JEE Main Exam 2022 - Session 2

28 Jul 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. A radioactive sample has concentration 64 times the allowed level. If half life of radioactive material in sample is 2 hours 30 minute then after how much time the sample is safe for human exposure.

A) 15 hours

B) 9 hours

C) 6 hours

D) 7.5 hours

Answer: 15 hours

Solution:

Activity of a radioactive sample after time t is given by, $A = A_0 2^{-\frac{t}{t_{\frac{1}{2}}}}$.

Therefore,

$t = t_{\frac{1}{2}} \log_2 \left(\frac{A_0}{A}\right)$

$\Rightarrow t = 2.5 \log_2 (64)$

$\Rightarrow t = 2.5 \times 6 = 15$ hours.

Q.2. A wave has equation as $y = 0.5 \sin \left(\frac{2\pi}{3}(400t - x)\right)$ with x in meters and t in seconds the wave speed is equal to

A) 100 m s^{-1}

B) $100\pi \text{ m s}^{-1}$

C) 400 m s^{-1}

D) 200 m s^{-1}

Answer: 400 m s^{-1}

Solution: From equation of wave, $\omega = \frac{800\pi}{3}$ and $k = \frac{2\pi}{3}$

So wave speed, $v = \frac{\omega}{k} = 400 \text{ m s}^{-1}$

Q.3. In a medium with relative permittivity 1 and relative permeability 4, the speed of light is

A) $0.5 \times 10^8 \text{ m s}^{-1}$

B) $1.5 \times 10^8 \text{ m s}^{-1}$

C) $2 \times 10^8 \text{ m s}^{-1}$

D) $1 \times 10^8 \text{ m s}^{-1}$

Answer: $1.5 \times 10^8 \text{ m s}^{-1}$

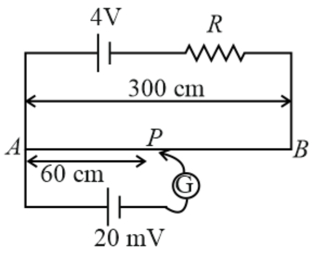
Solution: Refractive index of a medium is given by, $n = \sqrt{\varepsilon_r \mu_r}$. Therefore,

$n = \sqrt{1 \times 4} = 2$

Speed of light in the medium will be

$v = \frac{c}{n} = \frac{3 \times 10^8}{2} = 1.5 \times 10^8 \text{ m s}^{-1}$

Q.4. In a potentiometer setup shown point P is point of null deflection. If wire AB has a resistance of $20\ \Omega$, then value of resistance R is,



A) $380\ \Omega$

B) $480\ \Omega$

C) $680\ \Omega$

D) $780\ \Omega$

Answer: $780\ \Omega$

Solution: At null deflection, there will be no current through the $20\ \mathrm{mV}$ battery and $V_{AP} = 20\ \mathrm{mV}$

So, $\left(-\frac{\Delta V}{\Delta l}\right) = \frac{\left(20 \times 10^{-3}\right)}{60} = \frac{1}{3} \times 10^{-3}\ \mathrm{V\ cm^{-1}}$

Thus, $V_{AB} = \left(-\frac{\Delta V}{\Delta l}\right) \times 300 = 0.1\ \mathrm{V}$

Now,

$V_{AB} = \frac{R_{AB}E}{R_{AB}+R}$

$\Rightarrow 0.1 = \frac{20 \times 4}{20+R}$

$\Rightarrow R = 780\ \Omega$

Q.5. The force required to increase the length to twice of the original length of the wire is (Given that the cross-section of wire is $1\ \mathrm{cm}^2$ and Young's Modulus is $2 \times 10^{11}\ \mathrm{N\ m^{-2}}$)

A) $2 \times 10^7\ \mathrm{N}$

B) $1.5 \times 10^7\ \mathrm{N}$

C) $1 \times 10^7\ \mathrm{N}$

D) $2.5 \times 10^7\ \mathrm{N}$

Answer: $2 \times 10^7\ \mathrm{N}$

Solution: Final length of the wire is twice that of the original length. Therefore, $\Delta L = L$.

From Hooke's law, $Y = \frac{FL}{A\Delta L}$.

$\Rightarrow F = \frac{YA\Delta L}{L} = \frac{L}{L} \times YA = YA$

$\Rightarrow F = 2 \times 10^{11} \times 10^{-4} = 2 \times 10^7\ \mathrm{N}$

Q.6. Two identical capacitors of $40\ \mu\mathrm{F}$ each are connected in series. Now a dielectric slab of dielectric constant K is inserted in one of them such that the new equivalent capacitance comes out $24\ \mu\mathrm{F}$. Value of K is

A) 2

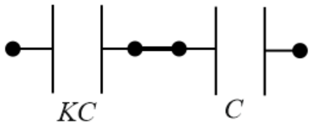
B) 1

C) 1.5

D) 3

Answer: 1.5

Solution:



When a dielectric is inserted between the plates of a capacitor, its capacitance becomes K times. Therefore, the equivalent capacitance in the series combination will be,

$C_{\text{net}} = \frac{KC \times C}{KC + C} \Rightarrow \frac{K40}{1+K} = 24$

$\Rightarrow K = 1.5$

Q.7. An apple falls from a height of $19.6 \mathrm{~m}$ from a tree and a man standing s distance away from the tree starts running with a speed of $2.5 \mathrm{~m} \mathrm{~s}^{-1}$ to catch the apple. To just catch the apple the distance s should be $\left(g = 9.8 \mathrm{~m} \mathrm{~s}^{-2}\right)$

A) $5 \mathrm{~m}$

B) $10 \mathrm{~m}$

C) $15 \mathrm{~m}$

D) $20 \mathrm{~m}$

Answer: $5 \mathrm{~m}$

Solution: The time taken by apple to reach ground

$t = \sqrt{\frac{2\mathrm{h}}{g}} = 2 \mathrm{~s}$

So, $s = vt = 2.5 \times 2$

$= 5 \mathrm{~m}$

Q.8. Dimensional formula of $\frac{B_0^2}{2\mu}$ is: (B_0 is magnetic field amplitude and μ is permeability)

A) $\left[\mathrm{M}^1 \mathrm{L}^2 \mathrm{T}^{-2}\right]$

B) $\left[\mathrm{M}^1 \mathrm{L}^{-1} \mathrm{T}^{-2}\right]$

C) $\left[\mathrm{M}^1 \mathrm{L}^3 \mathrm{T}^{-1}\right]$

D) $\left[\mathrm{M}^1 \mathrm{L}^2 \mathrm{T}^{-1}\right]$

Answer: $\left[\mathrm{M}^1 \mathrm{L}^{-1} \mathrm{T}^{-2}\right]$

Solution: $\frac{B_0^2}{2\mu}$ is the formula of energy density $\frac{dU}{dV}$

So, $\frac{dU}{dV} = \frac{\left[\mathrm{ML}^2 \mathrm{T}^{-2}\right]}{\left[\mathrm{L}^3\right]} = \left[\mathrm{ML}^{-1} \mathrm{T}^{-2}\right]$

Q.9. Efficiency of Carnot cycle is 50%. Now the efficiency is increased by 30%, when the temperature of sink is reduced by $40^{\circ} \mathrm{C}$. The temperature of the source is

A) $266.7 \mathrm{~K}$

B) $300 \mathrm{~K}$

C) $366.7 \mathrm{~K}$

D) $255 \mathrm{~K}$

Answer: $266.7 \mathrm{~K}$

Solution: Let the temperature of source is T_0 and that of sink is T.

$\Rightarrow 1-\frac{T}{T_0}=0.5=\frac{1}{2}$ $\quad \ldots(1)$ and for the second case,

$1-\frac{T-40}{T_0}=\frac{65}{100}=\frac{13}{20}$ $\quad \ldots(2)$

$\Rightarrow \frac{1}{2}+\frac{40}{T_0}=\frac{13}{20}$

$\Rightarrow \frac{40}{T_0}=\frac{3}{20}$

$\Rightarrow T_0=\frac{800}{3}\approx 266.7\ \mathrm{K}$

Q.10. With wavelength $560\ \mathrm{nm}$ fringe width comes out to be $72\ \mathrm{mm}$ in a YDSE. Now if wavelength is changed so that fringe width is $81\ \mathrm{mm}$, then the new wavelength is equal to

A) $490\ \mathrm{nm}$

B) $630\ \mathrm{nm}$

C) $800\ \mathrm{nm}$

D) $700\ \mathrm{nm}$

Answer: $630\ \mathrm{nm}$

Solution: As we know, fringe width is given by, $\beta=\frac{\lambda D}{d}$

Clearly, $\beta \propto \lambda$

Therefore, $\frac{\beta_1}{\beta_2}=\frac{\lambda_1}{\lambda_2}$

$\Rightarrow \frac{72}{81}=\frac{560}{\lambda_2}$

$\Rightarrow \lambda_2=630\ \mathrm{nm}$

Q.11. Radius of earth is shrunk by 2%. Now new acceleration due to gravity is,

A) increase by 4%

B) decrease by 4%

C) increase by 2%

D) decrease by 2%

Answer: increase by 4%

Solution: Acceleration due to gravity at distance r from the centre of earth is given by, $g=\frac{GMe}{r^2}$

So, $\left(\frac{dg}{g}\right)\times 100=-2\left(\frac{dr}{r}\right)\times 100$

Given: $\frac{dr}{r}\times 100=-2\%$

$\Rightarrow \left(\frac{dg}{g}\right)\times 100=-2\times(-2)\%=4\%$

So, g will increase by 4%.

Q.12. The value of modulation index so that modulated signal does not get distorted is,

A) $\mu \leq 1$

B) $\mu > 1$

C) $\mu = 0$

D) $\mu = 2$

Answer: $\mu \leq 1$

Solution: The modulation index of an amplitude modulated signal is defined as the measure or extent of amplitude variation about an un-modulated carrier.

For undistorted signal, $Am \leq Ac$

So, $\frac{Am}{Ac} \leq 1$

$\mu \leq 1$

Q.13. A train moves with a speed of 30 km h^{-1} with whistle frequency of 320 Hz, towards a wall. The frequency of echo heard by the driver is equal to (Speed of sound$= 330 \text{ m s}^{-1}$)

A) 336

B) 350

C) 300

D) 280

Answer: 336

Solution: Frequency as perceived by the wall is, $f' = \frac{v}{v - v_t} \times f_0$.

Now frequency of echo heard by the driver will be,

$f = \frac{v + v_t}{v} \times f'$

$== \frac{v + v_t}{v - v_t} \times f_0$

$= \left(\frac{330 + \frac{25}{3}}{330 - \frac{25}{3}} \right) \times 320$

$\approx 336 \text{ Hz}$

Q.14. $S1$: Net momentum of ideal gas depends on temperature.

$S2$: If initial v_{rms} of oxygen is v. Now temperature is doubled and oxygen molecule disassociates in two atoms, so new v_{rms} is $2v$.

A) $S1 \rightarrow$ True, $S2 \rightarrow$ True

B) $S1 \rightarrow$ False, $S2 \rightarrow$ False

C) $S1 \rightarrow$ True, $S2 \rightarrow$ False

D) $S1 \rightarrow$ False, $S2 \rightarrow$ True

Answer: $S1 \rightarrow$ False, $S2 \rightarrow$ True

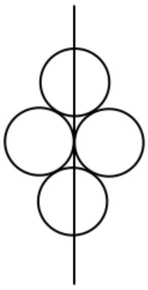
Solution: Net momentum of an ideal gas system is 0.

So, statement 1 is false

For statement 2 using, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

Temperature is doubled, and molar mass is halved, so v_{rms} speed is doubled.

Q.15. Each disk has mass m and diameter a. If moment of inertia of the system about the given axis is $\left(\frac{x}{4}ma^2\right)$. Find the value of x .



A) 3

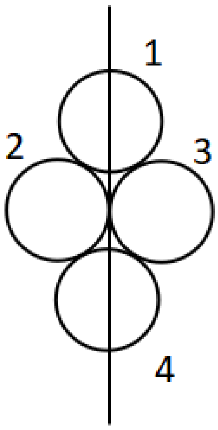
B) 7

C) 5

D) 4

Answer: 3

Solution:



Moment of inertia of the disc about an axis passing through the centre of mass and perpendicular to the plane of the disc is given by, $\frac{mr^2}{2} = \frac{ma^2}{8}$ and about an axis passing along its diameter is given by, $\frac{mr^2}{4} = \frac{ma^2}{16}$.

Therefore, $I_1 = I_4 = \frac{ma^2}{16}$.

Now using parallel axis theorem, $I_2 = I_3 = \left(\frac{ma^2}{16}\right) + \left(\frac{ma^2}{4}\right) = \frac{5ma^2}{16}$.

Hence,

$I_{net} = I_1 + I_2 + I_3 + I_4$

$= \frac{ma^2}{16} + \frac{5ma^2}{16} + \frac{5ma^2}{16} + \frac{ma^2}{16}$

$= \frac{3ma^2}{4}$

Q.16. A wire of resistance R is stretched so that the final length is double the original length, what is the final resistance?

A) $4R$

B) $2R$

C) R

D) $\frac{R}{2}$

Answer: $4R$

Solution: Length of the wire is doubled, i.e., $L' = 2L$

Equating the volumes, we get, $SL = (2L)S'$, i.e., $S' = \frac{S}{2}$.

Now, $R = \rho\frac{L}{S}$

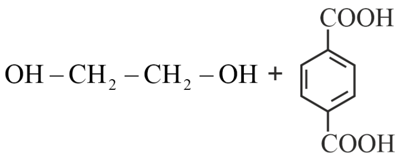
Hence, new resistance will be,

$$R' = \rho\left[\frac{L'}{S'}\right]$$
$$= \rho\frac{(2L)}{\left(\frac{S}{2}\right)}$$
$$= 4\rho\left(\frac{L}{S}\right)$$
$$= 4R$$

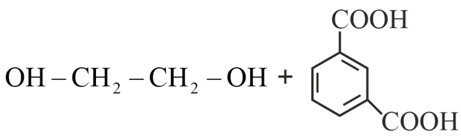
Section B: Chemistry

Q.17. Which of the following options represent the monomer of TERYLENE?

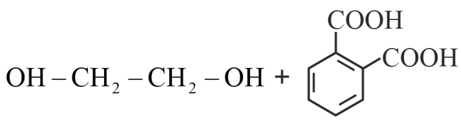
A)



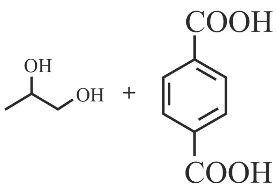
B)



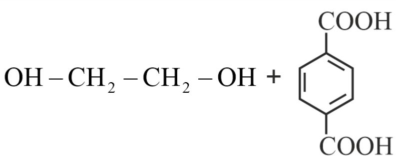
C)



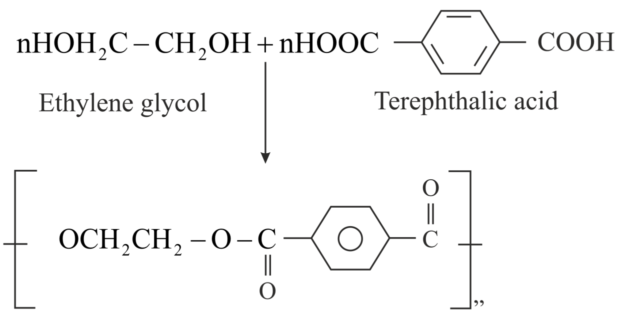
D)



Answer:



Solution:



Terylene or dacron

Q.18. Identify the number of paramagnetic species out of the following:

$Li_2,\ B_2,\ C_2,\ C_2^-,\ O_2,\ O_2^{2-},\ O_2^+$

A) 2

B) 3

C) 4

D) 6

Answer: 4

Solution: The species that contain unpaired electrons are paramagnetic.

Electronic configuration of Li_2 molecule is $(\sigma 1\ s)^2(\sigma^* 1\ s)^2(\sigma 2\ s)^2$.

The electronic configuration of B_2 molecule is

$(\sigma 1s)^2(\sigma^\star 1s)^2(\sigma 2s)^2(\sigma^\star 2s)^2\left(\pi 2p_x{}^1 = \pi 2p_y{}^1\right)$

The presence of unpaired electrons in the two π bonding molecular orbitals explains its paramagnetic nature.

The electronic configuration of C_2 $KK\,\sigma 2s^2\sigma^* 2s^2\pi 2p_x^2\pi 2p_y^2$

$O_2:\ KK\,(\sigma 2\,s)^2(\sigma 2\,s)^2\left(\sigma_{2pz}\right)^2\left(\pi_{2px}\right)^2 = \left(\pi_{2py}\right)^2\left(\pi^*{}_{2px}\right)^1 = \left(\pi^*{}_{2py}\right)^1$

Hence, O_2 molecule is paramagnetic.

The electronic configuration of C_2^- is $(\sigma 1s)^2(\sigma^\star 1s)^2(\sigma 2s)^2(\sigma^\star 2s)^2\left(\pi 2p_x{}^2 = \pi 2p_y{}^2\right)\sigma 2p_z{}^1$

Hence, it is paramagnetic substance.

$O_2^{2-}:\ KK\,(\sigma 2\,s)^2(\sigma 2\,s)^2\left(\sigma_{2pz}\right)^2\left(\pi_{2px}\right)^2 = \left(\pi_{2py}\right)^2\left(\pi^*{}_{2px}\right)^2 = \left(\pi^*{}_{2py}\right)^2$

$O_2^+:\ KK\,(\sigma 2\,s)^2(\sigma 2\,s)^2\left(\sigma_{2pz}\right)^2\left(\pi_{2px}\right)^2 = \left(\pi_{2py}\right)^2\left(\pi^*{}_{2px}\right)^1 = \left(\pi^*{}_{2py}\right)$

Hence, the above species paramagnetic.

Q.19. $MnO_4{}^{2-}$ disproportionates in acidic medium to form two compounds of manganese A and B. Oxidation state of Mn is less in B than in A. Spin only magnetic moment of B (in $B.M.$) is:

(Round of to nearest integer)

A) 3

B) 5

C) 6

D) 4

Answer: 4

Solution: Potassium permanganate Is prepared by fusion of MnO_2 with an alkali metal hydroxide and an oxidising agent like KNO_3. This produces the dark green K_2MnO_4 which disproportionates in a neutral or acidic solution to give permanganate.

$2\,MnO_2 + 4KOH + O_2 \rightarrow 2K_2MnO_4 + 2H_2O$

$3MnO_4{}^{2-} + 4H^- \rightarrow 2MnO_4{}^- + MnO_2 + 2H_2O$

The oxidation state of Mn in MnO_2 is Mn^{4+}

$Mn^{4+} = [Ar]3d^3$

Where, $n = 3$

$\mu_s = \sqrt{n\,(n+2)}$

$\Rightarrow \sqrt{3\,(3+2)}$

$\Rightarrow \sqrt{3\,(5)}$

$\Rightarrow \sqrt{15}$

$\Rightarrow 3.87 \approx 4\ B.M.$

Therefore, the spin only magnetic moment of Mn^{4+} ion is approximately $4\ B.M.$

Q.20. Which of the following metals has least tendency to evolve H_2 from acid?

A) Cu

B) Zn

C) Mn

D) Ni

Answer: Cu

Solution:

Element (M)	$\mathrm{E}^{\ominus}/\mathrm{V}$
Mn	-1.18
Ni	-0.25
Cu	0.34
Zn	-0.76

Copper does not react with dilute sulphuric acid, liberating hydrogen because copper is lower in electrochemical series than hydrogen

Q.21. Consider the following reaction:

$$\underset{1\ \mathrm{mol}}{\mathrm{X}} + \underset{1\ \mathrm{mol}}{\mathrm{Y}} + \underset{0.05\ \mathrm{mol}}{3\mathrm{Z}} \rightarrow \mathrm{XYZ}_3$$

Calculate the mass of XYZ_3 formed at the end of the reaction(in g).

[Given: molar masses of $\mathrm{X},\ \mathrm{Y}$ and Z are $10,\ 20,\ 30\ \mathrm{g/mol}$ respectively.]

A) 2

B) 3

C) 4

D) 5

Answer: 2

Solution:

$$\begin{array}{lccccc} & \mathrm{X} & +\mathrm{Y} & +3\mathrm{Z} & \rightarrow & \mathrm{XYZ}_3 \\ \text{initially} & 1 & 1 & 0.05 & & \\ \text{finally} & 1-\frac{0.05}{3} & 1-\frac{0.05}{3} & & & \frac{0.05}{3} \end{array}$$

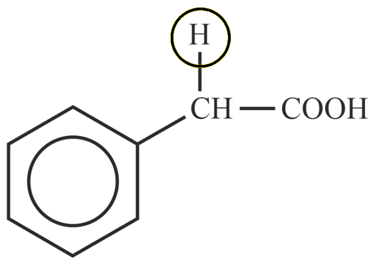
$$\text{Molar mass of } \mathrm{XYZ}_3 = 10 + 20 + 30 \times 3 = 120$$

Mass $= 120 \times \frac{0.05}{3}$

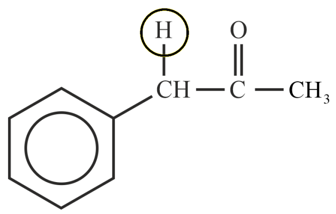
$= 2\ \mathrm{gm}$

Q.22. Which of the following indicated H-atom has lowest value of $\mathrm{pK_a}$?

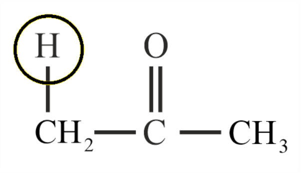
A)



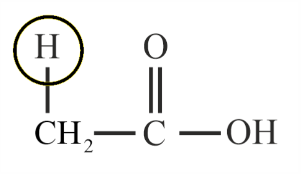
B)



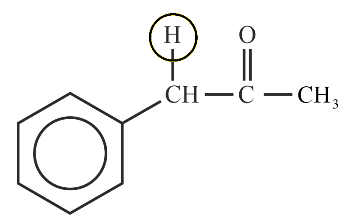
C)



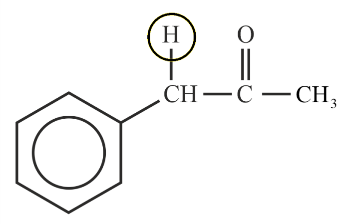
D)



Answer:



Solution: The smaller the value of pKa, the stronger the acid. The acidic nature of the compound depends on the stability of conjugate base.



The stability of conjugate base formed by above compound is more stable than other three compounds given.

Q.23. $10.2\ \mathrm{g}$ of ascorbic acid is dissolved in $150\ \mathrm{g}$ of CH_3COOH. The depression in freezing point of the resulting solution is $\mathrm{x} \times 10^{-1}\ \mathrm{K}$.

[Given $k_f = 3.9\ \mathrm{K\ kg\ mol^{-1}}$ of acetic acid and molar mass of ascorbic acid $= 176\ \mathrm{g\ mol^{-1}}$]

[Consider ascorbic acid and acetic acid remain undissociated]

(Round off to nearest integer)

A) 10

B) 15

C) 39

D) 10.2

Answer: 15

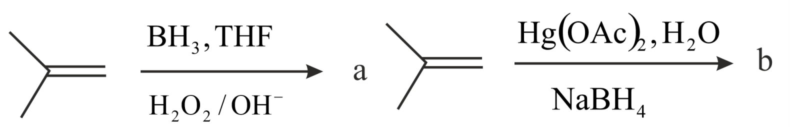
Solution: $|\Delta T_f| = k_f \cdot m$

$= 3.9 \times \frac{10.2}{176} \times \frac{1000}{150}$

$= \frac{39 \times 10.2}{176 \times 15}$

$= 1.5 = 15 \times 10^{-1}$

Q.24. Which of the following will give (i) Markovnikov product (ii) Antimarkovnikov product



A) a - Markovnikov; b - Markovnikov

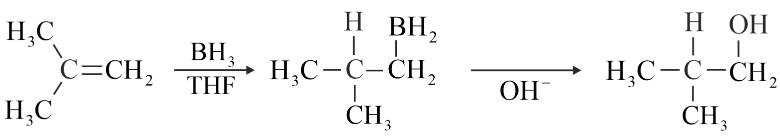
B) a - Markovnikov; b - Antimarkovnikov

C) a - Antimarkovnikov; b - Antimarkovnikov

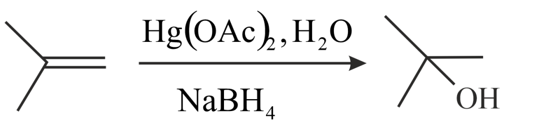
D) a - Antimarkovnikov; b - Markovnikov

Answer: a - Antimarkovnikov; b - Markovnikov

Solution: The hydroboration oxidation reaction mechanism can be considered as an anti-Markovnikov reaction where a hydroxyl group attaches itself to the carbon which is less substituted.



Alkene hydration using the oxymercuration-demercuration reaction pathway reliably produces the Markovnikov product without carbocation rearrangement as shown in the example below.



Markovnikov

Q.25.

	List 1		List 2
(A)	$Cd(s) + 2NiO(OH)(s) + 2H_2O(l) \rightarrow Cd(OH)_2(s) + 2Ni(OH)_2(s)$	(P)	Fuel cell
(B)	$Zn(Hg) + 2OH^- \rightarrow ZnO(s) + H_2O(l) + 2e^-$	(Q)	Primary battery
(C)	$PbSO_4 + 2e^- \rightarrow Pb(s) + SO_4^{2-}$	(R)	Discharging of secondary battery
(D)	$2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$	(S)	Charging of secondary battery

A) A-Q, B-R, C-P, D-S

B) A-Q, B-P, C-R, D-S

C) A-R, B-Q, C-S, D-P

D) A-P, B-Q, C-R, D-S

Answer: A-R, B-Q, C-S, D-P

Solution: Mercury cell, suitable for low current devices like hearing aids, watches, etc. consists of zinc - mercury amalgam as anode and a paste of HgO and carbon as the cathode. The electrolyte is a paste of KOH and ZnO. The electrode reactions for the cell are given below:

Anode: $Zn\,(Hg) + 2\,OH^- \rightarrow ZnO\,(s) + H_2O + 2e^-$

Cathode: $HgO + H_2O + 2e^- \rightarrow Hg\,(l) + 2\,OH^-$

Another important secondary cell is the nickel-cadmium cell which has longer life than the lead storage cell but more expensive to manufacture.

The overall reaction during discharge is:

$Cd\,(s) + 2\,Ni\,(OH)_3\,(s) \rightarrow CdO\,(s) + 2\,Ni\,(OH)_2\,(s) + H_2O\,(l)$

Fuel cell using H_2 and O_2 produces electricity.

Cathode: $O_2\,(g) + 2H_2O\,(l) + 4e^- \rightarrow 4\,OH^-\,(aq)$

Anode: $2H_2\,(g) + 4\,OH^-\,(aq) \rightarrow 4H_2O\,(l) + 4e^-$

Overall reaction being:

$2H_2\,(g) + O_2\,(g) \rightarrow 2H_2O\,(l)$

(ii) Cell reaction of lead storage battery during recharge -

The reverse of oxidation at anode during discharge is the reduction reaction at cathode during recharging.

$Pb^{2+}_{(aq)} + 2e^- \rightarrow Pb_{(s)} -$

$PbSO_{4(s)} \rightarrow Pb^{2+}_{(aq)} + SO^{2-}_{4(aq)}$

Final Reaction: $PbSO_{4(s)} + 2e^- \rightarrow Pb_{(s)} + SO^{2-}_{4(aq)}$

Q.26. The correct decreasing order for magnitude of electron gain enthalpy is:

A) $S > Se > Te > O$

B) $Te > Se > S > O$

C) $O > S > Se > Te$

D) $S > O > Se > Te$

Answer: $S > Se > Te > O$

Solution: Correct order of electron gain enthalpy is :-

$O < S > Se > Te$

$\Rightarrow S > Se > Te > O$

(Oxygen shows least electron gain enthalpy due to small size of atom)

Q.27. Match column I with column II.

	Column I		Column II
(I)	$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta}$	(p)	Cl_2
(II)	$KMnO_4 + HCl \rightarrow$	(q)	N_2
(III)	$Al + NaOH \rightarrow$	(r)	O_2
(IV)	$NaNO_3 \xrightarrow{\Delta}$	(s)	H_2

A) (I)-(p), (II)-(q), (III)-(r), (IV)-(s)

B) (I)-(q), (II)-(p), (III)-(s), (IV)-(r)

C) (I)-(p), (II)-(r), (III)-(s), (IV)-(q)

D) (I)-(q), (II)-(r), (III)-(p), (IV)-(s)

Answer: (I)-(q), (II)-(p), (III)-(s), (IV)-(r)

Solution: Thermal decomposition of ammonium dichromate $(NH_4)_2Cr_2O_7$ on heating gives N_2, Cr_2O_3 and H_2O as follows:

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$

$2KMnO_{4(s)} + 16HCl(aq) \rightarrow 2KCl(aq) + 2MnCl_{2(aq)} + 8H_2O(l) + 5Cl_{2(g)}$

The reaction of Aluminum with Sodium hydroxide $(Al + NaOH)$ gives a salt and Hydrogen gas.

$2Al(s) + 2NaOH(l) + 6H_2O(l) \rightarrow 2Na[Al(OH)_4](s) + 3H_2(l)$

On heating sodium nitrate it undergoes decomposition reaction and liberates oxygen and convert into sodium nitrite.

$2NaNO_3(heat) \rightarrow 2NaNO_2 + O_2$

Q.28. Which of the following reactions represent leaching?

A) $Al_2O_3 + NaOH \rightarrow$Products

B) $Cu_2S + O_2 \rightarrow$Products

C) $Fe_2O_3 + CO \rightarrow$Products

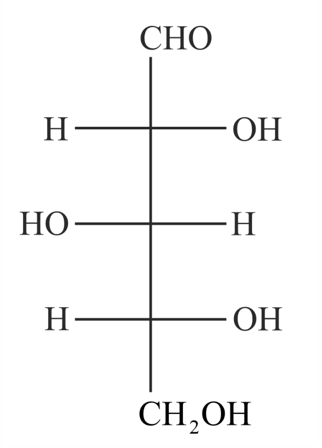
D) $FeS_2 + O_2 \rightarrow$Products

Answer: $Al_2O_3 + NaOH \rightarrow$Products

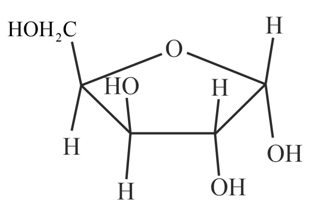
Solution: Leaching is often used if the ore is soluble in some suitable solvent.

Leaching of alumina from bauxite: Bauxite is the principal ore of aluminium. It usually contains SiO_2, iron oxides and titanium oxide (TiO_2) as impurities. Concentration is carried out by heating the powdered ore with a concentrated solution of $NaOH$ at $473-523$ K and $35-36$ bar pressure. This process is called digestion. This way, Al_2O_3 is extracted out as sodium aluminate.

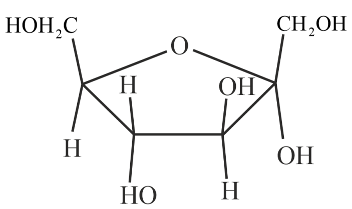
Q.29. Find the cyclic structure of pentose given below:



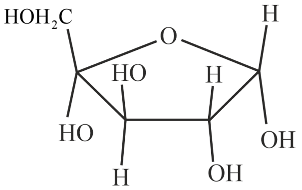
A)



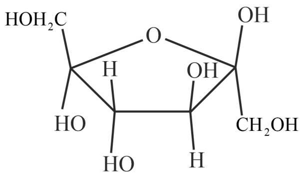
B)



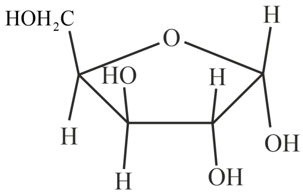
C)



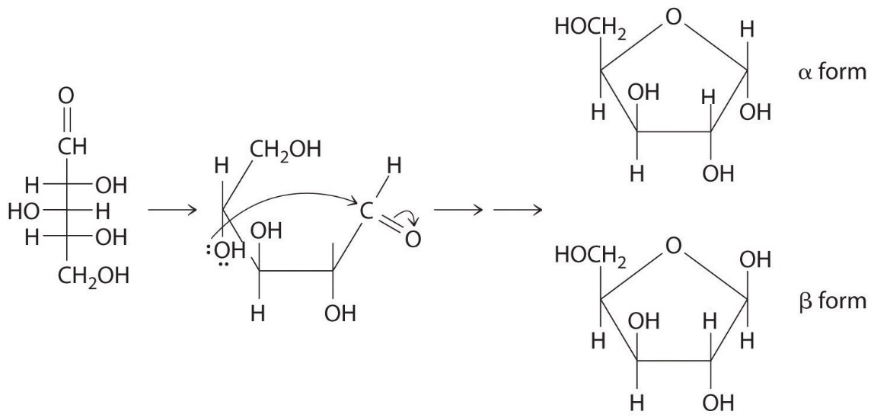
D)



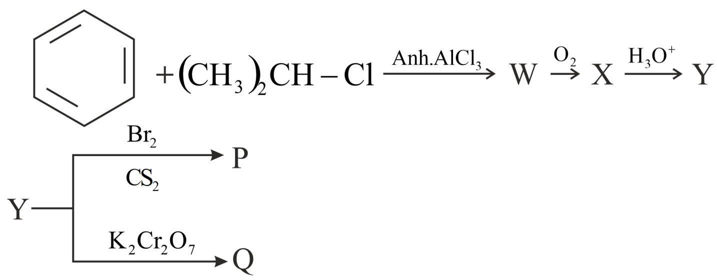
Answer:



Solution:

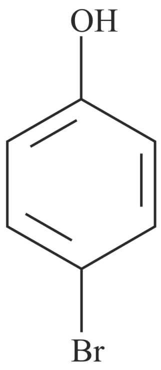


Q.30.



The compound P and Q respectively are:

A)



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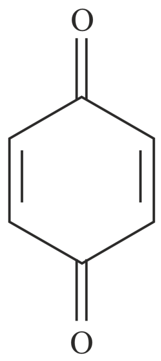
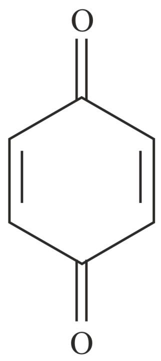


B)



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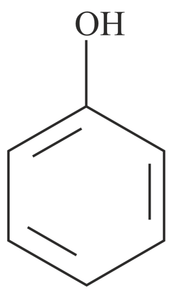
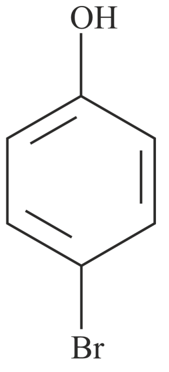


C)



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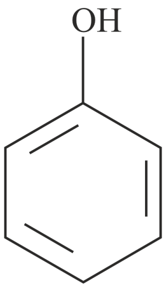
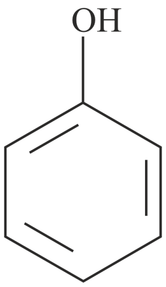


D)



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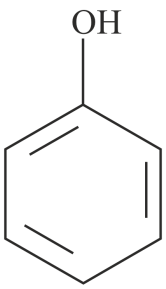
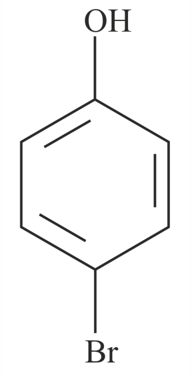


Answer:



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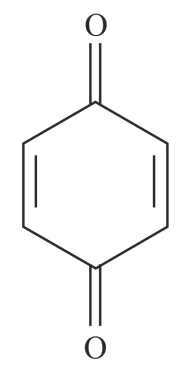
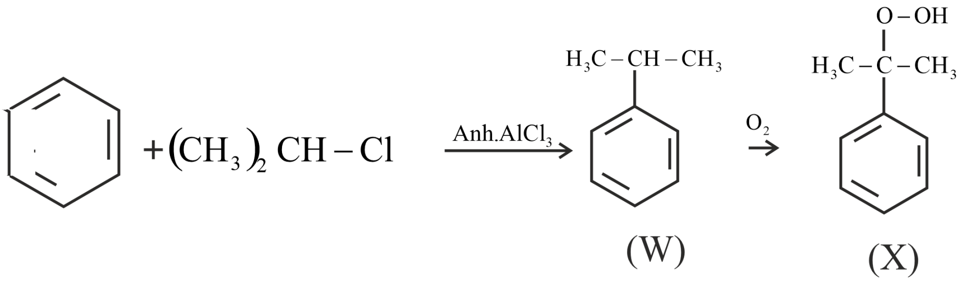
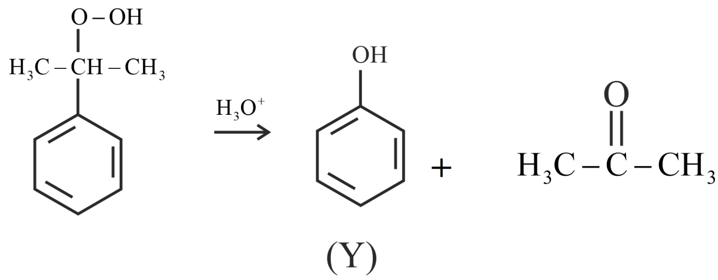
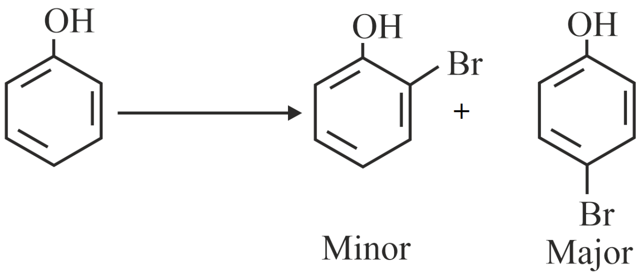
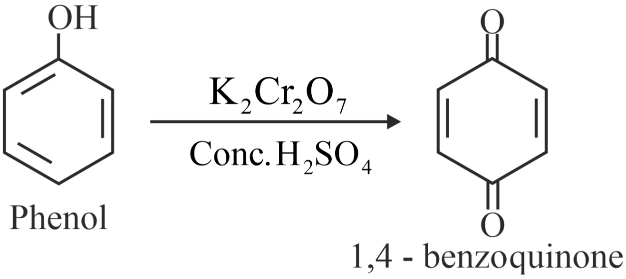


Solution:









Q.31. Choose the incorrect statement regarding Bohr’s theory.

A) The electron move around the nucleus in certain circular stable orbits without emitting radiation

B) The angular momentum of electrons in the Bohr orbit always remain constant

C) The angular momentum in third orbit is more than that of first orbit

D) The energy difference between consecutive levels remain same as n increases

Answer: The energy difference between consecutive levels remain same as n increases

Solution: An electron can move only in those orbits for which its angular momentum is integral multiple of $\frac{h}{2\pi}$. That means angular momentum is quantised. Radiation is emitted or absorbed only when transition of electron takes place from one quantised value of angular momentum to another. Therefore. Maxwell's electromagnetic theory does not apply here that is why only certain fixed orbits are allowed. Energy difference between consecutive levels decreases as n increases. So, as the n increases, energy between the consecutive levels will decrease.

Q.32. Which of the following salts are formed in Clark's method?

A) $Ca(OH)_2,\ Mg(OH)_2$

B) $CaCO_3,\ Mg(OH)_2$

C) $Ca(OH)_2,\ MgCO_3$

D) $CaCO_3,\ MgCO_3$

Answer: $CaCO_3,\ Mg(OH)_2$

Solution: Clark's method: In this method calculated amount of lime is added to hard water. It precipitates out calcium carbonate and magnesium hydroxide which can be filtered off.

$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2\,CaCO_3 \downarrow + 2H_2O$

$Mg(HCO_3)_2 + 2\,Ca(OH)_2 \rightarrow 2\,CaCO_3 \downarrow + Mg(OH)_2 \downarrow + 2H_2O$

Q.33. Calculate the pH of $0.2\ M$ butyric acid with $K_a = 2 \times 10^{-5}$.

[Given: $\log\ 2 = 0.3$]

A) 3.7

B) 2.7

C) 4.7

D) 11.3

Answer: 2.7

Solution: $HA \leftrightharpoons H^+ + A^-$

$C \quad - \quad -$

$C - C\alpha \quad C\alpha \quad C\alpha$

$K_a = C\alpha^2$

$CK_a = (C\alpha)^2$

$[H^+] = C\alpha = \sqrt{C.K_a}$

$= \sqrt{0.2 \times 2 \times 10^{-5}}$

$= 2 \times 10^{-3}$

$pH = -\log [H^+]$

$pH = -\log\ 2 \times 10^{-3}$

$= 3 - 0.3$

$= 2.7$

Q.34. Which of the following is an incorrect statement?

A) LiF is less soluble in water due to low hydration enthalpy

B) KO_2 is paramagnetic

C) Density of sodium is greater than potassium

D) H_2O_2 is diamagnetic

Answer: LiF is less soluble in water due to low hydration enthalpy

Solution: Due to the small size of Li^+ ion and F^-ions, the lattice energy of LiF is very high. LiF is covalent in nature. Therefore, LiF is insoluble in water.

The oxidation state of K in KO_2 is $+1$. It has 1 unpaired electron in $\pi^* 2p$ Molecular orbital ofO_2^- . Therefore, it is Paramagnetic.

Lithium has lesser atomic weight than sodium. So, lithium has lesser density than sodium. But in the case of 'K' and 'Na', d-orbitals present in Potassium, which increases the volume of 'K'.

Hydrogen peroxide is diamagnetic substance.

Section C: Mathematics

Q.35. The remainder when $7^{2022}+3^{2022}$ is divisible by 5 is

Answer: 3

Solution: $7^{2022}+3^{2022}=(50-1)^{1011}+(10-1)^{1011}$

$= {}^{1011}C_0(50)^{1011}(-1)^0+{}^{1011}C_1(50)^{1010}(-1)^1+\ldots {}^{1011}C_{1011}(50)^0(-1)^{1011}$
$+{}^{1011}C_0(10)^{1011}(-1)^0+{}^{1011}C_1(10)^{1010}(-1)^1+\ldots {}^{1011}C_{1011}(10)^0(-1)^{1011}$

$=5k-2$

Using binomial expansion we get the remainder as $-2+5=3$

Q.36. The sum of all the values of x satisfying $\cos^{-1}x-2\sin^{-1}x=\cos^{-1}2x$ is

A) 0

B) 1

C) $\frac{1}{2}$

D) $-\frac{1}{2}$

Answer: 0

Solution: $\cos^{-1}x-2\sin^{-1}x=\cos^{-1}2x$

$\frac{\pi}{2}-\sin^{-1}x-2\sin^{-1}x=\cos^{-1}2x$

$\frac{\pi}{2}-3\sin^{-1}x=\cos^{-1}2x$

$\frac{\pi}{2}-\cos^{-1}2x=3\sin^{-1}x$

$\sin^{-1}2x=3\sin^{-1}x$

$\sin^{-1}2x=\sin^{-1}\left(3x-4x^3\right)$

$2x=3x-4x^3$

$4x^3-x=0$

$x=0,\pm\frac{1}{2}$

Hence, sum of values is 0

Q.37. If $xdy=\left(\sqrt{x^2+y^2}+y\right)dx$ and $y(1)=0$, then $y(2)$ equals

A) $\frac{1}{2}$

B) $\frac{1}{4}$

C) $\frac{3}{2}$

D) $\frac{5}{2}$

Answer: $\frac{3}{2}$

Solution: Given $xdy=\left(\sqrt{x^2+y^2}+y\right)dx$

$\frac{dy}{dx}=\frac{\sqrt{x^2+y^2}+y}{x}$

Substituting $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$, we get

$v+x\frac{dv}{dx}=\sqrt{1+v^2}+v$

$\Rightarrow \int\frac{dv}{\sqrt{1+v^2}}=\int\frac{dx}{x}$

$\Rightarrow \ln\left(\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right)=\ln x+C$

Given $y(1)=0$,

so $C=0$

i.e. $y+\sqrt{x^2+y^2}=x^2$

Putting $x=2$, we get

$y+\sqrt{4+y^2}=4\Rightarrow 4+y^2=16-8y+y^2$

$\Rightarrow y=\frac{3}{2}$

Q.38.

If $a_{n+2}=\frac{2}{a_{n+1}}+a_n, a_1=1, a_2=2$, and $\left(\frac{a_1+\frac{1}{a_2}}{a_3}\right)\cdot\left(\frac{a_2+\frac{1}{a_3}}{a_4}\right)\cdots\left(\frac{a_{30}+\frac{1}{a_{31}}}{a_{32}}\right)=2^{\alpha}\ {}^{61}C_{31}$, then α is

A) -30

B) -31

C) -60

D) -61

Answer: -60

Solution: Given $a_{n+2}=\frac{2}{a_{n+1}}+a_n$

i.e. $a_{n+2}a_{n+1}-a_na_{n+1}=2$

i.e. $T_r=a_{r+1}a_r$ is an A.P. with common difference 2

Now, $T_1=a_1a_2=2,\ T_2=a_2a_3=4,\ \ldots,\ T_r=2r$

So $\left(\frac{a_1+\frac{1}{a_2}}{a_3}\right)\cdot\left(\frac{a_2+\frac{1}{a_3}}{a_4}\right)\cdots\left(\frac{a_{30}+\frac{1}{a_{31}}}{a_{32}}\right)=\prod_{i=1}^{30}\left(\frac{a_i+\frac{1}{a_{i+1}}}{a_{i+2}}\right)$

$=\prod_{i=1}^{30}\left(\frac{a_ia_{i+1}+1}{a_{i+1}a_{i+2}}\right)=\prod_{i=1}^{30}\frac{Tr+1}{T_{r+1}}$

$=\prod_{i=1}^{30}\left(\frac{2r+1}{2r+2}\right)=\frac{3\cdot5\cdot7\cdots61}{4\cdot6\cdot8\cdots62}=\frac{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdots61\cdot62}{2(4\cdot6\cdot8\cdots62)^2}$

$=\frac{62!}{2(4\cdot6\cdot8\cdots62)^2}=\frac{62!}{2^{61}\cdot(31!)^2}=\frac{62(61!)}{2^{61}\cdot31(31!)(30!)}$

$=2^{-60}\cdot{}^{61}C_{30}$

$=2^{-60}\cdot{}^{61}C_{31}$

Q.39. Let $z_1 \in C, |z_1 - 3| = \frac{1}{2}, z_2 \in C, |z_2 + |z_2 - 1|| = |z_2 - |z_2 + 1||$, then least value of $|z_1 - z_2|$ is

A) 0

B) $\frac{1}{2}$

C) $\frac{3}{2}$

D) $\frac{5}{2}$

Answer: $\frac{3}{2}$

Solution: Here $|z_1 - 3| = \frac{1}{2}$ represents a circle on argand plane with centre $(3, 0)$ and radius $\frac{1}{2}$

Given $|z_2 + |z_2 - 1||^2 = |z_2 - |z_2 + 1||^2$

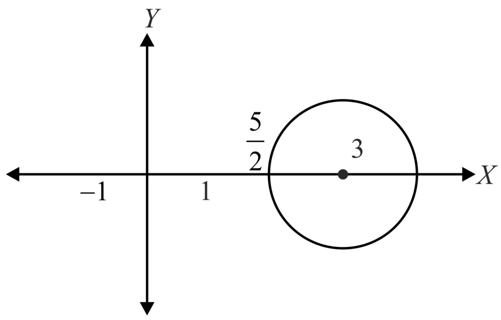
$\Rightarrow (z_2 + |z_2 - 1|)(\bar{z}_2 + |z_2 - 1|) = (z_2 - |z_2 + 1|)(\bar{z}_2 - |z_2 + 1|)$

$\Rightarrow z_2(|z_2 - 1| + |z_2 + 1|) + \bar{z}_2(|z_2 - 1| + |z_2 + 1|) = |z_2 + 1|^2 - |z_2 - 1|^2$

$\Rightarrow (z_2 + \bar{z}_2)(|z_2 + 1| + |z_2 - 1|) = 2(z_2 + \bar{z}_2)$

$\Rightarrow$ Either $z_2 + \bar{z}_2 = 0$ or $|z_2 + 1| + |z_2 - 1| = 2$

i.e. z_2 lies on imaginary axis or it lies on the line segment joining $(-1, 0)$ and $(1, 0)$



So, the minimum distance between z_1 & z_2 will be the distance between the points $(1, 0)$ & $\left(\frac{5}{2}, 0\right)$

Hence, $|z_1 - z_2|_{\min} = \frac{3}{2}$

Q.40. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B_0 = A^{98} + 2A^{49}, B_n = \text{adj}(B_{n-1})$, then $|B_4|$ is equal to

A) 3^{16}

B) 3^{32}

C) 2^{16}

D) 2^{32}

Answer: 3^{32}

Solution:

Given $A=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$

Now $A^2=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$

i.e. $A^{98}=I; A^{49}=A$

$\therefore B_0==A^{98}+2A^{49}=I+2A=\begin{bmatrix}1&2&0\\2&1&0\\0&0&3\end{bmatrix}$

Given $B_n=\operatorname{adj}(B_{n-1})$

$\therefore B_4=\operatorname{adj}(B_3)=\operatorname{adj}(\operatorname{adj}B_2)=\operatorname{adj}(\operatorname{adj}(\operatorname{adj}B_1))$

$=\operatorname{adj}(\operatorname{adj}(\operatorname{adj}(\operatorname{adj}B_0)))$

$\therefore |B_4|=|B_0|^{2^4}=|B_0|^{16}$

Now $|B_0|=\begin{bmatrix}1&2&0\\2&1&0\\0&0&3\end{bmatrix}=-9$

Hence, $|B_4|=|B_0|^{16}=9^{16}=3^{32}$

Q.41. If $y=\cos^{-1}\left(\frac{x^2-4x+2}{x^2+3}\right)$, then domain is:

A) $\left(-\infty,-\frac{1}{3}\right)$

B) $\left[-\frac{1}{4},\infty\right)$

C) $\left(-\frac{1}{3},\infty\right)$

D) $\left(-\frac{1}{4},\infty\right)$

Answer: $\left[-\frac{1}{4},\infty\right)$

Solution:

$f(x)=\cos^{-1}\left(\frac{x^2-4x+2}{x^2+3}\right)$

For domain $-1\leq\frac{x^2-4x+2}{x^2+3}\leq 1$

$\Rightarrow\ -x^2-3\leq x^2-4x+2\leq x^2+3$

$\Rightarrow\ 2x^2-4x+5\geq 0$ and $4x\geq -1$

$\Rightarrow\ x\in R$ and $x\in\left[-\frac{1}{4},\infty\right)$

$\therefore\ x\in\left[-\frac{1}{4},\infty\right)$

Q.42. Two tangent at points P and Q on parabola $y^2=2x-3$ meet at $R(0,1)$, then orthocentre of ΔPQR is:

A) $(0,1)$

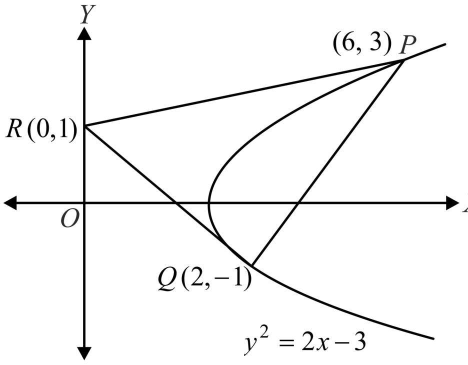
B) $(2,-1)$

C) $(6,3)$

D) $(2,1)$

Answer: $(2,-1)$

Solution:



Given parabola is $y^2 = 2x - 3 \Rightarrow y^2 = 2\left(x - \frac{3}{2}\right)$

Let a point on parabola be $P = \left(\left(\frac{3}{2} + \frac{t^2}{2}\right), t\right)$

Equation of tangent at P will be $yt = x + \frac{3}{2} + \frac{t^2}{2} - 3$

$\because$ $R(0,1)$ lies on this tangent

then $t = \frac{t^2}{2} - \frac{3}{2}$

i.e. $t = 3$ or -1.

So $P \equiv (6,3)$ & $Q \equiv (2,-1)$

Now triangle PQR is right angled at Q, so $(2,-1)$ is the orthocentre of the triangle.

Q.43. Let hyperbola $H : x^2 - y^2 = 1$ and ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and eccentricity of ellipse E is reciprocal of that of hyperbola H. If common tangent of H & E is $y = \sqrt{\frac{5}{2}}x + k$, then value of $4\left(a^2 + b^2\right)$ is equal to

Answer: 3

Solution: Given $H : x^2 - y^2 = 1, E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$e_H = \sqrt{2}$ & $e_E = \frac{1}{e_H} = \frac{1}{\sqrt{2}}$

For hyperbola $1 - \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$

Also given that the common tangent of H & E is $y = \sqrt{\frac{5}{2}}x + k \left(\text{i.e. } m = \sqrt{\frac{5}{2}}\right)$

We know that the condition for common tangent of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ is $a^2m^2 + b^2 = A^2m^2 - B^2$

Now, for common tangency: $\frac{5}{2}a^2 + b^2 = \frac{5}{2} - 1$

$\frac{5}{2} + \frac{b^2}{a^2} = \frac{3}{2a^2} \Rightarrow a^2 = \frac{1}{2}$

$\therefore$ $a^2 + b^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$\Rightarrow 4\left(a^2 + b^2\right) = 3$

Q.44. $A(\cos t, \sin t), B(\sin t, -\cos t)$ and $C(a,b)$ form an equilateral triangle, where $t \in [0, 2\pi]$ and orthocentre of this triangle lies on a circle with centre $\left(1, \frac{1}{3}\right)$ then $a^2 - b^2$ is equal to ______ .

Answer: 8

Solution: For an equilateral triangle the orthocentre and centroid coincide.

Here, centroid $(h, k) \equiv \left(\frac{\cos t+\sin t+a}{3}, \frac{\sin t-\cos t+b}{3}\right)$

$\Rightarrow 3h - a = \cos t + \sin t \qquad \ldots \text{(i)}$

$\Rightarrow 3k - b = \sin t - \cos t \qquad \ldots \text{(ii)}$

Eliminating t from above two equation (i) & (ii), we get

$\left(h - \frac{a}{3}\right)^2 + \left(k - \frac{b}{3}\right)^2 = \frac{2}{9}$

So, (h, k) lies on the circle whose centre is $\left(\frac{a}{3}, \frac{b}{3}\right)$.

i.e. $a = 3,\ b = 1$

Hence $a^2 - b^2 = 8$

Q.45. A password is to be made of 6 to 8 digits from either of the sets A, B, C, D, E or $1, 2, 3, 4, 5$ having at least one numeric. If possible password is equal to $k \times 5^6$, then k is equal to

Answer: 7073

Solution: Total available characters is 10 and total digits are 5.

So, for $6-$digit password, the number of ways $= 10^6 - 5^6$

For $7-$digit password, the number of ways $= 10^7 - 5^7$

For $8-$digit password, the number of ways $= 10^8 - 5^8$

$\therefore$ Total ways $= \left(10^6 + 10^7 + 10^8\right) - \left(5^6 + 5^7 + 5^8\right)$

$= 2^6 5^6 + 2^7 5^7 + 2^8 5^8 - 5^6 - 5^7 - 5^8$

$= 5^6 \left(2^6 + 5 \cdot 2^7 + 25 \cdot 2^8 - 1 - 5 - 25\right) = 7073 \cdot 5^6$

Q.46. The solution of differential equation $\sin^2(2x)\frac{dy}{dx} + \left(8\sin^2 2x + 4\sin 2x\right)y = 0$ $y\left(\frac{\pi}{4}\right) = e^{-\pi}$ then $y\left(\frac{\pi}{6}\right)$ is

A) $3e^{\frac{\pi}{3}}$

B) $3e^{-\frac{\pi}{3}}$

C) $\sqrt{3}e^{\frac{\pi}{3}}$

D) $\sqrt{3}e^{-\frac{\pi}{3}}$

Answer: $3e^{-\frac{\pi}{3}}$

Solution: $\sin^2(2x)\frac{dy}{dx}+\left(8\sin^2 2x+4\sin 2x\right)y=0$

$\Rightarrow \int\frac{dy}{y}=-\int(8+4\operatorname{cosec}2x)dx$

$\Rightarrow \ln y=-8x-2\ln|\operatorname{cosec}2x-\cot 2x|+c$

Given $y\left(\frac{\pi}{4}\right)=e^{-\pi}$, so

$-\pi+2\pi+2\ln 1=c\Rightarrow c=\pi$

Hence the particular solution of the differential equation will be

$\ln y+8x+2\ln|\operatorname{cosec}2x-\cot 2x|=\pi$

Now putting $x=\frac{\pi}{6}$, we get

$\ln y+\frac{4\pi}{3}+2\ln\left|\frac{1}{\sqrt{3}}\right|=\pi$

$\Rightarrow \ln\left(\frac{y}{3}\right)=-\frac{\pi}{3}$

$\Rightarrow y=3e^{-\frac{\pi}{3}}$

Q.47. Let $f(x)=\frac{5x^2}{2}+\frac{\alpha}{x^5},(x>0)$ and minimum value of $f(x)$ is 14, then α is equal to

A) 32

B) 64

C) 128

D) 256

Answer: 128

Solution: From weighted $\mathrm{A.M.}-\mathrm{G.M.}$ inequality, we know

$$\frac{5\left(\frac{x^2}{2}\right)+2\left(\frac{\alpha}{2x^5}\right)}{5+2}\geq\left(\left(\frac{x^2}{2}\right)^5\cdot\left(\frac{\alpha}{2x^5}\right)^2\right)^{\frac{1}{7}}$$

$\Rightarrow \frac{5x^2}{2}+\frac{\alpha}{x^5}\geq\frac{7}{2}(\alpha)^{\frac{2}{7}}$

Given that the least value of $\frac{5x^2}{2}+\frac{\alpha}{x^5}$ is 14

i.e. $\frac{7}{2}(\alpha)^{\frac{2}{7}}=14\Rightarrow(\alpha)^{\frac{1}{7}}=2\Rightarrow\alpha=128$