JEE Main Exam 2022 - Session 2

29 Jul 2022 - Shift 1 (Memory-Based Questions)

Section A: Physics

Q.1. Position of a particle x at time t are related as $t=\sqrt{2x+4}$. The velocity of the particle at $t=4\text{ s}$ is equal to (in S.I. units)

A) 4

B) 2

C) 1

D) 5

Answer: 4

Solution: $t=\sqrt{2x+4}\Rightarrow x=\frac{1}{2}\left(t^2-4\right)$

Velocity is the rate of change of position.

$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t}=v=t$

At $t=4\text{ s}, v=4\text{ m s}^{-1}$

Q.2. A projectile with kinetic energy E at point of projection is projected at angle 45°. Its kinetic energy at top most point is equal to

A) $\frac{E}{2}$

B) $\frac{3E}{2}$

C) $\frac{E}{4}$

D) $\frac{E}{3}$

Answer: $\frac{E}{2}$

Solution: Initial kinetic energy,

$K.E._i=\frac{1}{2}mv^2=E$

At the highest point, only the horizontal component remains. Therefore, speed at the highest point $v'=v\cos 45^\circ=\frac{v}{\sqrt{2}}$. Therefore, kinetic energy at the highest point,

$K.E._f=\frac{1}{2}m\left(v'\right)^2=\frac{1}{4}mv^2$

$K.E_f=\frac{E}{2}$

Q.3. Two rods of identical lengths and cross-sectional area are connected in series. If σ_1 and σ_2 is the thermal conductivity of material of two rods then equivalent conductivity of combination is equal to

A) $\frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

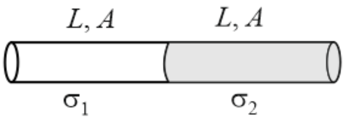
B) $\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

C) $\frac{\sigma_1\sigma_2}{\sigma_1-\sigma_2}$

D) $\frac{2\sigma_1\sigma_2}{\sigma_1-\sigma_2}$

Answer: $\frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

Solution:



The thermal resistance of a wire is given by, $R=\rho\frac{L}{A}=\frac{L}{\sigma A}$. Therefore,

$R_1=\frac{L}{\sigma_1 \mathrm{A}}$ and $R_2=\frac{L}{\sigma_2 \mathrm{A}}$

The wires are connected end to end i.e. they are in series. Therefore,

$R_{\text{net}}=R_1+R_2=\frac{L}{\mathrm{A}}\left(\frac{1}{\sigma_1}+\frac{1}{\sigma_2}\right)$

The combined length is $2L$, if the equivalent thermal conductivity is σ, then $R_{\text{net}}=R_1+R_2=\frac{2L}{\sigma \mathrm{A}}$

So, $\frac{2L}{\sigma \mathrm{A}}=\frac{L}{\mathrm{A}}\left(\frac{1}{\sigma_1}+\frac{1}{\sigma_2}\right)$

$\Rightarrow \sigma=\frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

Q.4. The range of a projectile at an angle θ is half of the maximum range if thrown at same speed. The angle of projection is given by

A) 60°

B) 30°

C) 15°

D) 70°

Answer: 15°

Solution: Range is given by, $R=\frac{u^2\sin 2\theta}{g}$.

The maximum range is at an angle of 45°,

$R_{max}=\frac{u^2}{g}$.

It is given that

$R=\frac{Rmax}{2}$

$\Rightarrow \frac{u^2\sin 2\theta}{g}=\frac{1}{2}\left(\frac{u^2}{g}\right)$

$\Rightarrow \sin 2\theta=\frac{1}{2}$

$\Rightarrow \theta=15^\circ$

Q.5. A travelling microscope has Vernier scale with $9\mathrm{MSD}=10\,\mathrm{VSD}$. If one main scale division (MSD) is equal to $1\ \mathrm{mm}$, then least count of travelling microscope is

A) $0.005\ \mathrm{m}$

B) $0.002\ \mathrm{m}$

C) $0.0001\ \mathrm{m}$

D) $0.0005\ \mathrm{m}$

Answer: $0.0001\ \mathrm{m}$

Solution: Least Count, $LC = 1\mathrm{MSD} - 1\,\mathrm{VSD}$

$$\Rightarrow LC = 1\mathrm{MSD} - \frac{9}{10}\mathrm{MSD}$$

$$\Rightarrow LC = \frac{1}{10}\mathrm{MSD} = \frac{1}{10} \times 0.001\ \mathrm{m}$$

$$\Rightarrow LC = 0.0001\ \mathrm{m}$$

Q.6. A cart is moving down a smooth incline of inclination α. What is the time period of a bob hanging from the roof of the cart with a light string?

A) $2\pi\sqrt{\frac{l}{g\ \cos\alpha}}$

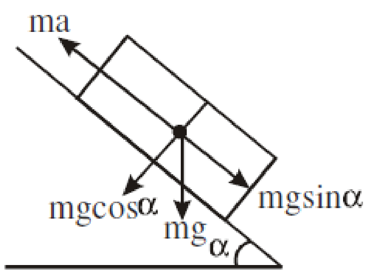
B) $2\pi\sqrt{\frac{l}{g}}$

C) $2\pi\sqrt{\frac{l}{g\ \sin\alpha}}$

D) $2\pi\sqrt{\frac{l}{g\ \cot\alpha}}$

Answer: $2\pi\sqrt{\frac{l}{g\ \cos\alpha}}$

Solution: Acceleration of vehicle down the incline will be $a = g\sin\alpha$
In the frame of the vehicle, we have to apply pseudo force. The Force diagram is shown in figure.
Component of mg along the inclined (i.e. $mg\sin\alpha$) will balance the pseudo force. Therefore, $g_{eff} = g\cos\alpha$



Hence,

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{g\cos\alpha}}$$

Q.7. If the length of wire is doubled and the radius is halved, then the young's modulus Y becomes

A) same

B) 8 times the original value

C) 4 times the original value

D) None of these

Answer: same

Solution: Young's modulus of a material is the property of the material. It does not depend on the dimensions of the wire, Therefore, it will remain the same.

Q.8. If one mole of monoatomic gas and three moles of diatomic gas are mixed, then the molar heat at constant volume is $\frac{\alpha^2 R}{4}$. The value of α is _______.

A) 3

B) 2

C) 5

D) 4

Answer: 3

Solution: Molar heat capacity of monatomic gas at constant volume is $C_V = \frac{3R}{2}$ and for diatomic gas is $C_V = \frac{5R}{2}$.

For the mixture,

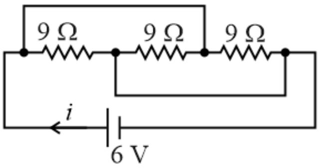
$$C_{V_{\text{mix}}} = \frac{\left(n_1 C_{V_1} + n_2 C_{V_2}\right)}{n_1 + n_2}$$

$$\Rightarrow C_{V_{\text{mix}}} = \frac{\left(1 \times \frac{3}{2} R + 3 \times \frac{5}{2} R\right)}{1+3}$$

$$\Rightarrow C_{V_{\text{mix}}} = \frac{9}{4} R$$

Therefore, $\alpha = 3$

Q.9. The value of current (in A) as shown is____



A) $2 \mathrm{~A}$

B) $3 \mathrm{~A}$

C) $4 \mathrm{~A}$

D) $5 \mathrm{~A}$

Answer: $2 \mathrm{~A}$

Solution: All the resistance are in parallel.

$$\Rightarrow R_{net} = \frac{9}{3} = 3 \ \Omega$$

$$\Rightarrow i = \frac{V}{R_{\text{net}}} = \frac{6}{3} = 2 \mathrm{~A}$$

Q.10. A wire of length $314 \mathrm{~cm}$ is made into a circular coil. Find its magnetic moment (in $\mathrm{A~m}^2$), if $I = 14 \mathrm{~A}$.(Take, $\pi = 3.14$)

A) $11 \mathrm{~A~m}^2$

B) $22 \mathrm{~A~m}^2$

C) $33 \mathrm{~A~m}^2$

D) $44 \mathrm{~A~m}^2$

Answer: $11 \mathrm{~A~m}^2$

Solution: Magnetic moment is given by, $\mu = I\left(\pi r^2\right)$

$$\Rightarrow \mu = I\pi\left(\frac{l}{2\pi}\right)^2$$

$$= 14 \times \pi\left(\frac{3.14}{2 \times 3.14}\right)^2$$

$$= 11 \mathrm{~A~m}^2$$

Q.11. The value of electric field at depletion layer in $p-n$ junction, if width is $6 \times 10^{-6} \mathrm{~m}$ and potential difference is $0.6 \mathrm{~V}$, is ____ $\times 10^5 \mathrm{~V~m}^{-1}$.

A) 1

B) 2

C) 3

D) 4

Answer: 1

Solution: As we know, $|E|=\left|\frac{\Delta V}{\Delta l}\right|$

$=\frac{0.6}{6\times10^{-6}}$

$=10^{5}\ \mathrm{V\ m^{-1}}$

Q.12. Find the ratio of energy of electron when it transitions from second to first energy state in comparison to highest state to first energy state of hydrogen atom.

A) $\frac{1}{4}$

B) $\frac{5}{36}$

C) $\frac{8}{9}$

D) $\frac{3}{4}$

Answer: $\frac{3}{4}$

Solution: Energy of electron(in Hydrogen) in n^{th} orbit is given by, $E_n=-\frac{13.6}{n^2}\ \mathrm{eV}$

So, $\Rightarrow E'(2\rightarrow1)=13.6\left(1-\frac{1}{2^2}\right)$

$=13.6\times\frac{3}{4}\ \mathrm{eV}$

$\Rightarrow E''(\infty\rightarrow1)=13.6\left(1-\frac{1}{\infty}\right)$

$=13.6\,\mathrm{eV}$

Therefore, $\frac{E'}{E''}=\frac{3}{4}$

Q.13. Assertion(A): Grease/oil stains can not be removed by water wash.

Reason(R): The angle of contact between water and oil is obtuse.

A) Both A and R are true and R is correct explanation of A

B) Both A and R are true but R is not correct explanation of A.

C) A is true and B is false

D) A is false and B is true

Answer: Both A and R are true and R is correct explanation of A

Solution: Angle of contact between oil and water is obtuse, therefore they do not stick and that's the reason grease/oil stains can not be removed by water wash.

Q.14. Assertion: Electric potential is constant inside & on the surface of a conductor.

Reason: Electric field just outside the conductor is perpendicular to its surface.

A) Only assertion is correct.

B) Only reason is correct

C) Both assertion & Reason are correct but reason is not the correct explanation of assertion.

D) Both assertion & Reason are correct and reason is the correct explanation of assertion.

Answer: Both assertion & Reason are correct but reason is not the correct explanation of assertion.

Solution: Electric field inside the conductor is zero as gaussian surface inside it doesn't contain any charge. Therefore, electric potential is constant inside the surface.

At the surface, due to property of conductor, it is a equipotential surface and therefore, electric field is perpendicular the surface of the conductor. Both assertion & Reason are correct but reason is not the correct explanation of assertion.

Q.15. Two objects of masses m and $3m$ are located at $\left(\hat{\imath}+2\hat{\jmath}+\hat{k}\right)$ and $\left(-3\hat{\imath}-2\hat{\jmath}+\hat{k}\right)$ respectively. Find position vector of COM of two objects.

A) $2\hat{\imath}+2\hat{\jmath}-\hat{k}$

B) $\hat{\imath}+\hat{\jmath}-\hat{k}$

C) $-2\hat{\imath}-\hat{\jmath}+\hat{k}$

D) $\hat{\imath}+2\hat{\jmath}-2\hat{k}$

Answer: $-2\hat{\imath}-\hat{\jmath}+\hat{k}$

Solution: As we know, centre of mass is given by, $\vec{r}_{\text{com}}=\frac{\sum m_i\vec{r}_i}{\sum m_i}$

$$\vec{r}_{\text{com}}=\frac{m\left(\hat{\imath}+2\hat{\jmath}+\hat{k}\right)+3m\left(-3\hat{\imath}-2\hat{\jmath}+\hat{k}\right)}{m+3m}$$

$$\vec{r}_{\text{com}}=\frac{-8m\hat{\imath}-4m\hat{\jmath}+4m\hat{k}}{4m}=-2\hat{\imath}-\hat{\jmath}+\hat{k}$$

Q.16. Value of acceleration due to gravity above the surface of Earth at height $h\,(h\ll R_e)$ is equal to acceleration due to gravity at depth d below Earth's surface, then $d=\alpha h$. Value of α is equal to ______.

A) 2

B) 3

C) 4

D) 5

Answer: 2

Solution: As, $g_{\text{above}}=g_{\text{below}}$

$$\Rightarrow g\left(1-\frac{2h}{Re}\right)=g\left(1-\frac{\alpha h}{Re}\right)$$

$$\Rightarrow\ a=2$$

Section B: Chemistry

Q.17. Ionic radius for A^{+} and B^{-} are $281\ pm$ and $180\ pm$ respectively forming a ccp structure. If B^{-} forms a ccp lattice and A^{+} fills the octahedral voids, then what is the value of edge length in pm?

A) $778\ pm$

B) $461\ pm$

C) $230\ pm$

D) $449\ pm$

Answer: $778\ pm$

Solution: For CCP lattice, given that cation in octahedral void.

Edge length = $a = ?$

$\left(r^{+} + r^{-}\right) = \frac{a}{2} = 281 + 180$

$a = 778\ pm$

Q.18. Product for the given reaction is:

$Zn + NaOH \rightarrow$

A) ZnO

B) $NaZnO_2$

C) $[ZnO_3]^{4-}$

D) $Zn(OH)_2$

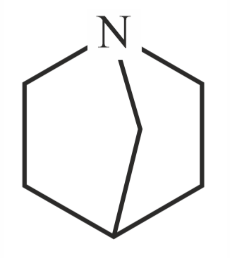
Answer: $NaZnO_2$

Solution: Hydrogen can be prepared by the reaction of zinc with aqueous alkali.

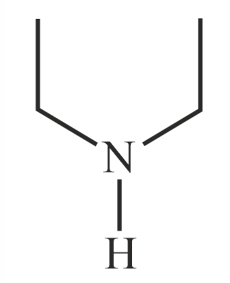
$Zn + 2\,NaOH \rightarrow Na_2ZnO_2 + H_2$

Q.19. Which of the following is the strongest Bronsted base?

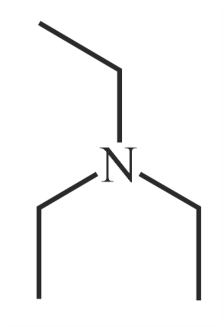
A)



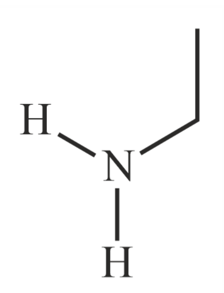
B)



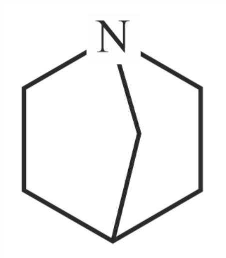
C)



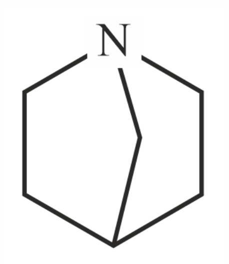
D)



Answer:



Solution: Cyclic amines are more basic than open chain amines of same degree. Because open chain amines undergo umbrella inversion which decreases the basic nature. Hence, among the given compounds, quinuclidene is more basic compound.



Q.20. Which pair among the following is colourless?

A) Sc^{3+}, Zn^{2+}

B) Ti^{2+}, Cu^{2+}

C) Fe^{3+}, Mn^{2+}

D) Fe^{3+}, Cu^{2+}

Answer: Sc^{3+}, Zn^{2+}

Solution: Transition metal ions exhibit colour due to the presence of incomplete d orbital. But the ions which have complete or vacant d orbital, are colourless.

The atomic number of Zn is 30. The electronic configuration of $Zn^{2+} = [Ar]3d^{10}4\ s^0$.

The atomic number of Sc is 21. The electronic configuration of $Sc^{3+} = [Ar]3\ d^0$.

Thus, both ions have completely filled or vacant d orbital respectively. Therefore, these ions are colourless.

Q.21. Consider a complex $[Fe(OH)_6]^{3-}$ which act as an inner orbital complex. If the CFSE value after ignoring pairing energy is represented as $-x\Delta_o$, then x is:

(Δ_o is splitting energy in octahedral complex)

A) 0

B) 2

C) 2.4

D) 1.2

Answer: 2

Solution: Electronic configuration in Fe^{3+}: d^5

Electronic configuration for inner orbital complex: $t^5{}_{2g}\, e^0{}_g$

$CFSE = [5(-0.4) + 0(0.6)]\Delta_o$

$= -2\Delta_o$

Q.22. Find limiting reagent and moles of NH_3 produced.

$\underset{10\,g}{N_2} + \underset{5\,g}{3H_2} \rightarrow 2NH_3$

A) $N_2,\ 1.42$

B) $N_2,\ 0.71$

C) $H_2,\ 1.42$

D) $H_2,\ 0.71$

Answer: $N_2,\ 0.71$

Solution: 28 g of N_2 should react with 6 g H_2

$1 \text{ mol } N_2 \rightarrow 2 \text{ mol } NH_3$

$\frac{10}{28} \text{mol of } N_2 \rightarrow 2 \times \frac{10}{28} \text{ mol of } NH_3$

$= 0.7$

Q.23. The magnitude of change in oxidation state of manganese in $KMnO_4$ in faintly alkaline or neutral medium is:

A) 4

B) 3

C) 5

D) 1

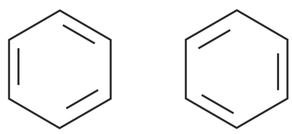
Answer: 3

Solution: $KMnO_4$ acts as an oxidising agent. In neutral or faintly alkaline medium alkaline $KMnO_4$ is reduced to MnO_2.

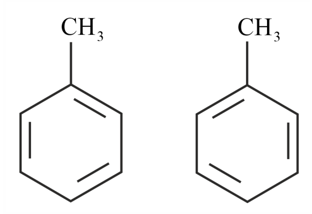
The change in oxidation state of manganese in this reaction is from $+7$ to $+4$.

Q.24. Which of the following pairs will give different products on ozonolysis?

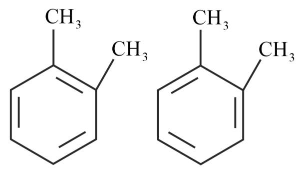
A)



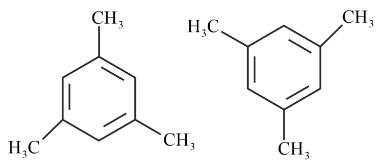
B)



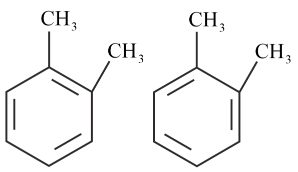
C)



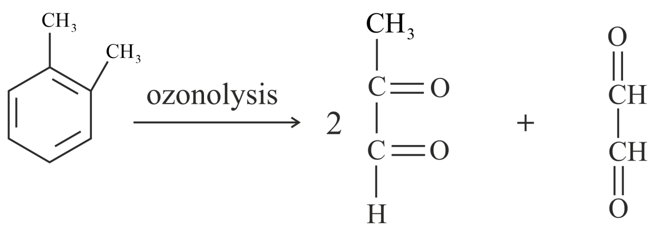
D)

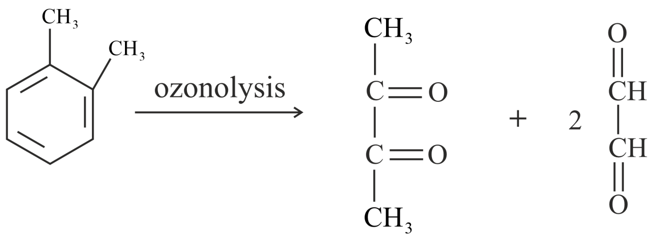


Answer:

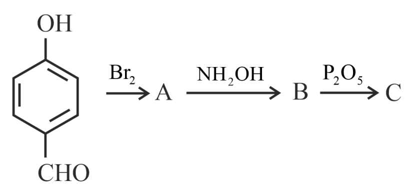


Solution:

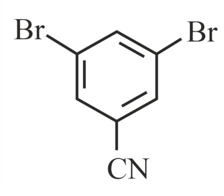




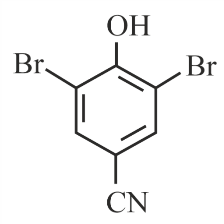
Q.25. Find C.



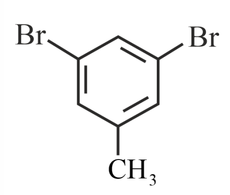
A)



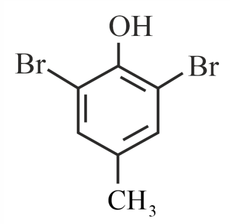
B)



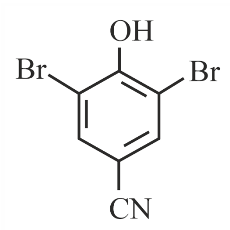
C)



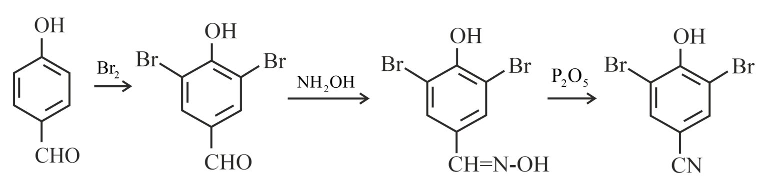
D)



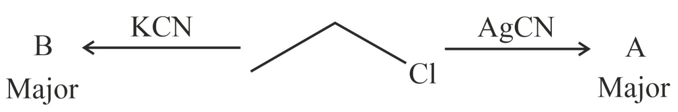
Answer:



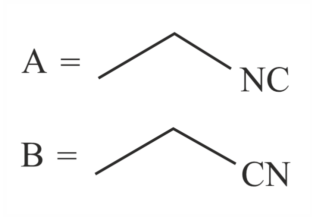
Solution:



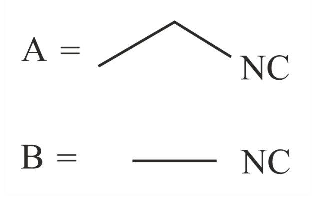
Q.26. Find A and B respectively.



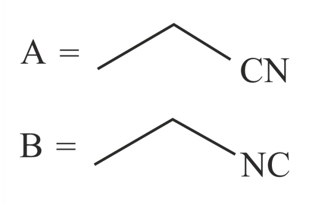
A)



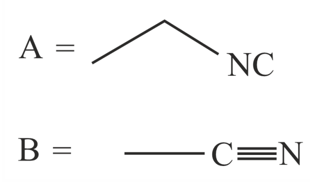
B)



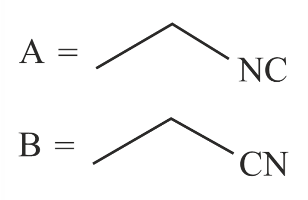
C)



D)



Answer:



Solution:

$$C_2H_5Cl \xrightarrow{\text{ethanolic KCN}} \underset{\substack{\text{Ethyl cyanide}\\ \text{(major product)}}}{C_2H_5CN} + \underset{\substack{\text{Ethyl isocyanide}\\ \text{(minor product)}}}{C_2H_5NC}$$

$$C_2H_5Cl \xrightarrow{\text{ethanolic AgCN}} \underset{\substack{\text{Ethyl isocyanide}\\ \text{(major product)}}}{C_2H_5NC} + \underset{\substack{\text{Ethyl cyanide}\\ \text{(minor product)}}}{C_2H_5CN}$$

Q.27. Which of the following is a hypnotic drug?

A) Seldane

B) Terpineol

C) Amytal

D) Histamine

Answer: Amytal

Solution: Derivatives of barbituric acid viz., veronal, amytal, nembutal, luminal and seconal constitute an important class of tranquilizers. These derivatives are called barbiturates. Barbiturates are hypnotic. i.e., sleep producing agents.

Q.28. K_{sp} of PbS is given as 9×10^{-30} at a given temperature. Its solubility is $x \times 10^{-15}$. Find the value of x.

A) 9

B) 3

C) 2

D) 4

Answer: 3

Solution: $PbS.(s) \rightleftharpoons Pb^{2+}(aq) + S^{2-}(aq)$

Let us consider the solubility of PbS is "S" M.

Thus,

Solubility product $= \left[Pb^{2+}(aq)\right]\left[S^{2-}(aq)\right]$

$Ksp = \left[Pb^{2+}(aq)\right]\left[S^{2-}(aq)\right] \quad \cdots \text{(i)}$

We know

Solubility product Ksp for PbS is 9×10^{-30}

Therefore, from equation (i)

$\Rightarrow S^2 = 9.04 \times 10^{-29}$

$\Rightarrow S = \left(9 \times 10^{-30}\right)^{\frac{1}{2}} = 3 \times 10^{-15} M$

Q.29. Which of the following pairs will have one of the compounds having odd number of electrons and will also contain a compound having expanded octet?

A) $BCl_3,\ H_2SO_4$

B) $NO,\ H_2SO_4$

C) $BCl_3,\ NO$

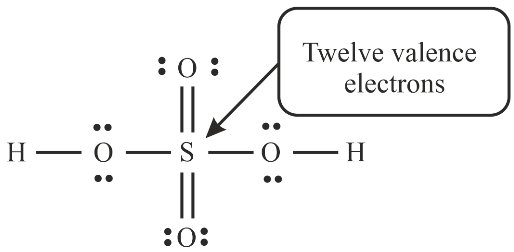
D) $NO,\ BCl_3$

Answer: $NO,\ H_2SO_4$

Solution: An odd electron bond means there is an odd number of the electron in the overall molecule. Or we can say that the molecule has unpaired electrons.

So, in NO, the valence electrons in nitrogen are 5, and valence electrons in oxygen are 6. So, by adding we get, $6 + 5 = 11$

In sulfuric acid (H_2SO_4), each oxygen has a full octet (eight valence electrons), whereas sulfur has an expanded octet (twelve valence electrons).



Q.30. Find the number of lone pairs present in following species:

$SCl_2,\ ClF_3,\ O_3,\ SF_6$

A) $4,\ 2,\ 2,\ 1$

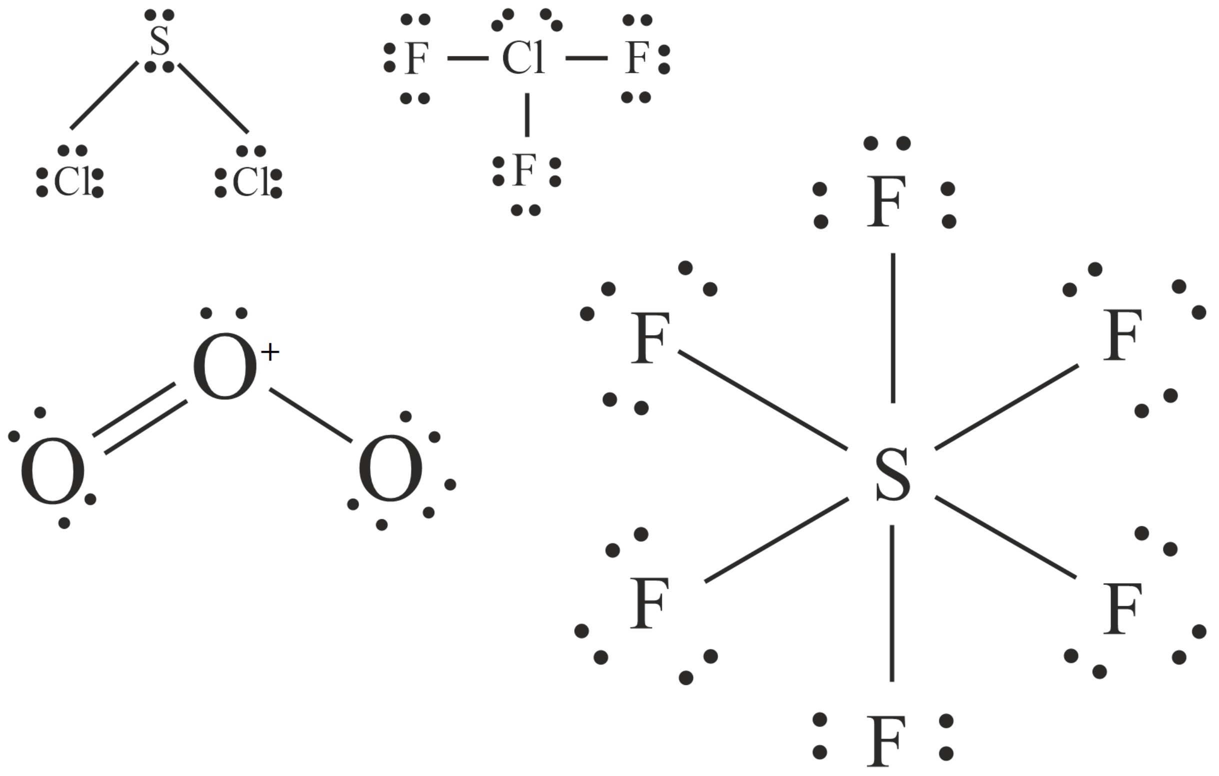
B) $6,\ 4,\ 4,\ 9$

C) $6,\ 2,\ 1,\ 0$

D) $8,\ 11,\ 6,\ 18$

Answer: $8, 11, 6, 18$

Solution:



Q.31. Identify the products formed in the following reaction:

Lithium nitrate $+ \ NaNO_3 \xrightarrow{\Delta}$ Products

A) Li_2O and $NaNO_2$

B) $LiNO_2$ and $NaNO_2$

C) Li_2O and Na_2O

D) $LiNO_2$ and Na_2O

Answer: Li_2O and $NaNO_2$

Solution: $4\,LiNO_3 \xrightarrow{heat} \underset{\text{Lithium oxide}}{2\,Li_2O} + 4\,NO_2 + O_2$

$2\,NaNO_3 \xrightarrow{heat} 2\,NaNO_2 + O_2$

Q.32. Which of the following are herbicides?

A) Sodium chlorate and sodium arsenite

B) Aldrin and dieldrin

C) Aldrin and sodium chlorate

D) Dieldrin and sodium arsenite

Answer: Sodium chlorate and sodium arsenite

Solution: Nowadays, the pesticide industry has shifted its attention to herbicides such as sodium chlorate $(NaClO_3)$, sodium arsinite (Na_3AsO_3) and many others.

Q.33. In a 5% w/V $NaCl$ solution, we add albumin of egg and stir well. The resultant colloidal solution is:

A) Lyophobic

B) Lyophilic

C) Emulsion

D) Precipitate

Answer: Lyophilic

Solution: The most appropriate method of making egg-albumin sol is Break an egg carefully and transfer the transparent part of the content to $100 \mathrm{~mL}$ of $5 \% \mathrm{~w} / \mathrm{V}$ saline solution and stir well. egg-albumin sol is a lyophilic sol.

Note: egg-albumin is the transparent part of egg. During preparation of sol, water should be cold. If we boil water, the sol will be coagulated.

Q.34. The first ionisation potential of Na, Mg and Si are respectively $496, 737$ and $786 \mathrm{~kJ} \mathrm{~mol}^{-1}$. The ionisation potential of Al will be closer to

A) $760 \mathrm{~kJ} \mathrm{~mol}^{-1}$

B) $575 \mathrm{~kJ} \mathrm{~mol}^{-1}$

C) $801 \mathrm{~kJ} \mathrm{~mol}^{-1}$

D) $419 \mathrm{~kJ} \mathrm{~mol}^{-1}$

Answer: $575 \mathrm{~kJ} \mathrm{~mol}^{-1}$

Solution: In the third period, Al comes after Mg and before Si. Since in Mg ($1 \mathrm{s}^2 2 \mathrm{s}^2 2 \mathrm{p}^6 3 \mathrm{s}^2$), the last electron is to be lost from a fully filled $3 \mathrm{s}$-orbital while in Al (ls^2 $2 \mathrm{s}^2$ $2 \mathrm{p}^6$ $3 \mathrm{s}^2$ $3 \mathrm{p}^1$) it is to be lost from a $3 \mathrm{p}$-orbital, therefore, $\Delta_{\mathrm{i}} \mathrm{H}_1$ of Al is lower than that of Mg ($737 \mathrm{~kJ} \mathrm{~mol}^{-1}$). Further, nuclear charge increases in going from Na to Al, therefore $\Delta_{\mathrm{i}} \mathrm{H}_1$ of Al is higher than that of Na($496 \mathrm{~kJ} \mathrm{~mol}^{-1}$) but lower than that of Mg ($737 \mathrm{~kJ} \mathrm{~mol}^{-1}$). Therefore, the appropriate value of ionisation potential of Al is $575 \mathrm{~kJ} \mathrm{~mol}^{-1}$.

Section C: Mathematics

Q.35. $\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\sin x+\cos x}$ is equal to:

A) $\tan^{-1}(2)$

B) $\tan^{-1}(2)-\frac{\pi}{4}$

C) $\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$

D) $\frac{\pi}{3}-\tan^{-1}(2)$

Answer: $\tan^{-1}(2)-\frac{\pi}{4}$

Solution: Let $I=\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\sin x+\cos x}=\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}+\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$

$=\int_0^{\frac{\pi}{2}} \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{3+3\tan^2\frac{x}{2}+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$

Let $\tan\frac{x}{2}=t \Rightarrow \sec^2\frac{x}{2}dx=2dt$

i.e. $I=\int_0^1 \frac{2dt}{2t^2+4t+4}=\int_0^1 \frac{dt}{(t+1)^2+1}$

$\Rightarrow \left[\tan^{-1}(t+1)\right]_0^1=\tan^{-1}2-\tan^{-1}1$

$=\tan^{-1}2-\frac{\pi}{4}$

Q.36. Let $z=2+3i$, then value of $(z)^5+(\bar{z})^5$ is:

A) 246

B) 244

C) 248

D) 234

Answer: 244

Solution: $(z)^5+(\bar{z})^5=(2+3i)^5+(2-3i)^5$

$=2\left[{}^5C_0\cdot 2^5+{}^5C_2\cdot 2^3(3i)^2+{}^5C_4\cdot 2^1\cdot(3i)^4\right]$ (applying binomial expansion)

$=2[32-720+810]=2\times 122=244$

Q.37. Let $\vec{a}=3\hat{\imath}+\hat{\jmath}$, $\vec{b}=\hat{\imath}+2\hat{\jmath}+\hat{k}$. If $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ and $\vec{b}$ is non parallel to $\vec{c}$, then value of λ is:

A) 5

B) -5

C) 1

D) -1

Answer: -5

Solution: Given, $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

and, $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{b} + \lambda\vec{c}$

We know that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \cdot \vec{c}\right)\vec{b} - \left(\vec{a} \cdot \vec{b}\right)\vec{c}$

$\Rightarrow \quad \left(\vec{a} \cdot \vec{c}\right)\vec{b} - \left(\vec{a} \cdot \vec{b}\right)\vec{c} = \vec{b} + \lambda\vec{c}$

$\Rightarrow \vec{a} \cdot \vec{b} = -\lambda$

$\therefore \quad \lambda = -(2+3) = -5$

Q.38. If $\lim_{x\to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \frac{2}{3}$, then which of the following option is incorrect?

A) $\alpha^2 + \beta^2 + \gamma^2 = 1$

B) $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$

C) $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$

D) $\alpha^2 - \beta^2 + \gamma^2 + 4 = 0$

Answer: $\alpha^2 + \beta^2 + \gamma^2 = 1$

Solution: Given $\lim_{x\to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \frac{2}{3}$

For $\frac{0}{0}$ form, we know that $\alpha + \beta = 0 \quad \ldots(1)$

Now $\lim_{x\to 0} \frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x^3} \times \frac{x^2}{\sin^2 x} = \frac{2}{3}$

Applying L'Hospital Rule,

$\lim_{x\to 0} \frac{\alpha e^x - \beta e^{-x} + \gamma \cos x}{3x^2} = \frac{2}{3}$

Again for $\frac{0}{0}$ form, we know $\alpha - \beta + \gamma = 0 \quad \ldots(2)$

Applying L'Hospital Rule twice again, we get

$\lim_{x\to 0} \frac{\alpha e^x + \beta e^{-x} - \gamma \sin x}{6x} = \frac{2}{3}$

$\lim_{x\to 0} \frac{\alpha e^x - \beta e^{-x} - \gamma \cos x}{6} = \frac{2}{3}$

$\Rightarrow \alpha - \beta - \gamma = 4 \quad \ldots(3)$

Solving (1), (2), (3), we get

$\alpha = 1,\ \beta = -1,\ \gamma = -2$

Q.39. If $A = \{1, 2, \cdots, 60\}$ and B is relation on A defined as $B = \{(x, y) : y = pq$ where p and q are primes $\geq 3\}$ then number of elements in B is:

A) 720

B) 660

C) 540

D) 600

Answer: 660

Solution: Given $y = pq$ where p & q are prime numbers ≥ 3 and $1 \leq pq \leq 60$

$\therefore$ y will be $3 \times 3, 3 \times 5, 3 \times 7, 3 \times 11, 3 \times 13, 3 \times 17, 3 \times 19, 5 \times 5, 5 \times 7, 5 \times 11, 7 \times 7$

i.e. total 11 possibilities

Now x can be $\{1, 2, \cdots, 60\}$

i.e. total 60 possibilities

Hence, the number of relations $= 60 \times 11 = 660$

Q.40. A matrix of 3×3 order, should be filled either by 0 or 1 and sum of all elements should be prime number. Then the number of such matrix is equal to ______ .

Answer: 282

Solution: Let the matrix be $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Since the elements can be either of 0 or 1, so

$a+b+c+d+e+f+g+h+i = 2/3/5/7$

For sum as 2, the number of cases will be $\frac{9!}{2!7!} = 36$

For sum as 3, the number of cases will be $\frac{9!}{3!6!} = 84$

For sum as 5, the number of cases will be $\frac{9!}{5!4!} = 126$

For sum as 7, the number of cases will be $\frac{9!}{2!7!} = 36$

Total required matrix will be 282

Q.41. Let $a_1, a_2, a_3, \cdots a_n$ are in A.P. and $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to ______ .

Answer: 16

Solution: Let the common difference of the given A.P. be d, then

$4 = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} \ldots$

$\frac{4}{2} = \quad \frac{a_1}{2^2} + \frac{a_2}{2^3} + \frac{a_3}{2^4} \ldots$

Subtracting above equations, we get,

$2 = \frac{a_1}{2} + \left(\frac{d}{2^2} + \frac{d}{2^3} \ldots\right)$

$\Rightarrow \frac{a_1}{2} + \frac{\frac{1}{4}}{1 - \frac{1}{2}} d = 2 \Rightarrow a_1 + d = 4$

Now, $a_2 = a_1 + d = 4 \Rightarrow 4a_2 = 16$

Q.42. If $f(x) = 3^{\left(x^2 - 2\right)^3} + 4$ and

$P : f(x)$ attains maximum value at $x = 0$

$Q : f(x)$ have point of inflection at $x = \sqrt{2}$.

$R : f(x)$ is increasing for $x > \sqrt{2}$, then which of the following statement are correct?

A) P and R

B) Q and R

C) P and Q

D) P, Q and R all

Answer: Q and R

Solution: $f(x)=3^{\left(x^2-2\right)^3}+4$

$f'(x)=3^{\left(x^2-2\right)^3}\times \ln 3\times 3\left(x^2-2\right)^2\times 2x$

For $f'(x)=0$, the critical points are at $0,\pm\sqrt{2}$

$x=0$ is a point of minima as the sign of $f'(x)$ changes from negative to positive.

$x=\pm\sqrt{2}$ are points of inflection as $f'(x)$ does not change sign

For $x>\sqrt{2},\ f'(x)>0$ hence $f(x)$ is increasing.

Q.43. If $\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\cdots+\frac{1}{100\cdot101\cdot102}=\frac{k}{101}$, then $34\,k$ is equal to ______ .

Answer: 286

Solution: $\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\cdots+\frac{1}{100\cdot101\cdot102}=\frac{k}{101}$

$\Rightarrow \frac{1}{2}\left(\frac{4-2}{2\cdot3\cdot4}+\frac{5-3}{3\cdot4\cdot5}+\cdots+\frac{102-100}{100\cdot101\cdot102}\right)=\frac{k}{101}$

$\Rightarrow \frac{1}{2}\left(\frac{1}{2\cdot3}-\frac{1}{3\cdot4}+\frac{1}{3\cdot4}-\frac{1}{4\cdot5}+\cdots+\frac{1}{100\cdot101}-\frac{1}{101\cdot102}\right)=\frac{k}{101}$

$\Rightarrow \frac{1}{2}\left(\frac{1}{2\cdot3}-\frac{1}{101\cdot102}\right)=\frac{k}{101}$

$\Rightarrow k=\frac{1}{2}\left(\frac{101}{6}-\frac{1}{102}\right)=\frac{10296}{6\cdot12\cdot102}=\frac{143}{17}$

Hence, $34k=286$

Q.44. Let $f(x)=|(x-1)|\cos|x-2|\sin|x-1|+|x-3|\left|x^2-5x+4\right|$. The number of points where the function is not differentiable is

A) 3

B) 4

C) 5

D) 6

Answer: 3

Solution: Given $f(x)=|(x-1)|\cos|x-2|\sin|x-1|+|x-3|\left|x^2-5x+4\right|$

$f(x)=|(x-1)|\cos|x-2|\sin|x-1|+|x-3|\,|(x-1)(x-4)|$

$f(x)=|(x-1)|\sin|x-1|\cdot\cos|x-2|+|x-3|\,|(x-1)|\,|(x-4)|$

Now $|(x-1)|\sin|x-1|\cdot\cos|x-2|=\begin{cases}(x-1)\sin(x-1)\cdot\cos(x-2) & x\geq 1\\ (1-x)\sin(1-x)\cdot\cos(x-2)=(x-1)\sin(x-1)\cdot\cos(x-2) & x<1\end{cases}$

Hence above function is differentiable

Now $|x-3|\,|(x-1)|\,|(x-4)|$ is non-differentiable at $x=1,3,4$

Q.45. Let A and B are two 3×3 non-zero real matrices and $AB=O$, then which of the following options is correct?

A) $AX=O$ has unique solution

B) $AX=O$ has infinite solutions

C) B is invertible

D) $(\operatorname{adj}(A))B$ is invertible

Answer: $AX=O$ has infinite solutions

Solution: $\because \quad AB = O \Rightarrow |A| = 0 = |B|$

So, A & B are not invertible as $|A| = |B| = 0$

$(\operatorname{adj}(A))B$ is not invertible as $|(\operatorname{adj}(A))B| = |\operatorname{adj}(A)|\,|B| = 0$

$AX = O$ has infinitely many solution as $|A| = 0$

Q.46. If $|x-1| \le y \le \sqrt{5-x^2}$, then the area of region bounded by the curves is:

A) $\frac{5\pi}{4} - \frac{1}{2}$

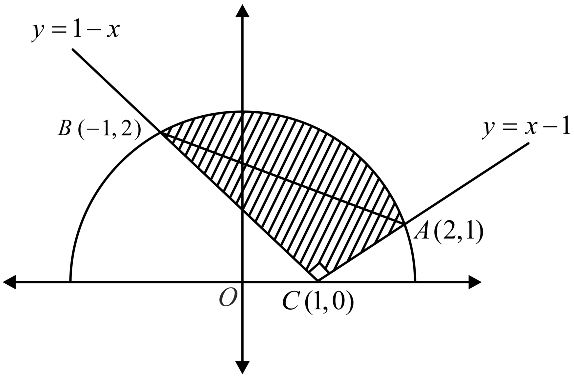
B) $\frac{5\pi}{4} - \frac{3}{2}$

C) $\frac{3\pi}{4} - \frac{1}{2}$

D) $\cos^{-1}\frac{1}{3} - \frac{1}{2}$

Answer: $\frac{5\pi}{4} - \frac{1}{2}$

Solution: Plotting region for $|x-1| \le y \le \sqrt{5-x^2}$, we get,



The intersection points of the curves $x^2 + y^2 = 5$ and $y = |x-1|$ will be $(-1, 2)$ & $(2, 1)$

Here chord AB subtends a right angle at the centre of the circle.
So, required area $=$ area of $\Delta ABC +$ area of segment of circle on chord AB

$= \frac{1}{2} AC \cdot BC +$ [area of quarter circle $-$ area of ΔAOB]

$= \frac{1}{2} \times \sqrt{2} \times 2\sqrt{2} + \left(\frac{90^\circ}{360^\circ} \times \pi(5)^2 - \frac{1}{2} \times \left(\sqrt{5}\right)^2\right)$

$= \frac{5\pi}{4} - \frac{1}{2}$

Q.47. The straight line $y = mx + c$ is a focal chord of parabola $y^2 = 4x$, which also touches the hyperbola $x^2 - y^2 = 4$, then the value of m is:

A) $m = \pm\frac{2}{\sqrt{3}}$

B) $m = \pm\frac{\sqrt{3}}{2}$

C) $m = \pm\frac{2}{3}$

D) $m = \pm\frac{3}{2}$

Answer: $m = \pm\frac{2}{\sqrt{3}}$

Solution: Since the focal chord passes through the focus of parabola $(1, 0)$

$\therefore \quad m + c = 0 \Rightarrow c = -m$

Also the focal chord touches $\frac{x^2}{4} - \frac{y^2}{4} = 1$

$\therefore \quad c^2 = 4m^2 - 4$

$\Rightarrow m^2 = \frac{4}{3}$

$\Rightarrow m = \pm\frac{2}{\sqrt{3}}$