NEET Important Questions with Solutions from Electromagnetic Induction and Alternating Currents

Q.1. An alternating current is given by $i=i_1\cos\omega t+i_2\sin\omega t$. The rms current is given by

A) $\frac{i_1+i_2}{\sqrt{2}}$

B) $\frac{|i_1+i_2|}{\sqrt{2}}$

C) $\sqrt{\frac{i_1^2+i_2^2}{2}}$

D) $\sqrt{\frac{i_1^2+i_2^2}{\sqrt{2}}}$

Answer: $\sqrt{\frac{i_1^2+i_2^2}{2}}$

Solution: Given current,

$i=i_1\cos\omega t+i_2\sin\omega t$

$\Rightarrow i=\sqrt{i_1^2+i_2^2}\left(\frac{i_1}{\sqrt{i_1^2+i_2^2}}\cos\omega t+\frac{i_2}{\sqrt{i_1^2+i_2^2}}\sin\omega t\right)\ \ldots(1)$ (multiplying and dividing by $\sqrt{i_1^2+i_2^2}$)

Now, let us assume that,

$\frac{i_1}{\sqrt{i_1^2+i_2^2}}=\sin\phi$ and $\frac{i_2}{\sqrt{i_1^2+i_2^2}}=\cos\phi$

Putting in the equation (1), we get,

$i=\sqrt{i_1^2+i_2^2}\ (\sin\phi\cos\omega t+\cos\phi\sin\omega t)$

$\Rightarrow i=\sqrt{i_1^2+i_2^2}\sin(\omega t+\phi)$

$\Rightarrow i=i_0\ \sin(\omega t+\phi)\ \ \ldots(2)$

Where, $i_0=\sqrt{i_1^2+i_2^2}=$ peak current.

We know that rms value of a sinusoidal AC is given by:

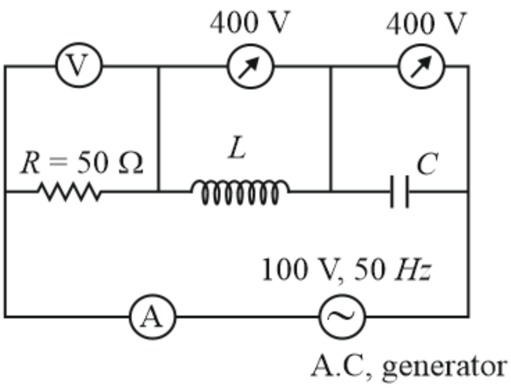
$i_{\text{rms}}=\frac{i_0}{\sqrt{2}}$

Hence, the rms value of the given AC,

$i_{\text{rms}}=\frac{\sqrt{i_1^2+i_2^2}}{\sqrt{2}}$

$\Rightarrow i_{\text{rms}}=\sqrt{\frac{i_1^2+i_2^2}{2}}$

Q.2. In the LCR series circuit, the voltmeter and ammeter readings are,



A) $E=100 \text{ V}, I=2 \text{ A}$

B) $E=100 \text{ V}, I=5 \text{ A}$

C) $E=300 \text{ V}, I=2 \text{ A}$

D) $E=300 \text{ V}, I=5 \text{ A}$

Answer: $E=100 \text{ V}, I=2 \text{ A}$

Solution: We know that for series LCR circuit, the potential difference across the inductor and the capacitor have phase difference of π.

V_L and V_C will be in opposite phase. So, they will cancel each other being equal in magnitude.

$\therefore$ Resultant potential difference=applied potential difference volts

Also, $Z=R$ as $X_L=X_C$

$I_m=\frac{V_m}{Z}=\frac{100}{50}=2 \text{ amp.}$

Q.3. A coil having an inductance of $\frac{1}{\pi}$ henry is connected in series with a resistance of 300Ω. If 20 volts from a 200 cycle source is impressed across the combination, the value of the phase angle between the voltage and the current is,

A) $\tan^{-1}\left(\frac{5}{4}\right)$

B) $\tan^{-1}\left(\frac{4}{5}\right)$

C) $\tan^{-1}\left(\frac{3}{4}\right)$

D) $\tan^{-1}\left(\frac{4}{3}\right)$

Answer: $\tan^{-1}\left(\frac{4}{3}\right)$

Solution: For an LR circuit, we have total impedance,

$Z=\sqrt{R^2+X_L{}^2}$

where, R is the resistance of the circuit,

X_L is the inductive reactance of the circuit.

Also, $X_L=2\pi fL$

$=\frac{1}{\pi}\times 2\pi\times 200=400\ \Omega$

where, f is the linear frequency,

L is the inductance of the circuit.

We know that, $\tan(\phi)=\frac{X_L}{R}$.

Putting in the values and solving, we get,

$\phi=\tan^{-1}\left(\frac{4}{3}\right)$.

Q.4. In an LR series circuit $\left(L=\frac{175}{11}\ \text{mH and } R=12\ \Omega\right)$, a variable emf source $\left(V=V_0\sin\omega t\right)$ of $V_{\text{rms}}=130\sqrt{2}$ V and frequency $50\ \text{Hz}$ is applied. The current amplitude in the circuit and phase of current with respect to voltage are, respectively $\left(\text{use, } \pi=\frac{22}{7}\right)$,

A) $14.14\ \text{A},\ 30^{\circ}$

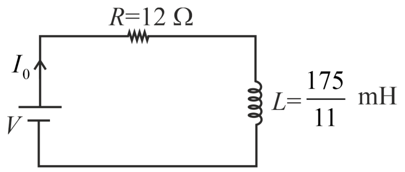
B) $10\sqrt{2}\ \text{A},\ \tan^{-1}\frac{5}{12}$

C) $10\ \text{A},\ \tan^{-1}\frac{5}{12}$

D) $20\ \text{A},\ \tan^{-1}\frac{5}{12}$

Answer: $20\ \text{A},\ \tan^{-1}\frac{5}{12}$

Solution:



Given, AC circuit has resistance, $(R)=12\ \Omega$ and inductance, $(L)=\frac{175}{11}\ \mathrm{mH}$ in series. The voltage of the source is, $V=V_0\sin\omega t$ and $V_{\text{rms}}=130\sqrt{2}\ \mathrm{V}$, frequency, $(f)=50\ \mathrm{Hz}$.

The impedance of circuit,

$(Z)=\sqrt{R^2+X_L^2}$.

Here,

$X_L=\omega L$ is inductive reactance.

$Z=\sqrt{(12)^2+\left(2\pi\times 50\times\frac{175}{11}\times 10^{-3}\right)^2}$

$Z=\sqrt{144+25}=13\ \Omega$

RMS current in the circuit,

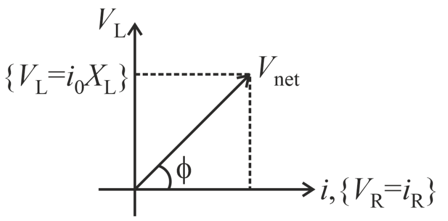
$(I_{\text{rms}})=\frac{V_{\text{rms}}}{Z}=\frac{130\sqrt{2}}{13}$

$I_{\text{rms}}=10\sqrt{2}\ \mathrm{A}$.

The peak current,

$(I_0)=\sqrt{2}I_{\text{rms}}$

$\Rightarrow I_0=20\ \mathrm{A}$.



From the above phasor diagram, the angle made by net voltage with the direction of current is ϕ,

$\therefore \tan\phi=\frac{V_L}{V_R}=\frac{I_0X_L}{I_0R}$

$\Rightarrow \tan\phi=\frac{X_L}{R}=\frac{5}{12}$

$\Rightarrow \phi=\tan^{-1}\left(\frac{5}{12}\right)$

Q.5. An alternating voltage E(in volt)$=200\sqrt{2}\sin(100t)$ is connected to a $1\ \mu\mathrm{F}$ capacitor through an AC ammeter. The reading of the ammeter shall be,

A) $10\ \mathrm{mA}$

B) $20\ \mathrm{mA}$

C) $40\ \mathrm{mA}$

D) $80\ \mathrm{mA}$

Answer: $20\ \mathrm{mA}$

Solution: The given circuit is purely capacitive. Therefore, the total impedance will be equal to the capacitive reactance.

$E=200\sqrt{2}\sin(100t)$

On comparing with the general equation, we get,

$E_{\text{peak}}=200\sqrt{2}\ \mathrm{V}$

$\omega=100\ \mathrm{rad\ s^{-1}}$.

We know that, for pure capacitive circuit,

$Z=X_C=\frac{1}{\omega C}$.

Also, AC measurement device measures rms value of the current,

$I_{\text{rms}}=\frac{E_{\text{rms}}}{Z}=\frac{E_{\text{peak}}}{\sqrt{2}Z}$.

On putting the values and solving, we get,

$I_{rms}=200\times\omega C=200\times100\times10^{-6}\ \mathrm{A}$

$=20\ \mathrm{mA}$.

Q.6. An LCR series circuit with $100\ \Omega$ resistance is connected to an AC source of $200\ \mathrm{V}$ and angular frequency 300 radians per second. When only the capacitance is removed, the current lags the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Then the current and power dissipated in LCR circuit are respectively

A) $1\ \mathrm{A},\ 200\ \mathrm{W}$

B) $1\ \mathrm{A}, 400\ \mathrm{W}$

C) $2\ \mathrm{A},\ 200\ \mathrm{W}$

D) $2\ \mathrm{A},\ 400\ \mathrm{W}$

Answer: $2\ \mathrm{A},\ 400\ \mathrm{W}$

Solution: Given,

Resistance in the circuit, $R = 100\ \Omega$

Applied AC voltage, $V_{\text{rms}} = 200\ \text{V}$

Now for the AC circuit consisting of inductor and resistors,

$\tan(60^\circ) = \frac{\omega L}{R} \quad \ldots.(1)$

Similarly, for the AC circuit consisting of a capacitor and resistor,

$\tan(60^\circ) = \frac{\frac{1}{\omega C}}{R} \quad \ldots..(2)$

So, from the equation (1) and (2),

$\omega L = \frac{1}{\omega C}$

Thus, there will be resonance in the circuit.

Then the net impedance in the circuit,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = R$$

Then the current, $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$.

$\Rightarrow I_{\text{rms}} = \frac{200}{100} = 2\ \text{A}$

Now the power will be, $P = V_{\text{rms}} I_{\text{rms}}$

$\Rightarrow P = 200 \times 2 = 400\ \text{W}$

Q.7. An LCR circuit contains $R = 50\ \Omega$, $L = 1\ \text{mH}$ and $C = 0.1\ \mu\text{F}$. The impedance of the circuit will be minimum for a frequency of:

A) $\frac{10^5}{2\pi}\ \text{Hz}$

B) $\frac{10^6}{2\pi}\ \text{Hz}$

C) $2\pi \times 10^5\ \text{Hz}$

D) $2\pi \times 10^6\ \text{Hz}$

Answer: $\frac{10^5}{2\pi}\ \text{Hz}$

Solution: Given,

$R = 50\Omega$

$L = 1\,\text{mH}$

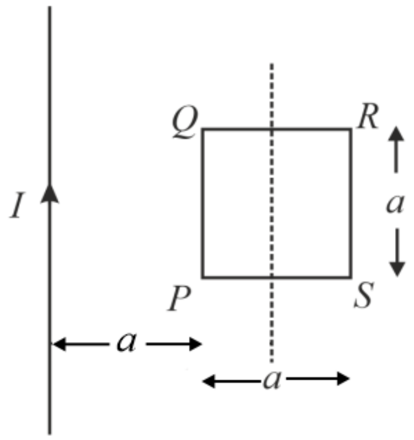
$C = 0.1\mu\text{F}$

We know that impedance of $L - C - R$ circuit will be minimum for a resonant frequency so,

$v_0 = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{1\times10^{-3}\times0.1\times10^{-6}}} = \frac{10^5}{2\pi}\text{Hz}$

Q.8. The figure shows a square loop $PQRS$ of side a and resistance r placed near an infinitely long wire carrying a constant current I. The sides PQ and RS are parallel to the wire. The wire and the loop are in the same plane. The loop is rotated by 180° about an axis parallel to the long wire and passing through the mid points of the side QR and PS. The total amount of charge which passes through any point of the loop during rotation is:



A) $\frac{\mu_0 I a}{2\pi r}\ln(2)$

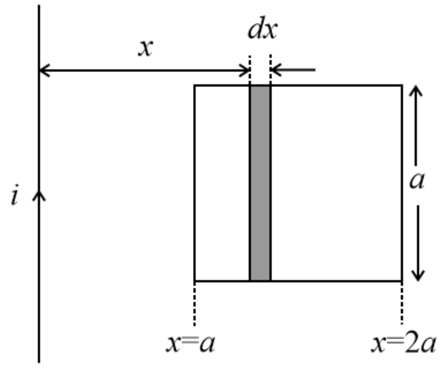
B) $\frac{\mu_0 I a}{\pi r}\ln(2)$

C) $\frac{\mu_0 I a^2}{2\pi r}$

D) Cannot be found because time of rotation not give.

Answer: $\frac{\mu_0 I a}{\pi r}\ln(2)$

Solution:



Magnetic flux density at a distance x from infinitely long current carrying wire, $B=\frac{\mu_0 I}{2\pi x}$
Flux linking with the differential area is, $d\phi = Ba\mathrm{d}x$

Total flux linking the loop initially,
$\phi=\int_a^{2a} Ba\mathrm{d}x=\int_a^{2a}\frac{\mu_0 I}{2\pi x}a\mathrm{d}x$
$=\left|\frac{\mu_0 Ia}{2\pi}\ln(x)\right|_a^{2a}=\frac{\mu_0 Ia}{2\pi}\ln(2)$
Upon rotating the loop, the total change in flux is $\Delta\phi=2\times\frac{\mu_0 Ia}{2\pi}\ln(2)=\frac{\mu_0 Ia}{\pi}\ln(2)$

Emf induced in loop, $|\varepsilon|=\frac{\Delta\phi}{\Delta t}$
Total charge that has moved, $q=i\,\Delta\,t=\frac{\mathrm{emf}}{r}\Delta t=\frac{1}{r}\frac{\Delta\phi}{\Delta t}\Delta t=\frac{\mu_0 Ia}{\pi}\ln(2)\frac{1}{r}$

Q.9. A copper disc of radius $0.1\ \mathrm{m}$ is rotated about its centre with 10 revolutions per second in a uniform magnetic field of $0.1\ \mathrm{tesla}$ with its plane perpendicular to the field. The emf induced across the radius of disc is

A) $\frac{\pi}{10}\ \mathrm{V}$

B) $\frac{2\pi}{10}\ \mathrm{V}$

C) $\pi\times10^{-2}\ \mathrm{V}$

D) $2\pi\times10^{-2}\ \mathrm{V}$

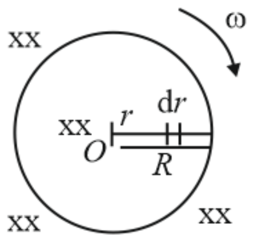
Answer: $\pi\times10^{-2}\ \mathrm{V}$

Solution: Given,

The magnitude of the magnetic field, $B=0.1\ \mathrm{T}$

The radius of disk, $R=0.1\ \mathrm{m}$

Rotational speed, $\omega=2\pi\times\text{revolution per second}=20\pi\ \mathrm{rad\ s^{-1}}$



Consider a small radial segment $\mathrm{d}r$ at distance r from the center.

The emf generated in this small elemental rod will be,

$\mathrm{d}E=vB\mathrm{d}r$, here v is the velocity of rod.

We know at any radial distance r, linear velocity will be,

$v=r\omega$

$\Rightarrow \mathrm{d}E=\omega Br\mathrm{d}r$

Since all these differential rods are connected in series, so net emf will be,

$\int \mathrm{d}E=\int_0^R \omega Br\mathrm{dr}$

$\Rightarrow E=\frac{\omega BR^2}{2}$

$\Rightarrow E=\frac{20\pi\times0.1\times(0.1)^2}{2}=\pi\times10^{-2}\ \mathrm{V}$

Q.10. Consider a conducting wire of length L bent in the form of a circle of radius R and another conductor of length $a\,(a<<R)$ is bent in the form of a square. The two loops are then placed in same plane such that the square loop is exactly at the centre of the circular loop. What will be the mutual inductance between the two loops?

A) $\mu_0\frac{\pi a^2}{L}$

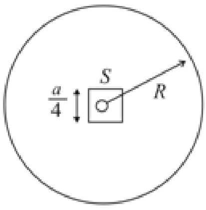
B) $\mu_0\frac{\pi a^2}{16L}$

C) $\mu_0\frac{\pi a^2}{4L}$

D) $\mu_0\frac{a^2}{4\pi L}$

Answer: $\mu_0\frac{\pi a^2}{16L}$

Solution: A conducting wire of length L bent in the form of a circle of radius R and another conductor of length a is bent in the form of a square. The two loops are then placed in same plane such that the square loop is exactly at the centre of circular loop as shown in the figure



For circular loop,

$2\pi R = L \ \ldots(\text{i})$

For square of side S,

$4S = a \ \ldots(\text{ii})$

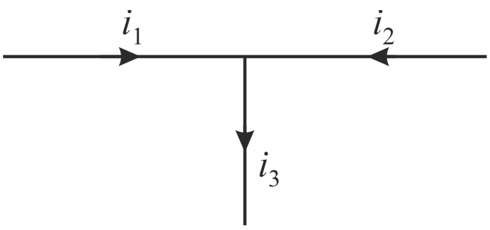
Flux linked, $\phi = \frac{\mu_0 I S^2}{2R}$

Mutual inductance, $M = \frac{\phi}{I} = \frac{\mu_0 S^2}{2R}$

Substituting R from Eq. (i) and S from Eq. (ii), we get

$M = \frac{\mu_0 a^2}{2(16)} \frac{2\pi}{L} = \frac{\mu_0 a^2 \pi}{16L}$

Q.11. In the given figure, if $i_1 = 3\sin\omega t$ and $i_2 = 4\cos\omega t$, then i_3 is



A) $5\sin(\omega t + 53°)$

B) $5\sin(\omega t + 37°)$

C) $5\sin(\omega t + 45°)$

D) $5\cos(\omega t + 53°)$

Answer: $5\sin(\omega t + 53°)$

Solution: This figure deals with the concept of charge conservation. According to the Kirchoff's current law, the amount of charge entering a junction is equal to the amount of charge leaving it.

We can see from the figure,
$i_3 = i_1 + i_2$
$= 3\ \sin(\omega t)\ +\ 4\ \cos(\omega t)$

On RHS, multiply and divide by 5, we get
$= 5\left(\frac{3}{5}\sin(\omega t) + \frac{4}{5}\cos(\omega t)\right)$

$= 5(\sin(\omega t)\cos 53^\circ + \cos(\omega t)\sin 53^\circ)\ \left[\because\ \cos 53^\circ = \frac{3}{5} \text{ and } \sin 53^\circ = \frac{4}{5}\right]$

$= 5\ \sin(\omega t + 53^\circ)$ [By using formula of $\sin(A+B)$]

Q.12. .A $8.0\ \Omega$ resistor, $5.0\ \mu\text{F}$ capacitor and $50.0\ \text{mH}$ inductor are connected in series with a variable frequency source with emf of $400\ \text{V}$ (RMS)completing the circuit. Calculate the power delivered to the circuit when the frequency is equal to one-half the resonant frequency.

A) $52\ \text{W}$

B) $57\ \text{W}$

C) $63\ \text{W}$

D) $69\ \text{W}$

Answer: $57\ \text{W}$

Solution: Average power in a RLC circuit is given by:

$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos(\phi)$

where V_{rms} and I_{rms} are the RMS value of voltage applied and current in the circuit respectively, also $\cos(\phi)$ is the power factor of the circuit.

The phase difference is ϕ.

As we know that the resonace frequency of RLC circuit is given by

$\Rightarrow \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\times10^{-3}\times5\times10^{-6}}}$

$\Rightarrow \omega = 2 \times 10^3\ \text{rad s}^{-1}$

Let ω_1 be the frequency that is one half the resonant frequency.

$\Rightarrow \omega_1 = \frac{\omega}{2}$

$\Rightarrow \omega_1 = 10^3\ \text{rad s}^{-1}$

$X_{\text{L}} = \omega_1 L$

$\Rightarrow X_{\text{L}} = 10^3 \times 50 \times 10^{-3}$

$\Rightarrow X_{\text{L}} = 50\ \Omega$

$X_{\text{C}} = \frac{1}{\omega_1 C}$

$\Rightarrow X_{\text{C}} = \frac{1}{10^3\times5\times10^{-6}}$

$\Rightarrow X_{\text{C}} = 200\ \Omega$

So the impedance of the circuit will be

$\therefore Z = \sqrt{R^2 + (X_{\text{L}} - X_{\text{C}})^2}$

$\Rightarrow Z = \sqrt{64 + (50 - 200)^2}$

$\Rightarrow Z \simeq 150\ \Omega$

Putting these values in the above equation we get the average power as:$\because \cos\ \phi = \frac{R}{Z}$

$\Rightarrow P_{\text{avg}} = \frac{(400)^2\times8}{(150)^2} = 56.8\ \text{W} \approx 57\ \text{W}$

Q.13. A $1.00\ \mu\text{F}$ capacitor is charged by a $40.0\ \text{V}$ power supply. The fully charged capacitor is then discharged through a $10.0\ \text{mH}$ inductor. Find the maximum current in the resulting oscillations.

A) $400\ \text{mA}$

B) $800\ \text{mA}$

C) $600\ \text{mA}$

D) $150\ \text{mA}$

Answer: $400\ \text{mA}$

Solution: For LC Oscillation maximum energy across the capacitor as well as the inductor will be the same. Hence $(U_C)_{\max} = (U_L)_{\max}$

$\frac{1}{2}CV^2{}_{\max} = \frac{1}{2}Li^2_{max}$

Then, $i_{max} = V_{max}\sqrt{\frac{c}{L}} = 4\sqrt{\frac{1\times10^{-6}}{10\times10^{-3}}}$

$= 0.400\ \text{A} = 400\ \text{mA}.$

Q.14. In an LR circuit, the value of L is $\left(\frac{0.4}{\pi}\right)$ henry and the value of R is $30\ \text{ohm}$. If in the circuit, an alternating emf of $200\ \text{volts}$ at $50\ \text{cycles per second}$ is connected, the impedance of the circuit and current will be,

A) $11.4\ \text{ohm}, 17.5\ \text{A}$

B) $30.7\ \text{ohm}, 6.5\ \text{A}$

C) $40.4\ \text{ohm}, 5\ \text{A}$

D) $50\ \text{ohm}, 4\ \text{A}$

Answer: $50\ \text{ohm}, 4\ \text{A}$

Solution: We know that for an LR circuit,

$X_L = \omega L = 2\pi f L$

where, ω is the angular frequency,

L is the inductance of the circuit,

f is the frequency.

Putting in the values, we get,

$2\pi \times 50 \times \frac{0.4}{\pi} = 40\ \Omega,\ R = 30\ \Omega.$

For an LR circuit, the total impedance is given by,

$\therefore Z = \sqrt{R^2 + X_L^2}.$

Putting in the values,

$Z = \sqrt{30^2 + 40^2} = 50\ \Omega.$

Current in the circuit is given by,

$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200}{50} = 4\ \text{A}.$

Q.15. An electric motor operating on a $50\ \text{V}$ DC supply draws a current of $12\ \text{A}$. If the efficiency of the motor is 30%, estimate the resistance of the windings of the motor.

A) $3.2\ \Omega$

B) $2.9\ \Omega$

C) $4.8\ \Omega$

D) $5.6\ \Omega$

Answer: $2.9\ \Omega$

Solution: We know that the efficiency of an electric generator is given by,

$\eta = \frac{V_{\text{output}}}{V_{\text{input}}}$

Given in the question that $\eta = 30\%,\ V_{\text{in}} = 50\ \text{V}$ and $I = 12\ \text{A}$.

$\Rightarrow V_{\text{output}} = 30\%\ \text{of}\ V_{\text{input}}$

$\Rightarrow V_{\text{output}} = \frac{30}{100} \times 50$

$\Rightarrow V_{\text{output}} = 15\ \text{V}$

Also for motors, the output voltage can be defined as

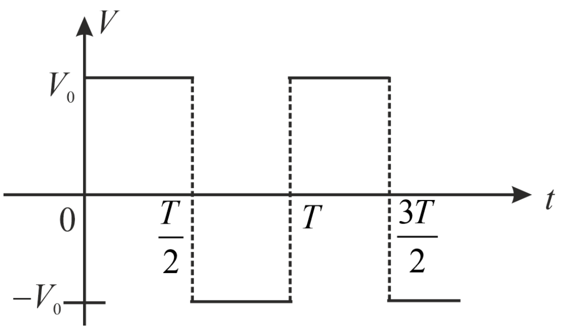
$V_{\text{output}} = V_{\text{input}} + IR$

$\Rightarrow 50 = 15 + 12 \times R$

$\Rightarrow 12 \times R = 35$

$\Rightarrow R = \frac{35}{12} = 2.9\ \Omega$

Q.16. The mean and the RMS value of an alternating voltage for half cycle, as shown in the figure, respectively, are,



A) V_0, V_0

B) $\frac{V_0}{2}, V_0$

C) $\frac{3V_0}{2}, \frac{V_0}{2}$

D) $\frac{V_0}{4}, \frac{V_0}{2}$

Answer: V_0, V_0

Solution: The mean value of the alternating voltage over half cycle is defined as the average value of voltage over a time period, $t = 0 \text{ to } t = \frac{T}{2}$.

For the given question, the mean value of voltage is given by,

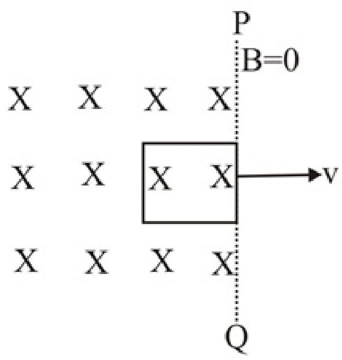
$$V_{mean} = \frac{\int_0^t v_0 \mathrm{d}t}{\int_0^t \mathrm{d}t} = \frac{\int_0^{\frac{T}{2}} v_0 \mathrm{d}t}{\int_0^{\frac{T}{2}} \mathrm{d}t} = V_0$$

The RMS value of alternating voltage is defined as the root mean square value of the alternating voltage over the time period, $t = 0 \text{ to } t = \frac{T}{2}$.

So, the RMS value of voltage in the given question is given by, $V_{rms} = \sqrt{\frac{\int_0^t V_0^2 \mathrm{d}t}{\int_0^t \mathrm{d}t}} = \sqrt{\frac{\int_0^{\frac{T}{2}} V_0^2 \mathrm{d}t}{\int_0^{\frac{T}{2}} \mathrm{d}t}} = V_0$

As we can see, both the mean value and RMS value of the voltage are same for the half cycle and are equal to V_0
.

Q.17. Figure shows a square loop of side 0.5 m and resistance 10Ω. The magnetic field on left side of line PQ has a magnitude $B = 1.0 \text{ T}$. The work done in pulling the loop out of the field uniformly in 2.0 s is



A) $3.125 \times 10^{-3} \text{ J}$

B) $6.25 \times 10^{-4} \text{ J}$

C) $1.25 \times 10^{-2} \text{ J}$

D) $5.0 \times 10^{-4} \text{ J}$

Answer: $3.125 \times 10^{-3} \text{ J}$

Solution: Given, side of the square loop is, $l = 0.5\ m$,

resistance of the loop is, $R = 10\ \Omega$,

Magnitude of the magnetic field, $B = 1\ T$,

time, $t = \ 2\ s$.

Force required to pull out the loop out of the field is given as;

$F = \frac{B^2 l^2 v}{R}$ (magnitude only)

Velocity with which the loop can be pulled is given

as;

$v = \frac{l}{t} = \frac{0.5}{2} = 0.25\ ms^{-1}$

So, we have;

$F = \frac{1^2 \times 0.5^2 \times 0.25^2}{10} = 0.00625\ N$,

So, the work done in pulling the loop uniformly is;

$|W| = Fl$

$\Rightarrow W = 0.00625 \times 0.5 = 0.003125\ \text{J}$

$\Rightarrow W = 3.125 \times 10^{-3}\ \text{J}$

Q.18. The peak value of an alternating e.m.f given by $E = E_0\ \cos\ \omega t$, is $10\ \text{V}$ and frequency is $50\ \text{Hz}$. At time $t = \left(\frac{1}{600}\right)$ sec, the instantaneous value of e.m.f is:

A) $10\ \text{Volt}$

B) $5\ \sqrt{3}\ \text{Volt}$

C) $5\ \text{Volt}$

D) $1\ \text{Volt}$

Answer: $5\ \sqrt{3}\ \text{Volt}$

Solution: Given,

General equation of AC voltage, $E = E_{\circ} \cos \omega t$.

The peak value of e.m.f, $E_{\circ} = 10\ \text{V}$

Frequency of voltage, $f = 50\ \text{Hz}$

So, the angular frequency, $\omega = 2\pi f$.

$\Rightarrow \omega = 2\pi \times 50 = 100\pi\ \text{rad s}^{-1}$

Now by putting the values in the general equation,

$E = 10 \cos\left(100\pi \times \frac{1}{600}\right) = 10 \cos\left(\frac{\pi}{6}\right)$

$\Rightarrow E = 5\sqrt{3}\ \text{V}$

Q.19. A coil of 1000 turns is wound on a book and this book is lying on the table. The vertical component of the earth's magnetic field is $0.6 \times 10^{-4}\ \text{T}$ and the area of the coil is $0.05\ \text{m}^2$. The book is turned over once about a horizontal axis in $0.1\ \text{s}$. This average emf induced in the coil is

A) $0.03\ \text{V}$

B) $0.06\ \text{V}$

C) $0 \mathrm{~V}$

D) $0.6 \mathrm{~V}$

Answer: $0.06 \mathrm{~V}$

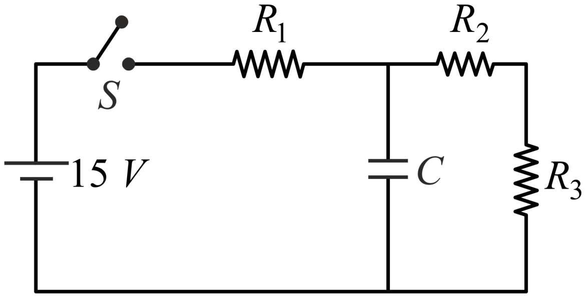
Solution: Induced emf is given by the rate of change of flux,

$e=\frac{-\Delta \phi}{\Delta t}=\frac{N B A\left(\cos 0^{\circ}-\cos 180^{\circ}\right)}{0.1}$

$e=\frac{2 N B A}{0.1}$

$e=\frac{2 \times 1000 \times 0.6 \times 10^{-4} \times 0.05}{0.1}=0.06 \mathrm{~V}$

Q.20. The figure shows a battery with emf $15 \mathrm{~V}$ in a circuit with $R_1=30 \Omega$, $R_2=10 \Omega$, $R_3=20 \Omega$ and capacitance $C=10 \mu \mathrm{F}$. The switch S is initially in the open position and is then closed at the time $t=0$. What will be the final steady-state charge on the capacitor?



A) $75 \mu \mathrm{C}$

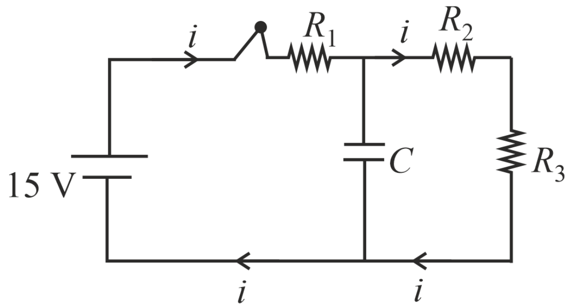
B) $50 \mu \mathrm{C}$

C) $10 \mu \mathrm{C}$

D) None of these

Answer: $75 \mu \mathrm{C}$

Solution: In a steady-state, the capacitor will behave as an open circuit, therefore, the resistance across the circuit will be,



All the resistances will be in series with each other and the equivalent resistance of the circuit will be given by,

$R_{eq} = R_1 + R_2 + R_3 = 30 + 10 + 20$

$R_{eq} = 60\ \Omega.$

The current through the circuit is given by,

$i = \frac{15}{R_{eq}} = \frac{15}{60} = 0.25\ \text{A}$

The capacitor is connected in parallel to the series combination of R_2 and R_3, therefore, the potential across the capacitor will be,

$V_{23} = i \times (R_2 + R_3) = 0.25 \times (10 + 20)$

$V_{23} = 7.5\ \text{V}.$

The charge stored in a capacitor is given by,

$Q = CV_{23} = 10 \times 7.5$

$Q = 75\ \mu\text{C}.$

Q.21. An induction coil stores $32\ \text{J}$ of magnetic energy and dissipates energy, as heat, at the rate of $320\ \text{W}$, when a current of $4\ \text{A}$ is passed through it. Find the time constant of the circuit, when the coil is joined across a battery.

A) $0.2\ \text{s}$

B) $0.1\ \text{s}$

C) $0.3\ \text{s}$

D) $0.4\ \text{s}$

Answer: $0.2\ \text{s}$

Solution: Magnetic energy, which is stored in an inductor, is given by,

$E=\frac{1}{2}LI^2$

As given in the question, $\frac{1}{2}LI^2=32$ and $I=4\text{ A}$.

$\Rightarrow \frac{1}{2}L(4)^2=32$

$\Rightarrow L=4\text{ H}$

The power dissipated in the inductor is given by,

$P=I^2R$

$\Rightarrow 320=\left(4\right)^2\times R$

$\Rightarrow R=\frac{320}{16}=20\ \Omega$

The time constant of an $L-R$ circuit will be,

$\tau=\frac{L}{R}=\frac{4}{20}=0.2\text{ s}$

Q.22. The equation of AC voltage is $E=220\sin\left(\omega t+\frac{\pi}{6}\right)$ V and the AC current is $I=10\sin\left(\omega t-\frac{\pi}{6}\right)$ A. The average power dissipated is:

A) 150 W

B) 550 W

C) 250 W

D) 50 W

Answer: 550 W

Solution: Given,

$E=220\ \sin\left(\omega t+\frac{\pi}{6}\right)$

$I=10\ \sin\left(\omega t-\frac{\pi}{6}\right)$

$E_0=220\text{ V}$

$I_0=10\text{ A}$

We know that $Z=\frac{E_0}{I_0}$

$\Rightarrow Z=\frac{220}{10}=22\ \Omega$

We can calculate the phase difference from the equations of the current and the voltage.

$\phi=\left[\frac{\pi}{6}-\left(-\frac{\pi}{6}\right)\right]=\frac{\pi}{3}$

Average power for an AC circuit is,

$P_a=V_{rms}I_{rms}\cos(\phi)$

$P_a=\frac{E_0}{\sqrt{2}}\times\frac{10}{\sqrt{2}}\times\cos\phi$

$=\frac{220}{\sqrt{2}}\times\frac{10}{\sqrt{2}}\times\cos\left(\frac{\pi}{3}\right)=550\text{ W}$

Q.23. When 100 V DC is supplied across a solenoid, a current of 1.0 A flows in it. When 100 V AC is applied across the same coil, the current drops to 0.5 A. If the frequency of ac source is 50 Hz, then the impedance and inductance of the solenoid are

A) $200\ \Omega$ and 0.55 H.

B) $100\ \Omega$ and 0.86 H.

C) $200\ \Omega$ and 1.0 H.

D) $100\ \Omega$ and 0.93 H.

Answer: $200\ \Omega$ and 0.55 H.

Solution: For DC supply, solenoid inductance will be zero. Therefore,

$R=\frac{V}{i}=\frac{100}{1}=100\ \Omega$

For AC supply, inductor with resistor will work effectively. Therefore,

$Z=\frac{V}{i}=\frac{100}{0.5}=200\ \Omega$

But,

$Z=\sqrt{R^2+(\omega L)^2}$

$200=\sqrt{(100)^2+4\pi^2(50)^2L^2}$

$\therefore\ L=0.55$ H

Q.24. A coil of area $10\ \text{cm}^2$ and 10 turns with a magnetic field directed perpendicular to the plane and is changing at the rate of 10^8 gauss per second. The resistance of the coil is $20\ \Omega$. The current in the coil will be

A) 5 amp.

B) 0.05 amp.

C) 0.5 amp.

D) 5×10^8 amp.

Answer: 5 amp.

Solution: Given,

the resistance in the circuit, $R=20\ \Omega$,

the rate of change in magnetic flux, $\frac{dB}{dt}=10^8$ gauss per sec $=10^4\ \text{T s}^{-1}$,

the area of the coil, $A=10\ \text{cm}^2=0.001\ \text{m}^2$,

the number of turns of the coil, $N=10$.

Now, the rate of change in the magnetic flux through the coil will be, $\frac{d\phi}{dt}=NA\frac{dB}{dt}$

$\Rightarrow\frac{d\phi}{dt}=10\times0.001\times10^4=100.$

According to Faraday's law of electromagnetic induction, EMF in the circuit will be,

$|\varepsilon|=\left|-\frac{d\phi}{dt}\right|=100\ \text{V}.$

Now, we know that $\varepsilon=iR$

$\Rightarrow i=\frac{\varepsilon}{R}=\frac{100}{20}=5$ amp.

Q.25. A square coil $10^{-2}\ \text{m}^2$ area is placed perpendicular to a uniform magnetic field of intensity $10^3\ \text{Wb m}^{-2}$. The magnetic flux through the coil is

A) $10 \mathrm{~Wb}$

B) $10^{-5} \mathrm{Wb}$

C) $10^{5} \mathrm{Wb}$

D) $100 \mathrm{~Wb}$

Answer: $10 \mathrm{~Wb}$

Solution: Given: The area of coil $A=10^{-2} \mathrm{~m}^{2}$ and magnetic field intensity $B=10^{3} \mathrm{~Wb} \mathrm{~m}^{-2}$.

The angle between magnetic field and area vector is $\theta=0^{\circ}$, the magnetic flux through the given coil is given by,

$\phi=B A \cos \theta$

$\phi=10^{-2} \times 10^{3} \times \cos 0^{\circ}$

$\phi=10 \mathrm{~Wb}$.

Practice more on Electromagnetic Induction and Alternating Currents