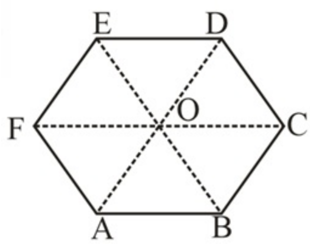
NEET Important Questions with Solutions from Kinematics

Q.1. $ABCDEF$ is a regular hexagon. What is the value of $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$?



A) 0

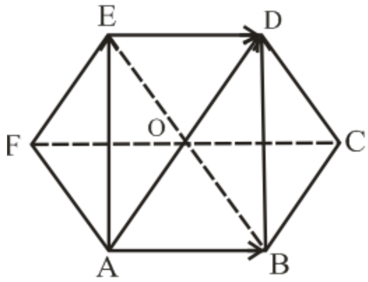
B) $2\overrightarrow{AO}$

C) $4\overrightarrow{AO}$

D) $6\overrightarrow{AO}$

Answer: $6\overrightarrow{AO}$

Solution:



The sum of the vectors, listed in the question, have to be expressed in terms of scalar multiple of $\overrightarrow{AO}$, as given in the choices of the answers.

Given equation is written as below,

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$

From the figure, we get,

$\overrightarrow{AO}+\overrightarrow{OD}=\overrightarrow{AD}=2\overrightarrow{AO}\ \ldots\left(1\right);\ \left(\because \overrightarrow{AO}=\overrightarrow{OD}\right)$

From the figure, we can apply triangle law of vector addition for ΔACD , we get

$\overrightarrow{AD}=\overrightarrow{AC}+\overrightarrow{CD}$

Also, $\overrightarrow{CD}=\overrightarrow{AF}$

$\Rightarrow \overrightarrow{AC}+\overrightarrow{AF}=\overrightarrow{AD}=2\overrightarrow{AO}\ \ldots\left(2\right)$

From the figure, we can apply triangle law of vector addition for the ΔAED, we get

$\overrightarrow{AE}+\overrightarrow{ED}=\overrightarrow{AD}$

Also, $\overrightarrow{ED}=\overrightarrow{AB}$

$\Rightarrow \overrightarrow{AE}+\overrightarrow{AB}=\overrightarrow{AD}=2\overrightarrow{AO}\ \ldots\left(3\right)$

Now adding equations (1), (2) and (3) we get,

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=6\overrightarrow{AO}$

Q.2. What is the ratio of potential energy with respect to ground, and kinetic energy at the topmost point of the projectile motion?

A) $\cos^2\theta$

B) $\sin^2\theta$

C) $\tan^2\theta$

D) $\cot^2\theta$

Answer: $\tan^2\theta$

Solution: Let the mass of the body of projectile, the initial speed and the angle of projection be $m,\ u$ and θ, respectively.

Potential energy (U) of the body at the topmost point will be mgH, where H is the maximum height of the projected body. $H=\frac{u^2\sin^2(\theta)}{2g}$.

$\Rightarrow U=mgH=mg\left(\frac{u^2\sin^2(\theta)}{2g}\right)$

$U=\frac{1}{2}mu^2\sin^2(\theta)\ \ldots(1)$

Now, we know that at the topmost point, the speed of projected body is,

$v=u\cos(\theta)$

So, kinetic energy is, $K=\frac{1}{2}mv^2=\frac{1}{2}m(u\cos(\theta))^2$

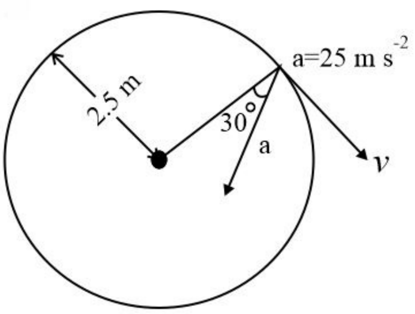
$K=\frac{1}{2}mu^2\cos^2(\theta)\ \ldots(2)$

Taking the ratio of (1) and (2), we have,

$\frac{U}{K}=\frac{\frac{1}{2}mu^2\sin^2\theta}{\frac{1}{2}mu^2\cos^2\theta}$

$U:K\ =\tan^2(\theta)$.

Q.3. The figure shows the total acceleration and velocity of a particle moving clockwise in a circle of radius $2.5\ \mathrm{m}$ at a given instant of time. At this instant, find (i) its radial acceleration, (ii) the speed of the particle and (iii) its tangential acceleration.



A) (i) $25\frac{\sqrt{3}}{2}\ \mathrm{m\,s^{-2}}$

(ii) $\left(125\frac{\sqrt{3}}{4}\right)^{\frac{1}{2}}\ \mathrm{m\,s^{-1}}$,

(iii) $12.5\ \mathrm{m\,s^{-2}}$

B) (i) $25\frac{\sqrt{3}}{4}\ \mathrm{m\,s^{-2}}$,

(ii) $\left(25\frac{\sqrt{3}}{4}\right)^{\frac{1}{2}}\ \mathrm{m\,s^{-1}}$,

(iii) $12.5\ \mathrm{m\,s^{-2}}$

C) (i) $25\frac{\sqrt{3}}{2}\ \mathrm{m\,s^{-2}}$,

(ii) $\left(125\frac{\sqrt{3}}{4}\right)\ \mathrm{m\,s^{-1}}$,

(iii) $6.25\ \mathrm{m\,s^{-2}}$.

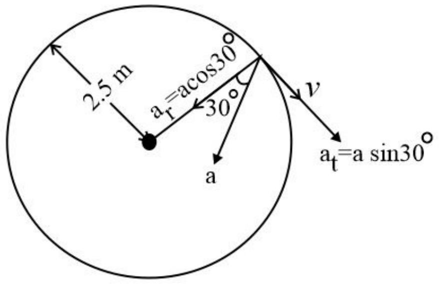
D) None of these.

Answer: (i) $25\frac{\sqrt{3}}{2}\ \mathrm{m\,s^{-2}}$

(ii) $\left(125\frac{\sqrt{3}}{4}\right)^{\frac{1}{2}}\ \mathrm{m\,s^{-1}}$,

(iii) $12.5\ \mathrm{m\,s^{-2}}$

Solution:



(i) Radial acceleration is,

$a_r = a\cos 30^\circ$

$= 25 \times \frac{\sqrt{3}}{2}\ \mathrm{m\,s^{-2}}$.

(ii) Speed of the particle is,

$a_r = \frac{v^2}{r}$

$\Rightarrow v = \sqrt{a_r \times r}$

$= \sqrt{\frac{25\sqrt{3}}{2} \times 2.5} = \sqrt{\frac{125\sqrt{3}}{4}}$

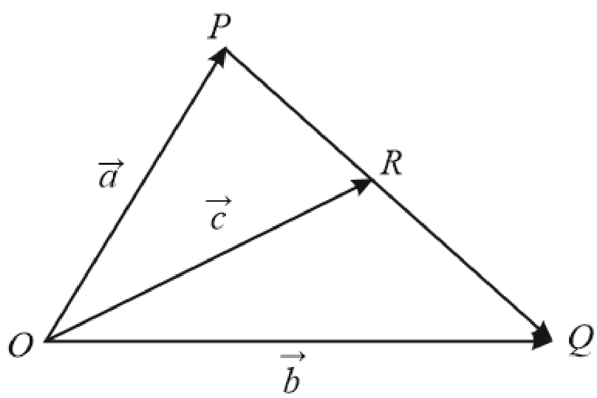
$\Rightarrow v = \left(125\frac{\sqrt{3}}{4}\right)^{\frac{1}{2}}\ \mathrm{m\,s^{-1}}$.

(iii) Tangential acceleration is,

$a_t = a\sin 30^\circ$

$= 25 \times \frac{1}{2} = 12.5\ \mathrm{m\,s^{-2}}$.

Q.4. The figure shows three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. Here, R is the midpoint of PQ. Then which of the following relations is correct?



A) $\vec{a} + \vec{b} = \vec{c}$

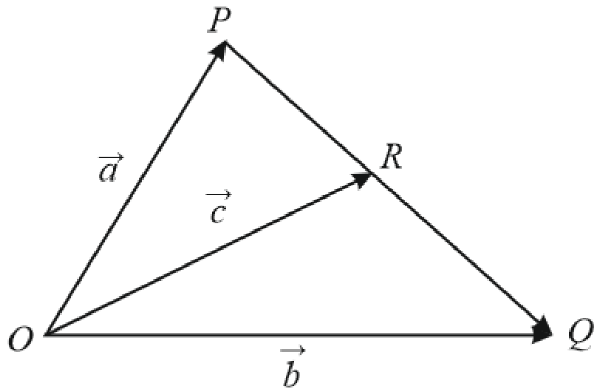
B) $\vec{a} + \vec{b} = 2\vec{c}$

C) $\vec{a} - \vec{b} = \vec{c}$

D) $\vec{a} - \vec{b} = 2\vec{c}$

Answer: $\vec{a} + \vec{b} = 2\vec{c}$

Solution:



In the above diagram, consider applying the triangle law of addition on ΔOPQ. Hence,

$$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ} \Rightarrow \vec{a} + \overrightarrow{PQ} = \vec{b}$$

$$\Rightarrow \overrightarrow{PQ} = \vec{b} - \vec{a} \quad \ldots\text{(i)}$$

Since it is given that R is the midpoint of the segment PQ,

$$\overrightarrow{RQ} = \frac{1}{2}\overrightarrow{PQ}$$

Now, applying triangle law on ΔORQ, we get,

$$\overrightarrow{OR} + \overrightarrow{RQ} = \overrightarrow{OQ} \Rightarrow \vec{c} + \frac{1}{2}\overrightarrow{PQ} = \vec{b}$$

$$\Rightarrow \overrightarrow{PQ} = 2\left(\vec{b} - \vec{c}\right) = 2\vec{b} - 2\vec{c} \quad \ldots\text{(ii)}$$

Substituting the equations (ii) in (i), we get,

$$\vec{b} - \vec{a} = 2\vec{b} - 2\vec{c} \Rightarrow \vec{a} + \vec{b} = 2\vec{c}$$

Q.5. Given, $\vec{a} + \vec{b} + \vec{c} = 0$. Out of the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ two are equal in magnitude. The magnitude of the third vector is $\sqrt{2}$ times that of either of the two have equal magnitude. The angles between the vectors are

A) 90°, 135° and 135°

B) 30°, 60° and 90°

C) 45°, 45° and 90°

D) 45°, 60° and 90°

Answer: 90°, 135° and 135°

Solution: The two vectors are equal in magnitude and the magnitude of the third vector is $\sqrt{2}$ times the other two. Since no other information is given here.

Let $\left|\vec{a}\right| = \left|\vec{b}\right| = a$ and $\left|\vec{c}\right| = \sqrt{2}a$.

$\vec{a} + \vec{b} + \vec{c} = 0$

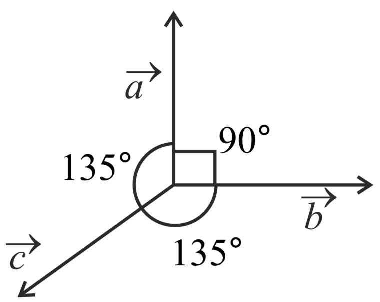
$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$

Resultant of $\vec{a}$ and $\vec{b}$ is equal and opposite to the third vector $\vec{c}$.

It means that $\Rightarrow \left|\vec{a} + \vec{b}\right| = \sqrt{2}a$ and makes an angle $45°$.

This is possible only when angle between $\vec{a}$ and $\vec{b}$ is 90°.

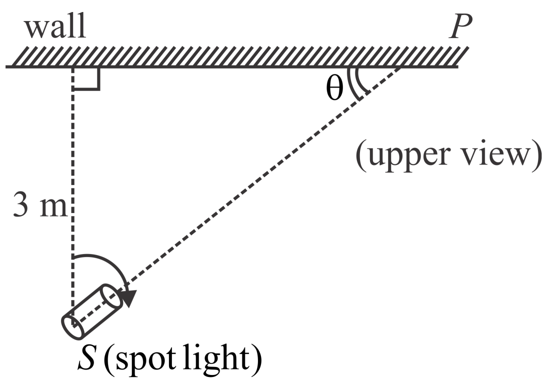
Hence, the situation will be as shown in the figure.



Hence, angle between $\vec{a}$ and $\vec{b}$ is 90°, angle between $\vec{b}$ and $\vec{c}$ is 135° and the angle between $\vec{a}$ and $\vec{c}$ will be 135°.

Remember that here the magnitude of $\vec{a}$ and $\vec{b}$ have been chosen to be equal, but we are free to do so with any of the two vectors here and correspondingly the magnitude of the third vector will be chosen.

Q.6. A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad s^{-1}. The spot of light P moves along the wall at a distance 3 m. What is the velocity of the spot P when $\theta = 45^\circ$?



A) 0.6 m s^{-1}

B) 0.5 m s^{-1}

C) 0.4 m s^{-1}

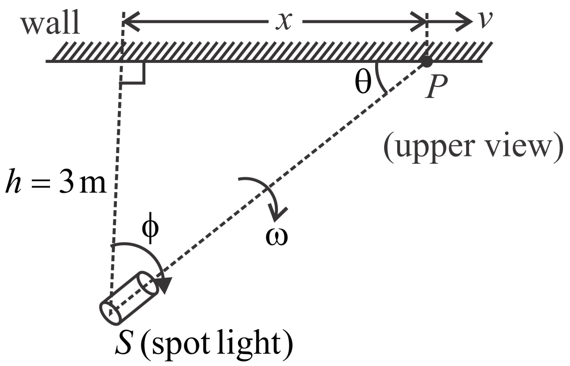
D) 0.3 m s^{-1}

Answer: $0.6 \mathrm{~m} \mathrm{~s}^{-1}$

Solution: Given here,
Angular velocity of spotlight, $\omega=0.1 \mathrm{~rad} \mathrm{~s}^{-1}$ and angle, $\theta=45^{\circ}$.

Let x be the distance between wall and P.

Let ϕ be the angle made by the spotlight with wall.



From above figure, we have

$\tan \phi=\frac{x}{h} \Rightarrow x=h \tan \phi$.

We know that velocity is given by $v=\frac{\mathrm{d} x}{\mathrm{~d} t}$, here t is time.

And angular velocity is given by $\omega=\frac{\mathrm{d} \phi}{\mathrm{d} t}$.

Then, $v=\frac{\mathrm{d} x}{\mathrm{~d} t}=h \times \sec ^{2} \phi \times \frac{\mathrm{d} \phi}{\mathrm{d} t}$

$\Rightarrow v=h \sec ^{2} \phi \times \omega$.

Now, when $\phi=\theta=45^{\circ}$ then, $v=3 \times \sec ^{2}\left(45^{\circ}\right) \times 0.1=0.3 \times 2$

$v=0.6 \mathrm{~m} \mathrm{~s}^{-1}$.

Hence, velocity of spot P is $0.6 \mathrm{~m} \mathrm{~s}^{-1}$.

Q.7. A man wants to swim across a river of width $200 \mathrm{~m}$ along the shortest path. If the speed of river stream is $3 \mathrm{~km} \mathrm{~h}^{-1}$ and speed of swimmer in still water is $5 \mathrm{~km} \mathrm{~h}^{-1}$, then the time of crossing the river is

A) $10 \mathrm{~min}$

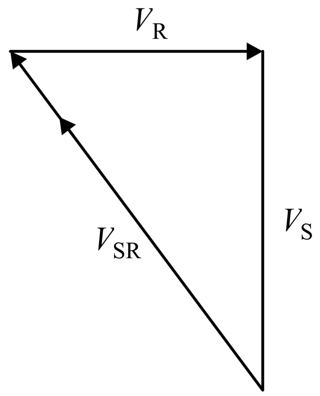
B) $15 \mathrm{~min}$

C) $3 \mathrm{~min}$

D) $6 \mathrm{~min}$

Answer: $3 \mathrm{~min}$

Solution: Speed of swimmer in still water = Speed of swimmer with respect to river.



$v_{\perp}=\sqrt{5^2-3^2}=4\ \mathrm{km\ h^{-1}}$

$\therefore \quad t=\frac{0.2}{4}\times 60\ \mathrm{min}=3\ \mathrm{min}$

Q.8. A constant torque of $1000\ \mathrm{N\ m}$ turns a wheel of moment of inertia $200\ \mathrm{kg\ m^2}$ about an axis through its centre. Its angular velocity after 3 s is (in $\mathrm{rad\ s^{-1}}$).

A) 1

B) 5

C) 15

D) 10

Answer: 15

Solution: Given,

Torque, $\tau=1000\ \mathrm{N\ m}$ and

Moment of inertia, $I=200\ \mathrm{kg\ m^2}$.

From first equation of kinematics, $\omega=\omega_0+\alpha t$

here, initial angular speed is,

$\omega_0=0$

$\Rightarrow \omega=\alpha t \ldots\text{(i)}$

Now the relation between torque, moment of inertia and angular acceleration is,

$\tau=I\alpha$

$1000=(200)\alpha$

$\alpha=5\ \mathrm{rad\ s^{-2}}$.

Substituting in equation (i),

$\omega=(5)(3)\quad (Given\ t=3\ \mathrm{sec})$

So, $\omega=15\ \mathrm{rad\ s^{-1}}$.

Q.9. A jet aeroplane travelling from east to west at a speed of $500\ \mathrm{km\ h^{-1}}$ ejects out gases of combustion at a speed of $1500\ \mathrm{km\ h^{-1}}$ with respect to the jet plane. What is the velocity of the gases with respect to an observer on the ground?

A) $1000\ \mathrm{km\ h^{-1}}$ in the direction east to west.

B) $1000\ \mathrm{km\ h^{-1}}$ in the direction west to east.

C) $2000\ \mathrm{km\ h^{-1}}$ in the direction west to east.

D) $2000\ \mathrm{km\ h^{-1}}$ in the direction east to west.

Answer: $1000\ \mathrm{km\ h^{-1}}$ in the direction west to east.

Solution: Take the unit vector $\hat{\imath}$ along the east direction.

Velocity of the airplane, $\overrightarrow{V_A} = -500\hat{\imath}$,

velocity of the gases with respect to airplane,

$\vec{V}_{GA} = 1500\ \hat{\imath}\ \mathrm{km\ h^{-1}}$

$\vec{V}_G = \vec{V}_{GA} + \vec{V}_A$

$\vec{V}_G = 1500\ \hat{\imath} + \left(-500\ \hat{\imath}\right)$

$\vec{V}_G = 1000\ \hat{\imath}\ \mathrm{km\ h^{-1}}$

Hence, the ejected gas from the reference frame of a person on ground, the velocity is $1000\ \mathrm{km\ h^{-1}}$, directed in the direction of the original motion of gas (opposite to that of the plane), i.e., west to east.

Q.10. When a ceiling fan is switched off, its angular velocity reduces to 50% while it makes 36 rotations. How many more rotations will it make before coming to rest? (Assume uniform angular retardation)

A) 18

B) 12

C) 36

D) 48

Answer: 12

Solution: Let the initial angular speed be ω_0 & angular retardation be α.

According to question angular speed after 36 revolution is

$\omega = 50\%$ of ω_0

$\Rightarrow \omega = \frac{\omega_0}{2}$.

Using $\omega^2 = \omega_0^2 - 2\alpha\theta$

$\Rightarrow \frac{\omega_0^2}{4} = \omega_0^2 - 72\alpha$

$\Rightarrow \alpha = \frac{3\omega_0^2}{4\times 72}$

$\Rightarrow \alpha = \frac{\omega_0^2}{96} \quad \ldots.(2)$.

Now, final angular velocity when fan stops is $\omega = 0$.

$\omega^2 = \omega_0^2 - 2\alpha\theta$

$\Rightarrow 0 = \frac{\omega_0^2}{4} - 2 \times \frac{\omega_0^2}{96} \times \theta$

$\Rightarrow \theta = 12\ \mathrm{rev.}$

Q.11. The equation of motion of a body is, $\frac{\mathrm{d}v(t)}{\mathrm{d}t} = 9 - 3v\left(t\right)$ where, $v(t)$ is the speed (in $\mathrm{m\ s^{-1}}$) at time t (in second). If the body was at rest at $t = 0$, then which of the following is correct?

A) The terminal speed is $3\ \mathrm{m\ s^{-1}}$.

B) Initial acceleration is $9 \mathrm{~m} \mathrm{~s}^{-2}$.

C) $v(t)=3\left(1-e^{-3 \mathrm{t}}\right)$

D) All of these are correct.

Answer: All of these are correct.

Solution: Given,

$\frac{\mathrm{d} v(t)}{\mathrm{d} t}=9-3 v(t)$.

We know acceleration is the rate of change of velocity,

$a=\frac{\mathrm{d} v}{\mathrm{~d} t}$
$a=9-3 v$

At terminal velocity, net force or acceleration is zero.

$a=0$
$9-3 v=0$
$9=3 v$
$v=3 \mathrm{~m} \mathrm{~s}^{-1}$

At $t=0$, $v=0$. So, initial acceleration is,

$\Rightarrow a=9-3 v$
$\Rightarrow 9-0$
$\Rightarrow 9 \mathrm{~m} \mathrm{~s}^{-2}$

Let us find velocity as a function of time. Now,

$a=9-3 v$
$\frac{\mathrm{d} v}{\mathrm{~d} t}=9-3 v$
$\frac{\mathrm{d} v}{9-3 v}=\mathrm{d} t$
$\int_0^v \frac{\mathrm{d} v}{9-3 v}=\int_0^{\mathrm{t}} \mathrm{d} t$
$-\frac{1}{3} \ln \frac{(9-3 v)}{9}=t$
$\frac{(9-3 v)}{9}=e^{-3 \mathrm{t}}$
$1-\frac{v}{3}=e^{-3 \mathrm{t}}$

$v(t)=3\left(1-e^{-3 \mathrm{t}}\right)$

Q.12. For a particle moving with speed v in a uniform circular motion, the acceleration at a point $P(R, \theta)$ in the first quadrant on the circle of radius R is (here, θ is measured from the x-axis)

A) $-\frac{v^2}{R} \cos \theta \hat{\mathrm{i}}+\frac{v^2}{R} \sin \theta \hat{\mathrm{j}}$.

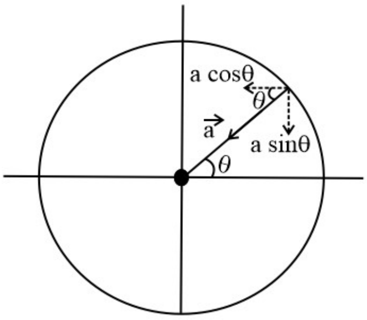
B) $-\frac{v^2}{R} \sin \theta \hat{\mathrm{i}}+\frac{v^2}{R} \cos \theta \hat{\mathrm{j}}$.

C) $-\frac{v^2}{R} \cos \theta \hat{\mathrm{i}}-\frac{v^2}{R} \sin \theta \hat{\mathrm{j}}$.

D) $\frac{v^2}{R} \hat{\mathrm{i}}+\frac{v^2}{R} \hat{\mathrm{j}}$.

Answer: $-\frac{v^2}{R} \cos \theta \hat{\mathrm{i}}-\frac{v^2}{R} \sin \theta \hat{\mathrm{j}}$.

Solution:



For a particle in a uniform circular motion, the radial acceleration towards the centre is,

$\left|\overrightarrow{\mathrm{a}}\right| = \frac{v^2}{R}.$

Resolve the net acceleration in components,

$\therefore\ \overrightarrow{\mathrm{a}} = \frac{v^2}{R}\left[\cos\theta\left(-\hat{\mathrm{i}}\right) + \sin\theta\left(-\hat{\mathrm{j}}\right)\right]$

or

$\overrightarrow{a} = -\frac{v^2}{R}\cos\theta\hat{\mathrm{i}} - \frac{v^2}{R}\sin\theta\hat{\mathrm{j}}.$

Q.13. A particle moves in an $x-y$ plane such that $x=t^2+2t$ and $y=2t$, where, t is time and $x,\ y$ are x and y coordinates of the particle, respectively. The trajectory of the path is

A) a circle

B) a straight line

C) a parabola.

D) None of these

Answer: a parabola.

Solution: To determine the trajectory, we need to find the relationship between y and x by eliminating t.

From the second equation, we get,

$t=\frac{y}{2}$

Substituting in the given equation,

$x=\left(\frac{y}{2}\right)^2+2\left(\frac{y}{2}\right)=\frac{y^2}{4}+y$

This is a quadratic equation in y and x, hence, it represents a parabola.

Q.14. A particle is moving in a circular path and its acceleration vector is making an angle of 30° with the velocity vector, then the ratio of centripetal acceleration to its tangential acceleration is

A) $\frac{1}{2}$

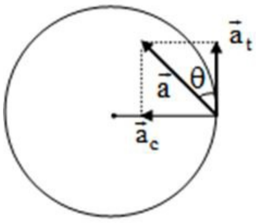
B) $\frac{\sqrt{3}}{2}$

C) $\frac{1}{\sqrt{3}}$

D) $\sqrt{3}$

Answer: $\frac{1}{\sqrt{3}}$

Solution:

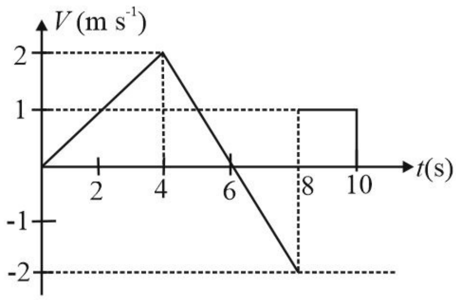


For a body undergoing circular motion, let net acceleration makes an angle θ with the tangent, then

$\tan\theta = \frac{a_c}{a_t}$

$\therefore \ \frac{a_c}{a_t} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

Q.15. The velocity-time graph of a body moving in a straight line is shown. The displacement of the body in 10 s is



A) $4\ \text{m}$

B) $6\ \text{m}$

C) $8\ \text{m}$

D) $10\ \text{m}$

Answer: $6\ \text{m}$

Solution: We know that the change in displacement is equal to the area under the velocity-time curve.

As initial displacement at time $t = 0$, the total displacement is equal to change in displacement.

Therefore, the total displacement is the vector sum of the area under velocity-time curve.

It is given as,

$s = A_1 + A_2 + A_3$

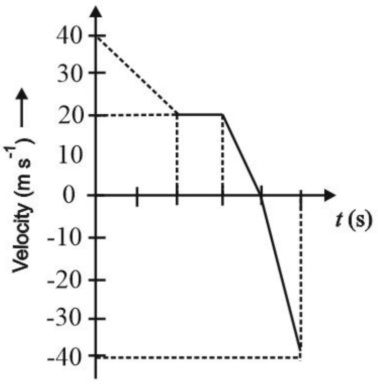
$\Rightarrow s = \text{area}\,(\text{Triangle}) - \text{area}\,(\text{Triangle}) + \text{area}\,(\text{Rectangle})$

$\Rightarrow s = \left(\frac{1}{2} \times 6 \times 2\right) - \left(\frac{1}{2} \times 2 \times 2\right) + (2 \times 1)$

$\Rightarrow s = (6) - (2) + (2)$

$\Rightarrow s = 6\ \text{m}$

Q.16. In the following velocity-time graph of a body, the distance travelled by the body and its displacement during 5 s (in metres) will respectively be



A) $75, 75$

B) $110, 70$

C) $110, 110$

D) $110, 40$

Answer: $110, 70$

Solution: The area under the velocity-time curve defines the displacement while the area under the speed-time curve defines the distance.

The total area under the velocity-time curve is equal to

$A_1 + A_2 + A_3 + A_4 \ \ldots(1)$

Where, A_1, A_2, A_3, A_4 are the area of trapezium, square, first triangle and second triangle, respectively.

Therefore, the total displacement is the area under velocity-time graph by taking sign into an account.

$s = \left[\frac{1}{2} \times (40 + 20) \times 2\right] + [20 \times 1] + \left[\frac{1}{2} \times 1 \times 20\right] - \left[\frac{1}{2} \times 1 \times 40\right]$

$\Rightarrow s = 60 + 20 + 10 - 20$

$\Rightarrow s = 70 \text{ m}$

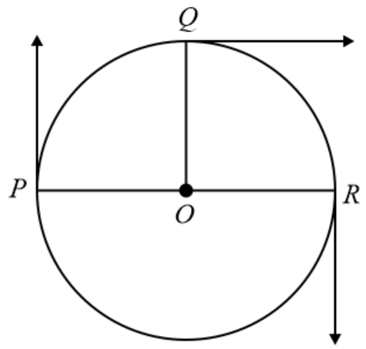
Hence, the total distance is the area under the velocity-time graph without taking sign into an account, i.e., it is the sum of magnitude of all areas.

$d = \left[\frac{1}{2} \times (40 + 20) \times 2\right] + [20 \times 1] + \left[\frac{1}{2} \times 1 \times 20\right] + \left[\frac{1}{2} \times 1 \times 40\right]$

$\Rightarrow d = \ 60 + 20 + 10 + 20$

$\Rightarrow d = 110 \text{ m}$

Q.17. Three point particles P, Q and R move in a circle of the radius r with different but constant speeds. They start moving at $t=0$ from their initial positions as shown in the figure. The angular velocities (in $\mathrm{rad\ s^{-1}}$) of P, Q and R are 5π, 2π and 3π respectively, in the same sense. The time interval after which they all meet is



A) $\frac{2}{3}$ s

B) $\frac{1}{6}$ s

C) $\frac{1}{2}$ s

D) $\frac{3}{2}$ s

Answer: $\frac{3}{2}$ s

Solution: The relative angular speed of P and Q particles is given by,

$5\pi - 2\pi = 3\pi$.

Now, we have to find when they will meet.

We know that time is given by,

$t = \frac{\theta_{\text{rel}}}{\omega_{\text{rel}}}$

Time after which the particles P and Q meet for the first time is

$t_1 = \frac{\left(\frac{\pi}{2}\right)}{3\pi} = \frac{1}{6}$ s.

second meet at

$t_2 = \frac{2\pi + \frac{\pi}{2}}{3\pi} = \frac{5}{6}$ s.

Third meet at

$t_3 = \frac{4\pi + \frac{\pi}{2}}{3\pi} = \frac{4.5}{3} = \frac{3}{2}$ s.

Fourth meet

$t_4 = \frac{6\pi + \frac{\pi}{2}}{3\pi} = \frac{13}{6}$ s.

The relative angular speed of P and R particles is given by,

$5\pi - 3\pi = 2\pi$.

They will meet at first time,

$t_5 = \frac{\pi}{2\pi} = \frac{1}{2}$ s

second time,

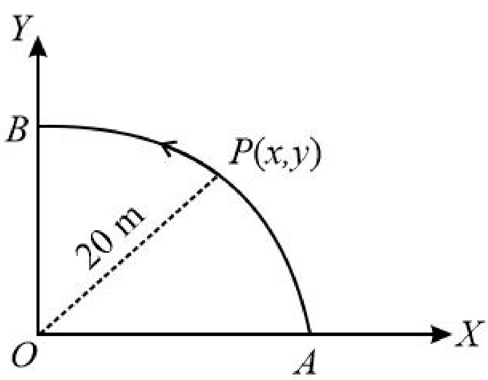
$t_6 = \frac{2\pi + \pi}{2\pi} = \frac{3}{2}$ s.

third time,

$t_7 = \frac{6\pi + \pi}{2\pi} = \frac{7}{2}$ s.

Therefore, all the particles will meet at $T = \frac{3}{2}$ s.

Q.18. A point P moves in a counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in m and t is in seconds. The radius of the path is $20\ \mathrm{m}$. The acceleration of P when $t = 2$ s is nearly



A) $13\ \mathrm{m\,s^{-2}}$.

B) $12\ \mathrm{m\,s^{-2}}$.

C) $7.2\ \mathrm{m\,s^{-2}}$.

D) $14\ \mathrm{m\,s^{-2}}$.

Answer: $14 \mathrm{~m} \mathrm{~s}^{-2}$.

Solution: Suppose the point P moves with linear speed v on a circular path.

Let a_{t} and a_{r} be the tangential and radial acceleration.

Its sweep length is,

$s=t^{3}+5$.

The linear speed of the particle, $v=\frac{\mathrm{d} s}{\mathrm{~d} t}=3 t^{2}$.

At $t=2 \mathrm{~s}$,

$v=3(2)^{2}=12 \mathrm{~m} \mathrm{~s}^{-1}$.

The tangential acceleration, $a_{\mathrm{t}}=\frac{\mathrm{d} v}{\mathrm{~d} t}=6 t$.

At $t=2 \mathrm{~s}$,

$a_{\mathrm{t}}=12 \mathrm{~m} \mathrm{~s}^{-2}$.

and centripetal acceleration is, $a_{\mathrm{r}}=\frac{v^{2}}{r}$.

$\therefore \; a_{\mathrm{r}}=\frac{\left(3 t^{2}\right)^{2}}{20}$

$=\frac{(12)^{2}}{20}=7.2 \mathrm{~m} \mathrm{~s}^{-2}. \quad(\text{at } t=2 \mathrm{~s})$

The net acceleration is, $a_{\text{net}}=\sqrt{\left(a_{\mathrm{t}}\right)^{2}+\left(a_{\mathrm{r}}\right)^{2}}$.

Put the values of a_{t} and a_{r},

$a_{\text{net}}=\sqrt{(12)^{2}+(7.2)^{2}}$

$=14 \mathrm{~m} \mathrm{~s}^{-2}$.

Q.19. A ball falls from a table top with initial horizontal speed V_{0} . In the absence of air resistance, which of the following statement is correct

A) The vertical component of the acceleration changes with time

B) The horizontal component of the velocity does not changes with time

C) The horizontal component of the acceleration is non zero and finite

D) The time taken by the ball to touch the ground depends on V_{0}

Answer: The horizontal component of the velocity does not changes with time

Solution: When the ball falls from a table top with initial speed v_{0} , its horizontal component of the velocity will remain unchanged with time because there is no air resistance.

Q.20. A car moving with a speed of $40 \mathrm{~km} \mathrm{~h}^{-1}$ can be stopped by applying brakes after at least $2 \mathrm{~m}$. If the same car is moving with a speed of $80 \mathrm{~km} \mathrm{~h}^{-1}$, then what is the minimum stopping distance?

A) $2 \mathrm{~m}$

B) $4 \mathrm{~m}$

C) $6 \mathrm{~m}$

D) $8 \mathrm{~m}$

Answer: $8 \mathrm{~m}$

Solution: Given that

$u=40\ \text{km h}^{-1}=\frac{100}{9}\ \text{m s}^{-1}$

And since the car stops, final velocity $(v)=0$

Now, by the third equation of motion,

$v^2=u^2+2ad$

First case, $0=\left(\frac{100}{9}\right)^2+2a\times 2$

$\Rightarrow a=-\left(\frac{100}{9}\right)^2\times\frac{1}{4}\ \ldots(1)$

As per the second case, $u=80\ \text{km h}^{-1}=\frac{200}{9}\ \text{m s}^{-1}$

$0=\left(\frac{200}{9}\right)^2+2ad$

$\Rightarrow 2ad=-\left(\frac{200}{9}\right)^2\ \ldots(2)$

Substitute the expression of acceleration, we get

$2\left(-\left(\frac{100}{9}\right)^2\times\frac{1}{4}\right)d=-\left(\frac{200}{9}\right)^2$

This gives

$d=8\ \text{m}.$

Q.21. Two seconds after projection, the projectile is travelling in a direction inclined at 30° to the horizontal and after one more second, it is travelling horizontally. The magnitude and direction of its initial velocity, respectively, are

A) $2\sqrt{20}\ \text{m s}^{-1},\ 60^\circ$

B) $20\sqrt{3}\ \text{m s}^{-1},\ 60^\circ$

C) $6\sqrt{40}\ \text{m s}^{-1},\ 30^\circ$

D) $40\sqrt{6}\ \text{m s}^{-1},\ 30^\circ$

Answer: $20\sqrt{3}\ \text{m s}^{-1},\ 60^\circ$

Solution: Let $u,\ v$ and θ be the initial velocity, velocity in mid-air or instantaneous velocity and the initial angle of projection, respectively.

$\text{Time of ascent} = \frac{u\sin\theta}{g} = 3\ \text{s}$

$u\sin\theta = 3g\ \ldots(1)$

Motion along y-axis,
$v_y = u_y + a_y t$

$v\sin 30^{\circ} = u\sin\theta - g(2)$

$\frac{v}{2} = 3g - 2g = g$

$v = 2g\ \ \ldots(2)$

Motion along $x-$axis, the horizontal component of velocity remains constant always, hence,

$v\cos 30^{\circ} = u\cos\theta\ \ \ldots(3)$

Assume, $g = 10\ \text{m s}^{-2}$.

After solving $(1),\ (2)$ and (3), we get,

$u = 20\sqrt{3}\ \text{m s}^{-1}$ and $\theta = 60^{\circ}$

Q.22. At what angle with the horizontal should a ball be thrown so that its range R is related to the time of flight as $R = 5T^2$? (Take $g = 10\,\text{m}\,\text{s}^{-2}$)

A) 30^o

B) 45^o

C) 60^o

D) 90^o

Answer: 45^o

Solution: We know that for projectile motion,

$R = \frac{u^2\sin 2\theta}{g}$.

& $T = \frac{2usin\theta}{g}$.

where, R = horizontal range,

u = initial velocity,

θ = projection angle,

g = acceleration due to gravity,

T = time of flight.

According to question,

$R = 5T^2$

$\frac{u^2\sin 2\theta}{g} = 5\left(\frac{2u\sin\theta}{g}\right)^2$

$\frac{2u^2\sin\theta\ \cos\theta}{g} = 5\frac{4u^2\sin^2\theta}{g^2}$

Upon solving, we get,

$\Rightarrow \tan\theta = \frac{2g}{4\times 5} = \frac{20}{20}$

$\Rightarrow \theta = 45^{\text{o}}$.

Q.23. If relation between distance and time is $s=a+bt+at^2$, and initial velocity and acceleration:

A) $b+2\ ct,\ 2c$

B) $b,\ 2c$

C) $2c,\ b$

D) $b+2c,\ 2c$

Answer: $b,\ 2c$

Solution: Here distance is given by the relation

$s=a+bt+ct^2\ldots.(i)$

Velocity is rate of change of displacement

Differentiate equation (i), we get

$v=\frac{ds}{dt}=0+b+2ct\ldots.(ii)$

Initial velocity at $t=0$

Substituting the value of t in equation (ii)

$v=0+b+2\times 0=b$

To get acceleration, differentiate equation (ii) w.r.t t

$a=\frac{dv}{dt}=2c$

Q.24. A particle starts moving along a line from zero initial velocity and comes to rest after moving distance d. During its motion it had a constant acceleration f over $\frac{2}{3}$ of the distance, and covered the rest of the distance with constant retardation. The time taken to cover the distance is

A) $\sqrt{\frac{2d}{3f}}$

B) $2\sqrt{\frac{d}{3f}}$

C) $\sqrt{\frac{3d}{f}}$

D) $\sqrt{\frac{3d}{2f}}$

Answer: $\sqrt{\frac{3d}{f}}$

Solution: Given that initial velocity $u=0$, total distance d.

Let v be the velocity at B.

Applying equation of motion, $v=u+at$.

When the particle travels from A to B in time t_1, then $v=0+ft_1=ft_1 \ldots(i)$.

Given that the distance travelled from A to B is $\frac{2d}{3}$, using the second equation of motion, we have $\frac{2d}{3}=0+\frac{1}{2}ft_1^2 \Rightarrow t_1=\sqrt{\frac{4d}{3f}} \ldots(ii)$

Let the acceleration of particle between B and C is a.

Now, when a particle travels from B to C, then using the first equation of motion, we have $\mathrm{v}'=\mathrm{v}-\mathrm{at}_2$. (Since the particle is retarding, thus a is negative)

$0=v-at_2 \Longrightarrow v=at_2 \ \ldots(iii\ )$ (The particle comes to rest)

From equation (i) and (iii), we have $a=\frac{ft_1}{t_2}$.

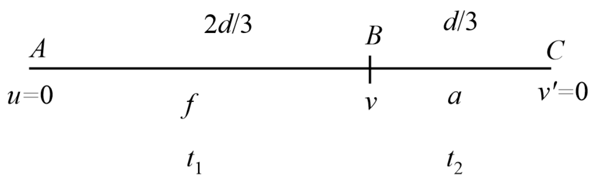
Using the third equation of motion, $v^2=u^2+2as$.

Distance covered is $\frac{d}{3}=\frac{v^2-v'^2}{2a} \Rightarrow \frac{d}{3}=\frac{v^2}{2a}=\frac{(ft_1)^2}{2\times\left(\frac{ft_1}{t_2}\right)}=\frac{ft_1t_2}{2} \ldots(iv)$

From equation (ii) and (iv), we get $\frac{d}{3}=\frac{ft_2}{2}\times\sqrt{\frac{4d}{3f}}$

$t_2=\sqrt{\frac{d}{3f}}$

Hence, the total time taken $t=t_1+t_2=\sqrt{\frac{4d}{3f}}+\sqrt{\frac{d}{3f}}=\sqrt{\frac{3d}{f}}$.



Q.25. A particle is projected from a point $(0,1)$ on the y-axis (assume $+y$ direction vertically upwards) aiming towards a point $(4,9)$. It fell on the ground along the x-axis in 1 s. Taking $g=10\ \mathrm{m\,s^{-2}}$ and all coordinates in meter, find the x-coordinate where it fell.

A) $(3,\ 0)$

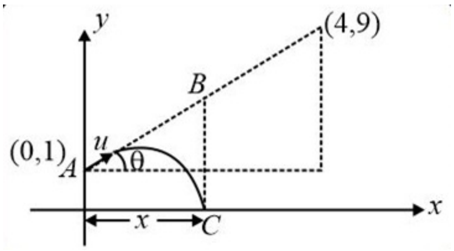
B) $(4,\ 0)$

C) $(2,\ 0)$

D) $\left(2\sqrt{5},0\right)$

Answer: $(2,\ 0)$

Solution:



From the above figure,
$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$\tan\theta = \frac{9-1}{4-0} = 2.$

Similarly,
$\sin\theta = \frac{2}{\sqrt{5}}$ and

$\cos\theta = \frac{1}{\sqrt{5}}.$

Now, using the equations of motion in the y direction, sign convention upwards is positive and downwards is negative.
$S_y = u_y t + \frac{1}{2} a_y t^2$

$\Rightarrow -1 = u\sin\theta\left(1\right) - \frac{1}{2} g(1)^2$
$\Rightarrow u\sin\theta = 4$
$\Rightarrow u\left(\frac{2}{\sqrt{5}}\right) = 4\left(\because \sin\theta = \frac{2}{\sqrt{5}}\right)$

$\Rightarrow u = 2\sqrt{5}\ \text{m s}^{-1}.$

Now, using the equation in the x direction,
$s_x = u_x t (\because a_x = 0)$
$x = u\cos\theta(1)$

$= \left(2\sqrt{5}\right) \times \frac{1}{\sqrt{5}} = 2\ \text{m}.$

Practice more on Kinematics