CBSE NCERT Solutions for Class 11 mathematics Chapter 15

Miscellaneous exercise on chapter 15

Q.1. The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Solution: Given: The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Let the remaining two observations be x and y

Therefore, the observations are 6, 7, 10, 12, 12, 13, x, y Mean, x¯=6+7+10+12+12+13+x+y8=9 ⇒60+x+y=72 ⇒x+y=12 ……(1)

Variance σ2=9.25=1n∑i=18xi-x¯2

9.25=18(-3)2+(-2)2+(1)2+(3)2+(3)2+(4)2+x2+y2-2×9(x+y)+2×(9)2

9.25=189+4+1+9+9+16+x2+y2-18(12)+162 [Using (1)] 9.25=1848+x2+y2-216+162
9.25=18x2+y2-6 ⇒x2+y2=80 ……(2)

From (1), we obtain

x2+y2+2xy=144 ……(3)

From (2) and (3), we obtain 2xy=64 ……(4) Subtracting 4from 2, we obtain x2+y2-2xy=80-64=16 ⇒x-y=±4 ……(5) Therefore, from 1 and 5, we obtain x=8 and y=4 when x-y=4 x=4 and y=8, when x-y=-4 Thus, the remaining observations are 4 and 8.

Q.2. The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12 and 14. Find the remaining two observations.

Solution: Given: The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12 and 14. Find the remaining two observations.

Let the remaining two observations be x and y.

The observations are 2, 4, 10, 12, 14, x, y Mean, x¯=2+4+10+12+14+x+y7=8 ⇒56=42+x+y ⇒x+y=14 ……(1)

If xi denotes the observations
then find the values for x1-x, x2-x,.........,x7-x
and add them
Variance,σ2=16=1n∑i=17xi-x¯2

16=17(-6)2+(-4)2+(2)2+(4)2+(6)2+x2+y2-2×8(x+y)+2×(8)2

16=1736+16+4+16+36+x2+y2-16(14)+2(64) [Using (1)] 16=17108+x2+y2-224+128 16=1712+x2+y2 ⇒x2+y2=112-12=100 x2+y2=100 ……(2)

From (1), we obtain

x2+y2+2xy=196 On squaring both sides of 1

From 1 and 2, we obtain 2xy=196-100 ⇒2xy=96 ……(3) Subtracting (3) from (2), we obtain x2+y2-2xy=100-96 ⇒(x-y)2=4 ⇒x-y=±2 ……(5) Therefore, On solving (1) and (5), we obtain x=8 and y=6 when x-y=2 x=6 and y=8 when x-y=-2 Thus, the remaining observations are 6 and 8.

Q.3. The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.

Solution: Let the observations be x1, x2, x3, x4, x5, and x6

It is given that mean is 8 and standard deviation is 4.

Mean, x¯=x1+x2+x3+x4+x5+x66=8 ……(1) If each observation is multiplied by 3 and the resulting observations are yi, then yi=3xi i.e., xi=13yi, for i=1 to 6 ∴ New mean, y¯=y1+y2+y3+y4+y5+y66 =3x1+x2+x3+x4+x5+x66 =3×8 [Using (1)] =24

Standard deviation, σ=1n∑i=16xi-x¯2

∴ (4)2=16∑i=16xi-x¯2 [Square both sides of the equation]

∑i=16xi-x¯2=96 ……(2) From the above, it can be observed that, y¯=3x¯ ⇒x¯=13y¯ and also we have xi=13yi Substituting the values of xi and x¯ in (2), we obtain ∑i=1613yi-13y¯2=96⇒13∑i=16yi-y¯2=96 ⇒∑i=16yi-y¯2=864 Therefore, variance of new observations =16×864=144 Hence, the standard deviation of new observations is 144=12

Q.4. Given that x¯ is the mean and σ2 is the variance of n observations x1, x2,…,xn. Prove that the mean and variance of the observations ax1, ax2, ax3,…,axn are ax and a2σ2, respectively, (a≠0).

Solution: The given n observations are x1, x2… xn

Mean =x¯=1n∑i=1nxi

Variance =σ2 ∴ σ2=1n∑i=1nxi-x¯2 ……(1)

New mean=y If each observation is multiplied by a and the new observations are yi, then yi=axi i.e., xi=1ayi 2

∴ y¯=1n∑i=1nyi=1n∑i=1naxi=an∑i=1nxi=ax¯ x¯=1n∑i=1nxi

Therefore, mean of the observations, ax1, ax2… axn, is ax¯
i.e., y=ax⇒x=1ay 3

Substituting the values of x and x¯ in 1 from 2 and 3 we obtain

σ2=1n∑i=1n1ayi-1ay¯2⇒σ2=1a2n∑i=1nyi-y¯2

⇒a2σ2=1n∑i=1nyi-y¯2 Thus, the variance of the observations, ax1, ax2… axn is a2σ2.

Q.5. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in the following case: If wrong item is omitted.

Solution: Number of observations (n)=20

Incorrect mean =10

Incorrect standard deviation =2 x¯=1n∑i=120xi 10=120∑i=120xi ⇒∑i=120xi=200 That is, incorrect sum of observations =200 Correct sum of observations =200-8=192 ∴ Correct mean = Correct sum 19=19219=10.1

Standard deviation, incorrect σ=1nincorrect∑i=1nxi2-1n2∑i=1nxi2

⇒2=1nincorrect∑i=1nxi2-incorrect1n∑i=1nxi2

⇒2=1nincorrect∑i=1nxi2-(incorrect x¯)2 as, 1n∑i=1nx=x¯ ⇒2=120× Incorrect ∑i=1nxi2-(10)2 ⇒4=120×Incorrect ∑i=1nxi2-100 ⇒120× Incorrect ∑i=1nxi2=104 ⇒ Incorrect ∑i=1nxi2=2080 Now, correct ∑i=1nxi2=Incorrect ∑i=1nxi2-(8)2 ⇒ correct ∑i=1nxi2=2080-64=2016 Correct Standard Deviation =1ncorrect∑i=1nxi2-(correct mean)2 ⇒ Correct Standard Deviation =119×2016-192192 ⇒ Correct Standard Deviation =201619-192192 ⇒ Correct Standard Deviation =144019=121019 ⇒ Correct Standard Deviation =12×3.16219=1.997

Q.6. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in the case: If it is replaced by 12.

Solution: Given,
number of observations n=20
Incorrect meanx=10 and
Incorrect standard deviation=2
We know that,
incorrect x=1nincorrect∑xii=1ni
⇒10=120incorrect∑xii=1n
⇒incorrect∑xii=1n =200
Thus, incorrect sum of observations=200

Finding correct sum of observations,
Since 8 is replaced by 12,

∴ Correct sum of observations =200-8+12=204

∴ Correct mean = Correct sum 20=20420=10.2

Finding incorrect Standard deviation,
incorrect σ=1nincorrect∑ i=1 nxi2-1n2incorrect∑i=1 nxi2=1nincorrect∑ i=1 nxi2-incorrect x¯2

⇒2=120 Incorrect ∑i=1nxi2-(10)2
On squaring both sides of the equation we have,

⇒4=120 Incorrect ∑i=1nxi2-(10)2

⇒120incorrect∑i=1nxi2=4+100=104⇒incorrect∑i=1nxi2=104×20=2080

⇒Incorrect∑i=1nxi2=2080

∴ Correct ∑i=1nxi2= Incorrect ∑i=1nxi2-(8)2+(12)2 =2080-64+144 =2160 ∴ Correct standard deviation = Correct ∑xi2n-(Correct mean)2 =216020-(10.2)2 =108-104.04 =3.96 =1.98

Q.7. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Solution: Given,
Standard deviation and mean of Mathematics is 12 and 42 respectively.

Standard deviation and mean of Physics is 15 and 32 respectively.

Standard deviation and mean of Chemistry 20 and 40.9 respectively. The coefficient of variation (C.V.) is given by Standard deviation Mean ×100 C.V. (in Mathematics) =1242×100=28.57 C.V.(in Physics) =1532×100=46.87 C.V.(in Chemistry) =2040.9×100=48.89 The subject with greater C.V. is more variable than others. Therefore, the highest variability in marks is in Chemistry and the lowest variability in marks is in Mathematics.

Q.8. The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

Solution: Number of observations (n)=100

Incorrect mean (x¯)=20

Incorrect standard deviation (σ)=3
We know that,
Incorrect meanx=1nIncorrect∑xii=1n⇒20=1100Incorrect∑xii=1100 ⇒Incorrect∑xii=1n=20×100=2000 ∴ Incorrect sum of observations=2000

Since, 21, 21 and 18 were three incorrect observations ∴ Correct sum of observations =2000-21-21-18=2000-60=1940 ∴ Correct Mean=Correctsum100-3=194097=20

Incorrect Standard deviationIncorrect(σ)=1nIncorrect∑xii=1n-1n2Incorrect∑i=1nxi2=1nIncorrect∑i=1nxi2-(x¯)2

⇒3=1100× Incorrect ∑xi2-(20)2

⇒9=1100×Incorrect ∑xi2-202

⇒ Incorrect∑xi2=100(9+400)=40900 Correct∑i=1nxi2=Incorrect∑i=1nxi2-(21)2-(21)2-(18)2 =40900-441-441-324 39694 ∴ Correct standard deviation= Correct ∑xi2n-(Correct mean)2 =3969497-(20)2 =409.216-400 =9.216 =3.036

Exercise 15.1

Q.1. Find the mean deviation about the mean for the data.
4, 7, 8, 9, 10, 12, 13, 17

3

Solution: The given data is
4, 7, 8, 9, 10, 12, 13, 17
We proceed step-wise and get the following:
Step 1 Mean of the data x¯=4+7+8+9+10+12+13+178=808=10
Step 2 The deviations of the respective observations from the mean x, i.e. xi-x¯, for each of the values.

Step 3 The absolute values of the deviations, i.e. |xi-x¯|, are
6, 3, 2, 1, 0,2, 3, 7
Step 4 The required mean deviation about the mean is M·D·x¯=∑i=18|xi-x-|8=6+3+2+1+0+2+3+78=248=3

Therefore, the mean deviation about the median for the given data is 3.

Q.2. Find the mean deviation about the mean for the data

Height in cms	Number of Boys
95-105	9
105-115	13
115-125	26
125-135	30
135-145	12
145-155	10

11.28

Solution: The following table is formed.

Height in cms	Number of boys fi	Mid-point xi	fixi	xi-x¯	fixi-x¯
95-105	9	100	900	25.3	227.7
105-115	13	110	1430	15.3	198.9
115-125	26	120	3120	5.3	137.8
125-135	30	130	3900	4.7	141
135-145	12	140	1680	14.7	176.4
145-155	10	150	1500	24.7	247

Here, N=∑i=16fi=100, ∑i=16fixi=12530 ∴x¯=1N∑i=16fixi=1100×12530=125.3 M.D. (x¯)=1N∑i=16fixi-x¯=1100×1128.8=11.28 Therefore, the mean deviation about the mean for the given data is 11.28.

Q.3. Find the mean deviation about median for the following data:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of Girls	6	8	14	16	4	2

10.342

Solution: The following table is formed.

Marks	Number of girls fi	Cumulative frequency c.f.	Mid-Point xi	xi-M	fixi-Med.
0-10	6	6	5	22.85	137.1
10-20	8	14	15	12.85	102.8
20-30	14	28	25	2.85	39.9
30-40	16	44	35	7.15	114.4
40-50	4	48	45	17.15	68.6
50-60	2	50	55	27.15	54.3
	50				517.1

The class interval containing the N2th or 25th item is 20-30. Therefore, 20-30 is the median class.

It is known that,

Median=l+N2-Cf×h

Here, l=20, C=14, f=14, h=10 and N=50 ∴ Median=20+25-1414×10=20+11014=20+7.85=27.85

Thus, mean deviation about the median is given by,

M.D.M=1N∑i=16fixi-M=150×517.1=10.34

Therefore, the mean deviation of the given data about the median is 10.342.

Q.4. Calculate the mean deviation about median age for the age distribution of 100 persons given below:

Age (in years)	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55
Number	5	6	12	14	26	12	16	9

7.35

Solution: The given data is not continuous. Therefore, it has to be converted into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval.

The table is formed as follows.

Age	Number fi	Cumulative frequency (c.f.)	
15.5-20.5	5	5	
20.5-25.5	6	11	
25.5-30.5	12	23	
30.5-35.5	14	37	
35.5-40.5	26	63	
40.5-45.5	12	75	
45.5-50.5	16	91	
50.5-55.5	9	100	
	100		

The class interval containing the Nth2 or 50th items is 35.5-40.5 Therefore, 35.5-40.5 is the median class (median class is the class interval whose cumulative frequency is just greater than or equal to N2)

It is known that,

Median=l+N2-Cf×h

Here, l=35.5, C=37, f=26, h=5, and N=100 ∴ Median=35.5+50-3726×5=35.5+13×526=35.5+2.5=38

Thus, mean deviation about the median is given by,

M.D. (M)=1N∑i=1nfixi-M=1100×735=7.35

Q.5. Find the mean deviation about the mean for the data. Write the answer upto 1 decimal place.
38, 70, 48, 40, 42, 55, 63, 46, 54, 44.
8.4

Solution: The given data is 38, 70, 48, 40, 42, 55, 63, 46, 54, 44.

Mean of the given data,

x¯=38+70+48+40+42+55+63+46+54+4410=50010=50

The deviations of the respective observations from the mean x¯, i.e. x-x¯, are
-12, 20,-2,-10,-8, 5, 13,-4, 4,-6.

The absolute values of the deviations, i.e. xi-x¯ are
12, 20, 2, 10, 8, 5, 13, 4, 4, 6.
The required mean deviation about the mean is M.D: x¯=∑i=110xi-x¯10=12+20+2+10+8+5+13+4+4+610=8410=8.4

Therefore, the mean deviation about the mean for the given data is 8.4.

Q.6. Find the mean deviation about the median for the data.

13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17
2.33

Solution: The given data is
13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Here, the numbers of observations are 12, which is even.

Arranging the data in ascending order, we obtain 10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18 Median, M=122thobservation+122+1thobservation2 =6thobservation+7thobservation2 =13+142=272=13.5

The deviations of the respective observations from the median, i.e. xi-M, are

-3.5, -2.5, -2.5, -1.5, -0.5, -0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5

The absolute values of the deviations, xi-M, are 3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5 The required mean deviation about the median is M.D.M=∑i=112xi-M12 =3.5+2.5+2.5+1.5+0.5+0.5+0.5+2.5+2.5+3.5+3.5+4.512 =2812=2.33. Therefore, the mean deviation about the median for the given data is 2.33.

Q.7. Find the mean deviation about the median for the data.

36, 72, 46, 42, 60, 45, 53, 46, 51, 49
7

Solution: The given data is

36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Here, the number of observations is 10, which is even. Arranging the data in ascending order, we obtain 36, 42, 45, 46, 46, 49, 51, 53, 60, 72 Median, M=102thobservation+102+1thobservation2 =5thobservation+6thobservation2 =46+492=952=47.5

The deviations of the respective observations from the median, i.e. xi-M, are

-11.5, -5.5, -2.5, -1.5, -1.5, 1.5, 3.5, 5.5, 12.5, 24.5

The absolute values of the deviations, xi-M, are
11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

Thus, the required mean deviation about the median is M.D.M=∑i=110xi-M10 =11.5+5.5+2.5+1.5+1.5+1.5+3.5+5.5+12.5+24.510 =7010=7.

Q.8. Find the mean deviation about the mean for the data.

xi	5	10	15	20	25
fi	7	4	6	3	5

6.32

Solution:

xi	fi	fixi	xi-x‾	fixi-x‾
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

N=∑i=15fi=25

∑i=15fixi=350 ∴ x‾=1N∑i=15fixi=125×350=14 ∴ MDx‾=1N∑i=15fixi-x-=125×158=6.32 Therefore, the mean deviation about the mean for the given data is 6.32.

Q.9. Find the mean deviation about the mean for the data

xi	10	30	50	70	90
fi	4	24	28	16	8

16

Solution:

xi	fi	fixi	xi-x‾	fixi-x‾
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

N=∑i=15fi=80, ∑i=15fixi=4000

∴ x‾=1N∑i=15fixi=180×4000=50 ∴ MDx‾=1N∑i=15fixi-x‾=180×1280=16 Therefore, the mean deviation about the mean for the given data is 16.

Q.10. Find the mean deviation about the median for the data

xi	5	7	9	10	12	15
fi	8	6	2	2	2	6

3.23

Solution: The given observations are already in ascending order.

Adding a column corresponding to cumulative frequencies of the given data, we obtain the following table.

xi	fi	c.f.
5	8	8
7	6	14
9	2	16
10	2	18
12	2	20
15	6	26

Here, N=26, which is even. Median is the mean of 13th and 14th observations. Both of these observations lie in the cumulative frequency 14, for which the corresponding observation is 7. ∴ Median=13thobservation+14thobservation2=7+72=7

The absolute values of the deviations from median, i.e. xi-M, are

xi-M	2	0	2	3	5	8
fi	8	6	2	2	2	6
fixi-M	16	0	4	6	10	48

∑i=16fi=26 and ∑i=16fixi-M=84 M.D.(M)=1N∑i=16fixi-M=126×84=3.23 Therefore, the mean deviation about the median for the given data is 3.23.

Q.11. Find the mean deviation about the median for the data

xi	15	21	27	30	35
fi	3	5	6	7	8

5.1

Solution: The given observations are already in ascending order.

Adding a column corresponding to cumulative frequencies of the given data, we obtain the following table.

xi	fi	c.f.
15	3	3
21	5	8
27	6	14
30	7	21
35	8	29

Here, N=29, which is odd. ∴ Median =29+12th observation=15th observation This observation lies in the cumulative frequency 21, for which the corresponding observation is 30. ∴ Median=30

The absolute values of the deviations from median, i.e. xi-M, are

xi-M	15	9	3	0	5
fi	3	5	6	7	8
fixi-M	45	45	18	0	40

∑i=15fi=29 and ∑i=15fixi-M=184 ∴ M.D.(M)=1N∑i=15fixi-M=129×184=5.1 Therefore, the mean deviation about the median for the given data is 5.1.

Q.12. Find the mean deviation about the mean for the data.

Income per day	Number of persons
0-100	4
100-200	8
200-300	9
300-400	10
400-500	7
500-600	5
600-700	4
700-800	3

157.92

Solution: The following table is formed.

Income per day	Number of person fi	Mid-point xi	
0-100	4	50	
100-200	8	150	
200-300	9	250	[illegible]
300-400	10	350	[illegible]
400-500	7	450	[illegible]
500-600	5	550	[illegible]
600-700	4	650	[illegible]
700-800	3	750	[illegible]
	50		[illegible]

Here, N=∑i=18fi=50, ∑i=18fixi=17900 ∴ x¯=1N∑i=18fixi=150×17900=358 M.D.(x¯)=1N∑i=1sfixi-x¯=150×7896=157.92 Therefore, the mean deviation about the median for the given data is 157.92.

Exercise 15.2

Q.1. Find the mean and variance for each of the data

6, 7, 10, 12, 13, 4, 8, 12

Solution: 6, 7, 10, 12, 13, 4, 8, 12

Mean, x¯=∑i=18xin=6+7+10+12+13+4+8+128=728=9

The following table for the above data with the deviation and variance is obtained.

xi	(xi-x¯)	(xi-x¯)2
6	-3	9
7	-2	4
10	-1	1
12	3	9
13	4	16
4	-5	25
8	-1	1
12	3	9
		74

Varianceσ2=1n∑i=18xi-x¯2=18×74=9.25

Q.2. The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circle.

Solution: The given data is not having continuous class-intervals. To make the class-intervals continuous we will subtract 0.5 from lower limit of each class interval and add 0.5 to upper limit of each class intervals.

Class	Frequency fi	Mid-point xi	yi=xi-42.54	yi2	fiyi	fiyi2
32.5-36.5	15	34.5	-2	4	-30	60
36.5-40.5	17	38.5	-1	1	-17	17
40.5-44.5	21	42.5	0	0	0	0
44.5-48.5	22	46.5	1	1	22	22
48.5-52.5	25	50.5	2	4	50	100
	100				25	199

Here, N=100, h=4. Let the assumed mean, A, be 42.5. Mean, x¯=A+∑i=15fiyiN×h=42.5+25100×4=43.5.

Varianceσ2=h2N2N∑i=15fiyi2-∑i=15fiyi2

=1610000100×199-(25)2

=1610000[19900-625] =1610000×19275 =30.84 ∴Standard deviation (σ)=5.55

Q.3. Find the mean and variance for the data.

First n natural numbers

Solution: The mean of first n natural numbers is calculated as follows.

Mean=Sum of all observationsNumber of observations

∴ Mean =n(n+1)2n=n+12

Varianceσ2=1n∑i=1nxi-x¯2

=1n∑i=1nxi-n+122

=1n∑i=1nxi2-1n∑i=1n2n+12xi+1n∑i=1nn+122 =1nn(n+1)(2n+1)6-n+1nn(n+1)2+(n+1)24n×n =(n+1)(2n+1)6-(n+1)22+(n+1)24 =(n+1)(2n+1)6-(n+1)24 =(n+1)4n+2-3n-312 =(n+1)(n-1)12 =n2-112.

Q.4. Find the mean and variance for the data

First 10 multiples of 3

Solution: The first 10 multiples of 3 are

3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Here, number of observations, n=10 Mean, x¯=∑i=110xi10=16510=16.5

The following table for the variance is obtained as per the given data.

xi	xi-x¯	xi-x¯2
3	-13.5	182.25
6	-10.5	110.25
9	-7.5	56.25
12	-4.5	20.25
15	-1.5	2.25
21	4.5	20.25
24	7.5	56.25
27	10.5	110.25
30	13.5	182.25
		742.5

Varianceσ2=1n∑i=110xi-x¯2=110×742.5=74.25

Q.5. Find the mean and variance for the data

xi	6	10	14	18	24	28	30
fi	2	4	7	12	8	4	3

Solution: The given data in tabular form is as follows.

xi	fi	fixi	xi-x¯	xi-x¯2	fixi-x¯2
6	2	12	-13	169	338
10	4	40	-9	81	324
14	7	98	-5	25	175
18	12	216	-1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
	40	760			1736

Here, N=40, ∑i=17fixi=760 ∴Mean x¯=∑i=17fixiN=76040=19 Variance=σ2=1N∑i=17fixi-x¯2=140×1736=43.4

Q.6. Find the mean and variance for the given data

xi	92	93	97	98	102	104	109
fi	3	2	3	2	6	3	3

Solution: The deviations and variances is obtained in tabular form for the given data as follows.

xi	fi	fixi	xi-x¯	xi-x¯2	fixi-x¯2
92	3	276	-8	64	192
93	2	186	-7	49	98
97	3	291	-3	9	27
98	2	196	-2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243
	22	2200			640

Here, N=22, ∑i=17fixi=2200 ∴ x¯=1N∑i=17fixi=122×2200=100 Varianceσ2=1N∑i=17fixi-x¯2=122×640=29.09 Therefore, variance for the given data is 29.09.

Q.7. Find the mean and standard deviation using short-cut method.

xi	60	61	62	63	64	65	66	67	68
fi	2	1	12	29	25	12	10	4	5

Solution: The data is obtained in tabular form as follows.

xi	fi	yi=xi-Ah=xi-641	yi2	fiyi	fiyi2
60	2	-4	16	-8	32
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80
	100	220		0	286

Let assumed meanA=64, h=width=61-60=1 and N=∑i=19fi=100 Therefore, Mean x¯=A+∑i=19fiyiN×h=64+0100×1=64+0=64

Variance,σ2=h2N2N∑i=19fiyi2-∑i=19fiyi2

=11002[100×286-0]

=2.86 ∴ Standard deviation(σ)=2.86=1.691

Q.8. Find the mean and variance for the following frequency distribution.

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Solution: Given data in tabular form:

Class	Frequency fi	Mid-point xi	yi=xi-Ah=xi-10530	yi2	fiyi	fiyi2
0-30	2	15	-3	9	-6	18
30-60	3	45	-2	4	-6	12
60-90	5	75	-1	1	-5	5
90-120	10	105	0	0	0	0
120-150	3	135	1	1	3	3
150-180	5	165	2	4	10	20
180-210	2	195	3	9	6	18
	30				2	76

Let assumed meanA=105, h=width of class-intervals=30 and N=∑i=17fi=30
Mean, x¯=A+∑i=17fiyiN×h=105+230×30=105+2=107

Varianceσ2=h2N2N∑i=17fiyi2-∑i=17fiyi2

=(30)2(30)230×76-(2)2

=2280-4 =2276 Therefore, the variance for the given data is 2276.

Q.9. Find the mean and variance for the following frequency distribution.

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Solution:

Given data in tabular form:

Class	Frequency fi	Mid-point xi	yi=xi-Ah=xi-2510	yi2	fiyi	fiyi2
0-10	5	5	-2	4	-10	20
10-20	8	15	-1	1	-8	8
20-30	15	25	0	0	0	0
30-40	16	35	1	1	16	16
40-50	6	45	2	4	12	24
	50				10	68

Let assumed meanA=25, h=width of class-intervals=10 and N=∑i=15fi=50
Mean, x¯=A+∑i=15fiyiN×h=25+1050×10=25+2=27

Varianceσ2=h2N2N∑i=15fiyi2-∑i=15fiyi2

=(10)2(50)250×68-(10)2

=125[3400-100]=330025 =132 Therefore, the variance for the given data is 132.

Q.10. Find the mean, variance and standard deviation using short-cut method

Height in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	3	4	7	7	15	9	6	6	3

Solution: Given data in tabular form:

Class	Frequency fi	Mid-point xi	yi=xi-92.55	yi2	fiyi	fiyi2
70-75	3	72.5	-4	16	-12	48
75-80	4	77.5	-3	9	-12	36
80-85	7	82.5	-2	4	-14	28
85-90	7	87.5	-1	1	-7	7
90-95	15	92.5	0	0	0	0
95-100	9	97.5	1	1	9	9
100-105	6	102.5	2	4	12	24
105-110	6	107.5	3	9	18	54
110-115	3	112.5	4	16	12	48
	60				6	254

Let assumed meanA=92.5, h=width of class intervals=5 and N=∑i=19fi=60
Mean, x¯=A+∑i=19fiyiN×h=92.5+660×5=92.5+0.5=93

Varianceσ2=h2N2N∑i=19fiyi2-∑i=19fi,yi2=(5)2(60)260×254-(6)2
=253600(15204)=105.52
∴ Standard deviation(σ)=105.52=10.27

Exercise 15.3

Q.1. From the data given below state which group is more variable, A or B?

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group A	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

Solution: Firstly, the standard deviation of group A is calculated as follows.

Marks	Group A fi	Mid-point xi	yi=xi-Ah=xi-4510	yi2	fiyi	fiyi2
10-20	9	15	-3	9	-27	81
20-30	17	25	-2	4	-34	68
30-40	32	35	-1	1	-32	32
40-50	33	45	0	0	0	0
50-60	40	55	1	1	40	40
60-70	10	65	2	4	20	40
70-80	9	75	3	9	27	81
	150				-6	342

Here, h=10, N=150, A=45 Mean=A+∑i=1nfiyiN×h=45+(-6)150×10=45-0.4=44.6

Variance of group A,
σ12=h2N2N∑i=17fiyi2-∑i=17fiyi2

=10022500150×342-(-6)2

=1225(51264) =227.84 ∴ Standard deviationσ1=227.84=15.09

The standard deviation of group B is calculated as follows.

Marks	Group B fi	Mid-point xi	yi=xi-4510	yi2	fiyi	fiyi2
10-20	10	15	-3	9	-30	90
20-30	20	25	-2	4	-40	80
30-40	30	35	-1	1	-30	30
40-50	25	45	0	0	0	0
50-60	43	55	1	1	43	43
60-70	15	65	2	4	30	60
70-80	7	75	3	9	21	63
	150				-6	366

Here, A=45, N=150, h=10
Mean,=A+∑i=17fiyiN×h=45+(-6)150×10=45-0.4=44.6

Variance of group B,
σ22=h2N2N∑i=17fiyi2-∑i=17fiyi2

=10022500150×366-(-6)2

=1225[54864]=243.84 ∴ Standard deviationσ2=243.84=15.61 Since the mean of both the groups is same, the group with greater standard deviation will be more variable. Thus, group B has more variability in the marks.

Q.2. From the prices of shares X and Y below, find out which is more stable in value:

X	35	54	52	53	56	58	52	50	51	49
Y	108	107	105	105	106	107	104	103	104	101

Solution: The group having more Coefficient of Variation will be more variable.
Coefficient of VariationC.V.=σx¯×100

Where,σ=Standard Deviationx¯=Mean

The prices of the share X are 35, 54, 52, 53, 56, 58, 52, 50, 51, 49 Here, the number of observations, N=10 ∴ Mean, x¯=1N∑i=110xi=110×510=51

The following table is obtained corresponding to share X.

xi	xi-x¯	xi-x¯2
35	-16	256
54	3	9
52	1	1
53	2	4
56	5	25
58	7	49
52	1	1
50	-1	1
51	0	0
49	-2	4
		350

Varianceσ12=1N∑i=110(xi-x¯)2=110×350=35 ∴ Standard deviationσ1=35=5.91 C.V.(Shares X)=σ1x×100=5.9151×100=11.58

The prices of share Y are

108, 107, 105, 105, 106, 107, 104, 103, 104, 101

∴ Mean, y¯=1N∑i=110yi=110×1050=105

The following table is obtained corresponding to share Y.

yi	yi-y¯	yi-y¯2
108	3	9
107	2	4
105	0	0
105	0	0
106	1	1
107	2	4
104	-1	1
103	-2	4
104	-1	1
101	-4	16
		40

Varianceσ22=1N∑i=110yi-y¯2=110×40=4 ∴ Standard deviationσ2=4=2 ∴ C.V. (Shares Y)=σ2y×100=2105×100=1.9=11.58 C.V. of prices of share X is greater than the C.V. of prices of share Y Thus, the prices of share Y are more stable than the prices of share X.

Q.3. An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

	Firm A	Firm B
No. of wage earners	586	648
Mean of monthly wages	₹5253	₹5253
Variance of the distribution of wages	100	121

Which firm A or B pays larger amount as monthly wages?

Solution: Monthly wages of firm A=₹5253.

Number of wage earners in firm A=586.

∴ Total amount paid=₹5253×586=₹3078258. Monthly wages of firm B=₹5253. Number of wage earners in firm B=648. ∴ Total amount paid=₹5253×648=₹3403944. Thus, firm B pays the larger amount as monthly wages as the number of wage earners in firm B are more than the number of wage earners in firm A.

Q.4. An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

	Firm A	Firm B
No. of wage earners	586	648
Mean of monthly wages	Rs 5253	Rs 5253
Variance of the distribution of wages	100	121

Which firm, A or B, shows greater variability in individual wages?

Solution: Given:

	Firm A	Firm B
No. of wage earners	586	648
Mean of monthly wages	₹5253	₹5253
Variance of the distribution of wages	100	121

Variance of the distribution of wages in firm Aσ12=100 ∴ Standard deviation of the distribution of wages in firm, Aσ1=Variance=100=10 Variance of the distribution of wages in firm Bσ22=121 ∴ Standard deviation of the distribution of wages in firm, Bσ2=Variance=121=11 The mean of monthly wages of both the firms is same i.e., 5253. Therefore, the firm
with greater standard deviation will have more variability. Thus, firm B has greater variability in the individual wages.

Q.5. The following is the record of goals scored by team A in a football session:

No. of goals scored	0	1	2	3	4
No. of matches	1	9	7	5	3

For the team B, mean number of goals scored per match was 2 with a standard deviation 1.25 goals. Find which team may be considered more consistent?

Solution: The mean and the standard deviation of goals scored by team A are calculated as follows.

No. of goals scored xi	No. of matches fi	fixi	xi2	fixi2
0	1	0	0	0
1	9	9	1	9
2	7	14	4	28
3	5	15	9	45
4	3	12	16	48
	25	50		130

Mean=∑i=15fixi∑i=15fi=5025=2 Thus, the mean of both the teams is same.

σ=1NN∑fixi2-∑fixi2

=12525×130-(50)2

=125750 =125×27.38 =1.09 The standard deviation of team B is 1.25 goals. The average number of goals scored by both the teams is same i.e., 2. Therefore, the team with lower standard deviation will be more consistent. Thus, team A is more consistent than team B.

Q.6. The sum and sum of squares corresponding to length x (in cm) and weight y (in gm) of 50 plant products are given below:

∑i=150xi=212, ∑i=150xi2=902.8, ∑i=150yi=261, ∑i=150yi2=1457.6

Which is more varying, the length or weight?

Solution: The sum and sum of squares corresponding to length x (in cm) of 50 plant products are given below:

∑i=150xi=212, ∑i=150xi2=902.8

Here, N=50 ∴ Mean, x¯=∑i=150xiN=21250=4.24

Varianceσ12=1N∑i=150xi-x¯2

=150∑i=150xi-4.242

=150∑i=150xi2-8.48xi+17.97 =150∑i=150xi2-8.48∑i=150xi+17.97×50 =150[902.8-8.48×(212)+898.5] =150[1801.3-1797.76] =150×3.54 =0.07 ∴ Standard deviation, σ1(Length)=0.07=0.26 ∴C.V. (Length)=StandarddeviationMean×100=0.264.24×100=6.13

The sum and sum of squares corresponding to weight y (in gm) of 50 plant products are given below:

∑i=150yi=261, ∑i=150yi2=1457.6

Mean, y¯=1N∑i=150yi=150×261=5.22

Varianceσ22=1N∑i=150yi-y¯2

=150∑i=150yi-5.222

=150∑i=150yi2-10.44yi+27.24 =150∑i=150yi2-10.44∑i=150yi+27.24×50 =150[1457.6-10.44×(261)+1362] =150[2819.6-2724.84] =150×94.76 =1.89 ∴Standard deviation, σ2(Weight)=1.89=1.37 ∴C.V.(Weight)=StandarddeviationMean×100=1.375.22×100=26.24 Thus, C.V. of weights is greater than the C.V. of lengths. Therefore, weights vary more than the lengths.