NEET Important Questions with Solutions from Atoms and Nuclei

Q.1. Two isotopes P and Q of atomic weight 10 and 20, respectively, are mixed in equal amounts by weight. After 20 days, their weight ratio is found to be $1:4$. Isotope P has a half-life of 10 days. The half-life of isotope Q is

A) zero

B) 5 days

C) 20 days

D) infinite

Answer: infinite

Solution: Number of nuclei are proportional to number of moles.

Number of the mole is, $\frac{m}{M}=\left(\frac{\text{Mass of substance}}{\text{Molar mass}}\right)$

The ratio of the initial number of atoms of P and Q is,

$$\frac{(N_0)_P}{(N_0)_Q}=\frac{m_1}{m_2}\times\frac{M_2}{M_1}=1\times\frac{20}{10}=2$$

The ratio of final number of atoms of P and Q is,

$$\frac{N_P}{N_Q}=\frac{m_{1,\text{final}}}{m_{2,\text{final}}}\times\frac{M_2}{M_1}=\frac{1}{4}\times\frac{20}{10}=\frac{1}{2}$$

Let n_P and n_Q be the number of half-lives of the samples of P and Q, respectively. Let T be the half-life of Q.

$n_P=\frac{\text{Age}}{\text{Half life}}=\frac{20}{10}=2$ and $n_Q=\frac{\text{Age}}{\text{Half life}}=\frac{20}{T}$

According to the radioactive law, number of undecayed nuclei left after n half-lives is, $N=\frac{N_0}{2^n}$

$$\therefore \frac{N_P}{N_Q}=\frac{(N_0)_P}{(N_0)_Q}\times\frac{2^{n_Q}}{2^{n_P}}$$

$$\frac{1}{2}=2\times\frac{2^{n_Q}}{2^2}$$

$$\Rightarrow 2^{n_Q}=1=2^0$$

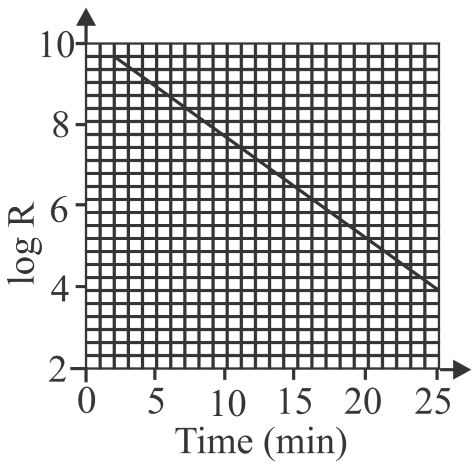
$$\Rightarrow n_Q=0$$

So,

$$\frac{20}{T}=0$$

$$\Rightarrow T=\infty$$

Q.2. The graph shows the log of activity $\log R$ of a radioactive material as a function of time t in minutes.



The half-life (in minute) for the decay is closest to

A) 2.1

B) 3.0

C) 3.9

D) 4.4

Answer: 3.0

Solution: Activity of a radioactive sample is given by

$R=-\frac{dN}{dt}=-\frac{d}{dt}N_0e^{-\lambda t}=\lambda N_0\cdot e^{-\lambda t}$

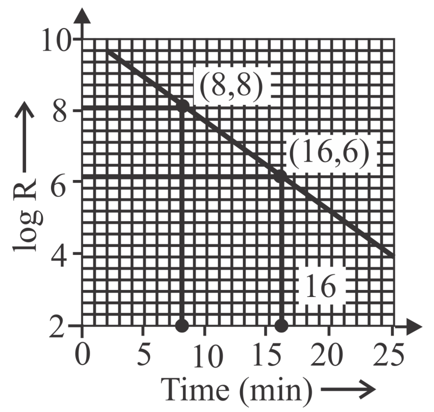
$\text{So}, \log R=\log(\lambda N_0)+\log\left(e^{-\lambda t}\right)$

$\Rightarrow \log R=-\lambda t+\log(\lambda N_0)$

This equation is form of $y=mx+C$

So, absolute value of slope of $\log R$ versus t graph gives decay constant λ.

Now, from graph,

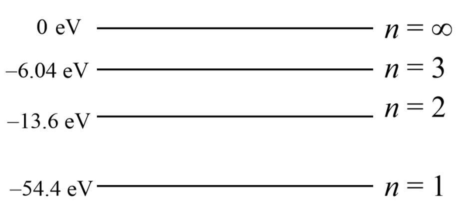


We get, slope $=\left|\frac{8-6}{8-16}\right|=\frac{1}{4}=\lambda$

So, half-life time period of sample is

$T_{1/2}=\frac{\log 2}{\lambda}=\frac{0.693}{1/4}\approx 3.0\ \text{min}$

Q.3. The energy level diagram for a hydrogen-like atom is shown in the figure. The radius of its first Bohr orbit is



A) $0.265\ \mathring{A}$

B) $0.53\ \mathring{A}$

C) $0.132\ \mathring{A}$

D) None of these

Answer: $0.265\ \overset{\circ}{\mathrm{A}}$

Solution: From Bohr's Model, the energy of the electron is:

$E_n = -13.6\frac{Z^2}{n^2}$ eV and radius of orbit is $r_n = 0.53\frac{n^2}{Z}\ \overset{\circ}{\mathrm{A}}$.

Given for a H-like atom, for $n=1$,

$E_1 = -54.4$ eV, so,

$-54.4 = -13.6\frac{Z^2}{1^2}$ or $Z=2$

Then radius of first Bohr orbit is

$r_1 = 0.53 \times \frac{1}{2} = 0.265\ \overset{\circ}{\mathrm{A}}$

Q.4. Which of the series of hydrogen atom spectrum lies in the visible region of electromagnetic spectrum?

A) Lyman series

B) Pfund series

C) Balmer series

D) Brackett series

Answer: Balmer series

Solution: Wavelength of Balmer series is given by

$= \frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{n_i^2}\right]$

When transition of electron takes place from $n_i = \infty$ to $n_f = 2$ wavelength of emitted photon is minimum, given by

$\frac{1}{\lambda_{\min}} = R_H\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right]$ or $\lambda_{\min} = 3646\overset{\circ}{\mathrm{A}}$

The range of the wavelength thus calculated is equivalent to the wavelength range at visible region of spectrum.

$\therefore$ Balmer series lie in the visible region of electromagnetic spectrum.

Q.5. M_{p} denotes the mass of a proton and M_{n} denotes that of a neutron. A given nucleus of binding energy B contains Z protons and N neutrons. The mass $M\ (N, Z)$ of the nucleus is given by (c is velocity of light),

A) $M\ (N, Z) = NM_{\mathrm{n}} + ZM_{\mathrm{p}} + Bc^2$

B) $M\left(N, Z\right) = NM_{\mathrm{n}} + ZM_{\mathrm{p}} - \frac{B}{c^2}$

C) $M\left(N, Z\right) = NM_{\mathrm{n}} + ZM_{\mathrm{p}} + \frac{B}{c^2}$

D) $M\ (N, Z) = NM_{\mathrm{n}} + ZM_{\mathrm{p}} - Bc^2$

Answer: $M\left(N, Z\right) = NM_{\mathrm{n}} + ZM_{\mathrm{p}} - \frac{B}{c^2}$

Solution: $Zp + Nn \rightarrow {}^{N+Z}_{Z}X + \text{energy}\ (B)$

We know that the binding energy, $BE = (\Delta m)c^2$
where, $\Delta \mathrm{m}$ is the mass defect which is given by,

$(\Delta m) = [ZM_\mathrm{p} + (A - Z)M_\mathrm{n}] - M\ (N, Z).$

Now, from the question,

$B = [(ZM_\mathrm{p} + NM_\mathrm{n}) - M\ (N, Z)]c^2$

$\Rightarrow \frac{B}{c^2} = (ZM_\mathrm{p} + NM_\mathrm{n}) - M\ (N, Z).$

Now,
$M\ (N, Z) = (ZM_\mathrm{p} + NM_\mathrm{n}) - \frac{B}{c^2}.$

Q.6. If ${}_{92}\mathrm{U}^{238}$ changes to ${}_{85}\mathrm{At}^{210}$ by a series of α- and β-decays, the number of α- and β-decays undergone, respectively, is

A) $7, 5$

B) $7, 7$

C) $5, 7$

D) $7, 9$

Answer: $7, 7$

Solution: In an α-decay $\left({}^{4}_{2}\mathrm{He}^{2+}\right)$, mass number decreases by 4 and atomic number decreases by 2.

In β^--decay $(n \rightarrow p^+ + e^-)$, mass number remains same while atomic number increases by 1.

Therefore, if ${}_{92}\mathrm{U}^{238}$ changes to ${}_{85}\mathrm{At}^{210}$, the change in mass number $\Delta m = 238 - 210 = 28$.

So, the number of α-decay $= \frac{\Delta m}{4} = \frac{28}{4} = 7$

The change in atomic number due to 7 α-decays $= 7 \times 2 = 14$

Number of β^--decay $= 14 - (92 - 85) = 7$

Q.7. The binding energies per nucleon for a deuteron and an α-particle are x_1 and x_2, respectively. What will be the energy Q released in the reaction ${}_1\mathrm{H}^2 + {}_1\mathrm{H}^2 \rightarrow {}_2\mathrm{He}^4 + Q$?

A) $4\left(x_1 + x_2\right)$

B) $4\left(x_2 - x_1\right)$

C) $2\left(x_1 + x_2\right)$

D) $2\left(x_2 - x_1\right)$

Answer: $4\left(x_2 - x_1\right)$

Solution: The binding energy of a nucleus $=$ binding energy per nucleon $\times$ total number of nucleons in the nucleus.

So, the BE of deuteron $= x_1 \times 2 = 2x_1$

and BE of α-particle $= x_2 \times 4 = 4x_2$

${}_1H^2 + {}_1\mathrm{H}^2 \rightarrow {}_2\mathrm{He}^4 + Q$

For the above reaction,

Total binding energy of the products $= 4x_2$

Similarly, total binding energy of the reactants $= 2x_1 + 2x_1 = 4x_1$

$Q = \mathrm{BE}_{\text{products}} - \mathrm{BE}_{\text{reactants}}$

$Q = 4x_2 - 4x_1$

$Q = 4(x_2 - x_1)$

Q.8. The activity of a radioactive sample is measured as 9750 counts per minute at $t = 0$ and as 975 counts per minute at $t = 5$ minutes. The decay constant is approximately,

A) 0.922 per minute.

B) 0.691 per minute.

C) 0.461 per minute.

D) 0.230 per minute.

Answer: 0.461 per minute.

Solution: According to radioactivity decay law, the number of decays per second is directly proportional to the number of active samples.

$\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda N$

$9750 = \lambda N_0$ .... (i)

$975 = \lambda N$ ... (ii)

Dividing (i) by (ii),

$\frac{N}{N_0} = \frac{1}{10}$.

We know that,

$\lambda = \frac{2.303}{t} \log \frac{N_0}{N}$

$= \frac{2.303}{5} \log 10$

$= 0.4606 = 0.461$ per minute.

Q.9. The shortest wavelength in Lyman series is $91.2\ \mathrm{nm}$. The longest wavelength of the series is

A) $121.6\ \mathrm{nm}$

B) $182.4\ \mathrm{nm}$

C) $234.4\ \mathrm{nm}$

D) $364.8\ \mathrm{nm}$

Answer: $121.6\ \mathrm{nm}$

Solution: The wavelength λ of lines is given by

$\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$

For Lyman series, the shortest wavelength is for n=∞ and longest is for $n=2$.

$\therefore \frac{1}{\lambda_s}= R\left(\frac{1}{1^2}\right) \quad \ldots(\mathrm{i})$

$\frac{1}{\lambda_L}= R\left(\frac{1}{1}-\frac{1}{2^2}\right)=\frac{3}{4}R \quad \ldots(\mathrm{ii})$

Dividing equation (ii) by (i), we get,

$\frac{\lambda_L}{\lambda_s}=\frac{4}{3}$

Given, λ_s =$91.2\ \mathrm{nm}$

$\Rightarrow \lambda_L$= $91.2\times\frac{4}{3}$ =$121.6\ \mathrm{nm}$

Q.10. A hydrogen atom is in the 4^{th} excited state, then:

A) the maximum number of emitted photons will be 10.

B) the maximum number of emitted photons will be 6.

C) it can emit three photons in ultraviolet region.

D) if an infrared photon is generated, then a visible photon may follow this infrared photon.

Answer: if an infrared photon is generated, then a visible photon may follow this infrared photon.

Solution: The hydrogen atom is in $n=5$ state.

$\therefore$ Maximum number of possible photons $=4$

To emit a photon in the ultraviolet region, it must jump to $n=1$, because only Lyman series lies in $U.V.$ region. Once it jumps to $n=1$ photon, it reaches its ground state and no more photons can be emitted. So only one photon in $U.V.$ range can be emitted.

If H atom emits a photon and then another photon of Balmer series, option D will be correct.

Q.11. A radioactive sample, at any instant, has its disintegration rate 5000 disintegration per minute. After 5 min, the rate is 1250 disintegration per minute. Then, the decay constant (per min) is

A) $0.4\ \ln(2)$

B) $0.2\ \ln(2)$

C) $0.1\ \ln(2)$

D) $0.8\ \ln(2)$

Answer: $0.4\ \ln(2)$

Solution: Rate of disintegration is the activity of the sample.

According to the question, initial activity at time $t=0$ is
$A_0=5000\ \text{disintegration per minute}$

And activity at time $t=5$ s is
$A=1250\ \text{disintegration per minute}$

Now, according to radioactive decay law,

$A=A_0e^{-\lambda t}$

where, λ is the decay constant.

Putting the values, we get,

$1250=5000e^{-5\lambda}$
$\Rightarrow e^{-5\lambda}=\frac{1}{4}$
$\Rightarrow e^{5\lambda}=4$

$\Rightarrow 5\lambda=2\ \ln(2)$
$\Rightarrow \lambda=\frac{2\ \ln(2)}{t}=\frac{2\ \ln(2)}{5}$
$\Rightarrow \lambda=\ 0.4\ \ln(2)\ \text{min}^{-1}$

Q.12. The initial ratio of active nuclei in two different samples is $2:3$. Their half-lives are $2\ \text{hours}$ and $3\ \text{hours}$, respectively. The ratio of their activities at the end of $12\ \text{hours}$ is

A) $1:6$

B) $6:1$

C) $1:4$

D) $4:1$

Answer: $1:4$

Solution: According to radioactive decay law, activity is given by,

$A=A_0 e^{-\lambda t}=\lambda N_0 e^{-\lambda t}\ \ldots(1)$
where, $A_0=\lambda N_0$
$A_0=$ Initial activity of the radioactive sample $(t=0)$
$A=$ Activity after time t of the radioactive sample
$N_0=$ Initial active nuclei of the radioactive sample $(t=0)$
$\lambda=$ Decay constant
Given:
Time, $t=12$ hours

Initial ratio of active nuclei in two given samples, $\frac{(N_0)_1}{(N_0)_2}=\frac{2}{3}$

Half lives,
$\left(T_{1/2}\right)_1=2$ hours (Half life of sample 1)
$\left(T_{1/2}\right)_2=3$ hours (Half life of sample 2)
Decay constant is given by, $\lambda=\frac{\ln(2)}{T_{1/2}}=\frac{0.693}{T_{1/2}}$

$\therefore \frac{\lambda_1}{\lambda_2}=\frac{\left(T_{1/2}\right)_2}{\left(T_{1/2}\right)_1}=\frac{3}{2}$

From equation (1),
$\frac{A_1}{A_2}=\frac{\lambda_1(N_0)_1}{\lambda_2(N_0)_2}\left[e^{-(\lambda_1-\lambda_2)t}\right]$
Substituting the given values in the above formula,

$\frac{A_1}{A_2}=\left(\frac{3}{2}\right)\left(\frac{2}{3}\right)e^{-\left(\frac{\ln(2)}{\left(T_{1/2}\right)_1}-\frac{\ln(2)}{\left(T_{1/2}\right)_2}\right)\times 12}$
$\Rightarrow \frac{A_1}{A_2}=e^{-\ln(2)\times\left(\frac{1}{2}-\frac{1}{3}\right)12}$
$\Rightarrow \frac{A_1}{A_2}=e^{-2\ln(2)}$
$\Rightarrow \frac{A_1}{A_2}=e^{-\ln(2^2)}$
$\Rightarrow \frac{A_1}{A_2}=\frac{1}{4}$

Q.13. The half-life of radioactive Radon is 3.8 days. The time at the end of which $\left(\frac{1}{20}\right)^{\text{th}}$ of the Radon sample will remain undecayed is (given $\log_{10} \mathrm{e}=0.4343$)

A) 13.8 days

B) 16.5 days

C) 33 days

D) 76 days

Answer: 16.5 days

Solution: Given:

Half-life of the sample, $t_{1/2}=3.8\ \text{days}$

Number of Radon nucleus remaining, $\frac{N_0}{20}$

where, N_0 is the initial number of Radon nucleus present.

Decay constant of the sample,

$\lambda=\frac{0.693}{t_1/_2}=\frac{0.693}{3.8}=0.182\ \text{days}^{-1}$

Time taken by the radioactive substance remain N number of nucleus.

$t=\frac{2.303}{\lambda}\log\left(\frac{N_0}{N}\right)$

$\Rightarrow t=\frac{2.303}{0.182}\log(20)=16.46\ \text{days}$

Q.14. In an inelastic collision, an electron excites a hydrogen atom from its ground state to a M-shell state. A second electron collides instantaneously with the excited hydrogen atom in the M-state and ionises it. At least how much energy the second electron transfers to the atom in the M-state?

A) $+3.4\ \text{eV}$

B) $+1.51\ \text{eV}$

C) $-3.4\ \text{eV}$

D) $-1.51\ \text{eV}$

Answer: $-1.51\ \text{eV}$

Solution: Given that the electron is in M-state. This corresponds to the principle of quantum number $n=3$.

From Bohr's model, the energy of a state with quantum number n is given by,

$E=-\frac{13.6}{n^2}$.

Thus, the energy of the electron in M-shell is,

we know that $E_{\text{m}}=-\frac{13.6}{(n)^2}$

$E_{\text{m}}=-\frac{13.6\ \text{eV}}{(3)^2}=-1.51\ \text{eV}$

In order to ionise the atom, the minimum energy required is $+1.51\ \text{eV}$.

Q.15. A radioactive nuclide is produced at the constant rate of n per second (say, by bombarding a target with neutrons). The expected number N of nuclei in existence t seconds after the number is N_0 is given by

A) $N=N_0e^{-\lambda t}$

B) $N=\frac{\text{n}}{\lambda}+N_0e^{-\lambda t}$

C) $N=\frac{\text{n}}{\lambda}+\left(N_0-\frac{\text{n}}{\lambda}\right)e^{-\lambda t}$

D) $N=\frac{\text{n}}{\lambda}+\left(N_0+\frac{\text{n}}{\lambda}\right)\text{e}^{-\lambda\text{t}}$

Answer: $N=\frac{\text{n}}{\lambda}+\left(N_0-\frac{\text{n}}{\lambda}\right)e^{-\lambda t}$

Solution: Rate of change of nuclei $=$Rate of production of nuclei $-$Rate of decay

$\Rightarrow \frac{\mathrm{d}N}{\mathrm{d}t}=\mathrm{n}-\lambda N \quad [\lambda=\text{Decay Constant}]$

$\Rightarrow \mathrm{d}N=\left(\mathrm{n}-\lambda N\right)\mathrm{d}t$

$\Rightarrow \frac{\mathrm{d}N}{(\mathrm{n}-\lambda N)}=\mathrm{d}t$

According to question, at $t=0$, $N=N_0$

Integrating both sides and putting the limits, we get,

$\int_{N_0}^{N}\frac{\mathrm{d}N}{\mathrm{n}-\lambda N}=\int_0^t \mathrm{d}t$

$\Rightarrow -\frac{1}{\lambda}\int_{N_0}^{N}\frac{-\lambda \mathrm{d}N}{\mathrm{n}-\lambda N}=\int_0^t \mathrm{d}t$

$\Rightarrow -\frac{1}{\lambda}[\log_e(\mathrm{n}-\lambda N)]_{N_0}^{N}=t$

$\Rightarrow -\frac{1}{\lambda}\left[\log_e\left(\frac{\mathrm{n}-\lambda N}{\mathrm{n}-\lambda N_0}\right)\right]=t$

$\Rightarrow \lambda t=\left[\log_e\left(\frac{\mathrm{n}-\lambda N_0}{\mathrm{n}-\lambda N}\right)\right]$

$\mathrm{e}^{\lambda \mathrm{t}}=\frac{\mathrm{n}-\lambda \mathrm{N}_0}{\mathrm{n}-\lambda \mathrm{N}}$

$\mathrm{n}-\lambda N=(\mathrm{n}-\lambda N_0)\mathrm{e}^{-\lambda \mathrm{t}}$

$N=\frac{\mathrm{n}}{\lambda}+\left(N_0-\frac{\mathrm{n}}{\lambda}\right)\mathrm{e}^{-\lambda \mathrm{t}}$

Q.16. Which one of the following statements is wrong in the context of $X-$rays generated from a $X-$ray tube?

A) Wavelength of characteristic $X-$rays decreases when the atomic number of the target increases.

B) Cut-off wavelength of the continuous $X-$rays depends on the atomic number of the target.

C) Intensity of the characteristic $X-$rays depends on the electrical power given to the $X-$ray tube.

D) Cut-off wavelength of the continuous $X-$rays depends on the energy of the electrons in the $X-$ray tube.

Answer: Cut-off wavelength of the continuous $X-$rays depends on the atomic number of the target.

Solution: The frequency of Characteristic X-ray is given by Moseley's Law

$\sqrt{f}=a\left(Z-b\right) \Rightarrow \left(\frac{1}{\lambda}\right)\propto(Z-b)^2$

So, wavelength of characteristic $X-$rays decreases when the atomic number of the target increases.

Now cut-off wavelength of continuous $X-$rays is given by

$\lambda_{\min}=\frac{hc}{eV}=\frac{hc}{E}$

So, it does not depend on the atomic number of the target, but it will depend on the energy of the electrons in the $X-$ray tube.

The intensity of $X-$rays depends on the number of electrons striking the target per second, which in turn depends on electrical power given to the $X-$ray tube.

Q.17. The Binding energy per nucleon of ${}_3^7Li$ and ${}_2^4He$ nuclei are $5.60\ \text{MeV}$ and $7.06\ \text{MeV}$, respectively. In the nuclear reaction ${}_3^7Li+{}_1^1H\rightarrow 2\,{}_2^4He+Q$, the value of energy Q released is

A) $19.6\ \text{MeV}$

B) $-2.4\ \text{MeV}$

C) $8.4\ \text{MeV}$

D) $17.3\ \text{MeV}$

Answer: 17.3 MeV

Solution:

Binding energy of ${}^4_2He = 4 \times 7.06 = 28.24 \text{ MeV}$
Binding energy of ${}^7_3Li = 7 \times 5.60 = 39.20 \text{ MeV}$

Given nuclear reaction is,
${}^7_3Li + {}^1_1H \rightarrow {}^4_2He + {}^4_2He + Q$

Total energy remains the same before and after the reaction.
Therefore, energy released, $Q = BE_{\text{after}} - BE_{\text{before}}$

$Q = 2 \times (28.24) - 39.20 = 17.28 \text{ MeV}.$

Q.18. A radioactive material decays by simultaneous emission of two particles with respective half-lives, 1620 and 810 years. The time (in years) after which one-fourth of the material remains is

A) 1080

B) 2430

C) 3240

D) 4860

Answer: 1080

Solution:

Decay constant λ of a radioactive sample is given by,

$\lambda = \frac{0.693}{t_{1/2}} \ldots (1)$

where, $t_{1/2}$ is the half-life of the sample.

Given half-lives of first and second processes as 1620 years and 810 years, respectively.

Let the decay constant for first and second processes be λ_1 and λ_2, respectively. So, using equation (1), we have,
$\lambda_1 = \frac{0.693}{1620} \text{ year}^{-1}$
and $\lambda_2 = \frac{0.693}{810} \text{ year}^{-1}$
In simultaneous decay, effective decay constant is given by,
$\lambda_{\text{net}} = \lambda_1 + \lambda_2 = \frac{0.693}{1620} + \frac{0.693}{810}$
$\Rightarrow \lambda_{\text{net}} = 0.693 \times \left(\frac{2430}{(1620) \times (810)}\right) \text{ year}^{-1}$

Hence, the net half-life of simultaneous decay is,
$T_{1/2} = \frac{0.693}{\lambda_{\text{net}}} = \frac{0.693}{0.693 \times \left(\frac{2430}{(1620) \times (810)}\right)}$
$\Rightarrow T_{1/2} = \frac{(1620) \times (810)}{2430} = 540 \text{ years}.$
Now, according to radioactive law, the number of active nuclei N after n half-lives is given by,

$N = \frac{N_0}{2^n}$
$\Rightarrow \frac{N}{N_0} = \frac{1}{2^n}$

where, N_0 and N are the number of initial active nuclei and number of active nuclei after time t (i.e., after n half-lives).

According to the question, $\frac{N}{N_0} = \frac{1}{4}$.

Putting in the above equation, we get,
$\Rightarrow \frac{1}{4} = \frac{1}{2^n}$
$\Rightarrow n = 2$
So, the required time will be
$t = n \times (T_{1/2})$
$\Rightarrow t = 2 \times (T_{1/2}) = 2 \times (540) = 1080 \text{ years}$

Q.19. The decay constant of a radioisotope is λ. If
A_1 and A_2 are its activities at times
t_1 and
t_2, respectively, then the number of nuclei decayed during the time $(t_2 - t_1)$ is

A) $A_1 t_1 - A_2 t_2$

B) $A_1 - A_2$

C) $\frac{(A_1 - A_2)}{\lambda}$

D) $\lambda (A_1 - A_2)$

Answer: $\frac{(A_1 - A_2)}{\lambda}$

Solution: Activity A of a radioactive nucleus is given by

$A = \lambda N$

where, λ is the decay constant and N is the number of active nuclei.

Given, at time $t = t_1$, activity $= A_1 = \lambda N_1$, where N_1 is the number of active nuclei at time t_1.
And, at time $t = t_2$, activity $= A_2 = \lambda N_2$, where N_2 is the number of active nuclei at time t_2.
So, $N_1 = \frac{A_1}{\lambda}$ and $N_2 = \frac{A_2}{\lambda}$
Thus, the number of molecules decayed during $(t_2 - t_1)$ is $N_1 - N_2$
$\therefore N_1 - N_2 = \frac{A_1}{\lambda} - \frac{A_2}{\lambda} = \frac{(A_1 - A_2)}{\lambda}$

Q.20. A radioactive element can undergo α and β type of disintegrations with half-lives
T_1 and T_2, respectively. Then, the half-life of the element is

A) $T_1 + T_2$

B) $T_1 T_2$

C) $T_1 - T_2$

D) $\frac{T_1 T_2}{T_1 + T_2}$

Answer: $\frac{T_1 T_2}{T_1 + T_2}$

Solution: This is the situation of simultaneous radioactive decay.

As the nucleus can undergo α and β type of disintegration, effective decay rate is the sum of the individual decay rate.
$\therefore \left(\frac{dN}{N}\right)_{\text{eff}} = \frac{dN_\alpha}{N} + \frac{dN_\beta}{N}$

Effective decay constant is given by,

$\lambda_{\text{eff}} = \lambda_\alpha + \lambda_\beta$

Now, decay constant is given by,

$\lambda = \frac{\ln 2}{t_1/_2}$
$\therefore \left(\frac{\ln 2}{t_1/_2}\right)_{\text{eff}} = \left(\frac{\ln 2}{T_1}\right) + \left(\frac{\ln 2}{T_2}\right)$
$\Rightarrow (t_{1/2})_{\text{eff}} = \frac{T_1 \times T_2}{T_1 + T_2}$

Q.21. If R is the Rydberg constant in cm^{-1}, then hydrogen atom does not emit any radiation of wavelength in the range of

A) $\frac{1}{R}$ to $\frac{4}{3R}$ cm

B) $\frac{7}{5R}$ to $\frac{19}{5R}$ cm

C) $\frac{4}{R}$ to $\frac{36}{5R}$ cm

D) $\frac{9}{R}$ to $\frac{144}{7R}$ cm

Answer: $\frac{7}{5R}$ to $\frac{19}{5R}$ cm

Solution: Wavelength of emitted radiation is given by

$\frac{1}{\lambda}=R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

For Lyman series,

$\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$ where $n=2,3,4,\ldots\ldots$

The range of wavelength in this series is given by

$\frac{1}{\lambda_{\max}}=R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)=\frac{3R}{4}\ \mathrm{cm}^{-1}$

and $\frac{1}{\lambda_{\min}}=R\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right)=R\ \mathrm{cm}^{-1}$

Thus for Lyman : $\left[\frac{1}{R}\text{ to }\frac{4}{3R}\ \mathrm{cm}\right]$

For Balmer series,

$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$, where $n=3,4,5,\ldots$

The range of wavelength in this series is given by

$\frac{1}{\lambda_{\max}}=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{5R}{36}\ \mathrm{cm}^{-1}$

and $\frac{1}{\lambda_{\min}}=R\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right)=\frac{R}{4}\ \mathrm{cm}^{-1}$

Thus, for Balmer: $\left[\frac{4}{R}\text{ to }\frac{36}{5R}\ \mathrm{cm}\right]$

For Paschen series,

$\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{n^2}\right)$ where $n=4,5,6,\ldots.$

The range of wavelength in this series is given by

$\frac{1}{\lambda_{\max}}=R\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=\frac{7R}{144}\ \mathrm{cm}^{-1}$

and $\lambda_{\min}=R\left(\frac{1}{3^2}-\frac{1}{\infty^2}\right)=\frac{R}{9}\ \mathrm{cm}^{-1}$

Thus, for Paschen : $\left[\frac{9}{R}\text{ to }\frac{144}{7R}\ \mathrm{cm}\right]$

For Brackett series,

$\frac{1}{\lambda}=R\left(\frac{1}{4^2}-\frac{1}{n^2}\right)$, where $n=5,6,7,\ldots.$

The range of wavelength in this series is given by

$\frac{1}{\lambda_{\max}}=R\left(\frac{1}{4^2}-\frac{1}{5^2}\right)=\frac{9R}{400}\ \mathrm{cm}^{-1}$

and $\frac{1}{\lambda_{\min}}=R\left(\frac{1}{4^2}-\frac{1}{\infty^2}\right)=\frac{R}{16}\ \mathrm{cm}^{-1}$

Thus, for Brackett : $\left[\frac{16}{R}\text{ to }\frac{400}{9R}\ \mathrm{cm}\right]$

For Pfund series,

$\frac{1}{\lambda}=R\left(\frac{1}{5^2}-\frac{1}{n^2}\right)$ where $n=6,7,8,\ldots.$

The range of wavelength in this series is given by

$\frac{1}{\lambda_{\max}}=R\left(\frac{1}{5^2}-\frac{1}{6^2}\right)=\frac{11R}{900}\ \mathrm{cm}^{-1}$

and $\frac{1}{\lambda_{\min}}=R\left(\frac{1}{5^2}-\frac{1}{\infty^2}\right)=\frac{R}{25}\ \mathrm{cm}^{-1}$

Thus, for Pfund: $\left[\frac{25}{R}\text{ to }\frac{900}{11R}\ \mathrm{cm}\right]$

Hence, wavelength range of $\left(\frac{7}{5R}\text{ to }\frac{19}{5R}\ \mathrm{cm}\right)$ does not emit.

Q.22. Suppose the potential energy between electron and proton at a distance r is given by $-\frac{ke^2}{3r^3}$. Application of Bohr's theory to hydrogen atom in this case shows that:

A) Energy in the n^{th} orbit is proportional to n^5

B) Energy is proportional to m^{-3} (m = mass of electron)

C) Energy in the n^{th} orbit is proportional to n^{-2}

D) Energy is proportional to m^3 (m= mass of electron)

Answer: Energy is proportional to m^{-3} (m = mass of electron)

Solution: Given,

$$\text{Potential energy}\left(U\right)=-\frac{ke^2}{3r^3} \quad ...(1)$$

The electrostatic force between an electron and a proton at a distance r is given by, $F=-\frac{\partial U}{\partial r}=\frac{ke^2}{r^4} \quad ...(2)$

According to Bohr's first postulate,

$$\frac{mv^2}{r}=F=\frac{ke^2}{r^4}$$

$$\Rightarrow K.E.=\frac{1}{2}mv^2=\frac{ke^2}{2r^3} \quad ...(3)$$

According to Bohr's second postulate,

$$mvr=\frac{nh}{2\pi} \quad ...(4)$$

From equation (3) and (4) -

$$\Rightarrow r=\frac{4\pi^2e^2mk}{n^2h^2} \quad ...(5)$$

We know that,

$$\text{Total energy}\,(E)=K.E+U$$

$$\Rightarrow E=\frac{\mathrm{ke}^2}{2r^3}-\frac{\mathrm{ke}^2}{3r^3}$$

$$\Rightarrow E=\frac{\mathrm{ke}^2}{6r^3}$$

Substituting the value of r, we will get -

$$\Rightarrow E=\frac{n^6h^6}{6(2\pi)^6m^3k^2e^4}$$

$$\Rightarrow E\propto\frac{1}{m^3}$$

Q.23. In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be

A) $\frac{16}{25}\lambda$

B) $\frac{25}{16}\lambda$

C) $\frac{20}{27}\lambda$

D) $\frac{27}{20}\lambda$

Answer: $\frac{20}{27}\lambda$

Solution: Energy in transition of electron from higher orbit to lower orbit is given by,

$\frac{hc}{\lambda} = \left(13.6\,\text{eV}\right) Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

From M orbit to L orbit:

$\frac{hc}{\lambda_1} = \left(13.6\,\text{eV}\right) Z^2 \left(\frac{1}{4} - \frac{1}{9}\right) \ldots(\text{i})$

From N orbit to L orbit:

$\frac{hc}{\lambda_2} = \left(13.6e\ \text{V}\right) Z^2 \left(\frac{1}{4} - \frac{1}{16}\right) \ldots(\text{ii})$

dividing (i) by (ii)

$\frac{\lambda_2}{\lambda_1} = \frac{5}{36} \times \frac{64}{12} = \frac{20}{27} \Rightarrow \lambda_2 = \frac{20}{27}\lambda_1$

Q.24. A radioactive nucleus with Z protons and N neutrons emits an $\alpha-$particle, $2\beta-$ particles and 2 gamma rays. The number of protons and neutrons in the nucleus left after the decay respectively, are:

A) $Z-3$, $\text{N}-1$

B) $Z-2$, $\text{N}-2$

C) $Z-1$, $\text{N}-3$

D) Z, $\text{N}-4$

Answer: Z, $\text{N}-4$

Solution: ${}_ZX^A \xrightarrow{\alpha} {}_{Z-2}X^{A-4} \xrightarrow{2\beta} {}_ZX^{A-4} \xrightarrow{2\gamma} {}_ZX^{A-4}$

$\text{P} = \text{Z}$

Neutron $= (\text{A}-4) - \text{Z}$

$= (\text{A}-\text{Z}) - 4$

$= \text{N}-4$

Q.25. A beam of 30 keV electrons strikes different targets in different experiments. The lowest wavelength cut-off of the continuous spectrum of X-rays generated by beam for any target is,

A) $1.0 \times 10^{-10}\,\text{m}$

B) $3.0 \times 10^{-10}\,\text{m}$

C) $4.14 \times 10^{-11}\,\text{m}$

D) dependent on the nature of the target.

Answer: $4.14 \times 10^{-11}\,\text{m}$

Solution: Given, energy of the electron beam, $E = 30\ \text{keV}$.

We know that for continuous spectrum of x-rays, we have, $E = \frac{hc}{\lambda}$

where, h is Planck's constant,

c is the speed of the light,

E is the energy of the wave,

λ is the wavelength associated with it.

$\Rightarrow \lambda = \frac{hc}{E}$

Putting in the values and solving, we get,

$\lambda = \frac{(6.626\times10^{-34})\times(3\times10^{8})}{(30\times10^{3})\times(1.6\times10^{-19})}$

$\Rightarrow \lambda = 4.14 \times 10^{-11}\ \text{m}.$

Practice more on Atoms and Nuclei