answer: Electrolysis of warm solution of barium hydroxide

Solution: Dihydrogen of high purity (> 99.95%) is obtained by electrolysis of warm aqueous barium hydroxide solution between nickel electrodes. In electrolysis, the reaction occurs at the anode and cathode :-



Hence, the high purity dihydrogen is obtained by electrolysis of warm aqueous barium hydroxide.

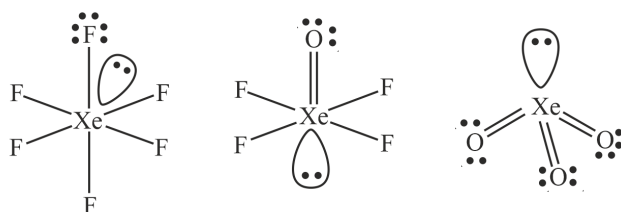
Q.34. Which of the following contain all the correct match of compounds and their hybridisation?



- A) (i), (ii) and (iv)
- B) (i), (iii) and (iv)
- C) (ii), (iii) and (iv)
- D) (i) and (ii) only

Answer: (ii), (iii) and (iv)

Solution:



$$\text{S.N} = \frac{1}{2}(8 + 4) = 6 = \text{sp}^3\text{d}^2$$



$$\frac{1}{2}(8) = 4 = \text{sp}^3$$



$$\frac{1}{2}(8 + 6) = 7 = \text{sp}^3\text{d}^3$$



$$\frac{1}{2}(8 + 2) = 5 = \text{sp}^3\text{d}$$



Section C: Mathematics

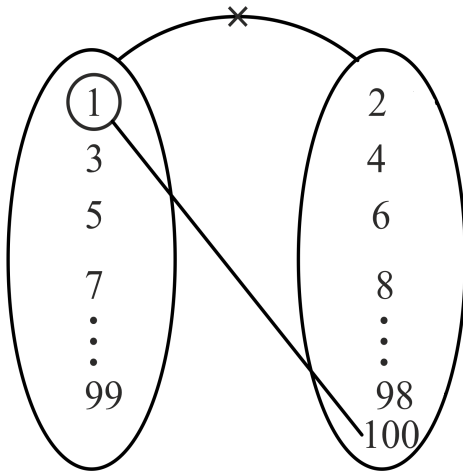
Q.35. The number of bijective function $f(1, 3, 5, 7, \dots, 99) \rightarrow (2, 4, 6, 8, \dots, 100)$ if $f(3) > f(5) > f(7) \dots > f(99)$ is

- A) ${}^{50}C_1$
- B) ${}^{50}C_2$
- C) $\frac{50!}{2}$
- D) ${}^{50}C_3 \times 3!$

Answer: ${}^{50}C_1$

Solution: Given, $f(1, 3, 5, 7, \dots, 99) \rightarrow (2, 4, 6, 8, \dots, 100)$

Now, let us assume $f(1) = 100$



Now as per diagram we have only 1 way for arranging $f(3) > f(5) > f(7) \dots > f(99)$,

Similarly if we choose $f(1) = 98$ then again we have only 1 way for arranging $f(3) > f(5) > f(7) \dots > f(99)$,

So we can see the arrangement $f(3) > f(5) > f(7) \dots > f(99)$ depends upon $f(1)$,

So number of ways for choosing $f(1)$ is ${}^{50}C_1$

Q.36. The value of $2 \sin \frac{\pi}{22} \sin \frac{3\pi}{22} \sin \frac{5\pi}{22} \sin \frac{7\pi}{22} \sin \frac{9\pi}{22}$ is equal to:

- A) $\frac{1}{16}$
- B) $\frac{5}{16}$
- C) $\frac{7}{16}$
- D) $\frac{3}{16}$

Answer: $\frac{1}{16}$



Solution: Given, $2 \sin \frac{\pi}{22} \sin \frac{3\pi}{22} \sin \frac{5\pi}{22} \sin \frac{7\pi}{22} \sin \frac{9\pi}{22}$

Now by we know that $\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$, so by using this we get $\sin \frac{\pi}{22} = \cos\left(\frac{\pi}{2} - \frac{\pi}{22}\right) = \cos \frac{5\pi}{11}$ and similarly $\sin \frac{3\pi}{22} = \cos \frac{4\pi}{11}$, $\sin \frac{5\pi}{22} = \cos \frac{3\pi}{11}$, $\sin \frac{7\pi}{22} = \frac{2\pi}{11}$, $\sin \frac{9\pi}{22} = \cos \frac{\pi}{11}$

$$\text{So, } 2 \sin \frac{\pi}{22} \sin \frac{3\pi}{22} \sin \frac{5\pi}{22} \sin \frac{7\pi}{22} \sin \frac{9\pi}{22}$$

$$= 2 \cos \frac{5\pi}{11} \cos \frac{4\pi}{11} \cos \frac{3\pi}{11} \cos \frac{2\pi}{11} \cos \frac{\pi}{11}$$

$$= 2 \cos \frac{1\pi}{11} \cos \frac{2\pi}{11} \cos \frac{4\pi}{11} \cos \frac{8\pi}{11} \cos \frac{16\pi}{11}$$

$$\{\text{As } \cos \frac{3\pi}{22} = -\cos\left(\pi - \frac{3\pi}{11}\right) = -\cos \frac{8\pi}{11} \text{ and } \cos \frac{5\pi}{11} = -\cos \frac{16\pi}{11}\}$$

$$= \frac{2 \sin \frac{\pi}{11} \cos \frac{\pi}{11} \cos \frac{2\pi}{11} \cos \frac{4\pi}{11} \cos \frac{8\pi}{11} \cos \frac{16\pi}{11}}{2 \sin \frac{\pi}{11}}$$

$$= \frac{2 \sin \frac{4\pi}{11} \cos \frac{4\pi}{11} \cos \frac{8\pi}{11} \cos \frac{16\pi}{11}}{2^2 \sin \frac{\pi}{11}}$$

$$= \frac{2 \sin \frac{8\pi}{11} \cos \frac{8\pi}{11} \cos \frac{16\pi}{11}}{2^3 \sin \frac{\pi}{11}}$$

$$= \frac{2 \sin \frac{16\pi}{11} \cos \frac{16\pi}{11}}{2^4 \sin \frac{\pi}{11}}$$

$$= \frac{\sin \frac{32\pi}{11}}{2^4 \sin \frac{\pi}{11}}$$

$$= \frac{1}{16} \frac{\sin\left(3\pi - \frac{\pi}{11}\right)}{\sin \frac{\pi}{11}} = \frac{1}{16}$$

Q.37. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{8\sqrt{2} - 8(\sin x + \cos x)}{\sqrt{2} - \sqrt{2} \sin 2x}$ is equal to

- A) 2
- B) 4
- C) 6
- D) 8

Answer: 2

Solution: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{8\sqrt{2} - 8(\sin x + \cos x)}{\sqrt{2} - \sqrt{2} \sin 2x}$ $\left(\frac{0}{0}\right)$ form

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-8(\cos x - \sin x)}{-\sqrt{2} \cdot 2 \cos 2x} \quad [\text{Applying L'Hospital Rule}] \quad \left(\frac{0}{0}\right) \text{ form}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{8(\sin x + \cos x)}{4\sqrt{2} \sin 2x} \quad [\text{Applying L'Hospital Rule again}]$$

$$= \frac{8\sqrt{2}}{4\sqrt{2}} = 2$$



Q.38. If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$ then $P(A | B') + P(A' | B) =$

- A) $\frac{5}{8}$
- B) $\frac{4}{9}$
- C) $\frac{29}{24}$
- D) 3

Answer: $\frac{29}{24}$

Solution: Given $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$

$$\text{So, } P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{1}{30}$$

$$\text{Now, } P(A | B') = \frac{P(A \cap B')}{P(B')} = \frac{\frac{1}{3} - \frac{1}{30}}{\frac{4}{5}} = \frac{3}{8}$$

$$\text{And } P(A' | B) = \frac{P(A' \cap B)}{P(B)} = \frac{\frac{1}{5} - \frac{1}{30}}{\frac{1}{5}} = \frac{5}{6}$$

$$\text{So, } P(A | B') + P(A' | B) = \frac{3}{8} + \frac{5}{6} = \frac{29}{24}$$

Q.39. Let $f(x) = [x^2 - 2x] + |5x - 7|$, where $[.]$ represents greatest integer function. If m be minimum value of $f(x)$ and M be maximum value of $f(x)$ in $[\frac{5}{4}, 2]$, then:

- A) $m = -1, M = 2$
- B) $m = 0, M = 3$
- C) $m = -1, M = 4$
- D) $m = -2, M = 2$

Answer: $m = -1, M = 2$

Solution: When $x \in [\frac{5}{4}, 2]$

$$x^2 - 2x \in (-1, 0]$$

$$\text{i.e. } [x^2 - 2x] = -1$$

$$\text{Now } |5x - 7| = \begin{cases} 7 - 5x & x < \frac{7}{5} \\ 5x - 7 & x \geq \frac{7}{5} \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 6 - 5x & \frac{5}{4} < x < \frac{7}{5} \\ 5x - 8 & \frac{7}{5} \leq x \leq 2 \end{cases}$$

$$\text{So, } m = -1 \text{ at } x = \frac{7}{5}$$

$$\text{and } M = 2 \text{ at } x = 2$$

Q.40. If the equation $x^3 + px^2 + qx + 1 = 0$ ($p < q$) has only one real root α , then α belongs to

- A) $(-2, -1)$
- B) $(-1, 0)$
- C) $(0, 1)$



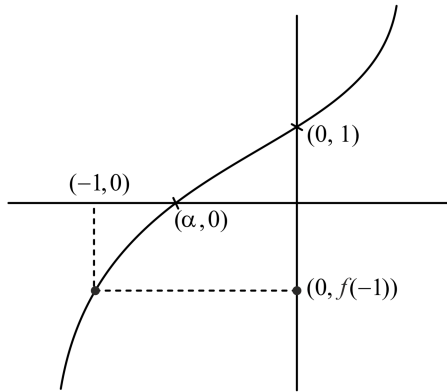
D) (1, 2)

Answer: (-1, 0)

Solution: Given, $x^3 + px^2 + qx + 1 = 0$ ($p < q$ or $p - q < 0$)

Now, $f(0) = 1$ & $f(-1) = -1 + p - q + 1 = p - q < 0$

$\therefore f(0) > 0$ & $f(-1) < 0$, on plotting the graph we get,



$\therefore f(x)$ must have root between $(-1, 0)$

Q.41. The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is ____ .

- A) 1
- B) 2
- C) 3
- D) 4

Answer: 2

Solution: $11^{1011} = (11^3)^{327} = (9k + 8)^{327}$

when divided by 9 is equivalent to the division of

$$8^{327} = (9 - 1)^{327}$$

i.e. remainder = $-1 + 9 = 8$

Now, $(1011)^{11} = (9m + 3)^{11}$

when divided by 9 is equivalent to the division of

$$3^{11} = (9)^5 \times 3$$

i.e. remainder = 3

$\therefore 11^{1011} + 1011^{11}$ has same remainder as $\frac{8+3}{9}$ i.e., 2

Q.42. Statement $p \rightarrow$ Ramu is innocent; $q \rightarrow$ Ramu is not honest; $r \rightarrow$ Ramu is rich

Then statement "Ramu is innocent and not honest if and only if he is rich" is represented by:

- A) $(p \vee q) \rightarrow r$
- B) $(p \wedge q) \rightarrow r$
- C) $(p \wedge q) \leftrightarrow r$
- D) $(p \vee q) \leftrightarrow r$

Answer: $(p \wedge q) \leftrightarrow r$



Solution: $p \rightarrow$ Ramu is innocent

$q \rightarrow$ Ramu is not honest

$r \rightarrow$ Ramu is rich

$\therefore$ Ramu is innocent and not honest is represented by $p \wedge q$

So, Ramu is innocent and not honest if and only if he is rich is represented by $(p \wedge q) \leftrightarrow r$

Q.43. If $A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$ and $A^n = \begin{bmatrix} 1 & 48 & 2064 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$ then $n + a + b$ is equal to:

- A) 24
- B) 25
- C) 26
- D) 2736

Answer: 26

Solution: Given, $A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$ and $A^n = \begin{bmatrix} 1 & 48 & 2064 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$

$$\text{Let } \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = X$$

So, $A = I + X$

$$\text{Now solving } X^2 = \begin{bmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } X^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$\text{Now, } A^n = (I + X)^n$$

$$= I + {}^nC_1 \cdot X + {}^nC_2 \cdot X^2 + {}^nC_3 \cdot X^3 + \dots$$

$$= I + nX + \frac{n(n-1)}{2} X^2$$

$$= \begin{bmatrix} 1 & na & na + \frac{n(n-1)}{2} ab \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Now comparing with } A^n = \begin{bmatrix} 1 & 48 & 2064 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$$

We can see clearly, $na = 48$, $nb = 96$ and

$$\frac{na(nb-b)}{2} + na = 2064$$

On simplifying above values we get,

$$\Rightarrow b = 12, a = 6 \text{ and } n = 8$$

$$\therefore n + a + b = 12 + 6 + 8 = 26$$

Q.44. Focus of ellipse $\frac{x^2}{9} + \frac{y^2}{7} = 1$ and hyperbola $\frac{x^2}{\frac{1}{4}} - \frac{y^2}{\alpha^2} = 1$ coincide. Then, latus rectum of hyperbola is

- A) 5
- B) 6
- C) 7



D) 8

Answer: 7

Solution: If ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{l^2} - \frac{y^2}{m^2} = 1$ have same foci,

Then $ae_1 = le_2$ or $(ae_1)^2 = (le_2)^2$ where e_1 & e_2 are eccentricity of ellipse and hyperbola respectively,

$$\text{So, } a^2 - b^2 = l^2 + m^2$$

Now putting the given values we get,

$$\Rightarrow 9 - 7 = \frac{1}{4} + \alpha^2 \Rightarrow \alpha^2 = \frac{7}{4}$$

$$\text{Length of latus rectum} = \frac{2m^2}{l} = \frac{2\alpha^2}{\frac{1}{2}} = 4 \left(\frac{7}{4} \right) = 7$$

Q.45. Eight numbers 3, 5, 7, $2k$, 12, 15, 21, 27 are in increasing order and if mean deviation about median is 6. Then median of data is

A) 10

B) 12

C) 20

D) 24

Answer: 12

Solution: We know that median of data for even number of observation is given by mean of $\left(\frac{n}{2}\right)^{th}$ & $\left(\frac{n}{2} + 1\right)^{th}$ observation,

$$\text{So, median of given data} = \frac{2k+12}{2} = k + 6$$

Now Mean Deviation about median is given as 6

$$\begin{aligned} \text{Now by formula we get, } (M) &= \frac{(k+3)+(k+1)+(k-1)+(6-k)+(9-k)+(15-k)+(21-k)}{8} \\ &= \frac{54-2k}{8} = 6 \Rightarrow k = 6 \end{aligned}$$

Hence, median = $k + 6 = 12$

Q.46. Area enclosed by the curve $y^2 + x^4 = x^2$ is

A) $\frac{2}{3}$

B) $\frac{4}{3}$

C) $\frac{8}{3}$

D) $\frac{10}{3}$

Answer: $\frac{4}{3}$



Solution: Since powers of x & y are even so $y^2 = x^2 - x^4$ will be symmetric across x & y axis.

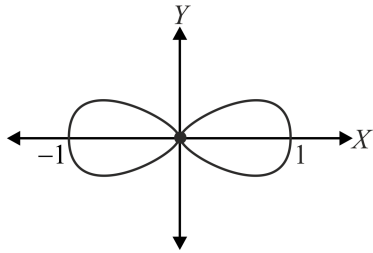
Now $y^2 = x^2 - x^4 = x^2(1 - x^2) = x^2(1 - x)(1 + x)$ will touch axis at

$$y = 0 \Rightarrow x = 0, 1, -1$$

$$\text{Also we know } y^2 \geq 0 \Rightarrow x^2(x + 1)(1 - x) \geq 0$$

$$\Rightarrow x \in [-1, 1]$$

Plotting the graph of $y = \pm |x| \sqrt{1 - x^2}$, we get,



$$\text{Therefore, required area} = 4 \int_0^1 x \sqrt{1 - x^2} dx$$

$$= \left[-\frac{4(1-x^2)^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 = \frac{4}{3}$$

Q.47. The shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ is:

- A) $\frac{\sqrt{29}}{2}$
- B) $3\sqrt{29}$
- C) $\sqrt{29}$
- D) $2\sqrt{29}$

Answer: $2\sqrt{29}$



Solution: Given lines are, $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = \lambda$ (Say)

$$\text{and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} = \mu \text{ (Say)}$$

Vector form of lines are $\vec{r}_1 = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k})$, and $\vec{r}_2 = -\hat{i} - \hat{j} - \hat{k} + \mu(7\hat{i} - 6\hat{j} + \hat{k})$, respectively,

$$\text{So, } \vec{a}_1 = 3\hat{i} + 5\hat{j} + 7\hat{k}, \vec{a}_2 = -\hat{i} - \hat{j} - \hat{k}$$

$$\vec{b}_1 = \hat{i} - 2\hat{j} + \hat{k}, \vec{b}_2 = 7\hat{i} - 6\hat{j} + \hat{k}$$

$$\text{So, } \vec{a}_2 - \vec{a}_1 = -\hat{i} - \hat{j} - \hat{k} - (3\hat{i} + 5\hat{j} + 7\hat{k})$$

$$= -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$$

$$= \hat{i}((-2) + 6) - \hat{j}(1 - 7) + \hat{k}((-6) + 14)$$

$$= 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\text{So, } (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (-4\hat{i} - 6\hat{j} - 8\hat{k}) \cdot (4\hat{i} + 6\hat{j} + 8\hat{k})$$

$$= -16 - 36 - 64$$

$$= -116$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(4)^2 + (6)^2 + (8)^2} = \sqrt{16 + 36 + 64}$$

$$= \sqrt{116}$$

∴ The shortest distance between given lines is,

$$\left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \left| \frac{-116}{\sqrt{116}} \right|$$

$$= \frac{116}{\sqrt{116}} = \sqrt{116}$$

$$= 2\sqrt{29}$$

Q.48. The value of $\int_{-3}^{101} ([\sin \pi x] + e^{[\cos 2\pi x]}) dx$ is, where $[.]$ represents greatest integer function

A) $\frac{52}{e}$

B) $-52 + \frac{50}{e}$

C) $-48 + \frac{48}{e}$

D) $-30 + \frac{30}{e}$

Answer: $\frac{52}{e}$



Solution: Given, $\int_{-3}^{101} ([\sin \pi x] + e^{[\cos 2\pi x]}) dx$

Now let $I_1 = \int_{-3}^{101} [\sin \pi x] dx$ and $I_2 = \int_{-3}^{101} e^{[\cos 2\pi x]} dx$

Now checking periodicity of $[\sin \pi x]$ we get,

$$0 < x < 1 \Rightarrow [\sin \pi x] = 0$$

$$1 < x < 2 \Rightarrow [\sin \pi x] = -1$$

$[\sin \pi x] \rightarrow$ periodic with period 2

$$\begin{aligned} \text{So, } I_1 &= 52 \int_0^2 [\sin \pi x] dx \\ &= 52 \left[\int_0^1 0 + \int_1^2 -1 dx \right] = -52 \end{aligned}$$

Now checking periodicity of $[\cos 2\pi x]$ we get,

$$0 < x < \frac{1}{4} \Rightarrow [\cos 2\pi x] = 0$$

$$\frac{1}{4} < x < \frac{1}{2} \Rightarrow [\cos 2\pi x] = -1$$

$$\frac{1}{2} < x < \frac{3}{4} \Rightarrow [\cos 2\pi x] = -1$$

$$\frac{3}{4} < x < 1 \Rightarrow [\cos 2\pi x] = 0$$

So, $[\cos 2\pi x]$ has periodicity of 1

$$\text{So, } I_2 = 104 \left(\int_0^{\frac{1}{4}} e^0 dx + \int_{\frac{1}{4}}^{\frac{3}{4}} e^{-1} dx + \int_{\frac{3}{4}}^1 e^0 dx \right)$$

$$\Rightarrow I_2 = 104 \left(\frac{1}{4} + \frac{1}{e} \times \frac{1}{2} + \frac{1}{4} \right)$$

$$\Rightarrow I_2 = 52 + \frac{52}{e}$$

$$\text{So, } I_1 + I_2 = \frac{52}{e} \text{ or } \int_{-3}^{101} ([\sin \pi x] + e^{[\cos 2\pi x]}) dx = \frac{52}{e}$$

Q.49. Let $f(x)$ be a quadratic polynomial whose leading coefficient is 1. If $f(0) = p$, $p \neq 0$, $f(1) = \frac{1}{3}$ such that $f(x) = 0$ and $f \circ f \circ f \circ f \circ f(x) = 0$ have a common root, then $f(-3)$ is equal to

A) 25

B) $\frac{25}{2}$

C) 9

D) $\frac{7}{2}$

Answer: 25



Solution: Let $f(x) = x^2 + bx + p$

If α is the common root, then $f(\alpha) = 0$ and $f(f(f(\alpha))) = 0$

i.e. $f(f(f(0))) = 0 \Rightarrow f(f(p)) = 0$

Now $f(1) = 1 + b + p = \frac{1}{3} \Rightarrow b = -p - \frac{2}{3}$

$$f(p) = p^2 - \left(p + \frac{2}{3}\right)p + p = \frac{p}{3}$$

$$\Rightarrow f\left(\frac{p}{3}\right) = 0$$

i.e. $\frac{p^2}{9} - \left(p + \frac{2}{3}\right)\frac{p}{3} + p = 0 \Rightarrow p = \frac{7}{2}$ and $b = \frac{-25}{6}$

$$\text{So } f(x) = x^2 - \frac{25}{6}x + \frac{7}{2}$$

$$\text{Hence, } f(-3) = 9 + \frac{25}{2} + \frac{7}{2} = 25$$

Q.50. Solution of differential equation $\frac{dy}{dx} = \frac{4y^3 + 2x^2y}{3y^2x + x^3}$, $y(1) = 1$ is

A) $y^3 + x^2y = 2x^4$

B) $3y^3 + 2x^2y = 8x^4$

C) $y^3 + x^2y = 8x^4$

D) $y^3 - x^2y = 2x^4$

Answer: $y^3 + x^2y = 2x^4$

Solution: Given,

$$\frac{dy}{dx} = \frac{4y^3 + 2x^2y}{3y^2x + x^3}, y(1) = 1$$

Since it is a homogenous equation,

$$\text{So let } y = vx, \text{ then } \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{4v^3 + 2v}{3v^2 + 1}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v^3 + v}{3v^2 + 1} \Rightarrow \int \frac{3v^2 + 1}{v^3 + v} dv = \int \frac{1}{x} dx$$

$$\Rightarrow \int \frac{3v^2 + 1}{v^3 + v} dv = \int \frac{1}{x} dx \Rightarrow \ln |v^3 + v| = \ln |x| + \ln c$$

$$\Rightarrow \left(\frac{y}{x}\right)^3 + \left(\frac{y}{x}\right) = cx$$

Also given $y(1) = 1$

$$\text{So, } \left(\frac{1}{1}\right)^3 + \left(\frac{1}{1}\right) = c \times 1 \Rightarrow c = 2$$

$$\therefore y^3 + x^2y = 2x^4$$

