JEE Main Exam 2022 - Session 2

29 Jul 2022 - Shift 2 (Memory-Based Questions)

Section A: Physics

Q.1. An unpolarised light of intensity $2I_0$, when passed through two polaroids P & Q which are inclined at an angle of 30° with each other. What will be the intensity of light coming out of polariser Q.

A) I_0

B) $2I_0$

C) $\frac{3I_0}{2}$

D) $\frac{3I_0}{4}$

Answer: $\frac{3I_0}{4}$

Solution: When unpolarized light is passed through a polarizer, its intensity becomes half. Therefore, $I_1 = \frac{2I_0}{2} = I_0$.

Using Malus law for the second polarizer: $I_2 = I_1 \cos^2 30^\circ = I_0 \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3I_0}{4}$

Q.2. Find the torque about the point $\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ of a force $6\hat{\imath} + 4\hat{k}$ acting through the point $\left(-\hat{\imath} + \hat{\jmath} + \hat{k}\right)$. (All quantities are in S.I. unit.)

A) $\hat{\imath} - 8\hat{\jmath} - 2\hat{k}$

B) $2\hat{\imath} - 10\hat{\jmath} + 8\hat{k}$

C) $-4\hat{\imath} - 10\hat{\jmath} + 6\hat{k}$

D) $2\hat{\imath} - 7\hat{\jmath} + 8\hat{k}$

Answer: $-4\hat{\imath} - 10\hat{\jmath} + 6\hat{k}$

Solution: $\vec{\tau} = \vec{r} \times \vec{F}$

$\Rightarrow \vec{\tau} = \left(\left(-\hat{\imath} + \hat{\jmath} + \hat{k}\right) - \left(\hat{\imath} + 2\hat{\jmath} + 4\hat{k}\right)\right) \times \left(6\hat{\imath} + 4\hat{k}\right)$

$\Rightarrow \vec{\tau} = \left(-2\hat{\imath} - \hat{\jmath} - 3\hat{k}\right) \times \left(6\hat{\imath} + 4\hat{k}\right)$

$\Rightarrow \vec{\tau} = -4\hat{\imath} - 10\hat{\jmath} + 6\hat{k}$

Q.3. If the momentum of the body is increased by 20%, then percentage increase in its kinetic energy of the body is

A) 40%

B) 10%

C) 20%

D) 44%

Answer: 44%

Solution: A body of mass m, moving with momentum p and kinetic energy K are related as follows,

$K=\frac{p^2}{2m}$.

As the momentum of the body is increased by 20%, then its momentum becomes,

$\left(p+\frac{20}{100}p\right)=1.2\,p$

and the mass of the body remains unchanged. So, kinetic energy is directly related to the square of momentum.

Hence, new kinetic energy (K') will be,

$K'=\frac{(1.2\,p)^2}{2m}$

$K'=1.44\text{ K}$.

So, the percentage change in kinetic energy

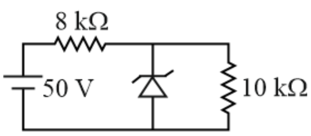
$=\frac{K'-K}{K}\times 100$

$=\frac{1.44K-K}{K}\times 100$

$=(1.44-1)\times 100$

$=44\%$.

Q.4. Find the current through Zener diode in the circuit shown (in mA) if voltage across Zener diode is 10 V.



A) 4

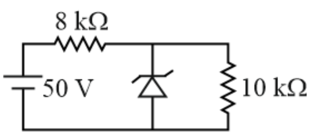
B) 2

C) 5

D) 3

Answer: 4

Solution:



Voltage across $10\text{ k}\Omega$ resistance will be equal to the voltage across the diode.

$\Rightarrow V_{10\text{k}\Omega}=10\text{ V}$

So $i_{10\text{k}\Omega}=\frac{V_{10\text{k}\Omega}}{R}=\frac{10}{10\times 10^3}=1\text{ mA}$

Now the voltage across the $8\text{ k}\Omega$ resistance will be, $V_{8\text{k}\Omega}=50-10=40\text{ V}$

$\Rightarrow i_{8\text{k}\Omega}=\frac{40}{8\times 10^3}=5\text{ mA}$

Using Kirchhoff's junction law,

$\Rightarrow i_{\text{zener}}=i_{8\text{k}\Omega}-i_{10\text{k}\Omega}=4\text{ mA}$

Q.5. A mass m is taken from earth surface to a height $3R$. Then find the change in potential energy of the mass.

A) $\frac{GMem}{R}$

B) $\frac{2GMem}{R}$

C) $\frac{3GMem}{4R}$

D) $\frac{3GMem}{R}$

Answer: $\frac{3GMem}{4R}$

Solution: Gravitational potential energy as a distance r from the centre of the earth is given by, $U=-\frac{GMem}{r}$.

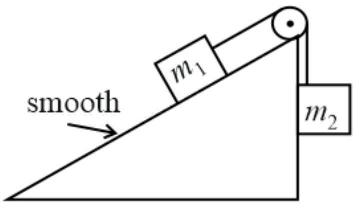
$\Rightarrow U_i=-\frac{GMem}{Re}$ and

$U_f=-\frac{GMem}{(Re+3Re)}=\frac{-GMem}{4Re}$

The change in the potential energy of the mass, $U_f-U_i=\frac{3GMem}{4Re}$.

Q.6. If the shown system is in equilibrium then force applied by the inclined plane on the block m_1 is equal to

Given, $m_1=5\ \text{kg},\ m_2=3\ \text{kg}$



A) $30\ \text{N}$

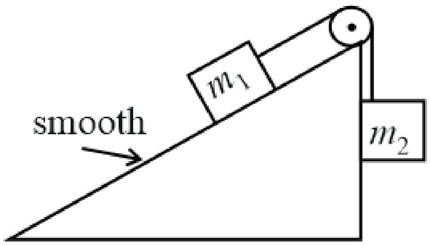
B) $50\ \text{N}$

C) $40\ \text{N}$

D) $60\ \text{N}$

Answer: $40\ \text{N}$

Solution:



As the system is in equilibrium,

$m_1 g\sin\theta=m_2 g$

$\Rightarrow 5g\sin\theta=3g$

$\Rightarrow \theta=37^\circ$

So, force given by the inclined plane is

$N=m_1 g\cos\theta$

$\Rightarrow N=5\times10\times\frac{4}{5}=40\ \text{N}$

Q.7. When a light ray is incident from air on an interface at an angle of incidence of 45° the deviation of light is 15°, then find the refractive index of the second medium.

A) $\sqrt{3}$

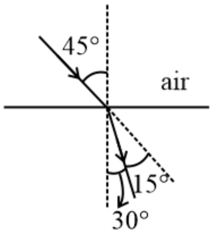
B) 1.5

C) $\sqrt{2}$

D) 1.6

Answer: $\sqrt{2}$

Solution:



From the diagram, as the deviation angle is 15°, angle of refraction is 30°.

Using Snell's Law,

$1 \times \sin 45^\circ = \mu \sin 30^\circ$

$\Rightarrow \mu = \sqrt{2}$

Q.8. A body is dropped from a height h. If t_1 and t_2 be the times in covering first half and the next half distances respectively, then the relation t_1 and t_2 is

A) $t_1 = t_2$

B) $t_2 = \sqrt{2} t_1$

C) $t_2 = \left(\sqrt{2} - 1\right) t_1$

D) $t_1 = \frac{t_2}{2}$

Answer: $t_2 = \left(\sqrt{2} - 1\right) t_1$

Solution: Using $t = \sqrt{\frac{2H}{g}}$

For first half distance, $t_1 = \sqrt{\frac{2\frac{h}{2}}{g}} = \sqrt{\frac{h}{g}}$

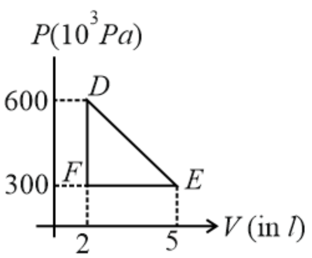
For second half, if the time for complete fall is t, then $t_2 = t - t_1$

$t = \sqrt{\frac{2h}{g}}$. Therefore,

$t_2 = \sqrt{\frac{2h}{g}} - \sqrt{\frac{h}{g}} = \sqrt{\frac{h}{g}}\left(\sqrt{2} - 1\right)$

$\Rightarrow t_2 = \left(\sqrt{2} - 1\right) t_1$

Q.9. Find the total work done from D to E and E to F.



A) -450 J

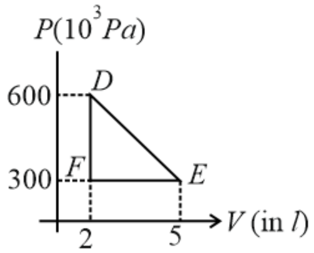
B) 450 J

C) 900 J

D) 1350 J

Answer: 450 J

Solution:



$W_{DEF} = W_{DE} + W_{EF}$

$= \left[\frac{1}{2} \times \left(300 \times 10^3\right) \times \left(3 \times 10^{-3}\right) + \left(300 \times 10^3\right) \times \left(3 \times 10^{-3}\right)\right] + \left[-\left(300 \times 10^3\right) \times \left(3 \times 10^{-3}\right)\right]$

$= \frac{1}{2} \times \left(300 \times 10^3\right) \times \left(3 \times 10^{-3}\right)$

$= 450 \text{ J}$

Q.10. At a location, vertical component of Earth's magnetic field is 1.6×10^{-5} T. If dip angle at that location is 37°, then find the magnitude of magnetic field at that point.

A) 2×10^{-5} T

B) 2.66×10^{-5} T

C) 4×10^{-6} T

D) 2.66×10^{-7} T

Answer: 2.66×10^{-5} T

Solution: As we know, vertical component of magnetic field is given by, $B_v = B \sin(\theta)$

$\Rightarrow B = \frac{1.6 \times 10^{-5}}{3} \times 5$

$= \frac{8}{3} \times 10^{-5}$

$= 2.66 \times 10^{-5}$ T

Q.11. Two radioactive substance have equal masses at $t = 0$. Decay constant of two substances are 25λ and 16λ. If ratio of remaining mass after a time $\left[\frac{1}{\alpha\lambda}\right]$ is e^{-1}, then find the value of α.

A) 9

B) 10

C) 3

D) 2

Answer: 9

Solution: As we know, $N = N_0 e^{\lambda t}$. Multiplying with molar mass both the side we get,

$M = M_0 e^{-\lambda t}$

So, $\frac{Ma}{Mb} = \frac{M_0 e^{-25t}}{M_0 e^{-16t}} = \frac{1}{e}$

$\Rightarrow e^{9\lambda t} = e$

$\Rightarrow t = \frac{1}{9\lambda}$

Therefore, $\alpha = 9$

Q.12. Two wires X & Y are placed at a distance of 5 cm apart and parallel to each other. If length of wire X is 50 cm and other wire Y has length 5 m, then find the force between them. (Both wires carry equal current of 1 A each)

A) 43.2×10^{-7} N

B) $2 \times 10^{-6}\ \mathrm{N}$

C) $12.4 \times 10^{-6}\ \mathrm{N}$

D) $5 \times 10^{-7}\ \mathrm{N}$

Answer: $2 \times 10^{-6}\ \mathrm{N}$

Solution: As, $5\ \mathrm{m}$ wire is relatively long compared to $0.5\ \mathrm{m}$ long wire, we can write,

$$F = \frac{\mu_0 I_1 I_2}{2\pi d} \times l_2$$

$$= \frac{\left(2\times10^{-7}\right)\times1\times1\times0.5}{0.05}$$

$$= 2 \times 10^{-6}\ \mathrm{N}$$

Q.13. Find the ratio of momentum of α particle and a proton if both are accelerating through same potential difference.

A) $2\sqrt{2} : 1$

B) $\sqrt{2} : 1$

C) $1 : 1$

D) $1 : 2\sqrt{2}$

Answer: $2\sqrt{2} : 1$

Solution: Momentum of a particle in terms of kinetic energy can be written as, $p = \sqrt{2mK} = \sqrt{2mqV}$

So,

$$\frac{p\alpha}{pp} = \frac{\sqrt{2m\alpha q\alpha V}}{\sqrt{2mpqpV}}$$

$$= \frac{\sqrt{4m\times2e}}{\sqrt{me}}$$

$$= 2\sqrt{2} : 1$$

Q.14. Two identical charged conducting spheres A & B, having equal charges are kept at some separation, their net force is F. Now an uncharged identical sphere C is first touched with A and then with B and kept at center of A and B, net force on C is,

A) $\frac{3F}{4}$

B) $\frac{3F}{8}$

C) F

D) $2F$

Answer: $\frac{3F}{4}$

Solution: Let initially charge on sphere is q_0, So $F=\frac{Kq_0^2}{r^2}$

When the uncharged sphere is first touched with A and then with B, then charge on A will become $\frac{q_0}{2}$ and that on B is $\frac{3q_0}{4}$ and charge on C will be $\frac{3q_0}{4}$.

Considering rightward direction to be positive, net force acting will be,

$$F'=\frac{\left(k\frac{q_0}{2}\times\frac{3q_0}{4}\right)}{\left(\frac{r}{2}\right)^2}-\frac{\left(k\frac{3q_0}{4}\times\frac{3q_0}{4}\right)}{\left(\frac{r}{2}\right)^2}$$

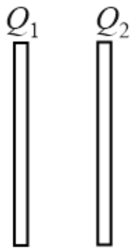
$$=+\frac{3kq_0^2}{2r^2}-\frac{9kq_0^2}{4r^2}$$

$$=-\frac{3kq_0^2}{4r^2}$$

$$\Rightarrow F'=\frac{-3}{4}F$$

Clearly, magnitude of the force $F'=\frac{3}{4}F$ and direction will be towards left.

Q.15. Two identical metal plates are given positive charge Q_1 and $Q_2(<Q_1)$ respectively. If they are now brought closer together to form a parallel plate capacitor with capacitance C. What will be the potential difference between the plates?



A) $\frac{Q_1-Q_2}{2C}$

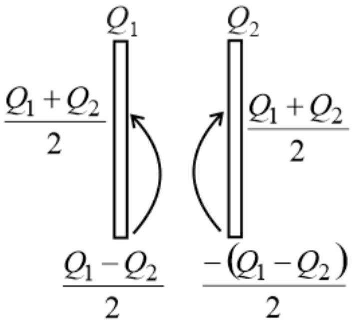
B) $\frac{2Q_1+2Q_2}{2C}$

C) $\frac{Q_1-Q_2}{C}$

D) $\frac{Q_1+Q_2}{C}$

Answer: $\frac{Q_1-Q_2}{2C}$

Solution: Due to induction charges that will appear on each surface will be as shown in figure.



Clearly, $\Delta V=\frac{\left(\frac{Q_1-Q_2}{2}\right)}{C}$

Q.16. A wire is cut into 2 parts of length x and y. Now x length part is stretched up to twice its length w. The part with its length equals to w has resistance twice that of wire part of length y. Then the relation between x and y is,

A) $y=x$

B) $x=2y$

C) $y=2x$

D) $x=4y$

Answer: $y=2x$

Solution: As we know resistance of a wire with uniform cross-section is given by, $R=\frac{\rho L}{A}$.

As the wire with length x is stretched twice, hence its cross-section area will be halved.

Therefore,

$\frac{\rho(2x)}{\frac{A}{2}}=2\times\frac{\rho(y)}{A}$

$\Rightarrow 4x=2y$

$\Rightarrow y=2x$

Section B: Chemistry

Q.17. Number of chlorine atoms in Bithionol is:

A) 2

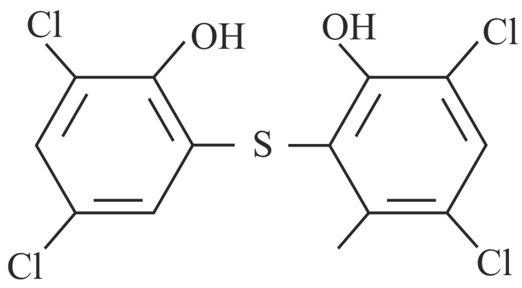
B) 4

C) 6

D) 0

Answer: 4

Solution: Bithionol (the compound is also called bithional) is added to soaps to impart antiseptic properties.



Bithionol

Q.18. Which of the following is not a natural polymer?

A) Protein

B) Starch

C) Rubber

D) Rayon

Answer: Rayon

Solution: Natural polymers
These polymers are found in plants and animals. Examples are proteins, cellulose, starch, some resins and rubber.

Semi synthetic polymers are those polymers that are prepared by chemically treating natural polymers. Rayon is a regenerated cellulosic fibre.

Q.19. Ethanol on reaction with $\text{conc. } H_2SO_4$ gives A, which on further reaction with Baeyer's reagent will give:

A) ethan$-1,\ 2-$diol

B) formaldehyde

C) formic acid

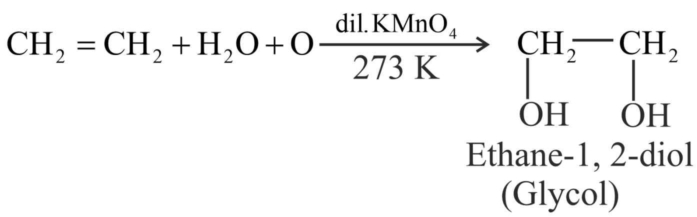
D) ethanoic acid

Answer: ethan$-1,\ 2-$diol

Solution: Ethanol undergoes dehydration by heating it with concentrated H_2SO_4 at $443\ K$.

$$C_2H_5OH \xrightarrow[443\ K]{H_2SO_4} CH_2=CH_2+H_2O$$

Alkenes on reaction with cold, dilute, aqueous solution of potassium permanganate (Baeyer's reagent) produce vicinal glycols. Decolorisatlon of $KMnO_4$ solution is used as a test for unsaturation.



Q.20. How many among the following are sp^3d^2 hybridized?

$BrF_5,\ [ICl_4]^-,\ ICl_3,\ ICl_5,\ SF_6,\ PCl_5$

A) 5

B) 4

C) 6

D) 2

Answer: 4

Solution: $S.N.=\frac{1}{2}$(Valence electrons. + Number of monovalent atoms $\pm$ Anion / cation)

$BrF_5=\frac{1}{2}(7+5)=6=sp^3d^2$

$[ICl_4]^-=\frac{1}{2}(7+4+1)=6=sp^3d^2$

$ICl_3=\frac{1}{2}(7+3)=5\ =sp^3d$

$ICl_5=\frac{1}{2}(7+5)=6\ =sp^3d^2$

$PCl_5=\frac{1}{2}(5+5)=5\ =sp^3d$

Q.21. Which of the following ions has lowest value of hydration enthalpy in magnitude?

A) Cr^{2+}

B) Mn^{2+}

C) Fe^{2+}

D) CO^{2+}

Answer: Mn^{2+}

Solution:

Element (M)	$\Delta_{hyd}H^{o}\left(M^{2+}\right)$
Cr	-1925
Mn	-1862
Fe	-1998
Co	-2079

Experimental Hydration Enthalpy $=$ theoretical Hydration Enthalpy $+$ CFSE

CFSE calculation for aqua complexes:

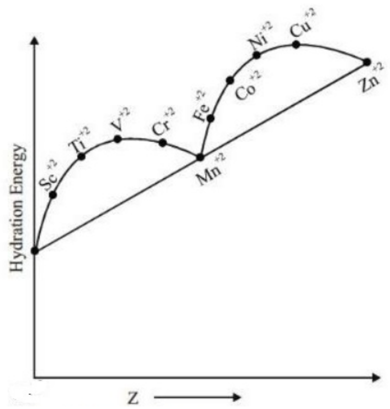
$Fe^{2+} = 3d^{6} = -0.4\,\Delta_0$

$Co^{2+} = 3d^{7} = -0.90\,\Delta_0$

$Cr^{2+} = 3d^{3} = -1.2\,\Delta_0$

$Mn^{2+} = 3d^{5} = 0$

Hence, Mn^{2+} has lowest hydration enthalpy.



Q.22. Which among the following oxoacids of sulphur has peroxide bond?

A) $H_2S_2O_8$

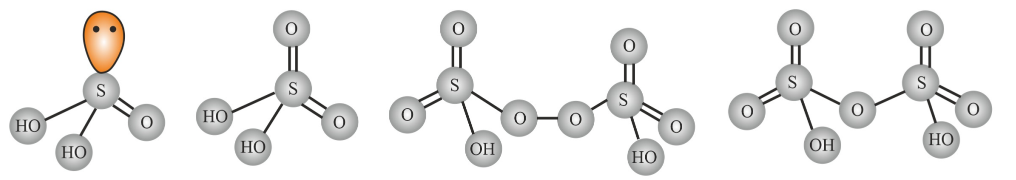
B) $H_2S_2O_7$

C) H_2SO_3

D) H_2SO_4

Answer: $H_2S_2O_8$

Solution:



Sulphurous acid (H_2SO_3) Sulphuric acid (H_2SO_4) Peroxodisulphuric acid $(H_2S_2O_8)$ Phyrosulphuric acid (Oleum) $(H_2S_2O_7)$

Q.23. Which of the following is harmful to plant leaves?

A) NO

B) NO_3^-

C) NO_2

D) NO_4^-

Answer: NO_2

Solution: At high concentration levels, nitrogen dioxide is potentially toxic to plants, can injure leaves and reduce growth and yield.

Q.24. If the order of critical temperature for He, O_2, N_2 and CH_4 is $He < N_2 < O_2 < CH_4$, then which of the given gases would be least adsorbed by charcoal?

A) He

B) O_2

C) N_2

D) CH_4

Answer: CH_4

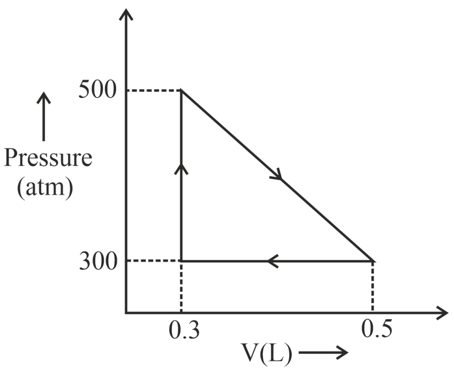
Solution: Critical Constants for Some Substances

Substance	T_c/K
He	5.3
N_2	126.0
O_2	154.3
CH_4	191

If a gas like O_2, He, CH_4, N_2 is taken in a closed vessel containing powdered charcoal, it is observed that the pressure of the gas in the enclosed vessel decreases. The gas molecules concentrate at the surface of the charcoal, i.e., gases are adsorbed at the surface. More the critical temperature, more is the adsorption.

Q.25. Find the work done in the above $P - V$ plot?

(Take $1 \text{ L atm} = 100 \text{ J}$)



A) 2 kJ

B) -2 kJ

C) 4 kJ

D) -4 kJ

Answer: -2 kJ

Solution: Work done = Area under the $P - V$ graph

$= -\frac{1}{2}(200)(0.2)$

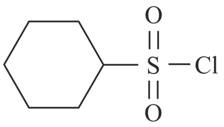
$= -20 \text{ L atm}$

$= -20 \times 100 \text{ J}$

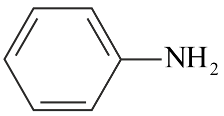
$= -2000 \text{ J} = -2 \text{ kJ}$

Q.26. Which of the following is structure of Hinsberg reagent?

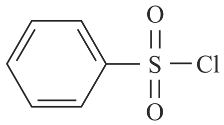
A)



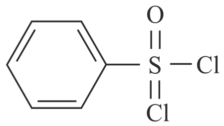
B)



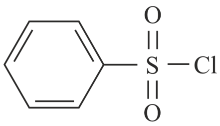
C)



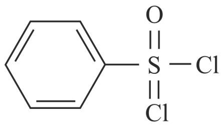
D)



Answer:



Solution: Benzenesulphonyl chloride $(C_6H_5SO_2Cl)$, which is also known as Hinsberg's reagent, reacts with primary and secondary amines to form sulphonamides.



Q.27. The correct order of energy for the following is:

(a)	$n = 4$	$l = 2$	$m = -1$	$s = +\frac{1}{2}$
(b)	$n = 4$	$l = 1$	$m = 1$	$s = -\frac{1}{2}$
(c)	$n = 3$	$l = 2$	$m = -1$	$s = +\frac{1}{2}$
(d)	$n = 3$	$l = 1$	$m = 1$	$s = +\frac{1}{2}$

A) a < b < c < d

B) d < c < b < a

C) a < b < d < c

D) b < d < c < a

Answer: d < c < b < a

Solution: $n + l = 6 \Rightarrow 4d$

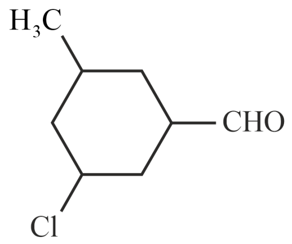
$n + l = 5 \Rightarrow 4p \text{ or } 3d$

$n + l = 4 = 3p$

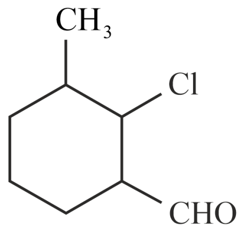
$3p < 3d < 4p < 4d$

Q.28. Which of the following is structure of 2-chloro-3-methylcyclohexane carbaldehyde?

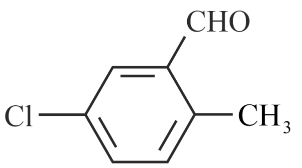
A)



B)



C)



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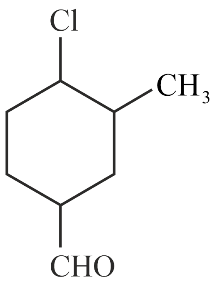
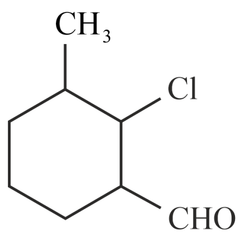
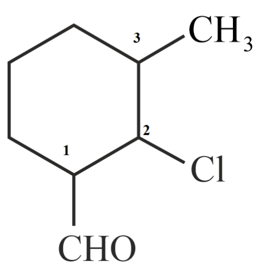


Answer:



Solution:



In the above molecule cyclohexane is the parent chain. The $-\mathrm{CHO}$, group is the major functional group and it cannot be included in the parent chain, hence, the name carbaldehyde is used.

Q.29. Which among the following is added to cement to make it sufficiently harder?

A) Gypsum

B) Alumina

C) Silicates

D) Calcium oxide

Answer: Gypsum

Solution: Gypsum $(CaSO_4 . 2H_2O)$ is added to cement to regulate its setting rate as cement becomes hard over a period of time when it is mixed with water.

Q.30. If $400\ mL$ of $0.2\ M\ H_2SO_4$ is mixed with $600\ mL$ of $0.1\ M\ HCl$, then find the pH of the resulting solution.

A) 0.14

B) 0.86

C) 1.4

D) 0.66

Answer: 0.66

Solution: $[H^+] = \frac{400 \times 0.4 + 600 \times 0.1}{1000}$

$= \frac{160+60}{1000} = \frac{220}{1000} = 0.22\ M$

$pH = -\log(0.22)$

$= 0.657$

$\simeq 0.66$

Q.31. Which of the following ions give blue colour in borax bead test and the formula of the compound formed is:

A) $Fe^{2+},\ Fe(BO_2)_2$

B) $Cu^{2+},\ Cu(BO_2)_2$

C) $Ni^{2+},\ Ni(BO_2)_2$

D) $Mn^{2+},\ Mn(BO_2)_2$

Answer: $Cu^{2+},\ Cu(BO_2)_2$

Solution: When hot glassy mass is brought in contact with a coloured salt and heated again in flame, B_2O_3 displaces the volatile oxides and combines with basic oxides to form metaborates. These metaborates have characteristic colours

$CuSO_4 + B_2O_3 \rightarrow CuO . B_2O_3 + SO_3$

$CuO . B_2O \rightarrow Cu(BO_2)_2\ (Blue)$

Q.32. x g of O_2 is mixed with $20\ g$ of Ne. The non-reactive mixture is having total pressure $25\ bar$. If partial pressure of Ne is $20\ bar$. Find value of x (in g).

A) $4\ g$

B) $8\ g$

C) $2\ g$

D) $5\ g$

Answer: $8\ g$

Solution: $P_{O_2} = 25 - 20 = 5\ \text{bar}$

$P_{O_2} = x_{O_2} \times P_{Total};$

$$5 = \frac{\frac{x}{32}}{\frac{x}{32} + \frac{20}{20}} \times 25$$

$$\frac{1}{5} = \frac{\frac{x}{32}}{\frac{x}{32} + 1}$$

On solving; $x = 8\ g$

Q.33. Consider an electrochemical cell,

$$Cu(s) \left| \begin{matrix} Cu^{+2}(aq) \\ (10^{-3} M) \end{matrix} \right|\left| \begin{matrix} Ag^{+}(aq) \\ (10^{-2} M) \end{matrix} \right| Ag(s)$$

If the standard electrode potential for the above cell is $0.43\ V$, then find out the EMF of the cell at the given concentration (in eV).

(Take $\frac{2.303\ RT}{F} = 0.06$)

A) 0.4

B) 0.2

C) 0.8

D) 0.6

Answer: 0.4

Solution: $Cu \rightarrow Cu^{2+} + 2e^-$

$\underline{2\,Ag^+ + 2e^- \rightarrow 2\,Ag}$

$Cu + 2\,Ag^+ \rightarrow Cu^{2+} + 2\,Ag$

$$E_{cell} = 0.43 - \frac{0.06}{2} \log \frac{[10^{-3}]}{[Ag^+]^2}$$

$$E_{cell} = 0.43 - \frac{0.06}{2} \log_{10} 10$$

$= 0.43 - 0.03$

$= 0.40$

Section C: Mathematics

Q.34. The value of $\sum_{r=1}^{20}\left(r^2+1\right)\cdot r!$ is

A) $22!-2\cdot(20)!$

B) $(22)!-2\,(21)!$

C) $(22)!$

D) $2\,(21)!$

Answer: $(22)!-2\,(21)!$

Solution: $\sum_{r=1}^{20}\left(r^2+1\right)r!=\sum_{r=1}^{20}[(r+1)\,(r+2)r!-3\,(r+1)r!+2r!]$

$=\sum_{r=1}^{20}((r+2)!-3\,(r+1)!+2r!)$

$=\sum_{r=1}^{20}((r+2)!-(r+1)!)-2\sum_{r=1}^{20}((r+1)!-r!)$

$=(22!-2!)-2\,(21!-1!)=22!-2\times 21!-2+2$

$=(22)!-2\,(21)!$

Q.35. If $\left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|=14$ and $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{b}\times\vec{c}\right)+\left(\vec{b}\times\vec{c}\right)\cdot\left(\vec{c}\times\vec{a}\right)+\left(\vec{c}\times\vec{a}\right)\cdot\left(\vec{a}\times\vec{b}\right)=168$ and $\vec{a},\vec{b},\vec{c}$ are coplanar, concurrent and make equal angles with each other, then $\left|\vec{a}\right|+\left|\vec{b}\right|+\left|\vec{c}\right|$ is equal to:

A) 14

B) 16

C) 10

D) 12

Answer: 16

Solution: Given that $\vec{a},\vec{b},\vec{c}$ are coplanar and make equal angle $\theta=60^\circ$ with each other.

We know $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{b}\times\vec{c}\right)=\left|\vec{a}\times\vec{b}\right|\left|\vec{b}\times\vec{c}\right|\cos 0^\circ$ (as $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{c}$ are parallel)

i.e. $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{b}\times\vec{c}\right)=\left|\vec{a}\right|\left|\vec{b}\right|\sin 60^\circ\left|\vec{b}\right|\left|\vec{c}\right|\sin 60^\circ=14\times\frac{3}{4}\left|\vec{b}\right|=\frac{21}{2}\left|\vec{b}\right|$

Similarly $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{b}\times\vec{c}\right)+\left(\vec{b}\times\vec{c}\right)\cdot\left(\vec{c}\times\vec{a}\right)+\left(\vec{c}\times\vec{a}\right)\cdot\left(\vec{a}\times\vec{b}\right)$

$=\frac{21}{2}\left(\left|\vec{a}\right|+\left|\vec{b}\right|+\left|\vec{c}\right|\right)$

i.e. $\frac{21}{2}\left(\left|\vec{a}\right|+\left|\vec{b}\right|+\left|\vec{c}\right|\right)=168$

$\Rightarrow\ \left|\vec{a}\right|+\left|\vec{b}\right|+\left|\vec{c}\right|=16$

Q.36. The number of solution of the equation $2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}$ is/are

A) 1

B) 0

C) 3

D) Infinite

Answer: 1

Solution: We know $-2 \leq 2\cos\left(\frac{x^2+x}{6}\right) \leq 2$ and

$4^x + 4^{-x} \geq 2$

For equality to hold $4^x + 4^{-x} = 2$ and $2\cos\left(\frac{x^2+x}{6}\right) = 2$

Now $4^x + 4^{-x} = 2$ when $x = 0$

i.e. $\cos\left(\frac{x^2+x}{6}\right) = 1$ for $x = 0$

So there exist only one solution $x = 0$.

Q.37. A perpendicular drawn from $(1, 2, 3)$ to the plane $x + 2y + z = 14$ and intersect plane at Q. R be a point on plane such that PR makes an angle 60° with the plane, then area of ΔPQR is

A) $\sqrt{3}$ sq.units

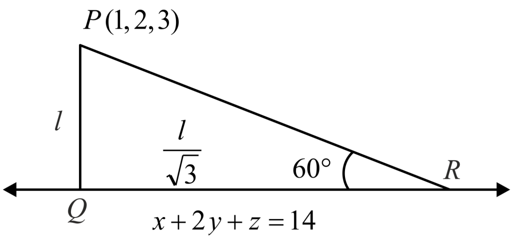
B) 3 sq.units

C) $\frac{\sqrt{3}}{2}$ sq.units

D) 4 sq.units

Answer: $\sqrt{3}$ sq.units

Solution: On plotting the given values in diagram we get,



In ΔPQR, $cot60^\circ = \frac{QR}{PQ}$

$\Rightarrow (QR) = PQ \cdot \cot 60^\circ = \frac{l}{\sqrt{3}}$

Now using distance of a point from a plane formula we get, $l = \left|\frac{1+4+3-14}{\sqrt{1+4+1}}\right| = \sqrt{6}$

Area of $\Delta PQR = \frac{1}{2} l \cdot \frac{l}{\sqrt{3}} = \frac{6}{2\sqrt{3}} = \sqrt{3}$

Q.38. Let $\vec{a}, \vec{b}$ are two vectors and $\vec{a} \cdot \vec{b} = 3$, $\left|\vec{a} \times \vec{b}\right|^2 = 75$, and $\left|\vec{a} + \vec{b}\right|^2 = \left|\vec{a}\right|^2 + 2\left|\vec{b}\right|^2$, then $\left|\vec{a}\right|^2$ is equal to ______ .

Answer: 14

Solution: Given, $\vec{a}\cdot\vec{b}=3, \left|\vec{a}\times\vec{b}\right|^2=75$, and $\left|\vec{a}+\vec{b}\right|^2=\left|\vec{a}\right|^2+2\left|\vec{b}\right|^2$, then $\left|\vec{a}\right|^2$ is equal to ______ .

Now we know that $\left|\vec{a}+\vec{b}\right|^2=|a|^2+2\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2=\left|\vec{a}\right|^2+2\left|\vec{b}\right|^2 \text{ (given)}$

$\Rightarrow \left|\vec{b}\right|^2=2\left(\vec{a}\cdot\vec{b}\right)=2\times3=6$

Also, $\left|\vec{a}\times\vec{b}\right|^2+\left|\vec{a}\cdot\vec{b}\right|^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2\sin^2\theta+\left|\vec{a}\right|^2\left|\vec{b}\right|^2\cos^2\theta$

$\Rightarrow \left|\vec{a}\times\vec{b}\right|^2+\left|\vec{a}\cdot\vec{b}\right|^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2$

$\Rightarrow \quad 75+9=6\left|\vec{a}\right|^2$

$\Rightarrow \left|\vec{a}\right|^2=\frac{84}{6}=14$

Q.39. If sum and product of mean and variance in a binomial distribution are 82.5 and 1350 respectively, then n is equal to ______ . (where n is number of trial in binomial distribution).

Answer: 96

Solution: Given, sum of mean and variance is 82.5

Product of roots is 1350

Now, Mean and variance are the roots of

$x^2-(\text{sum of roots})x+\text{product of roots}=0$

$\Rightarrow x^2-82.5x+1350=0$

$\Rightarrow x=60 \text{ or } 22.5$

So, mean $=np=60$ and variance $=npq=22.5$

$\Rightarrow \quad q=\frac{22.5}{60}=\frac{3}{8}$

So, $p=\frac{5}{8}$ and $n=\frac{60}{\frac{5}{8}}=96$

Q.40. The number of numbers lying between 1024 and 23146 which are divisible by 55 and made from $2,3,4,5,6$ without repetition, is ______ .

Answer: 6

Solution: For divisibility by 55, the number should be divisible by 5 & 11.

i.e. the unit digit of the required number should be 5

The numbers from 1024 to 23146 will be either 4 or 5 digits.

CASE I:

When number has 4 digits

Let the number be $abcd$

Here d is fixed as 5. So the other 3 digits can be

$(6, 4, 3), (3, 4, 6), (2, 3, 6), (6, 3, 2), (3, 2, 4)$ or $(4, 2, 3)$ only

So the number of possible numbers$= 6$

CASE II:

When number has 5 digits.

Let the number be $abcde$.

Here e is fixed at 5. So the other 4 digits must be from $2, 3, 4, 6$

No such number is possible because even the least number formed is greater than 23146.

Hene, the total number of required numbers $= 6$

Q.41. If $f(x) = \begin{cases} \frac{\log_e(1+5x)-\log_e(1+ax)}{x}, & x \neq 0 \\ 10, & x = 0 \end{cases}$ is continuous at $x = 0$, then

A) -5

B) 5

C) 10

D) -10

Answer: -5

Solution: For $f(x)$ to be continuous

$\lim_{x \to 0} f(x) = f(0)$

$\Rightarrow \lim_{x \to 0} \frac{\ln(1+5x)-\ln(1+ax)}{x} = 10$

$\Rightarrow \lim_{x \to 0} \frac{5\ln(1+5x)}{5x} - \frac{a\ln(1+ax)}{a} = 10$

$\Rightarrow \quad 5 - a = 10$

$\Rightarrow \ a = -5$

Q.42. There are two urns U_1 and U_2 containing 3 red 4 black and 3 white balls in first urn and 3 red 5 black and 4 white balls in second one. One ball is drawn from U_1 and put into U_2. At random another ball is drawn from U_2 and found to be black. The probability that the ball drawn from U_1 was red is

A) $\frac{3}{10}$

B) $\frac{1}{6}$

C) $\frac{5}{18}$

D) $\frac{1}{3}$

Answer: $\frac{5}{18}$

Solution: Let, $E \rightarrow$ ball drawn fromU_2 is black after putting ball from U_1

Let, $E_R \rightarrow$ Red ball transferred from U_1 to U_2

Let, $E_W \rightarrow$ White ball transferred from U_1 to U_2

Let, $E_B \rightarrow$Black ball transferred from U_1 to U_2

Now to Find $P\left(E_R|E\right)$

We will use baye's theorem,

$$P\left(E_R|E\right)=\frac{P(E|E_R)\cdot P(E_R)}{P(E|E_R)P(E_R)+P(E|E_W)P(E_W)+P(E|E_B)P(E_B)}$$

Here,

$$P\left(E_R\right)=\frac{3}{10}, P\left(E_W\right)=\frac{3}{10}, P\left(E_B\right)=\frac{4}{10}$$

$$P\left(E|E_R\right)=\frac{5}{13}, P\left(E|E_W\right)=\frac{5}{13}, P\left(E|E_B\right)=\frac{6}{13}$$

Putting all the values in the formula we get,

$$\Rightarrow P\left(E_R \mid E\right)=\frac{\frac{5}{13}\times\frac{3}{10}}{\frac{15}{130}+\frac{24}{130}+\frac{15}{130}}=\frac{15}{54}=\frac{5}{18}$$

Q.43. Let equation of circle is $(x-2)^2+(y+1)^2=\frac{169}{4}$ and a chord AB of length $12\,\mathrm{cm}$, and two tangent at A and B meets at P. Then distance of chord from the point P is:

A) $\frac{72}{5}$

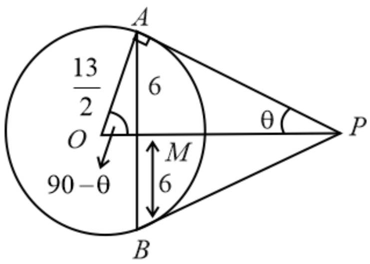
B) $\frac{36}{5}$

C) $\frac{72}{25}$

D) $\frac{144}{5}$

Answer: $\frac{72}{5}$

Solution: Plotting the diagram of given value we have,



Now given, radius $=\frac{\sqrt{169}}{\sqrt{4}}=\frac{13}{2}$

Now by diagram M is the midpoint of AB so, $AM=6$

Now in $\Delta AMP,\ PM=6\cot\theta$

And in ΔOAM

$$\sin\left(90^\circ-\theta\right)=\frac{6}{\frac{13}{2}}$$

$$\cos\theta=\frac{12}{13}$$

$$\therefore\ PM=6\times\frac{12}{5}=\frac{72}{5}$$