NEET Important Questions with Solutions from Electrostatics

Q.1. A pair of parallel metal plates are kept with a separation d. One plate is at a potential $+V$ and the other is at ground potential. A narrow beam of electrons enters the space between the plates with a velocity v_0 and in a direction parallel to the plates. What will be the angle of the beam with the plates after it travels an axial distance L?

A) $\tan^{-1}\left(\frac{eVL}{mdv_0}\right)$

B) $\tan^{-1}\left(\frac{eVL}{mdv_0^2}\right)$

C) $\sin^{-1}\left(\frac{eVL}{mdv_0}\right)$

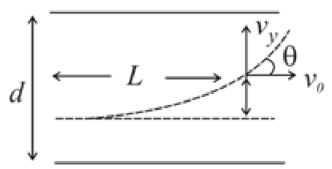
D) $\cos^{-1}\left(\frac{eVL}{mdv_0^2}\right)$

Answer: $\tan^{-1}\left(\frac{eVL}{mdv_0^2}\right)$

Solution: Distance between the parallel plates $= d$

The axial distance of beam from the centre of parallel plates $= L$

As, time, $t = \frac{\text{distance}}{\text{velocity}} = \frac{L}{v_0}$



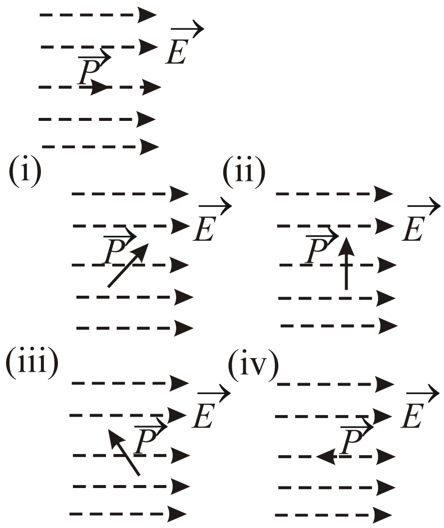
y-component of velocity, $v_y = \left(\frac{qE}{m}\right)t = \frac{e}{m} \cdot \frac{V}{d} \times \frac{L}{v_0}$

$\Rightarrow v_y = \frac{eVL}{mdv_0}$

$\therefore\ \tan\theta = \frac{v_y}{v_z} = \frac{eLV}{mdv_0 \cdot v_0} = \frac{eVL}{mdv_0^2}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{eVL}{mdv_0^2}\right)$

Q.2. An electric dipole of dipole moment $\vec{p}$ is oriented parallel to a uniform electric field $\vec{E}$, as shown. It is rotated to one of the four orientations shown below. Rank the final orientations according to the change in the potential energy of the dipole-field system, most negative to most positive.



A) (i), (ii), (iv) and (iii).

B) (iv), (iii), (ii), and (i).

C) (i), (ii), (iii) and (iv).

D) (iii), (ii), and (iv). tie, then (i).

Answer: (i), (ii), (iii) and (iv).

Solution: The potential energy of an electric dipole inside a uniform electric field is given by,

$U=-\vec{p}\cdot\vec{E}=-pE\cos\theta$

where $\vec{p}$ is dipole moment, $\vec{E}$ is the electric field and θ is the angle between $\vec{p}$ and $\vec{E}$.

Initially, $\vec{p}$ is parallel to $\vec{E}$, so $\theta=0^\circ$.

$\therefore$ Initial potential energy is

$U=-pE\cos\theta=-pE.$

Now let us consider each of the given figures.

Figure (i): $\theta=\theta_1$ is acute

$\therefore\ U_1=-pE\cos\theta_1$ is negative

Change in potential energy is

$\Delta U_1=U_1-U=-pE(\cos\theta_1-1)$

Clearly, ΔU_1 is positive. $\ldots(1)$

Figure (ii):

Here, $\theta=90^\circ$

So, potential energy in this situation is

$U_2=-pE\cos 90^\circ=0$

$\therefore$ Change in potential energy from the initial position

$\Delta U_2=U_2-U=+pE\ \ldots(2)$

Figure (iii): Here, angle $\theta=\theta_3$ is obtuse

So, potential energy in this situation

$\Delta U_3=-pE\cos\theta_3$ is $+\text{ve}$.

Change in potential energy from the initial position

$\therefore\ \Delta U_3=U_3-U=-pE\cos\theta_3+pE\ \ldots(3)$

ΔU_3 is positive.

Figure (iv): Here, $\theta=180^\circ$

The potential energy in this situation

$\therefore\ U_4=-pE\cos\theta=pE$

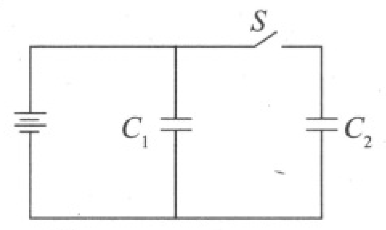
Change in potential energy from the initial position

$\therefore\ \Delta U_4=U_4-U=2pE.$

So the required order is $\Delta U_1<\Delta U_2<\Delta U_3<\Delta U_4$.

$\therefore$ Required ranking is (i), (ii), (iii) and (iv).

Q.3. Two capacitors of equal capacitance $(C_1 = C_2)$ are shown in the figure. Initially, while the switch S is open, one of the capacitors is uncharged and the other carries charge Q_0. The energy stored in the charged capacitor is U_0. Sometimes after the switch is closed, the capacitors C_1 and C_2 carry charges Q_1 and Q_2, respectively, the voltage across the capacitors are V_1 and V_2, and the energies stored in the capacitors are U_1 and U_2. Which of the following statements is incorrect?



A) $Q_0 = \frac{1}{2}(Q_1 + Q_2)$

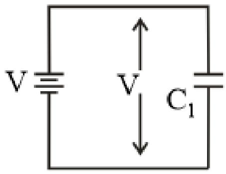
B) $Q_1 = Q_2$

C) $V_1 = V_2$

D) $U_0 = U_1 + U_2$

Answer: $U_0 = U_1 + U_2$

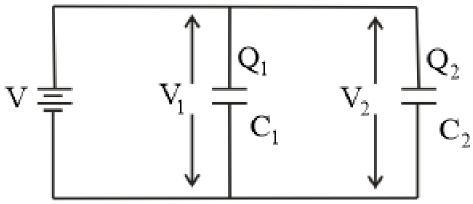
Solution: When S is open,



$Q_0 = C_1 V$

$U_0 = \frac{1}{2} C_1 V^2$

After the switch is closed, both capacitors would be in parallel.



So, $V_1 = V_2 = V$ (due to parallel combination)

$Q_1 = C_1 V_1$

$Q_2 = C_2 V_2 \text{ (as } V_1 = V_2 = V) \ (C_1 = C_2)$

$Q_1 = Q_2 = Q_0$

$U_1 = \frac{1}{2} C_1 V_1^2 = U_0$

$U_2 = \frac{1}{2} C_2 V_2^2 = U_0$

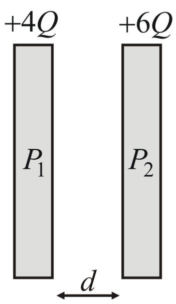
$2U_0 = U_1 + U_2$

$U_0 \neq U_1 + U_2$

$Q_1 + Q_2 = 2Q_0$

$Q_0 = \frac{1}{2}(Q_1 + Q_2)$

Q.4. Two identical conducting very large plates P_1 and P_2 having charges $+4Q$ and $+6\ Q$ are placed very closed to each other at separation d. The plate area of either face of the plate is A. The potential difference between plates P_1 and P_2 is



In the above question, if plates P_1 and P_2 are connected by a thin conducting wire, then the amount of heat produced will be

A) $\frac{Q^2}{A\varepsilon_0} d$

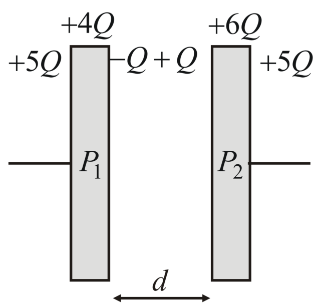
B) $\frac{5Q^2}{A\varepsilon_0} d$

C) $\frac{2Q^2}{A\varepsilon_0}d$

D) None of these

Answer: None of these

Solution: From the above questions charges on different plates will be,



When we connected both the plates with the wires, the charges will start flowing from one plate to the other and the final charges are as shown in the figure. Energy stored in the capacitor will be zero finally, i.e., $U_f = 0$, but initially, the energy will be stored in the electric field between the charges which can be given by,

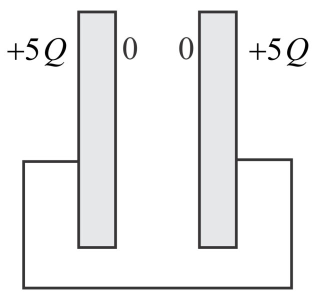
$$E = \frac{Q}{2A\varepsilon_0} + \frac{Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

So, the initial energy in the plates will be,

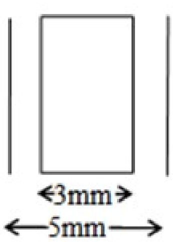
$$U_{\mathrm{i}} = \frac{1}{2}\varepsilon_0 E^2 \times Ad = \frac{1}{2} \times \varepsilon_0 \times \left(\frac{Q}{A\varepsilon_0}\right)^2 \times Ad$$

The energy lost in heat will be,

$$\Delta H = U_{\mathrm{i}} - U_{\mathrm{f}} = \frac{Q^2}{2A\varepsilon_0}d$$



Q.5. The separation between the plates of a parallel plate capacitor is $5\ \mathrm{mm}$. This capacitor, having air as the dielectric medium between the plates, is charged to a potential difference $25\ \mathrm{V}$ using a battery. The battery is then disconnected and a dielectric slab of thickness $3\ \mathrm{mm}$ and dielectric constant $K = 10$ is placed between the plates, as shown. The potential difference between the plates after the dielectric slab has been introduced is



A) 18.5 V

B) 13.5 V

C) $11.5 \mathrm{~V}$

D) $6.5 \mathrm{~V}$

Answer: $11.5 \mathrm{~V}$

Solution: The capacitor is charged by a battery $25 \mathrm{~V}$. Let the magnitude of surface charge density on each plate be σ. Before inserting the dielectric slab, field strength between the plates,

$E=\frac{\sigma}{\varepsilon_0}=\frac{V}{d}$

or $E=\frac{\sigma}{\varepsilon_0}=\frac{25}{5\times10^{-3}}=5000 \mathrm{~N/C}$

The capacitor is disconnected from the battery but the charge on it will not change so σ has the same value. When a dielectric slab of thickness $3mm$ is placed between the plates, the air thickness between the plates will be $5-3=2 \mathrm{~mm}$. Electric field strength in air will have the same value $(5000 \mathrm{~N/C})$ but inside the dielectric, it will be $\frac{5000}{K}=\frac{5000}{10}=500 \mathrm{~N/C}$

So potential difference

$V=E_{air}d_{air}+E_{med}d_{med}$

$V=5000\times\left(2\times10^{-3}\right)+500\times\left(3\times10^{-3}\right)$

$V=11.5 \mathrm{~V}$

Q.6. Two capacitors C_1 and C_2 are charged to $120 \mathrm{~V}$ and $200 \mathrm{~V}$, respectively. It is found that by connecting them together the potential on each one can be made zero. Then

A) $3C_1=5C_2$

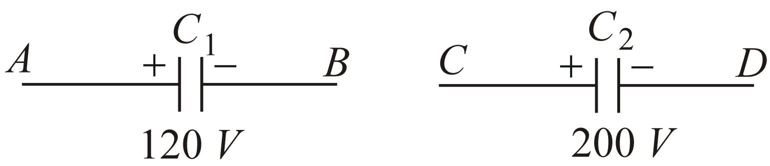
B) $3C_1+5C_2=0$

C) $9C_1=4C_2$

D) $5C_1=3C_2$

Answer: $3C_1=5C_2$

Solution: Initial state:



Charge on C_1

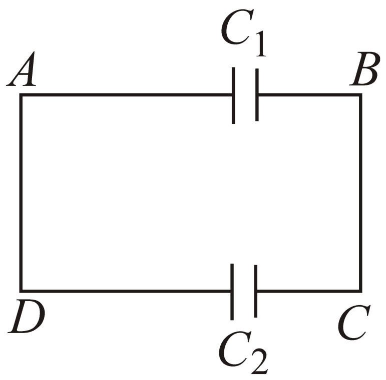
$Q_1 = C_1 V_1 = 120 C_1$

Charge on C_2

$Q_2 = C_2 V_2 = 200 C_2$

Final state :

As final common potential is zero, so point A and D must be connected together and points B and C must be connected together.



Net charge on the system in this case is,

$Q_{\text{net}} = Q_2 - Q_1$

$= 200 C_2 - 120 C_1$

As common potential is given by,

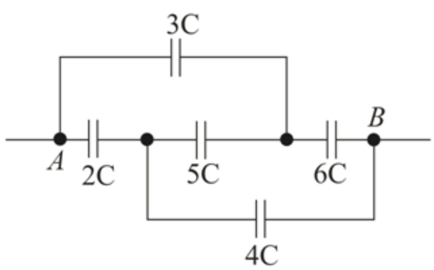
$V = \frac{Q_{\text{net}}}{C_1 + C_2}$

$\therefore 0 = \frac{200 C_2 - 120 C_1}{C_1 + C_2}$

$\Rightarrow 200 C_2 = 120 C_1$

$\Rightarrow 5 C_2 = 3 C_1$

Q.7. The capacitance of a system of capacitors between points A and B, as shown in the given figure is,



A) $\frac{10}{3} C$

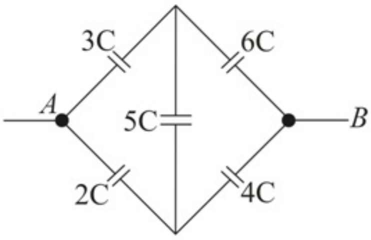
B) C

C) $2C$

D) $\frac{C}{2}$

Answer: $\frac{10}{3}C$

Solution: Rearranging the circuit,

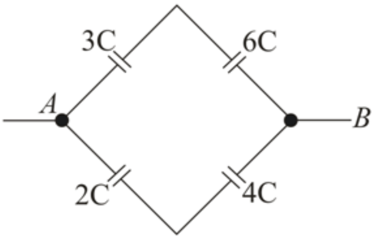


$\frac{3C}{6C} = \frac{2C}{4C} = \frac{1}{2}$

Hence, this is a balanced Wheatstone bridge.

$5C$ can be removed.

The new circuit diagram is,



Now, $3C$ is in series with $6C$ and $2C$ is in series with $4C$.

For series combination,

$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}$

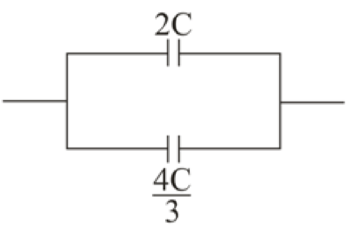
C_{eq} of $3C$ and $6C$ is,

$= \frac{3C \times 6C}{3C + 6C} = 2C$

Similarly
$C_{\text{eq}} = 2C \text{ and } 4C$

$C_{\text{eq}} = \frac{2C \times 4C}{2C + 4C} = \frac{4}{3}C$

The final circuit is,

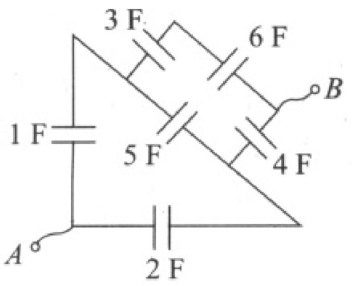


Now, these capacitors are in parallel,

$C_{\text{eq}} = C_1 + C_2 = 2C + \frac{4C}{3}$

$C_{\text{eq}} = \frac{10C}{3}$

Q.8. In the figure shown below, the equivalent capacitance between A and B is,



A) 3.75 F

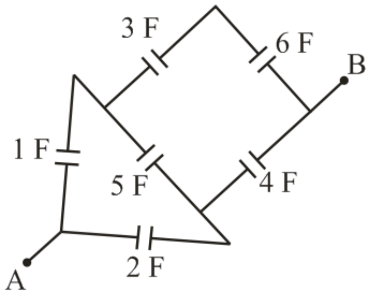
B) 2 F

C) 21 F

D) 16 F

Answer: 2 F

Solution:



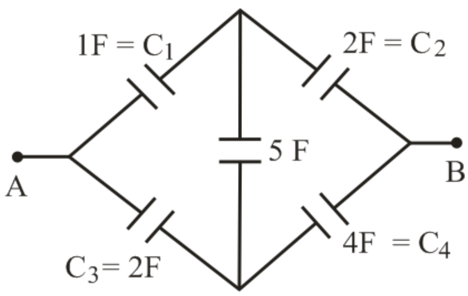
3 F and 6 F are in series.

Let C' be C_{eq} of 3 F and 6 F.

$C_{\text{series}} = \frac{C_1 \times C_2}{C_1 + C_2}$

$C' = \frac{3 \times 6}{3+6} = 2 \text{ F}$

On rearranging the circuit diagram,



As,

$\frac{C_1}{C_2} = \frac{C_3}{C_4} = \frac{1}{2}$

This circuit is a balanced Wheatstone bridge. Hence, 5 F can be removed.

C_1 and C_2 are in series,

$C_{12} = \frac{1 \times 2}{1+2} = \frac{2}{3} \text{ F}$

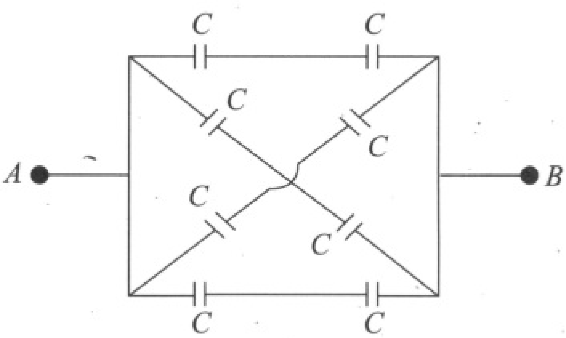
C_3 and C_4 are also in series combination,

$C_{34} = \frac{2 \times 4}{2+4} = \frac{4}{3} \text{ F}$

Now, C_{12} and C_{34} are in parallel.

$C_{eq} = C_{12} + C_{34} = \frac{2}{3} + \frac{4}{3} = 2 \text{ F}$

Q.9. In the given circuit, the capacity between the points A and B will be,



A) C

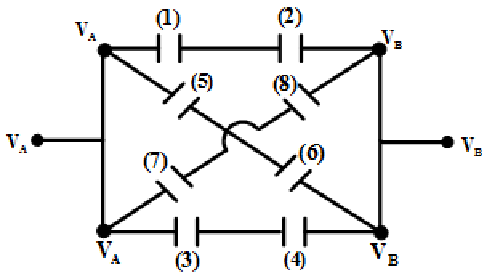
B) $2C$

C) $3C$

D) $4C$

Answer: $2C$

Solution:



Here,
$(1)\&(2)$ and $(3)\&(4)$ are in series as the charges on them will be equal. Similarly,
$(5)\ \&\ (6)$ and
$(7)\ \&\ (8)$ are also in series.

For series combination,

$\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}$
So, for
(1) and (2),

$\frac{1}{C_{\text{eq}}}=\frac{1}{C}+\frac{1}{C}=\frac{2}{C}$

$C_{\text{eq}}=\frac{C}{2}$

Also, the combination of
$(1\ \&\ 2)$, $(5\ \&\ 6)$, $(7\ \&\ 8)$, $(3\ \&\ 4)$ are equal and are in parallel as the potential difference across them is $V_{\text{A}}-V_{\text{B}}$.

For parallel combination,

$C_{\text{eq}}=$Sum of all the combination.

Now,

$C_{\text{eq}}=C_{12}+C_{34}+C_{56}+C_{78}$

$=4\times\frac{C}{2}=2C$

$C_{\text{eq}}=2C$.

Q.10. Three identical spheres, each having a charge q and radius R, are kept in such a way that each touches the other two. The magnitude of the electric force on any sphere due to the other two is

A) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

B) $\frac{\sqrt{3}}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

C) $\frac{\sqrt{3}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

D) $\frac{\sqrt{5}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

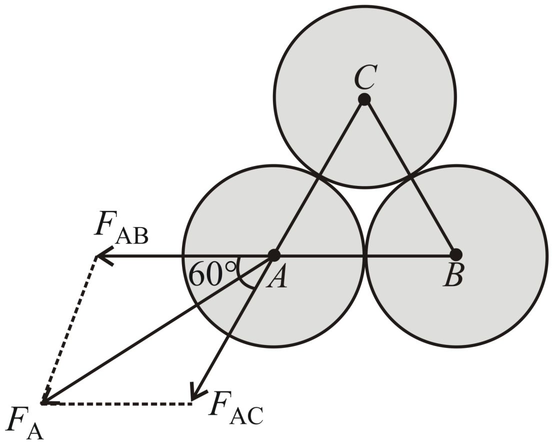
Answer: $\frac{\sqrt{3}}{16\pi\varepsilon_0}\left(\frac{q}{R}\right)^2$

Solution: Given that

Charge on the identical spheres $= q$

The radius of the sphere $= R$

The electric force on any sphere due to the other two is F_{A}



Force on A due to B,
$F_{\mathrm{AB}} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2}$

Also force on sphere A due to sphere C,
$F_{\mathrm{AC}} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2}$

Now as angle between $\vec{F}_{\mathrm{AB}}$ and $\vec{F}_{\mathrm{AC}}$ is 60^{o} and $\left|\vec{F}_{\mathrm{AB}}\right| = \left|\vec{F}_{\mathrm{AC}}\right| = F$

$\therefore F_{\mathrm{A}} = \sqrt{F^2 + F^2 + 2FF\cos 60^{\mathrm{o}}} = \sqrt{3}F$

$\Rightarrow F_{\mathrm{A}} = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{3}}{4}\left(\frac{q}{R}\right)^2$

Q.11. Three concentric metallic spherical shells of radii R, $2R$ and $3R$ are given charges Q_1, Q_2 and Q_3 respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then the ratio $Q_1 : Q_2 : Q_3$ of the charges given to the shells is

A) $1:2:3$

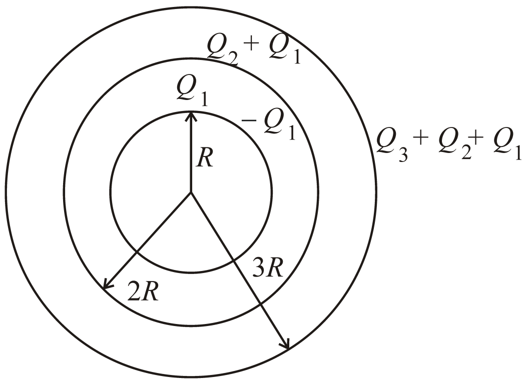
B) $1:3:5$

C) $1:4:9$

D) $1:8:18$

Answer: $1:3:5$

Solution: The induced charges on the surface of spheres is given as shown in figure below,



Due to charge Q_1 on sphere of radius R, there will be induced charge on inner surface of $2R$ which will be $-Q_1$ and on outer surface it will be $+Q_1$. The net charge on outer surface of $2R$ radius sphere will be (Q_1+Q_2). Similarly, induced charge on inner surface of $3R$ will be $-(Q_1+Q_2)$ and on outer surface it will be $+(Q_1+Q_2+Q_3)$.

The surface charge density of the outer surfaces of the sphere is same. The surface charge density is defined as charge per unit area. So,

$$\frac{Q_3+Q_2+Q_1}{4\pi(3R)^2}=\frac{Q_2+Q_1}{4\pi(2R)^2}=\frac{Q_1}{4\pi R^2}$$

$$\Rightarrow\frac{Q_3+Q_2+Q_1}{9}=\frac{Q_2+Q_1}{4}$$

and $\frac{Q_2+Q_1}{4}=Q_1$

$$\Rightarrow Q_2+Q_1=4Q_1$$

$$\Rightarrow Q_2=3Q_1$$

Also,

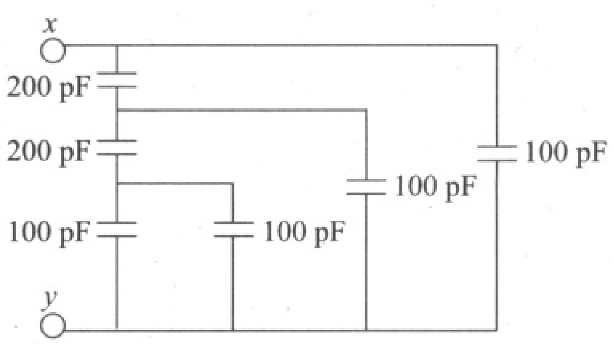
$$\frac{Q_3+3Q_1+Q_1}{9}=\frac{Q_1}{1}$$

$$\Rightarrow Q_3+4Q_1=9Q_1$$

$$\Rightarrow Q_3=5Q_1$$

Hence, $Q_1:Q_2:Q_3=1:3:5$

Q.12. The equivalent capacitance between the terminals x and y, as shown in the below figure, will be



A) 100 pF

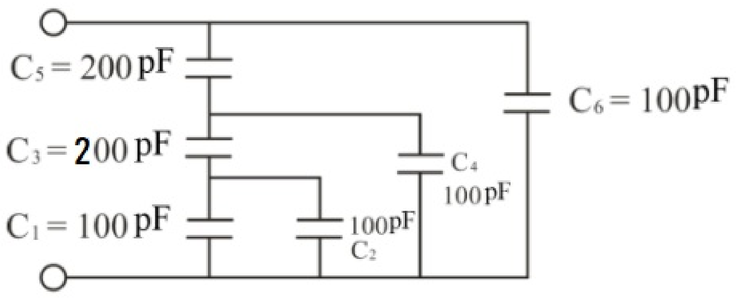
B) 200 pF

C) 300 pF

D) $400\ \mathrm{pF}$

Answer: $200\ \mathrm{pF}$

Solution:



C_1 and C_2 are in parallel,

$C_{12}=C_1+C_2=100+100=200\ \mathrm{pF}$

Now, C_{12} is in series with C_3,

$C'=\frac{C_{12}\times C_3}{C_{12}+C_3}=100\ \mathrm{pF}$

Where C' is equivalent to C_{12} and C_3.

Now, C' is in parallel with C_4,

$C''=C'+C_4=100+100=200\ \mathrm{pF}$

Where C'' is equivalent to C' and C_4.

Now, C'' is in series with C_5,

$C'''=\frac{C''\times C_5}{C''+C_5}=100\ \mathrm{pF}$

C''' is equivalent to C'' and C_5.

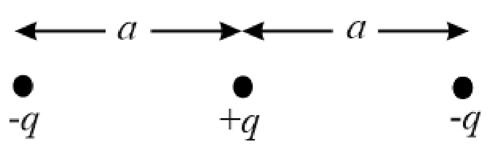
C''' is in parallel with C_6,

$C_{\mathrm{eq}}=C'''+C_6=100+100$

The final equation is,

$C_{\mathrm{eq}}=200\ \mathrm{pF}$.

Q.13. The three point charges shown in the figure lie along a straight line. The energy needed to exchange the position of the central positive charge with one of the negative charges is,



A) $\frac{q^2}{8\pi\varepsilon_0 a}$

B) $\frac{3q^2}{8\pi\varepsilon_0 a}$

C) $\frac{q^2}{4\pi\varepsilon_0 a}$

D) $\frac{-q^2}{8\pi\varepsilon_0 a}$

Answer: $\frac{q^2}{4\pi\varepsilon_0 a}$

Solution: The energy of the given system of charge is given by the sum of the electrostatic potential energy of each pair among the three charges.

Hence, the initial electrostatic potential energy of the system will be,

$$E_{\text{i}}=\frac{kq^2}{2a}+\left(-\frac{kq^2}{a}\right)+\left(-\frac{kq^2}{a}\right)$$

Now, if the position of the central positive charge is exchanged with one of the negative terminal charges, the final potential energy of the system will again be given by the sum of the electrostatic potential energy of each pair among the three charges.

Hence, the final electrostatic energy of the system will be,

$$E_{\text{f}}=\frac{kq^2}{a}+\left(-\frac{kq^2}{a}\right)+\left(-\frac{kq^2}{2a}\right)=-\frac{kq^2}{2a}$$

The energy required to do the work for the rearrangement of the charges is the difference between the final and the initial potential energy of arrangement of the system.

Hence, the energy required is,

$$E=E_{\text{f}}-E_{\text{i}}=-\frac{kq^2}{2a}-\left[\frac{kq^2}{2a}+\left(-\frac{kq^2}{a}\right)+\left(-\frac{kq^2}{a}\right)\right]=\frac{q^2}{4\pi\varepsilon_0 a}$$

Q.14. Two equally charged and identical metal spheres A and B repel each other with a force F. The spheres are kept fixed with a distance r between them. A third identical, but uncharged sphere C is brought in contact with A and then placed at the mid-point of the line joining A and B. The magnitude of the net electric force on C is

A) F

B) $\frac{3F}{4}$

C) $\frac{F}{2}$

D) $\frac{F}{4}$

Answer: F

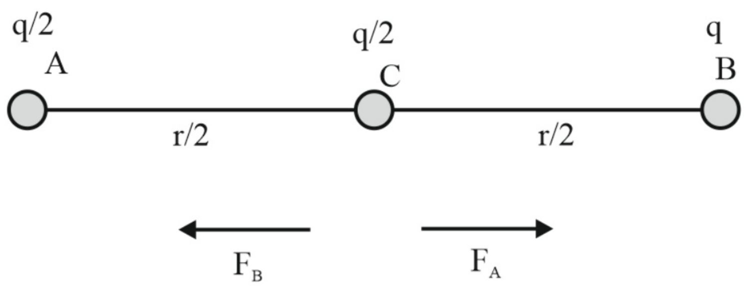
Solution: According to Coulomb's law,

$F=\frac{kq_1q_2}{r^2}$.

Therefore, the force between A and B was

$F=\frac{kq^2}{r^2}$.

When A and C are brought in contact, redistribution of charges will take place and as both are identical, both will end up with equal charge $\frac{q}{2}$ each.

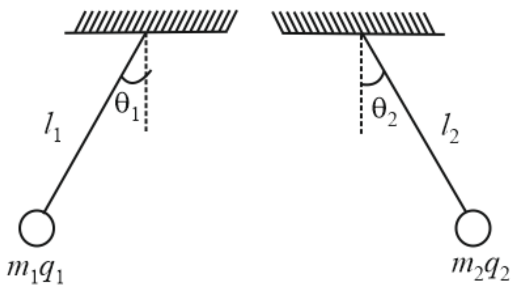


$F_{\text{net}_C}=F_B-F_A$ (since force exerted by B and A are opposite).

$F_{\text{net}_C}=\frac{kq(\frac{q}{2})}{(\frac{r}{2})^2}-\frac{k(\frac{q}{2})(\frac{q}{2})}{(\frac{r}{2})^2}$

$F_{\text{net}_C}=\frac{2kq^2}{r^2}-\frac{kq^2}{r^2}=2F-F=F.$

Q.15. Two small spheres with mass m_1 and m_2 hang from massless insulating threads of length l_1 and l_2. The two spheres carry charges q_1 and q_2, respectively. The spheres are hung such that they are on the same horizontal level and the threads are inclined to the vertical at angles θ_1 and θ_2. Which of the condition is required if $\theta_1=\theta_2$?



A) If $m_1=m_2$

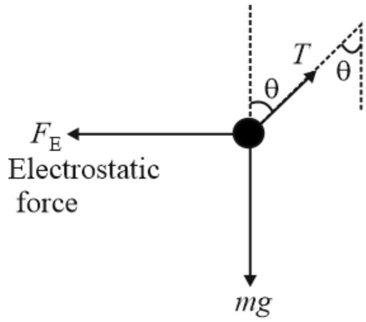
B) $|q_1|=|q_2|$

C) $l_1=l_2$

D) $\frac{q_1}{m_1}=\frac{q_2}{m_2}$

Answer: If $m_1=m_2$

Solution:



Consider the above diagram as the free body diagram for the left sphere of mass m_1, the angle of deflection θ_1 from the vertical and charge q_1. Let the tension in the string be T_1.

Let the electrostatic force between the two spheres be denoted by F_e.

Since the sphere is in equilibrium,

$\Rightarrow T_1 \cos\theta_1 = m_1 g$.

and $T_1 \sin\theta_1 = F_e$.

Dividing these two equations, we get,

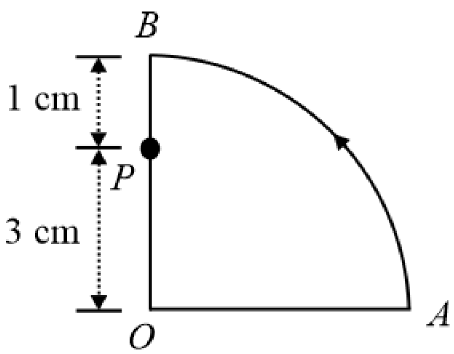
$\frac{F_e}{m_1 g} = \tan\theta_1 \quad —(1)$

Solving similarly for the right sphere, we get,

$\frac{F_e}{m_2 g} = \tan\theta_2 \quad —(2)$

The magnitude of the electrostatic force on either of the two spheres will be equal irrespective of the charges on them. Hence, from the equation 1 and equation 2, for θ_1 and θ_2 to be equal, the two spheres have to be of the same mass. Hence, the required condition is $m_1 = m_2$.

Q.16. A point charge $5\ \mathrm{C}$ is placed at a point P as shown in the figure. A unit positive charge is taken from A to B along the circular path as shown. Then, the net work done by electrostatic forces is, (O is the centre of the circular path and $k = 9 \times 10^9\ \mathrm{N\ m^2\ C^{-2}}$.)



A) $+400k$

B) $+4k$

C) $-400k$

D) $-4k$

Answer: $-400k$

Solution: The quarter of the circle that the moving charges traces has a radius $4\ \mathrm{cm}$.

The work done by the electrostatic force is given by the difference between the initial and final electrostatic potential energies of the system of charge.

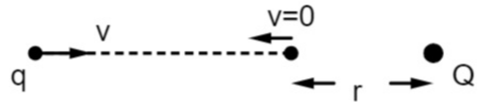
We know that the electrostatic potential energy of a system of two charges q_1 and q_2 separated by a distance r is given by $U = \frac{kq_1q_2}{r}$.

Initial distance $= \sqrt{OP^2 + OA^2} = \sqrt{3^2 + 4^2} = 5\ \mathrm{cm}$

Final distance $= 1\ \mathrm{cm}$

Hence, the work done on the moving charge due to the electrostatic force acting on it is,
$W = U_{\mathrm{i}} - U_{\mathrm{f}} = \frac{k(5)(1)}{\left(\frac{5}{100}\right)} - \frac{k(5)(1)}{\left(\frac{1}{100}\right)} = -400k$

Q.17. A charged particle q is shot towards another charged particle Q, which is fixed, with speed v. It approaches Q up to a closest distance r and then returns. If q was given a speed $2v$, the closest distance of approach would be,



A) r

B) $2r$

C) $\frac{r}{2}$

D) $\frac{r}{4}$

Answer: $\frac{r}{4}$

Solution: Let the mass of the projected charge be m. The initial kinetic energy of the projected charge is completely converted to electrostatic potential energy as it reaches the closest distance of approach.

Using energy conservation for the first case,

$\frac{1}{2}mv^2 = \frac{kqQ}{r}$

Let the closest distance of approach when projected with speed $2v$ be r'. Then, by applying energy conservation for this case,

$\frac{1}{2}m(2v)^2 = \frac{kqQ}{r'}$.

Comparing the two equations, we get, $r' = \frac{r}{4}$.

Q.18. Two equally charged, identical metal spheres A and B repel each other with a force F. The spheres are kept fixed with a distance r between them. A third identical, but uncharged sphere C is brought in contact with A and then placed at the mid-point of the line joining A and B. The magnitude of the net electric force on C is

A) F

B) $\frac{F}{4}$

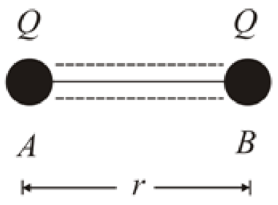
C) $\frac{F}{2}$

D) $4F$

E) $3F$

Answer: F

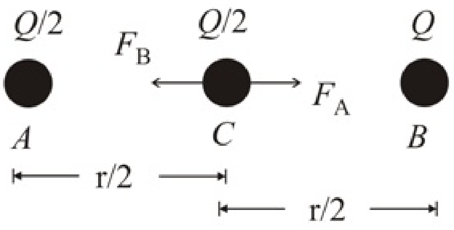
Solution: Initially



$F=\frac{kQ^2}{r^2}$

Finally, when an uncharged sphere is kept in touch with the sphere of charge q, the net charge on both sphere become

$\frac{q+0}{2}=\frac{q}{2}$



The force on C due to A is given by

$F_A=\frac{k(\frac{Q}{2})^2}{(\frac{r}{2})^2}$

$F_A=\frac{kQ^2}{r^2}$

The force on C due to B is given by

$F_B=\frac{kQ(\frac{Q}{2})}{(\frac{r}{2})^2}$

$F_B=\frac{2kQ^2}{r^2}$

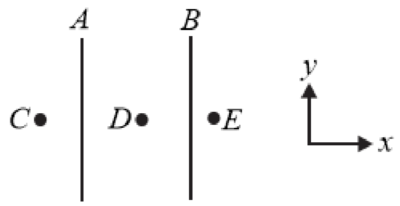
$\therefore$ Net force on C is given by,

$F_{\text{net}}=F_B-F_C$

$F_{\text{net}}=(2-1)\frac{kQ^2}{r^2}$

$F_{\text{net}}=F$

Q.19. Two infinite plane sheets A and B are shown in the figure. The surface charge densities on A and B are $\left(\frac{2}{\pi}\right) \times 10^{-9}\ \mathrm{C\ m^{-2}}$ and $\left(\frac{-1}{\pi}\right) \times 10^{-9}\ \mathrm{C\ m^{-2}}$ respectively. C, D and E are three points where electric fields (in $\mathrm{N\ C^{-1}}$) are E_c, E_D and E_E respectively.



A) $\overrightarrow{\mathrm{E_c}} = 18\hat{\imath}$

B) $\overrightarrow{\mathrm{E_D}} = 54\hat{\imath}$

C) $\overrightarrow{\mathrm{E_D}} = 18\hat{\imath}$

D) $\overrightarrow{\mathrm{E_E}} = -18\hat{\imath}$

Answer: $\overrightarrow{\mathrm{E_D}} = 54\hat{\imath}$

Solution: The magnitude of the electric field of sheet $A = E_\mathrm{A} = \frac{\sigma_\mathrm{A}}{2\varepsilon_0}$

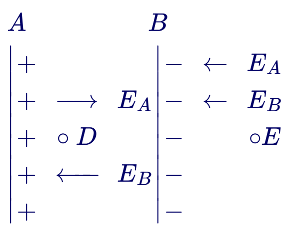
$$\Rightarrow E_\mathrm{A} = \frac{\left(\frac{2}{\pi}\right)\times 10^{-9}}{2\times\left(\frac{1}{4\pi\times 9\times 10^{9}}\right)} = 36\ \mathrm{N\ C^{-1}}$$

The magnitude of the electric field of sheet $B = E_\mathrm{B} = \frac{\sigma_\mathrm{B}}{2\varepsilon_0}$

$$\Rightarrow E_\mathrm{B} = \frac{\left(\frac{1}{\pi}\right)\times 10^{-9}}{2\times\left(\frac{1}{4\pi\times 9\times 10^{9}}\right)} = 18\ \mathrm{N\ C^{-1}}$$

The sheet A is positively charged, the electric field will be outwards.

The sheet B is negatively charged, the electric field will be inwards.



At C

$E_\mathrm{C} = -E_\mathrm{A}\hat{\imath} + E_\mathrm{B}\hat{\imath} = -36\hat{\imath} + 18\hat{\imath} = -18\hat{\imath}$

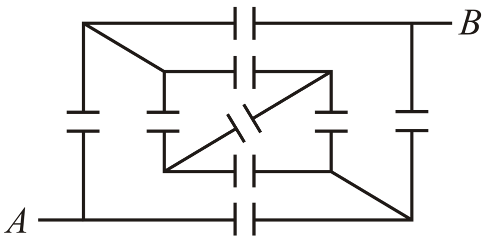
At D

$E_\mathrm{D} = E_\mathrm{A}\hat{\imath} + E_\mathrm{B}\hat{\imath} = 36\hat{\imath} + 18\hat{\imath} = 54\hat{\imath}$

At E

$E_\mathrm{E} = E_\mathrm{A}\hat{\imath} - E_\mathrm{B}\hat{\imath} = 36\hat{\imath} - 18\hat{\imath} = 18\hat{\imath}$

Q.20. The effective capacitance between points A and B is (the capacitance of each of the capacitors is C)



A) C

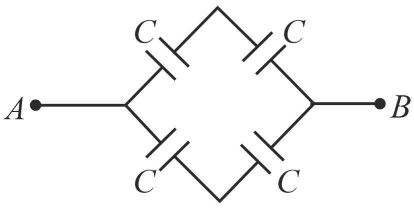
B) $\frac{C}{2}$

C) $\frac{36C}{17}$

D) $\frac{42C}{17}$

Answer: C

Solution: This is the case of a balanced Wheatstone bridge. The middle five capacitors will have no charge and will be useless.



Therefore, the equivalent capacitance of the upper and lower arms of the circuit will be,

$C_1 = \frac{C \times C}{C+C} = \frac{C}{2}$; both capacitors are in series.

Both upper and lower arm will be in parallel combination, therefore equivalent capacitance between A and B will be,

$C_{\mathrm{AB}} = \frac{C}{2} + \frac{C}{2} = C.$

Q.21. Eight small drops, each of radius r and having same charge q are combined to form a big drop. The ratio between the potentials of the bigger drop and the smaller drop is

A) $8 : 1$

B) $4 : 1$

C) $2 : 1$

D) $1 : 8$

E) $1 : 1$

Answer: $4 : 1$

Solution: Let the radius of the big drop is R and radius of small drops is r,

$\therefore$ The volume of big drop $= 8\times$ volume of the small drop

or $\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$

or $R = 2\,r$

The potential of the small drop,

$V_{\text{small}} = \frac{1}{4\pi\varepsilon}\frac{q}{r}$

The potential of the big drop,

$V_{\text{big}} = \frac{1}{4\pi\varepsilon}\frac{Q}{R} = \frac{1}{4\pi\varepsilon}\frac{8q}{2r} = \frac{1}{4\pi\varepsilon}\frac{4q}{r}$

(q is a charge on the small drop)

$\therefore\ V_{\text{big}} = 4\text{V}_{\text{small}}$

$\therefore\ \frac{V_{\text{big}}}{V_{\text{small}}} = \frac{4}{1}$

Q.22. Two charges, each equal to q, are kept at $x = -\ a$ and $x = a$ on the $x-$axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $(y\ \ll a)$ along the $y-$axis, the net force acting on the particle is proportional to

A) y

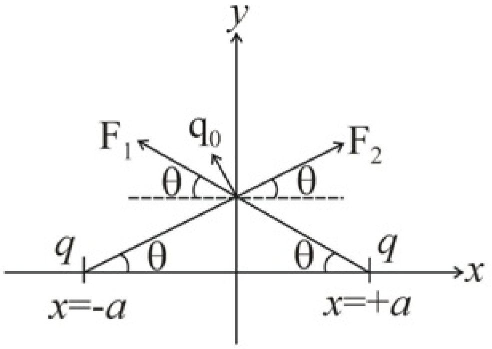
B) $-y$

C) $\frac{1}{y}$

D) $-\frac{1}{y}$

Answer: y

Solution:



From above diagram, we can see that on charge $q_0\left(=\frac{q}{2}\right)$, force due to charge q kept at $x=+a$ and $x=-a$ is F_1 and F_2.

From coulomb's law-

$$\left|\vec{F}_1\right|=\left|\vec{F}_2\right|=\frac{kqq_0}{\left(\sqrt{y^2+a^2}\right)^2}=\frac{kq^2}{2(y^2+a^2)}$$

Components of forces F_1 and F_2 along $x-$axis will cancel each other.

$$\sum F_x=F_1\cos\theta-F_2\cos\theta=\left(\frac{kq^2}{2(y^2+a^2)}-\frac{kq^2}{2(y^2+a^2)}\right)\cos\theta$$

$$\sum F_x=0$$

Along $y-\text{direction}$,

$$\sum F_y=F_1\sin\theta+F_2\sin\theta$$

$$\sum F_y=\left(\frac{kq^2}{2(y^2+a^2)}+\frac{kq^2}{2(y^2+a^2)}\right)\sin\theta=$$

$$\sum F_y=\left\{\frac{kq^2}{2(y^2+a^2)}+\frac{kq^2}{2(y^2+a^2)}\right\}\frac{y}{\left(\sqrt{y^2+a^2}\right)}$$

$$\sum F_y=\frac{kq^2y}{2(y^2+a^2)^{\frac{3}{2}}}$$

Since $y<<a$,

$$\sum F_y=\frac{kq^2y}{2a^3}$$

Net force$(F_{\text{net}})=\sqrt{F_x^2+F_y^2}=\sqrt{0+\left(\frac{kq^2y}{2a^3}\right)^2}$

$$\Rightarrow F_{\text{net}}=\frac{kq^2y}{2a^3}$$

$$\Rightarrow F_{\text{net}}\propto y$$

Q.23. A capacitor of capacitance $10\ \mu\text{F}$ is charged to a potential of $100\ \text{V}$. Now connecting it in parallel with an uncharged capacitor, the resultant potential difference becomes $40\ \text{V}$. The capacitance of this capacitor is

A) $2.5\ \mu\text{F}$

B) $5\ \mu\text{F}$

C) $10\ \mu\text{F}$

D) $15\ \mu\text{F}$

Answer: $15\ \mu\text{F}$

Solution: Let the charged capacitor be C_1 and uncharged capacitor be C_2.

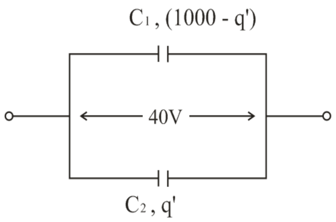
We know that, for a capacitor, $q = CV$.

So, the initial charge on C_1 is,

$q = 10 \times 100 = 1000\ \mu\text{C}$.

On connecting it with uncharged capacitor (C_2), let charge q' be transferred to the uncharged capacitor.

So, the final charge on C_1 is,$= (1000 - q')\ \mu\text{C}$.



When two capacitors will be in parallel and voltage across them will be same. Hence, both the capacitors have a potential difference of $40\ \text{V}$ across them.

Again applying, $q = CV$ on capacitor C_1,

$(1000 - q') = 10 \times 40$

$\Rightarrow q' = 600\ \mu\text{F}$.

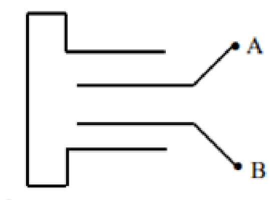
For capacitor C_2,

$q' = C_2(40)$

$\Rightarrow 600 = C_2(40)$

$\Rightarrow C_2 = 15\ \mu\text{F}$.

Q.24. Find the effective capacity between A and B.



A) $\frac{3}{2}C$

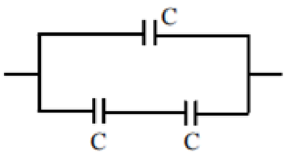
B) 3C

C) $\frac{3}{5}\text{C}$

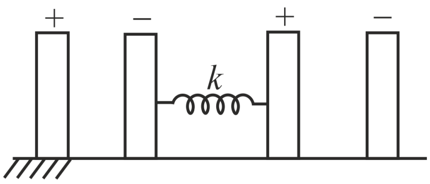
D) $\frac{2}{3}\text{C}$

Answer: $\frac{3}{2}C$

Solution: Equivalent circuit is
$C_{\text{eff}} = \frac{C}{2} + C = \frac{3}{2}C$



Q.25. Two charged capacitors have their outer plates fixed and inner plates connected by a spring of force constant k. The magnitude of charge on each capacitor is q and sign of charge is shown in figure. Find the extension in the spring at equilibrium.



A) $\frac{q^2}{2A\varepsilon_0 k}$

B) $\frac{q^2}{4A\varepsilon_0 k}$

C) $\frac{q^2}{A\varepsilon_0 k}$

D) zero

Answer: $\frac{q^2}{2A\varepsilon_0 k}$

Solution: We know that spring force $F = kx$ will balance electrostatic force of attraction.
Therefore, $\frac{q^2}{2A\varepsilon_0} = kx$
$\Rightarrow x = \frac{q^2}{2A\varepsilon_0 k}$

Practice more on Electrostatics