JEE Main Exam 2022 - Session 2

28 Jul 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. Time period of a simple pendulum on Earth's surface is $4 \mathrm{~s}$ and at height h above the earth's surface is $6 \mathrm{~s}$. Find the value of h, if $R=6400 \mathrm{~km}$.

A) $3200 \mathrm{~km}$

B) $1200 \mathrm{~km}$

C) $16000 \mathrm{~km}$

D) $6400 \mathrm{~km}$

Answer: $3200 \mathrm{~km}$

Solution: $T=2 \pi \sqrt{\frac{l}{g}} \Rightarrow T \propto \frac{1}{\sqrt{g}}$

$$\Rightarrow \frac{T_1}{T_2}=\sqrt{\frac{g_2}{g_1}}=\sqrt{\frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}}$$

$$\Rightarrow \frac{4}{6}=\frac{R}{R+h}$$
$$\Rightarrow 2R+2h=3R$$

$$\Rightarrow h=\frac{R}{2}$$

$$=\frac{6400}{2}$$
$$=3200 \mathrm{~km}$$

Q.2. A bullet of mass $200 \mathrm{~g}$ with initial kinetic energy equal to $90 \mathrm{~J}$ hits a block and after moving for $1 \mathrm{~s}$, its kinetic energy reduces to $40 \mathrm{~J}$. After how long further the bullet will stop? (Assume constant retardation)

A) $2 \mathrm{~s}$

B) $4 \mathrm{~s}$

C) $9 \mathrm{~s}$

D) $12 \mathrm{~s}$

Answer: $2 \mathrm{~s}$

Solution: Initial kinetic energy $\frac{1}{2} m v_0^2=90 \mathrm{~J}$

$\Rightarrow v_0=30 \mathrm{~m} \mathrm{~s}^{-1}$

Let the retardation is a, then after $1 \mathrm{~s}$

$\frac{1}{2} m(30-a \times 1)^2=40 \mathrm{~J}$

$\Rightarrow 30-a=20$

$\Rightarrow a=10 \mathrm{~m} \mathrm{~s}^{-2}$

So, it will move further for time t then,

$0=20-a t$
$\Rightarrow 0=20-10 t$
$\Rightarrow t=2 \mathrm{~s}$

Q.3. The energy of a photon is 5 times the work function and in second case the energy of photon is 10 times the work function. The ratio of maximum speed in first case to second case for the electron with maximum kinetic energy is

A) $\frac{2}{3}$

B) $\frac{1}{3}$

C) $\frac{3}{2}$

D) $\frac{3}{1}$

Answer: $\frac{2}{3}$

Solution: According to the Einstein's photoelectric equation, $K_{max} = h\nu - \phi$

$\Rightarrow KE_1 = h\nu_1 - \phi = 4\phi$

And $KE_2 = h\nu_2 - \phi = 9\phi$

$$\Rightarrow \frac{KE_1}{KE_2} = \frac{4}{9}$$

$$\Rightarrow \left(\frac{v_1}{v_2}\right)^2 = \frac{4}{9}$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{2}{3}$$

Q.4. The efficiency of a Carnot engine is $\eta = \frac{\alpha\beta}{\sin\theta}\log\left(\frac{\beta x}{kT}\right)$, where x is length, T is temperature and k is Boltzmann constant, then

A) β has dimension of force

B) $\frac{x}{\alpha}$ has dimension of power

C) α and β have same dimensions

D) $\frac{\beta}{\eta}$ is dimensionless

Answer: β has dimension of force

Solution: Given that, $\eta = \frac{\alpha\beta}{\sin\theta}\log\left(\frac{\beta x}{kT}\right)$. Since expression inside the logarithm is dimensionless

$[\beta x] = [kT]$

$$[\beta] = \frac{[kT]}{[x]} = \frac{[E]}{[x]} = \frac{\left[\mathrm{ML^2T^{-2}}\right]}{[\mathrm{L}]} = \left[\mathrm{MLT^{-2}}\right]$$

Therefore, β has dimensions of force.

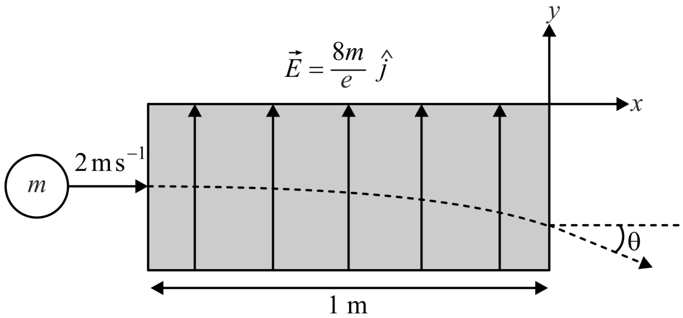
Since efficiency is a dimensionless ratio,

$$[\alpha][\beta] = \left[\mathrm{M^0L^0T^0}\right]$$

$$\Rightarrow [\beta] = \frac{1}{[\alpha]}$$

$$\frac{[x]}{[\alpha]} = [\beta][x] = \left[\mathrm{ML^2T^{-2}}\right]$$

Q.5. An electron moving along x-axis with speed of $2 \mathrm{~m~s}^{-1}$ in electric field $\vec{E} = \frac{8\mathrm{m}}{e}\hat{\jmath}$, crosses a region of width $1 \mathrm{~m}$ (m is mass of electron while e is charge on electron). The angle of deviation of electron in the region is



A) $\tan^{-1}(\sqrt{2})$

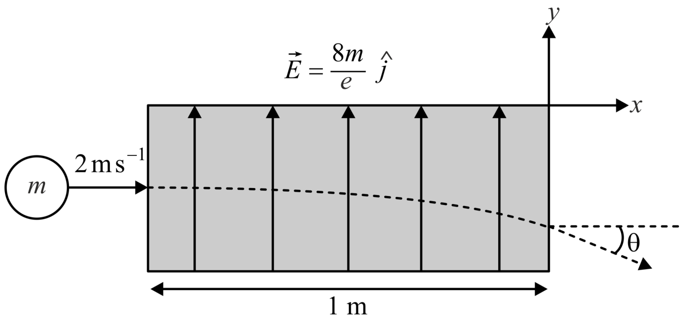
B) $\tan^{-1}(2)$

C) $\tan^{-1}(3)$

D) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer: $\tan^{-1}(2)$

Solution:



Time taken by the electron to cross,

$t = \frac{1}{Vx} = \frac{1}{2} \mathrm{~s}$

Acceleration along y axis

$a_g = \frac{F}{m} = \frac{Ee}{m} = \frac{8m}{e} \times \frac{e}{m} = 8 \mathrm{~m~s}^{-2}$

V_y of the electron will be,

$V_y = a_y t = 8 \times \frac{1}{2} = 4 \mathrm{~m~s}^{-1}$

Therefore,

$\tan\theta = \frac{Vy}{Vx} = \frac{4}{2} = 2$

$\Rightarrow \theta = \tan^{-1}(2)$

Q.6. Magnetic field at the centre of a circular loop carrying some current is B_1 while at a distance of $\sqrt{3}R$ (R is radius of loop) on the axis from the centre, magnetic field is B_2. Then $B_1 : B_2$ is equal to

A) $\frac{1}{8}$

B) $\frac{8}{1}$

C) $\frac{4}{1}$

D) $\frac{3}{1}$

Answer: $\frac{8}{1}$

Solution: Magnetic field at the centre of the loop i.e. $B_1 = \frac{\mu_0 i}{2R}$

Magnetic field on the axis of the loop is,

$$B_2 = \frac{\mu_0 i R^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}} = \frac{\mu_0 i \times R^2}{2\left(R^2 + 3R^2\right)^{\frac{3}{2}}} = \frac{\mu_0 i \times R^2}{2 \times 8R^3} = \frac{\mu_0 i}{2 \times 8R}$$

Therefore, the ratio $\frac{B_1}{B_2} = \frac{8}{1}$

Q.7. The area of cross section of a wire is $0.4\ \text{mm}^2$ and its length is equal to $0.5\ \text{m}$. A mass of $2\ \text{kg}$ suspended at the end is revolved around in vertical circle. The speed at lowest point is $5\ \text{ms}^{-1}$. Change in length at lowest point is $\left(Y = 2 \times 10^{11}\ \text{Nm}^{-2}\right)$

A) $0.3250\ \text{mm}$

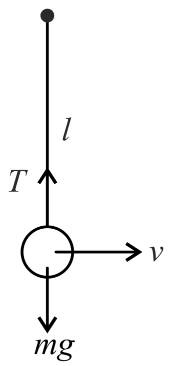
B) $1.5\ \text{mm}$

C) $0.75\ \text{mm}$

D) $0.5\ \text{mm}$

Answer: $0.75\ \text{mm}$

Solution:



At the bottom point, $T - mg = \frac{mv^2}{l}$

$$\Rightarrow \quad T = mg + \frac{mv^2}{l}$$

$$\text{Stress} = \frac{T}{A}$$

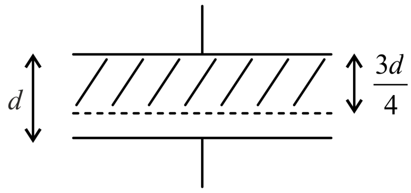
Using Hooke's law.

$$Y = \frac{Tl}{A \Delta l}$$

$$\Rightarrow \Delta l = \left(mg + \frac{mv^2}{l}\right)\frac{l}{AY} = \frac{m\left[gl + v^2\right]}{AY} = \frac{2[5+25]}{0.4 \times 10^{-6} \times 2 \times 10^{11}}$$

$$\Rightarrow \Delta l = 0.75\ \text{mm}$$

Q.8. A dielectric of dielectric constant k is inserted in a parallel plate capacitor of capacitance C as shown in the figure. Find the new capacitance



A) $\frac{4kC}{k+3}$

B) $\frac{kC}{k+3}$

C) $(k+3)C$

D) $\frac{(k+3)}{4}C$

Answer: $\frac{4kC}{k+3}$

Solution: Initial capacitance $C=\frac{\varepsilon_0 A}{d}$

Capacitance of the capacitor with dielectric $C_1=\frac{4k\varepsilon_0 A}{3d}$

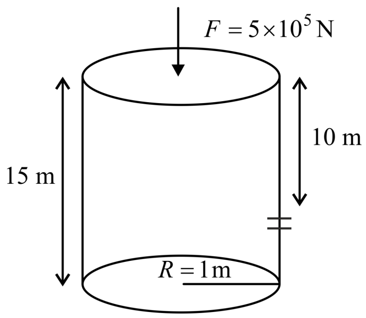
Capacitance of the capacitor without dielectric $C_2=\frac{4\varepsilon_0 A}{d}$

Since, C_1 and C_2 are in series

$$C_{eq}=\frac{C_1C_2}{C_1+C_2}=\frac{\frac{4k\varepsilon_0 A}{3d}\times\frac{4\varepsilon_0 A}{d}}{\frac{4\varepsilon_0 A}{d}\left(\frac{k}{3}+1\right)}$$

$$\Rightarrow C_{eq}=\frac{4k\varepsilon_0 A}{(k+3)d}=\frac{4kC}{k+3}$$

Q.9. Find the velocity of efflux of water for the given figure.



A) $22.8\ \mathrm{m\ s^{-1}}$

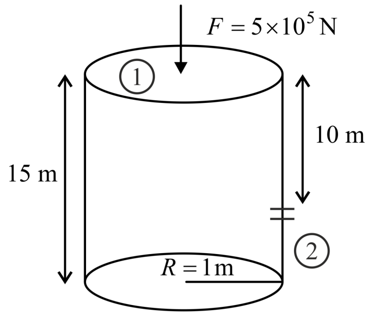
B) $24.8\ \mathrm{m\ s^{-1}}$

C) $26.8\ \mathrm{m\ s^{-1}}$

D) $28.8\ \mathrm{m\ s^{-1}}$

Answer: $22.8\ \mathrm{m\ s^{-1}}$

Solution:



Applying Bernoulli's equation at point 1 and 2,

$$p_1+\rho g h_1+\frac{1}{2}\rho v_1^2=p_2+\rho g h_2+\frac{1}{2}\rho v_2^2$$

$$\Rightarrow\left(P_0+\frac{F}{\pi R^2}\right)+\rho g\left(h_1-h_2\right)=P_0+\frac{1}{2}\rho v_2^2$$

$$\Rightarrow v_2=\sqrt{\frac{2F}{\pi R^2\rho}+2g\left(h_1-h_2\right)}$$

$$v_2=\sqrt{\frac{2\times\left(5\times10^5\right)}{3.14\times1^2\times1000}+2\times10\times10}$$

$$\approx 22.8\ \mathrm{m\ s^{-1}}$$

Q.10. Statement 1: A resistance of $80\ \Omega$ is divided in four parts then its equivalent resistance if connected in parallel will be $R_{eq}=5\ \Omega$.

Statement 2: If two resistances of $2R$ and $3R$ are connected in parallel, then heat generated will be in the ratio $3:2$.

A) Statement 1 is True 2 is False

B) Statement 2 is True 1 is False

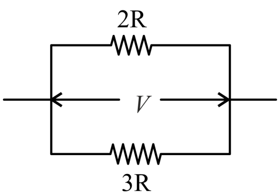
C) Both statements are true

D) Both statements are false

Answer: Both statements are true

Solution: If $80\ \Omega$ is divided in four parts then resistance of each part will be $20\ \Omega$. Now if these resistors are connected in parallel, then equivalent resistance will be $\frac{20\ \Omega}{4}=5\ \Omega$.

Now,



Voltage drop in parallel connection will remain same, therefore heat generated will be,

$$\frac{H_1}{H_2}=\frac{\frac{V^2}{2R}}{\frac{V^2}{3R}}=\frac{3}{2}.$$

Hence, both the statements are correct.

Q.11. Work done by a gas during expansion at constant pressure is $150\ \mathrm{J}$ and degree of freedom is 8, then find the heat given to the system.

A) $650\ \mathrm{J}$

B) $750\ \mathrm{J}$

C) 850 J

D) 950 J

Answer: 750 J

Solution: As we know, work done by the gas is, $W = p \Delta V = nR \Delta T = 150 \text{ J}$

Now heat given to the system will be, $Q = nC_p \Delta T$

$$= n\left(\frac{f+2}{2}\right)R\Delta T$$

$$= \left(\frac{8+2}{2}\right)150 = 5 \times 150 = 750 \text{ J}$$

Q.12. Two bulbs of rating $(100 \text{ W}, 200 \text{ V})$ and $(60 \text{ W}, 200 \text{ V})$ are connected in series with 220 V, find the power consumed by 100 W bulb.

A) 15 W

B) 16 W

C) 17 W

D) 18 W

Answer: 17 W

Solution: Resistance of the first bulb, $R_1 = \frac{V^2}{P_1} = \frac{200^2}{100} = 400 \ \Omega.$

Resistance of the second bulb will be,

$$R_2 = \frac{V^2}{P_2} = \frac{200^2}{60} = \frac{2000}{3} \ \Omega.$$

Equivalent resistance in the series combination will be, $R_{eq} = R_1 + R_2 = 400 \ \Omega + \frac{2000}{3} \ \Omega = \frac{3200}{3} \ \Omega$

Current in the circuit, $I = \frac{220}{Req} = \frac{220 \text{ V}}{\frac{3200}{3} \ \Omega} = \frac{33}{160} \text{ A}$

Hence, power generated in resistor will be

$$= I^2 \times R_1$$
$$= \left(\frac{33}{160}\right)^2 \times 400$$
$$\approx 17 \text{ W}$$

Q.13. Density of a rod AB is given by $\rho = \rho_0\left(1 - \frac{x^2}{L^2}\right)$, where x is the distance from end A. Find the location of centre of mass from the end A.

A) $\frac{3L}{8}$

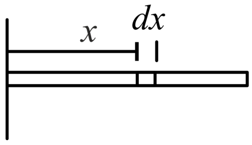
B) $\frac{5L}{8}$

C) $\frac{L}{8}$

D) $\frac{3L}{4}$

Answer: $\frac{3L}{8}$

Solution:



Mass of the element, $\mathrm{d}m=\rho(Adx)=\rho_0 A\left[1-\frac{x^2}{L^2}\right]\mathrm{d}x$

Now,

$$X_{\mathrm{cm}}=\frac{\int x\ \mathrm{d}m}{\int \mathrm{d}m}=\frac{\rho_0 A\int_0^L\left[x-\frac{x^3}{L^2}\right]\mathrm{d}x}{\rho_0 A\int_0^L\left(1-\frac{x^2}{L^2}\right)\mathrm{d}x}$$

$$=\frac{\left[\frac{L^2}{2}-\frac{L^4}{4L^2}\right]}{\left[L-\frac{L^3}{3L^2}\right]}$$

$$\Rightarrow\quad X_{\mathrm{cm}}=\frac{3L}{8}$$

Q.14. Find the heat energy required to double the RMS velocity of $14\ \mathrm{g}$ nitrogen at $27\ ^{\circ}\mathrm{C}$.

A) $9348\ \mathrm{J}$

B) $9368\ \mathrm{J}$

C) $9448\ \mathrm{J}$

D) $9748\ \mathrm{J}$

Answer: $9348\ \mathrm{J}$

Solution: RMS velocity is given by, $\sqrt{\frac{3RT}{M}}$. To double the RMS velocity, temperature will become four times of the initial temperature.

Therefore, $\Delta T=T_f-T_i=4T_i-T_i=3T_i=3\times(27+273)=900\ \mathrm{K}$Now heat energy required will be,

$Q=nC_v\Delta T$

$=\left(\frac{14}{28}\right)\times\frac{5}{2}\times 8.31\times 900$

$\approx 9348\ \mathrm{J}$

Section B: Chemistry

Q.15. Find the molarity of Na_2SO_4 formed upon mixing of $0.2\ M,\ 2\ L\ H_2SO_4$ and $0.1\ M,\ 2\ L\ NaOH$.

A) $0.05\ M$

B) $0.03\ M$

C) $0.04\ M$

D) $0.025\ M$

Answer: $0.025\ M$

Solution:

$$\begin{array}{lcccc} & H_2SO_4 + & 2NaOH \rightarrow & Na_2SO_4 + & 2H_2O \\ \text{initial moles} & 0.4 & 0.2 & - & - \\ \text{Final moles} & 0.4-0.1=0.3 & & 0.1 & \end{array}$$

2 moles $NaOH \rightarrow 1$ mole of Na_2SO_4

$\therefore\ 0.2$ moles gives $\frac{0.2}{2}=0.1$ moles

Total volume $=4\ L$

$M_{Na_2SO_4}=\frac{0.1}{4}$

Q.16. Correct order of metallic character is

A) $Na > Mg > Be > Si > P$

B) $Be > Na > Mg > Si > P$

C) $Na < Mg < Si < Be < P$

D) $P > Mg > Na > Si > Na$

Answer: $Na > Mg > Be > Si > P$

Solution: Moving down a group of metals, metallic nature increases due to increasing size. Going across a period, the metallic character decreases.

Q.17. Match the compounds/species given in column I with hybridisation of central metal in column II

	Column I		Column II
(a)	$[Ni(CO)_4]$	(i)	sp^3
(b)	$[Ni(CN)_4]^{2-}$	(ii)	dsp^2
(c)	$[CoF_6]^{3-}$	(iii)	sp^3d^2
(d)	$[Co(CN)_6]^{3-}$	(iv)	d^2sp^3

A) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

B) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

C) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

D) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

Answer: (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

Solution: Ni is in zero oxidation state in $Ni(CO)_4$ so the electronic configuration of Ni is $3d^8 4s^2$. As CO is a strong ligand, it pushes all the electrons in the $3d$ orbital, therefore the hybridisation of $Ni(CO)_4$ is sp^3 and it has tetrahedral geometry. It is diamagnetic due to the absence of unpaired electrons.

In $\left[Ni(CN)_4\right]^{2-}$, there is Ni^{2+} ion for which the electronic configuration in the valence shell is $3d^8 4s^0$.

In presence of strong field CN^- ions, all the electrons are paired up.

The empty, $3d$, $3s$ and two $4p$ Orbitals undergo dsp^2 hybridization to make bonds with CN^- ligands in square planar geometry.

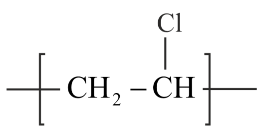
The atomic number of Co is 27 and its valence shell electronic configuration is $3d^7 4s^2$.

Co is in $+3$ oxidation state in the complex $[CoF_6]^{3-}$.

$[CoF_6]^{3-}$ is $sp^3 d^2$ hybridized and it is octahedral in shape.

Another Co^{3+} complex, $\left[Co(CN)_6\right]^{3-}$, is diamagnetic and has no unpaired electrons. The hybrid orbitals used to form this complex are $d^2 sp^3$.

Q.18. Match Column I with Column II.

	Column I		Column II
$a.$	Natural rubber	$1.$	Thermosetting
$b.$		$2.$	Fiber
$c.$	Nylon $6, 6$	$3.$	Elastomer
$d.$	Bakelite	$4.$	Thermoplastic

A) a-1, b-2, c-3, d-4

B) a-3, b-4, c-2, d-1

C) a-2, b-1, c-3, d-4

D) a-4, b-2, c-1, d-3

Answer: a-3, b-4, c-2, d-1

Solution: The Thermosetting Polymers tend to have a cross-linked $3D$ structure.

Bakelite is an example of Thermosetting Polymer.

Polymer fibers are a subset of man-made fibers, which are based on synthetic chemicals (often from petrochemical sources) rather than arising from natural materials by a purely physical process. These fibers are made from: polyamide nylon.

Thermoplastics are a class of polymers that can be softened and melted by the application of heat, and can be processed either in the heat-softened state (e.g. by thermoforming) or in the liquid state (e.g. by extrusion and injection molding). polyvinyl chloride is an example of it.

Elastomer, any rubbery material composed of long chainlike molecules, or polymers, that are capable of recovering their original shape after being stretched to great extents - hence the name elastomer, from "elastic polymer."

Q.19. The temperature at which rms speed of gas molecules becomes double of its value at 0^oC is

A) 819^oC

B) 760^oC

C) 273^oC

D) 224^oC

Answer: 819^oC

Solution: $2\ v_{rms}(0^{o}C) = v_{rms}(t^{o}C)$

$$2\sqrt{\frac{3R(273)}{M}} = \sqrt{\frac{3RT}{M}}$$

$$4 \times 273 = T$$

$$T = 1092\ K = 819^{o}C$$

Q.20. In the reaction, $N_2 + O_2 \rightleftharpoons 2NO$
If initial moles of N_2 and O_2 are $2\ \text{mol}$ and $1\ \text{mol}$, then find the value of K_c. The moles of O_2 at equilibrium is $0.6\ \text{mol}$.

A) 0.37

B) 0.27

C) 0.67

D) 0.87

Answer: 0.67

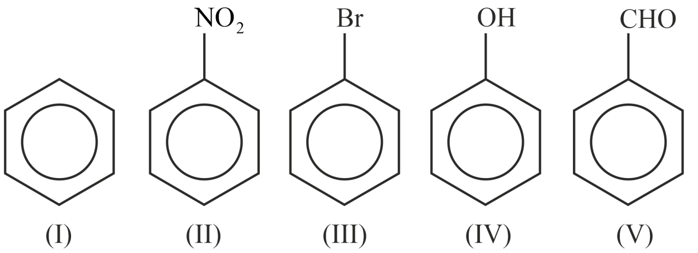
Solution:

$$\begin{array}{lccc} & N_2 & +O_2 & \rightleftharpoons 2NO \\ \text{Inital moles} & 2 & 1 & \\ \text{Final moles} & 2-0.4 & 1-\underbrace{}_{0.6}^{0.4} & 0.4\times 2 \\ & =1.6 & & 0.8 \end{array}$$

$$K_c = \frac{[NO]^2}{[N_2][O_2]}$$

$$= \frac{\left(\frac{0.8}{V}\right)^2}{\left(\frac{1.6}{V}\right)\left(\frac{0.6}{V}\right)} = 0.67$$

Q.21. The correct order of nitration in the following:



A) (II) < (V) < (I) < (III) < (IV)

B) (II) < (V) < (III) < (I) < (IV)

C) (V) < (II) < (IV) < (I) < (III)

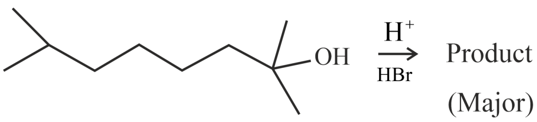
D) (IV) < (III) < (II) < (I) < (V)

Answer: (II) < (V) < (III) < (I) < (IV)

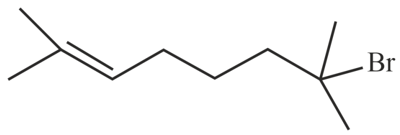
Solution: Nitration is an electrophilic aromatic substitution reaction. The $+M$ effect increases the rate of nitration whereas the $-M$ effect decreases the rate of nitration.

$$-OH(+M),\ -NO_2(-M),\ -Br(+M\ \text{but dominant}\ -I)\ \text{and}\ -CHO(-M)$$

Q.22.



A)



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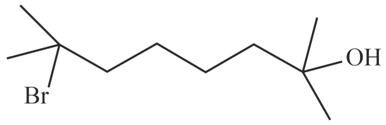
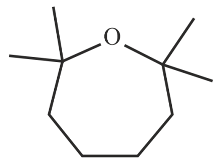
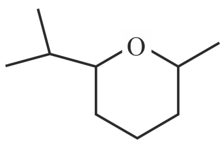


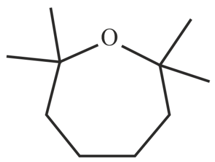
C)



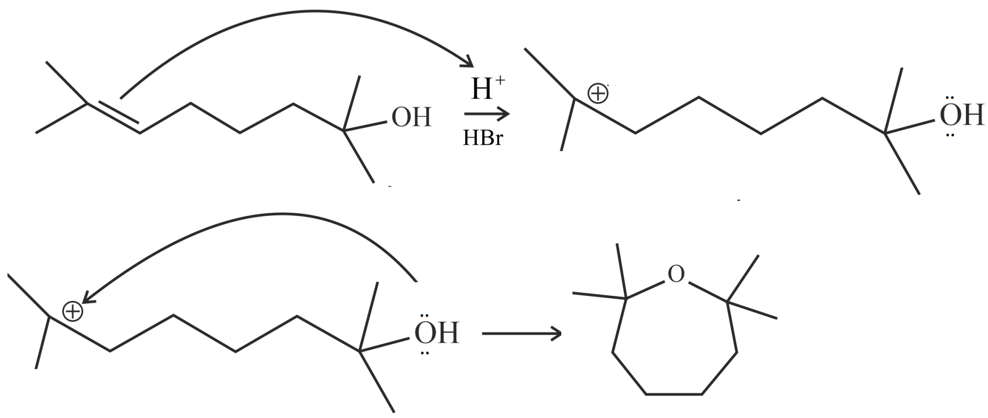
D)



Answer:

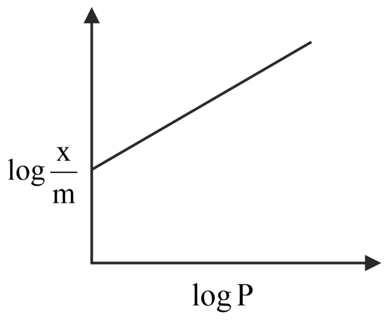


Solution:

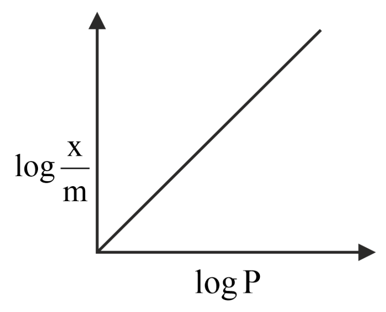


Q.23. Which of the following is correct as per Freundlich adsorption isotherm?

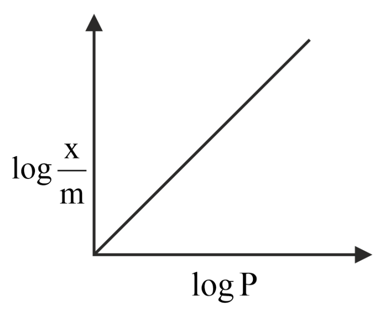
A)



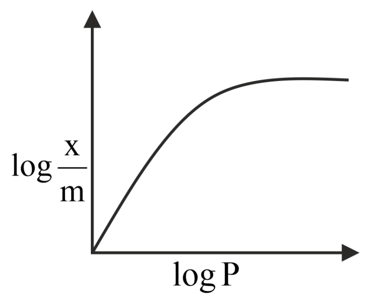
B)



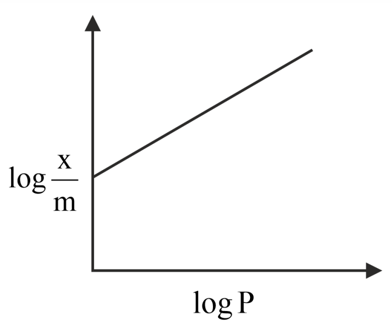
C)



D)



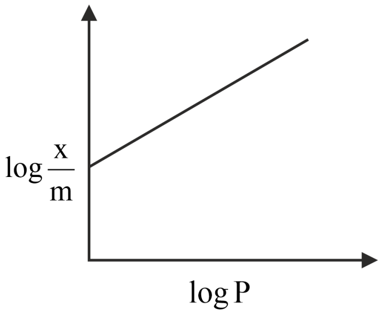
Answer:



Solution: According to Freundlich adsorption isotherm, the extent of adsorption at a given temperature is directly related to pressure of the gas. Hence,

$\frac{x}{m} = K(P)^{\frac{1}{n}}$

$\log\frac{x}{m} = \log K + \frac{1}{n}\log P$



Q.24. I_2 reacts with concentrated HNO_3 as per the following reaction:

$I_2 + \text{conc. } HNO_3 \rightarrow$

Products of the reaction are:

A) $HIO_4,\ NO_2,\ H_2O$

B) $HIO_3,\ N_2O,\ H_2O$

C) $HIO_3,\ NO_2,\ H_2O$

D) $HIO_4,\ N_2O,\ H_2O$

Answer: $HIO_3,\ NO_2,\ H_2O$

Solution: Concentrated nitric acid also oxidises non-metals and their compounds. Iodine is oxidised to iodic acid,

$I_2 + 10\,HNO_3 \rightarrow 2\,HIO_3 + 10\,NO_2 + 4H_2O$

Q.25. A and B are two volatile liquids which upon mixing forms an ideal solution. The vapour pressure of solution formed by mixing A and B is 0.8 atm. It is given that mole fraction of A in liquid phase is 0.2 and mole fraction of A in vapour phase is 0.5. Calculate the vapour pressure of pure liquid A (in atm).

A) 4

B) 2

C) 6

D) 3

Answer: 2

Solution: $P_A = P^o{}_A\chi_A$ (According to Raoult's law)

$P_A = P_T Y_A$ (According to Dalton's law)

So, $P_A = P^o{}_A\chi_A = Y_A P_T$

$P^o_A(0.2) = 0.5 \times 0.8$

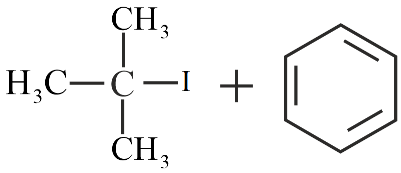
$P^o_A = 2$ atm

Q.26. $PhO - C(CH_3)_3 \xrightarrow{HI} P\ (\text{aromatic}) + Q$

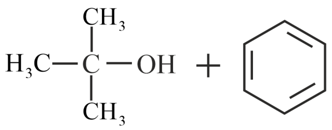
$P\ (\text{aromatic}) \xrightarrow{Zn/\Delta} R$

Q and R respectively are

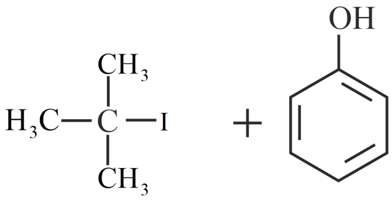
A)



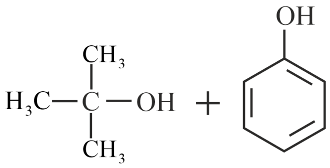
B)



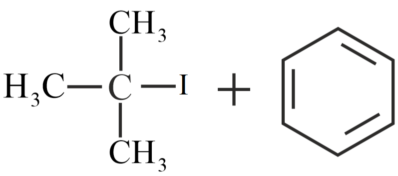
C)



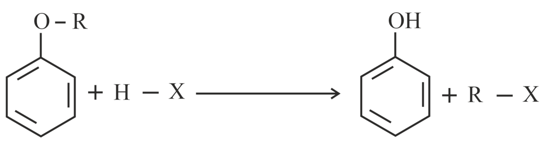
D)



Answer:

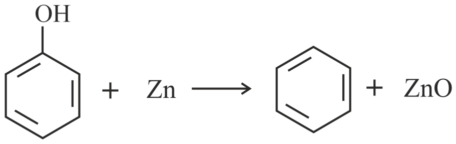


Solution: Alkyl aryl ethers are cleaved at the alkyl-oxygen bond due to the more stable aryl-oxygen bond. The reaction yields phenol and alkyl halide.



$R = -C(CH_3)_3$ and $X = I$

Phenol is converted to benzene on heating with zinc dust.



Q.27. Statement I: During nitration of aniline all products(ortho, meta and para) are formed.

Statement II: The solution mixture is highly acidic in nature.

A) Both statement I and II are correct

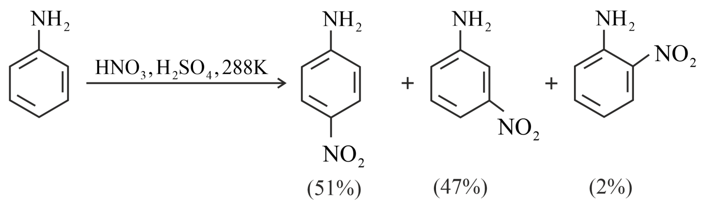
B) Statement I is correct and statement II is incorrect

C) Statement I is incorrect and statement II is correct

D) Both statement I and II are incorrect

Answer: Both statement I and II are correct

Solution: Direct nitration of aniline yields tarry oxidation products in addition to the nitro derivatives. Moreover, in the strongly acidic medium, aniline is protonated to form the anilinium ion which is meta directing. That is why besides the ortho and para derivatives, significant amount of meta derivative is also formed.



Q.28. How many of the following are state functions?

Internal energy, heat, enthalpy, volume, entropy, Gibbs free energy, pressure

A) 5

B) 4

C) 6

D) 3

Answer: 6

Solution: A property whose value doesn't depend on the path taken to reach that specific value is known to as state functions or point functions. State functions are the values which depend on the state of the substance like temperature, pressure or the amount or type of the substance, as a matter of fact, state functions do not depend on how the state was reached or established.

Among the given functions Internal energy, enthalpy, volume, entropy, Gibbs free energy and pressure are the state functions.

Q.29. Consider the following statements:

Statement I: Reduction of metal oxide in liquid state is more easier as compared to the reduction in solid state.

Statement II: In liquid state, we get more negative value of ΔG due to increase in entropy in liquid state.

A) Both statement I and II are correct

B) Statement I is correct and statement II is incorrect

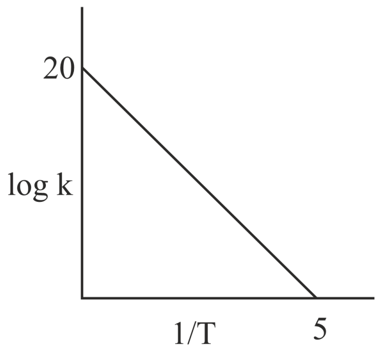
C) Both statement I and II are incorrect

D) Statement I is incorrect and statement II is correct

Answer: Both statement I and II are correct

Solution: The entropy is higher if the metal is in liquid state than when it is in solid state. The value of entropy change (ΔS) of the reduction process is more on positive side when the metal formed is in liquid state and the metal oxide being reduced is in solid state. Thus the value of $\Delta_f G$ becomes more on negative side and the reduction becomes easier.

Q.30. The plot of $\log\ k$ vs $\frac{1}{T}$ for a reaction is given as:



Where k is rate constant of reaction and T is temperature in kelvin. Find out the energy of activation of the reaction in cal mol^{-1}.
[Take $R = 2\ \text{cal mol}^{-1}\ K^{-1}$]

A) $18.424\ \text{cal / mol}$

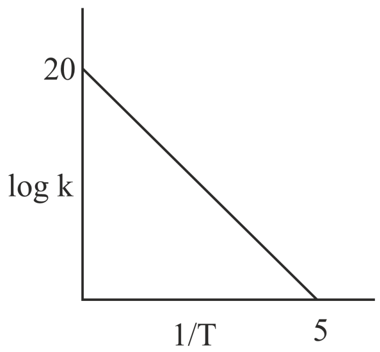
B) $23.03\ \text{cal / mol}$

C) $8\ \text{cal / mol}$

D) $9.212\ \text{cal / mol}$

Answer: $18.424\ \text{cal / mol}$

Solution:



According to Arrhenius equation,

$$k = Ae^{\frac{-Ea}{RT}}$$

$$\log\ k = \log\ A - \frac{Ea}{2.303R}\left(\frac{1}{T}\right)$$

Slope$= -\frac{Ea}{2.303R} = \frac{-\Delta y}{\Delta x}$

$$= \frac{20}{5}$$

$$Ea = 4 \times 2.303 \times 2$$

$$= 18.424\ \text{cal / mol}$$

Q.31. $P_4 + SOCl_2 \rightarrow$

Products of the reaction are:

A) $PCl_3,\ SO_2,\ S_2Cl_2$

B) $PCl_5,\ SO_3,\ S_2Cl_2$

C) $PCl_5,\ SO_2,\ Cl_2$

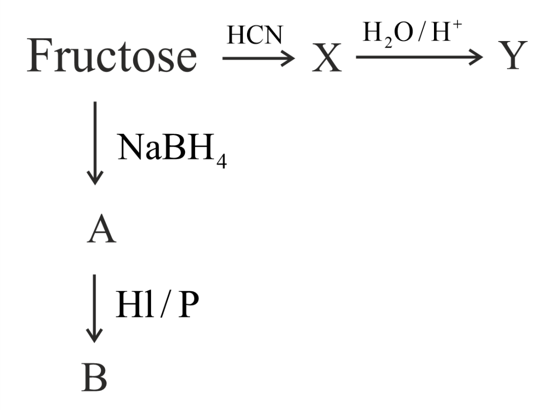
D) $PCl_3,\ SO_2,\ Cl_2$

Answer: PCl_3, SO_2, S_2Cl_2

Solution: The action of thionyl chloride with white phosphorus produces phosphorus trichloride. sulphur dioxide and sulphur monochloride.

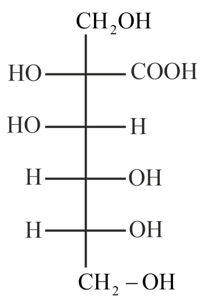
$P_4 + 8\,SOCl_2 \rightarrow 4\,PCl_3 + 4\,SO_2 + 2S_2Cl_2$

Q.32.



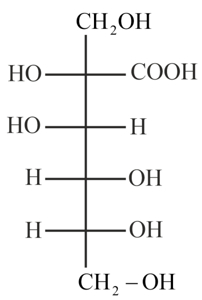
Y and B respectively are:

A)



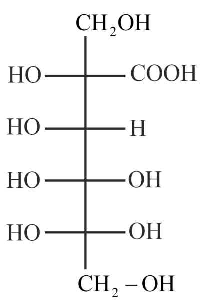
n– hexane

B)



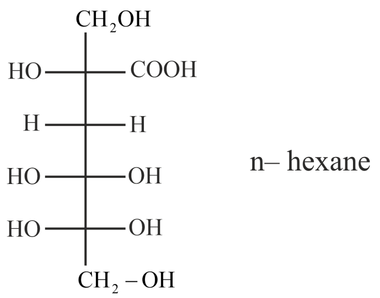
n– pentane

C)

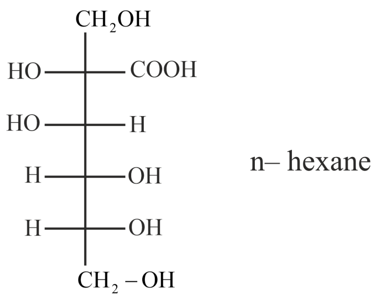


n– pentane

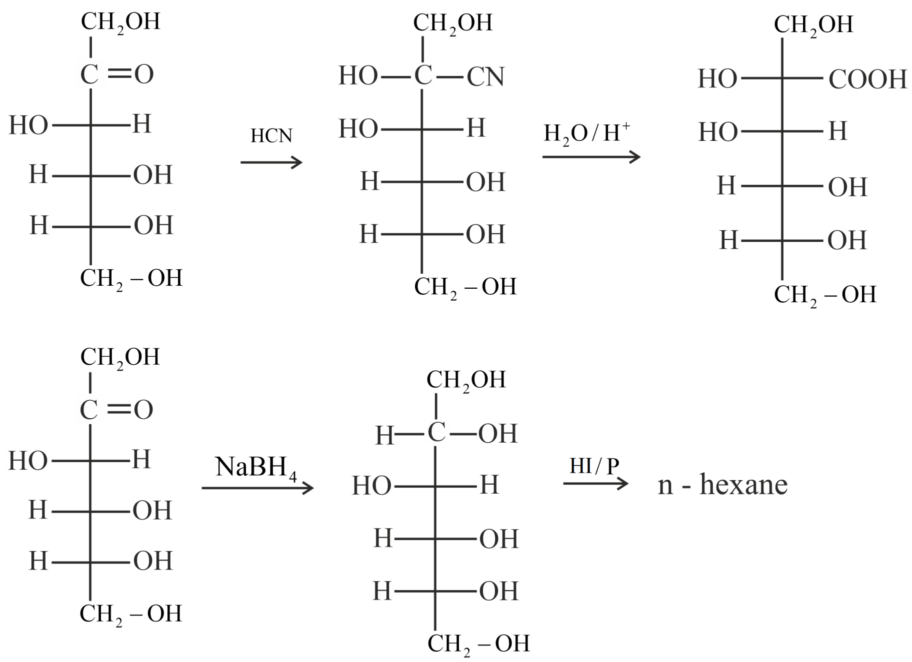
D)



Answer:



Solution:



Section C: Mathematics

Q.33. If $P(B \mid A)=\frac{5}{7}; P(A \mid B)=\frac{7}{9}$ and $P(A \cap B)=\frac{1}{9}$. Given $S_1 \equiv P\left(A' \cup B\right)=\frac{5}{6}$ and $S_2 \equiv P\left(A' \cap B'\right)=\frac{1}{18}$, then

A) Both S_1 and S_2 are correct

B) S_1 is true and S_2 is false

C) S_1 is false and S_2 is true

D) Both S_1 and S_2 are False

Answer: Both S_1 and S_2 are False

Solution: Given: $P\left(\frac{B}{A}\right)=\frac{P(A \cap B)}{P(A)}=\frac{5}{7}$ & $P\left(\frac{A}{B}\right)=\frac{P(A \cap B)}{P(B)}=\frac{7}{9}$

As $P(A \cap B)=\frac{1}{9}$, we get

$P(A)=\frac{7}{45}$ & $P(B)=\frac{1}{7}$

We know $P\left(A' \cap B'\right)=1-P(A \cup B)$

$=1-\left(\frac{7}{45}+\frac{1}{7}-\frac{1}{9}\right)=\frac{256}{315}$

$\therefore$ S_2 is false.

Now $P\left(A' \cup B\right)=1-(P(A)-P(A \cap B))$

$=1-\left(\frac{7}{45}-\frac{1}{9}\right)=\frac{43}{45}$

$\therefore$ S_1 is false.

Q.34. Absolute maximum value of $f(x)=\tan^{-1}(\sin x-\cos x)$ is

A) 0

B) $\tan^{-1}\frac{1}{\sqrt{2}}-\frac{\pi}{4}$

C) $\frac{\pi}{4}$

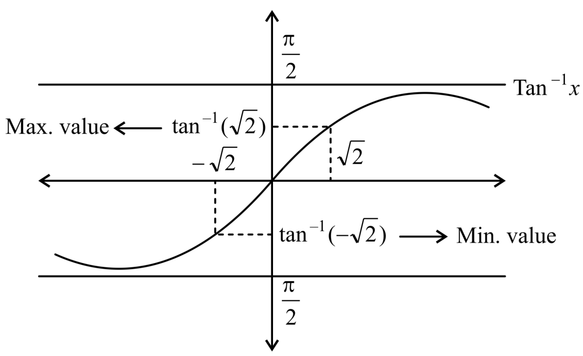
D) $\tan^{-1}\sqrt{2}$

Answer: $\tan^{-1}\sqrt{2}$

Solution: We know that $\sin x-\cos x \in\left[-\sqrt{2}, \sqrt{2}\right]$

and $\tan^{-1} x$ is an increasing function in R.

So maximum value of $\tan^{-1}(\sin x-\cos x)$ is $\tan^{-1}\sqrt{2}$.



Q.35. A class have B boys and G girls, 3 boys and 2 girls selected at random and number of ways of selecting 3 boys and 2 girls are 168. Then $B+3G$ is equal to _____ .

Answer: 17

Solution: Given, B boys and G girls

Also given total ways of selecting 3 boys and 2 girls which will be ${}^{B}C_3 \times {}^{G}C_2 = 168$

$\Rightarrow \frac{B(B-1)(B-2)}{6} \times \frac{G(G-1)}{2} = 168$

$\Rightarrow B(B-1)(B-2)G(G-1) = 7 \times 6 \times 4 \times 3 \times 2 \times 2$

$\Rightarrow B(B-1)(B-2)G(G-1) = 8 \times 7 \times 6 \times 3 \times 2$

Now on comparing both side we get, $B = 8$ and $G = 3 \Rightarrow B + 3G = 8 + 9 = 17$

Q.36. Find the values of λ for which the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{\lambda^2}$ and $\frac{x-3}{1} = \frac{y-2}{\lambda^2} = \frac{z-1}{2}$ are coplanar.

A)

Answer:

Solution: The lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar

if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

The given lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{\lambda^2}$ and $\frac{x-3}{1} = \frac{y-2}{\lambda^2} = \frac{z-1}{2}$ are coplanar.

$\therefore \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 3-1 & 2-2 & 1-(-3) \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 2 & 0 & 4 \\ 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \end{vmatrix} = 0$

$\Rightarrow 2\left(4 - \lambda^4\right) - 0 + 4\left(\lambda^2 - 2\right) = 0$

$\Rightarrow -2\lambda^4 + 4\lambda^2 = 0$

$\Rightarrow \lambda^2\left(\lambda^2 - 2\right) = 0$

$\Rightarrow \lambda^2 = 0$ or $\lambda^2 - 2 = 0$

$\Rightarrow \lambda = 0$ or $\lambda = \pm\sqrt{2}$

Thus, the values of λ are $0, -\sqrt{2}$and $\sqrt{2}$.

Q.37. From point $(2, 0)$ tangents are drawn on $2y^2 = -x$. These tangents also touches the circle $(x-5)^2 + y^2 = r^2$. The value of $17r^2$ is:

A) 1

B) 12

C) 9

D) 4

Answer: 9

Solution: Given, From point $(2,0)$ tangents are drawn on $2y^2=-x$. These tangents also touches the circle $(x-5)^2+y^2=r^2$. The value of $17r^2$ is:

Now equation of tangents when point is outside will be $y=mx-\frac{1}{8m}$, now $(2,0)$ will lie here so $m=\pm\frac{1}{4}$, putting the value of m we get,

$x-4y=2$ and $4y+x=2$

Using distance of centre from tangent equal to radius, we get

$\left|\frac{5-0-2}{\sqrt{17}}\right|=r$ or $\left|\frac{5+0-2}{\sqrt{17}}\right|=r$

$\therefore\ r^2=\frac{9}{17}$

$\Rightarrow\ 17r^2=9$

Q.38. Let $f(x)=\lim_{n\to\infty}\frac{\cos 2\pi x-x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$, is continuous at

A) $\mathbb{R}-\{1\}$

B) $\mathbb{R}-\{-1,1\}$

C) $\mathbb{R}-\{0,1\}$

D) $\mathbb{R}-\{0\}$

Answer: $\mathbb{R}-\{-1,1\}$

Solution: Given $f(x)=\lim_{n\to\infty}\frac{\cos 2\pi x-x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$

$=\lim_{n\to\infty}\frac{\cos 2\pi x-\left(x^2\right)^n\sin(x-1)}{1+x\left(x^2\right)^n-\left(x^2\right)^n}$

$=\lim_{n\to\infty}\frac{\left(x^2\right)^{-n}\cos 2\pi x-\sin(x-1)}{\left(x^2\right)^{-n}+x-1}$

For $-1<x<1$, as $0<x^2<1\Rightarrow\lim_{n\to\infty}x^{2n}\to 0$

i.e. $f(x)=\cos 2\pi x$

For $x>1$ or $x<-1$, $\lim_{n\to\infty}x^{-2n}\to 0$

i.e. $f(x)=-\frac{\sin(x-1)}{x-1}$

For $x=\pm1,\ f(x)=\begin{cases}1 & \text{if } x=1\\ \frac{(1+\sin 2)}{-1} & \text{if } x=-1\end{cases}$

i.e. $\lim_{x\to1^+}f(x)=-1,\lim_{x\to1^-}f(x)=1$

So f is discontinuous at $x=1$

$\lim_{x\to-1^+}f(x)=1,\lim_{x\to-1^-}f(x)=-\frac{\sin 2}{2}$

So $f(x)$ is discontinuous at $x=-1$

Q.39. Let $f(x)=ax^2+bx+c$ and $f(1)=3, f(-2)=\lambda, f(3)=4$, then value of λ for which $f(0)+f(1)+f(-2)+f(3)=14$ is ______ .

Answer: 4

Solution: We know $f(0)=c$

$f(1)=a+b+c=3\ \ldots(i)$

$f(-2)=4a-2b+c=\lambda\ \ldots(ii)$

and $f(3)=9a+3b+c=4\ \ldots(iii)$

Also given $f(0)+f(1)+f(-2)+f(3)=14$

$\Rightarrow\ c+3+\lambda+4=14\Rightarrow c=7-\lambda$

Subtracting (ii) from (iii) we get,

$a+b=\frac{4-\lambda}{5}$

Substituting in (i), we get

$\frac{4-\lambda}{5}+7-\lambda=3\Rightarrow\lambda=4$

Q.40. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}+\frac{y}{x^2-1}=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$, and $y(2)=\frac{1}{\sqrt{3}}$, then $\sqrt{7}y(8)$ is equal to:

A) $19-6\ln 3$

B) $19+6\ln 3$

C) $9-6\ln 3$

D) $9+6\ln 3$

Answer: $19-6\ln 3$

Solution: $\frac{dy}{dx}+\frac{y}{x^2-1}=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$ is a linear differential equation.

Here $\mathrm{I.F.}=e^{\int\frac{dx}{x^2-1}}=e^{\frac{1}{2}\ln\left(\frac{x-1}{x+1}\right)}=\sqrt{\frac{x-1}{x+1}}$

General solution will be $y\sqrt{\frac{x-1}{x+1}}=\int\frac{x-1}{x+1}dx+C$

$y\sqrt{\frac{x-1}{x+1}}=x-2\ln(x+1)+c$

Given $y(2)=\frac{1}{\sqrt{3}}$

$\Rightarrow\ c=2\ln 3-\frac{5}{3}$

i.e. $y\sqrt{\frac{x-1}{x+1}}=x-2\ln(x+1)+2\ln 3-\frac{5}{3}$

Putting $x=8$, we get

$\frac{\sqrt{7}y(8)}{3}=8-4\ln 3+2\ln 3-\frac{5}{3}$

$\Rightarrow\ \sqrt{7}y(8)=19-6\ln 3$

Q.41. If $f(x)=xe^{x(1-x)}$, then $f(x)$ is

A) increasing on $\left[-\frac{1}{2},1\right]$

B) decreasing on R

C) increasing on R

D) decreasing on $\left[-\frac{1}{2},1\right]$

Answer: increasing on $\left[-\frac{1}{2}, 1\right]$

Solution: We have,

$f(x) = xe^{x(1-x)}$

$\Rightarrow \ f'(x) = e^{x(1-x)} + xe^{x(1-x)}(1-2x)$

$\Rightarrow f'(x) = e^{x(1-x)} + xe^{x(1-x)}(1-2x)$

We put $f'(x) = 0 \left(\because e^{x(1-x)} \neq 0\right)$

$\Rightarrow 1 + x - 2x^2 = 0$

$\Rightarrow (1-x)(2x+1) = 0$

$\Rightarrow x = 1$ or $x = \frac{-1}{2}$

For $x \in \left[-\frac{1}{2}, 1\right], f'(x) > 0$

Hence, $f(x)$ is increasing on $\left[\frac{-1}{2}, 1\right]$

Q.42. The differential equation of the circle passing through $(0, 2)$ and $(0, -2)$ is:

A) $x^2 + 2xyy' - y^2 + 4 = 0$

B) $2xyy' + y^2 - 4 + x^2 = 0$

C) $2xyy' - x^2 - y^2 - 4 = 0$

D) $2xyy' + x^2 + y^2 + 4 = 0$

Answer: $x^2 + 2xyy' - y^2 + 4 = 0$

Solution: Let the equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

Now given circle passes through $(0, 2)$ & $(0, -2)$ now putting these values we will get, $f = 0$ and $c = -4$

So, equation of circle will be $C \equiv x^2 + y^2 + 2gx - 4 = 0$

Now forming Differential equation we get,

$\frac{x^2+y^2-4}{x} = -2g$

Differentiating both sides, we get

$x\left(2x + 2yy'\right) - \left(x^2 + y^2 - 4\right) = 0$

$\Rightarrow \ x^2 + 2xyy' - y^2 + 4 = 0$

Q.43. The value of $\int_0^{\frac{\pi}{2}} \frac{60\sin(6x)}{\sin x} dx$ is _____ .

Answer: 104

Solution: Given, $60 \int_0^{\frac{\pi}{2}} \frac{\sin(6x)}{\sin x} dx$

$= 60\left(\int_0^{\frac{\pi}{2}} \frac{\sin 6x - \sin 4x}{\sin x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin 4x - \sin 2x}{\sin x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sin x} dx\right)$

Using formula of $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$ we get,

$= 60\left(\int_0^{\frac{\pi}{2}} 2\cos 5x dx + \int_0^{\frac{\pi}{2}} 2\cos 3x dx + \int_0^{\frac{\pi}{2}} 2\cos x dx\right)$

$= 60\left(\frac{2}{5} - \frac{2}{3} + \frac{2}{1}\right) = 60\left(\frac{6-10+30}{15}\right) = \frac{26}{15} \times 60 = 104$

Q.44. If A is symmetric and B is skew symmetric matrix, then which of the following is not correct?

A) $A^4 + B^4$ is symmetric

B) $AB - BA$ is symmetric

C) $A^5 - B^5$ is skew symmetric

D) $AB + BA$ is skew symmetric

Answer: $A^5 - B^5$ is skew symmetric

Solution: Given $A^T = A$ and $B^T = -B$

We know that $\left(M^T\right)^n = (M^n)^T$

Now $\left(A^4 + B^4\right)^T = A^4 + (-B)^4 = A^4 + B^4$

i.e. $A^4 + B^4$ is symmetric

$(AB - BA)^T = B^T \cdot A^T - A^T \cdot B^T = -BA + AB$
i.e. $AB - BA$ is symmetric

$\left(A^5 - B^5\right)^T = A^5 - (-B)^5 = A^5 + B^5$
i.e. $A^5 - B^5$ is neither symmetric nor skew symmetric

$(AB + BA)^T = B^T A^T + A^T B^T = -(AB + BA)$
i.e. $AB + BA$ is skew symmetric