

# **JEE Main**

**Shift 1**



## Physics

Q.1. Kinetic energy of electron, proton and a particle is given as  $K$ ,  $2K$  and  $4K$  respectively, then which of the following gives the correct order of de-Broglie wavelengths of electron, proton and a particle

- A)  $\lambda_p > \lambda_\alpha > \lambda_e$       B)  $\lambda_\alpha > \lambda_p > \lambda_e$       C)  $\lambda_e > \lambda_p > \lambda_\alpha$       D)  $\lambda_e > \lambda_\alpha > \lambda_p$

**Answer:**  $\lambda_e > \lambda_p > \lambda_\alpha$

**Solution:** The de Broglie wavelength of a particle is given by

$$\lambda = \frac{h}{mv} \dots \dots \dots (1)$$

The kinetic energy ( $K$ ) is given by

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}} \dots \dots \dots (2)$$

From equations (1) and (2), it can be written that

$$\lambda = \frac{h}{\sqrt{2mK}} \dots \dots \dots (3)$$

Thus, the de Broglie wavelengths of the given particles can be calculated as follows-

$$\lambda_e = \frac{h}{\sqrt{2m_e K}} \dots \dots \dots (4)$$

$$\lambda_p = \frac{h}{\sqrt{2m_p(2K)}}$$

$$= \frac{h}{\sqrt{4m_p K}} \dots \dots \dots (5)$$

$$\lambda_\alpha = \frac{h}{\sqrt{2(4m_p)(4K)}}$$

$$= \frac{h}{\sqrt{32m_p K}} \dots \dots \dots (6)$$

Mass of electron is  $\frac{1}{1837}$  times the mass of a proton. So, for electron the wavelength will be the maximum.

Hence, from equations (4), (5) and (6), it can be concluded that  $\lambda_e > \lambda_p > \lambda_\alpha$ .

Q.2. If the height of a tower used for LOS communication is increased by 21%. The percentage change in range is

- A) 5%      B) 10%      C) 15%      D) 12%

**Answer:** 10%

**Solution:** Range of Line of sight (LOS) communication is given by formula,

$$\text{Range, } R = \sqrt{2h_T R_e}$$

where,  $h_T$  is height of the tower

and  $R_e$  is radius of the earth

Let  $R'$  be the final range of LOS after 21% increase in the height of the tower.

Then, the final height of tower will be  $1.21h_T$ .

$$\text{So, Final range, } R' = \sqrt{2(1.21h_T)R_e}$$

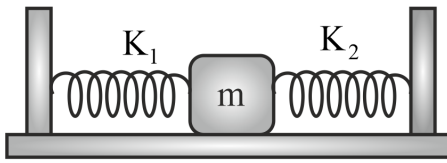
$$\Rightarrow R' = 1.1R$$

$$\text{Percentage change in } R \text{ is given by } \frac{\Delta R}{R} \times 100 = \frac{R' - R}{R} \times 100 = \frac{0.1R}{R} \times 100 = 10.$$

Therefore, the % change in range is 10%.



Q.3. For the oscillations exhibited by the spring block system on the smooth surface along the spring, the time period is equal to



- A)  $2\pi\sqrt{\frac{m(K_1+K_2)}{K_1K_2}}$       B)  $2\pi\sqrt{\frac{m(K_1+K_2)}{2K_1K_2}}$       C)  $2\pi\sqrt{\frac{m}{K_1+K_2}}$       D)  $\pi\sqrt{\frac{m}{K_1+K_2}}$

**Answer:**  $2\pi\sqrt{\frac{m}{K_1+K_2}}$

**Solution:** Here, when we slightly disturb the spring block system, the left spring extends the same amount as right spring would do. So, springs are connected in parallel.

Equivalent spring constant for a set of springs in parallel is given by,

$$K_{\text{equivalent}} = K_1 + K_2 + K_3 + \dots$$

So, equivalent spring constant for two springs connected in parallel is given by,

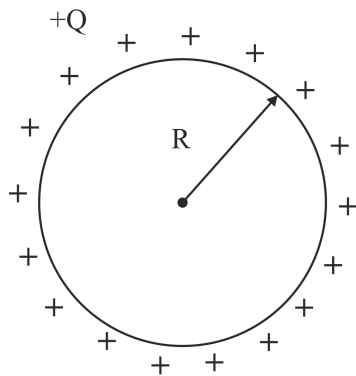
$$K_{\text{equivalent}} = K_1 + K_2$$

Time period of oscillations exhibited by the spring block system is given by,

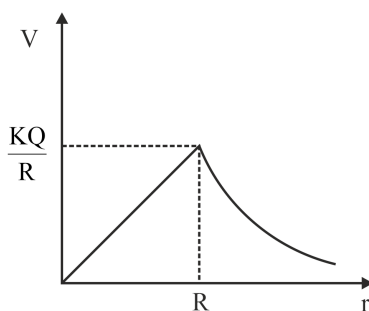
$$T = 2\pi\sqrt{\frac{m}{K_{\text{equivalent}}}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{K_1+K_2}}$$

Q.4. Pick the correct graph between potential  $V$  at distance  $r$  from centre for the uniformly charged spherical shell of radius  $R$ .

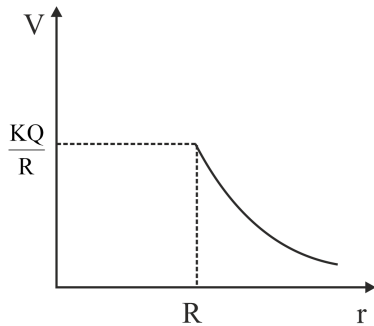


A)

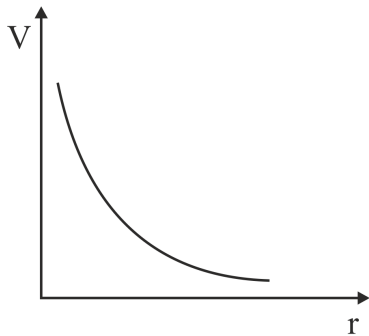




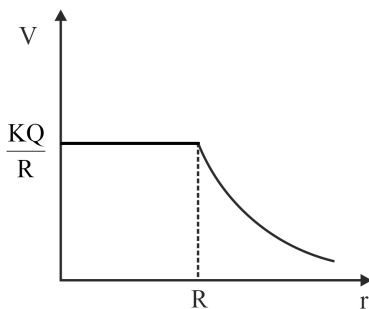
B)



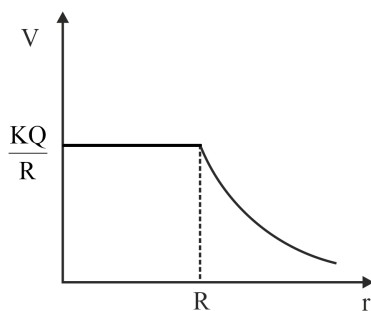
C)



D)



Answer:



**Solution:** For a special charged shell, the total charge resides on its surface.

The potential inside the shell will be the same as that on the surface. As one goes away from the surface of the spherical shell, the potential changes according to the following formula-

$$V(r) = K \frac{Q}{r}$$

where,  $K$  is Coulomb's constant.

Thus, starting from the centre of the shell, the potential remains constant up to  $r = R$  and beyond this point, the potential is inversely proportional to the distance.

Hence, this is the correct option.



Q.5. Assertion (A): Earth has atmosphere and moon doesn't.

Reason (R): Escape speed on moon is less than that of Earth.

- A) (A) and (R) are correct and (R) is the correct explanation of (A).  
B) (A) and (R) are correct and (R) is not the correct explanation of (A).  
C) (A) is true but (R) is false.  
D) (A) and (R) both are false.

**Answer:** (A) and (R) are correct and (R) is the correct explanation of (A).

**Solution:** The escape velocity is given by below formula,

$$v_e = \sqrt{2gR}$$

From above formula, we can observe that escape velocity is dependent on acceleration due to gravity.

$$\text{As, } g_{\text{moon}} < g_{\text{earth}}$$

$$(v_e)_{\text{moon}} < (v_e)_{\text{earth}}$$

The RMS velocity of the molecules of gas at the surface temperature of moon is greater than the escape velocity on the surface of moon. Therefore, the gas molecules escape and hence moon cannot hold an atmosphere.

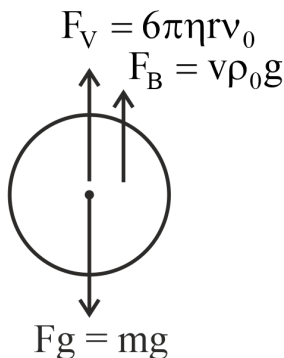
Q.6. A ball of mass  $m$  and radius  $r$  and density  $\rho$  is dropped in a liquid of density  $\rho_0$ . After moving for sometime, the speed of the ball becomes constant and equal to  $v_0$ . The coefficient of viscosity of the liquid is

- A)  $\frac{mg}{6\pi r v_0} \left(1 - \frac{\rho_0}{\rho}\right)$       B)  $\frac{mg}{6\pi r v_0} \left(1 + \frac{\rho_0}{\rho}\right)$       C)  $\frac{mg}{3\pi r v_0} \left(1 + \frac{\rho_0}{\rho}\right)$       D)  $\frac{mg}{3\pi r v_0} \left(1 - \frac{\rho_0}{\rho}\right)$

**Answer:**  $\frac{mg}{6\pi r v_0} \left(1 - \frac{\rho_0}{\rho}\right)$



**Solution:** When an object moves through a liquid, it is acted upon by different forces, as depicted by the following diagram-



Here,  $F_g$  is the gravitational force,  $F_v$  is the viscous force and  $F_B$  is the buoyant force.

The formula to calculate the gravitational force on the ball is given by

$$F_g = mg \dots \dots \dots (1)$$

where,  $m$  is the mass and  $g$  is the acceleration due to gravity. This force acts vertically downward.

The formula to calculate the viscous force on the ball is given by

$$F_v = 6\pi\eta r v_0 \dots \dots \dots (2)$$

where,  $\eta$  is the coefficient of viscosity,  $r$  is the radius of the ball and  $v_0$  is the final constant velocity, also known as the terminal velocity. This force acts upward.

The formula to calculate the buoyant force on the ball is given by

$$F_B = V\rho_0 g \dots \dots \dots (3)$$

where,  $V$  is the volume of the ball and  $\rho_0$  is the density of the liquid. This force also acts upward.

The formula to calculate the density ( $\rho$ ) of the material of the ball is given by

$$\rho = \frac{m}{V} \dots \dots \dots (4)$$

With the help of the diagram shown, construct the equilibrium condition for the ball.

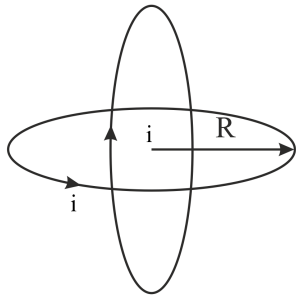
$$mg = 6\pi\eta r v_0 + V\rho_0 g \dots \dots \dots (5)$$

Use equation (4) and equation (5) and solve to obtain the expression for the coefficient of viscosity of the liquid.

$$\begin{aligned} mg &= 6\pi\eta r v_0 + \frac{m}{\rho} \rho_0 g \\ \Rightarrow 6\pi\eta r v_0 &= mg - \frac{m}{\rho} \rho_0 g \\ &= mg \left( 1 - \frac{\rho_0}{\rho} \right) \\ \Rightarrow \eta &= \frac{mg}{6\pi r v_0} \left( 1 - \frac{\rho_0}{\rho} \right) \end{aligned}$$



Q.7. Two identical current carrying coils with same centre are placed with their planes perpendicular to each other as shown. If  $i = \sqrt{2}$  A and radius of coil is  $R = 1$  m, then magnetic field at centre  $C$  is equal to



- A)  $\mu_0$                       B)  $\frac{\mu_0}{2}$                       C)  $2\mu_0$                       D)  $\sqrt{2}\mu_0$

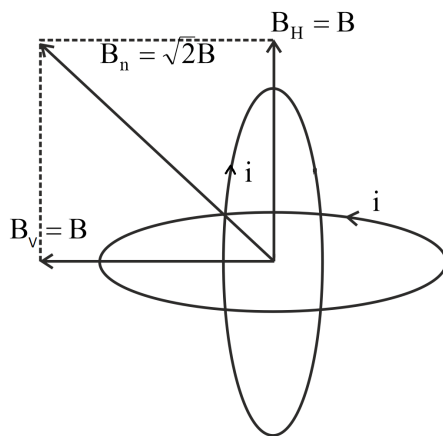
**Answer:**  $\mu_0$

**Solution:** The formula to calculate the magnetic field ( $B$ ) at the centre of a current carrying circular loop is given by

$$B = \frac{\mu_0 i}{2R} \dots \dots \dots (1)$$

where,  $\mu_0$  is the permeability of free space,  $i$  is the current and  $R$  is the radius of the loop.

Let's consider the following diagram:



Here,  $B_V$ ,  $B_H$  are the magnetic field due to the vertical and the horizontal loop, which by magnitude are equal.

The resultant magnetic field ( $B_n$ ) at the centre is, then, given by

Substitute the expression for the magnetic field from equation (1) into equation (2) and solve to calculate the required magnetic field at the centre.

$$\begin{aligned} B_n &= \sqrt{2}B \\ &= \sqrt{2} \left( \frac{\mu_0 \times \sqrt{2} \text{ A}}{2 \times 1 \text{ m}} \right) \\ &= \mu_0 \text{ A m}^{-1} \end{aligned}$$

Q.8. The amount of heat supplied to a gas in a system is equal to 1000 J, the system in return does 200 J of work on the surrounding. Find change in internal energy of the gas.

- A) 800 J                      B) 1200 J                      C) 1000 J                      D) 1100 J

**Answer:** 800 J



**Solution:** From first law of thermodynamics, we know that,

$$\Delta Q = \Delta U + \Delta W$$

Also, given that heat is supplied to the system. Hence,  $\Delta Q = +1000 \text{ J}$

It is also given that the 200 J of work is done on the surroundings. So, the work done by the system is +200 J.

$$\text{Therefore, } 1000 = \Delta U + 200$$

$$\Rightarrow \Delta U = 1000 - 200 = 800 \text{ J}$$

**Q.9.** On a planet  $\rho$  (mass density) is same as that of Earth while mass of planet is twice than that of Earth. Ratio of weight of a body on surface of the planet to that on Earth is equal to:

A) 1

B)  $\frac{1}{2^3}$

C)  $\frac{1}{2^3}$

D) 2

**Answer:**  $\frac{1}{2^3}$

**Solution:** We know that acceleration due to gravity on any surface of the planet is given by below formula,

$$g = \frac{GM}{R^2} \text{-----(i)}$$

We also know that,  $W = mg$

For a given body, mass remains constant whether it is on Earth or any planet. So, ratio of weight of a body on surface of the planet to that on Earth is same as ratio of acceleration due to gravity on the surface of the planet to that on Earth.

$$\text{Since for a planet, } \rho = \frac{M}{V}$$

$$M = \frac{4}{3}\pi R^3 \rho$$

In above equation, as  $\rho$  is constant (given)

$$R \propto M^{\frac{1}{3}} \text{-----(ii)}$$

From (i) and (ii),

$$g \propto \frac{M}{M^{\frac{2}{3}}}$$

$$\Rightarrow g \propto M^{\frac{1}{3}}$$

So, the ratio is  $\frac{1}{2^3}$  (Since, given that ratio of mass of planet to Earth as 2 : 1)

**Q.10.** A block of mass 100 g is placed on smooth surface, moves with acceleration of  $a = 2x$ , then the change in kinetic energy can be given as  $\left(\frac{x^n}{10}\right)$ , find the value of  $n$ .

**Answer:** 2





**Solution:** The acceleration ( $a$ ) of a particle is given by

$$a = v \frac{dv}{dx} \dots (1)$$

Substitute  $2x$  for  $a$  into equation (1) and simplify to obtain the change in velocity of the particle.

$$\begin{aligned} 2x &= v \frac{dv}{dx} \\ \Rightarrow v dv &= 2x dx \\ \Rightarrow \int_u^v v dv &= 2 \int_0^x x dx \\ \frac{1}{2} (v^2 - u^2) &= x^2 \dots (2) \end{aligned}$$

The formula to calculate the change in kinetic energy of the particle ( $\Delta K$ ) can be written as

$$\Delta K = \frac{1}{2} m (v^2 - u^2) \dots (3)$$

From equations (2) and (3), it follows that

$$\Delta K = mx^2 \dots (4)$$

Substitute the value of the mass of the particle in equation (4) to obtain the required change in kinetic energy.

$$\begin{aligned} \Delta K &= 0.1x^2 \\ &= \frac{x^2}{10} \dots (5) \end{aligned}$$

Comparing equation (5) with the given expression, it can be concluded that  $n = 2$ .

- Q.11. A car is moving with speed of  $15 \text{ m s}^{-1}$  towards a stationary wall. A person in the car press the horn and experience the change in frequency of 40 Hz due to reflection from the stationary wall. Find the frequency of horn. (Use  $v_{\text{sound}} = 330 \text{ m s}^{-1}$ )

**Answer:** 420



**Solution:** The formula to calculate the frequency ( $f'$ ) as heard by an observer with respect to the frequency ( $f$ ) produced from a source is given by

$$f' = \frac{v \pm v_o}{v \mp v_s} f \dots \dots \dots (1)$$

where,  $v$  is the speed of sound in air,  $v_o$  is the speed of the observer and  $v_s$  is the speed of the source.

In the first situation, the source is the moving car and the observer is the wall. Hence, the frequency ( $f_w$ ) as heard on the

wall can be written as

$$f_w = \frac{v-0}{v-v_c} f$$

$$= \frac{v}{v-v_c} f \dots \dots \dots (2)$$

where,  $v_c$  is the speed of the car.

In the second situation, when the sound reflects back from the wall, the source of sound is the wall and the observer is the car.

Hence, the frequency ( $f_c$ ) of sound as heard by the car after the reflection is given by

$$f_c = \frac{v+v_c}{v-0} f_w$$

$$= \frac{v+v_c}{v} f_w \dots \dots \dots (3)$$

From equation (2) and (3), it follows that

$$f_c = \frac{v+v_c}{v} \times \frac{v}{v-v_c} f$$

$$= \frac{v+v_c}{v-v_c} f \dots \dots \dots (4)$$

It is given in the problem that

$$f_c - f = 40 \dots \dots \dots (5)$$

Substitute the expression from equation (4) into equation (5) and solve to calculate the frequency of the horn.

$$\frac{v+v_c}{v-v_c} f - f = 40$$

$$\Rightarrow \frac{2v_c}{v-v_c} f = 40$$

$$\Rightarrow f = 40 \frac{v-v_c}{2v_c} = 40 \times \frac{330-15}{2 \times 15} \text{ m s}^{-1} = 420 \text{ m s}^{-1}$$

**Q.12.** If the length of a conductor is increased by 20 percent and cross-sectional area is decreased by 4 percent, then calculate the percentage change in the resistance of the conductor.

**Answer:** 25

**Solution:** Resistance of a conductor depends on its length ( $l$ ), resistivity ( $\rho$ ) and cross-sectional area ( $A$ ) by below formula,

$$R = \rho \frac{l}{A}$$

As length of a conductor is increased by 20 percent, final length ( $l'$ ) =  $l + \frac{20}{100}l = 1.2l$

Also, as the area of cross-section is decreased by 4 percent, final cross-sectional area ( $A'$ ) =  $A - \frac{4}{100}A = 0.96A$

Let the final resistance of conductor be  $R'$ .

$$\text{Then, } R' = \rho \frac{l'}{A'}$$

$$\Rightarrow R' = \frac{1.2l}{0.96A} = 1.25 R$$

Percentage change in resistance of the conductor is given by,

$$\frac{\Delta R}{R} \times 100 = \frac{R' - R}{R} \times 100 = \frac{1.25 R - R}{R} \times 100 = 25$$



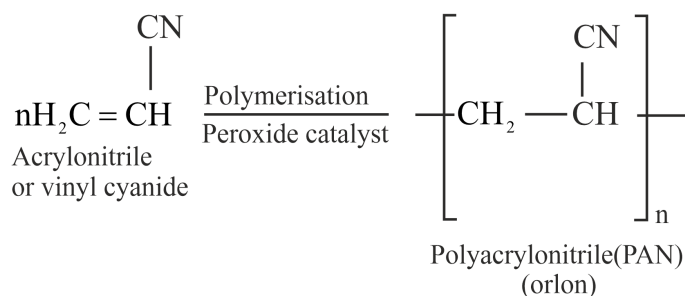
## Chemistry

Q.13. Polymer which is named as orlon is:

- A) Polyamide                      B) Polyacrylonitrile                      C) Polycarbonate                      D) Polyethylene

**Answer:** Polyacrylonitrile

**Solution:** Orlon is also known as PAN(Polyacrylonitrile). Acrylonitrile is actually a monomer which just undergoes the free radical polymerization and then further is dried and redissolved in the other solvent and further is after spinning produces Orlon.

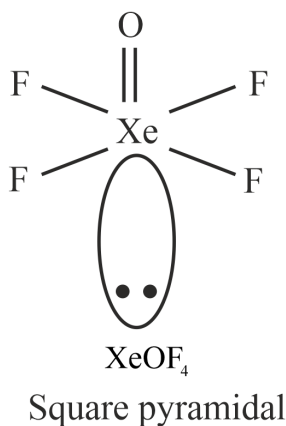


Q.14. Which of the following have square pyramidal structure?

- A) XeOF<sub>4</sub>                      B) BrF<sub>4</sub>                      C) XeF<sub>4</sub>                      D) XeO<sub>3</sub>

**Answer:** XeOF<sub>4</sub>

**Solution:** XeOF<sub>4</sub> has square pyramidal shape.



Xenon is the central atom in this structure. Hybridization in this molecule is

sp<sup>3</sup>d<sup>2</sup>. Fluorine generally binds with one atom and oxygen generally binds with two atoms. This structure involves six atoms. So, Xenon must be the central atom. Xenon uses 4 of its electrons to make bonds with 4 fluorine atoms. 2 of the valence electrons of xenon make double bonds with oxygen and the remaining two are as a lone pair.

Q.15. Match column I (Deficiency) with column II (Disease)

Column I (Deficiency)		Column II (Disease)	
(P)	Vitamin A	(1)	Scurvy
(Q)	Vitamin-B <sub>2</sub> (Riboflavin)	(2)	Xerophthalmia
(R)	Vitamin-B <sub>1</sub> (Thiamine)	(3)	Cheilosis
(S)	Vitamin-C	(4)	Beriberi

- A) P – 2, Q – 3, R – 4, S – 1    B) P – 3, Q – 2, R – 1, S – 4    C) P – 2, Q – 3, R – 1, S – 4    D) P – 1, Q – 3, R – 4, S – 2

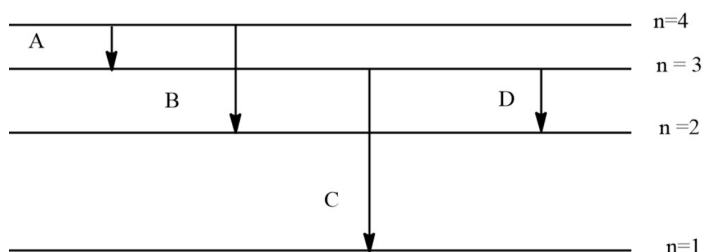
**Answer:** P – 2, Q – 3, R – 4, S – 1



**Solution:** Deficiency of Vitamin B<sub>1</sub> leads to BeriBeri, Cheilosis is caused due to lack of Vitamin-B<sub>2</sub> in the body. The deficiency of vitamin C results in a rare disease called Scurvy. Xerophthalmia is a condition in which our eyes fail to produce tears. It is an eye disease caused by vitamin A deficiency.

Therefore, option A is correct.

Q.16. Shortest wavelength will be for which of the following transition.



- A) Transition A                      B) Transition B                      C) Transition C                      D) Transition D

**Answer:** Transition C

**Solution:** For an electron to undergo electronic transition from lower energy level to higher energy level, energy is required. If an electron jumps from higher energy level to lower energy level then, energy is given out in the form of photons.

In the question, we can see that all transitions are taking place from higher energy levels to lower energy levels. Hence, photons are emitted out in each of the transitions.

For calculating wavelength of a photon, we have the formula,

$$\frac{1}{\lambda} = R \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For shortest wavelength,  $\left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$  should be maximum.

The transition  $n = 3$  to  $n = 1$  has the highest value of  $\left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$  that is 0.8889.

Therefore,  $n = 3$  to  $n = 1$  transition will have the shortest wavelength.

Q.17. Which of the following have highest electron gain enthalpy difference.

- A) F, Ne                      B) Ar, F                      C) Ne, Cl                      D) Ne, F

**Answer:** Ne, Cl

**Solution:** Chlorine has the maximum electron gain enthalpy ( $-349 \text{ kJ mol}^{-1}$ ). Halogens have the highest electron gain enthalpy (electron affinity). Neon has the highest positive ( $+116 \text{ kJ mol}^{-1}$ ) electron gain enthalpy.

The electron gain enthalpy difference of Ne, F is  $424 \text{ kJ mol}^{-1}$

The electron gain enthalpy difference of Ne, Cl is  $465 \text{ kJ mol}^{-1}$

Therefore, option C is correct.

Q.18. Which of the following is used for setting of cement?

- A) Gypsum                      B) Limestone                      C) Clay                      D) Silica

**Answer:** Gypsum

**Solution:** Setting of Cement: When mixed with water, the setting of cement takes place to give a hard mass. This is due to the hydration of the molecules of the constituents and their rearrangement. The purpose of adding gypsum is only to slow down the process of setting of the cement so that it gets sufficiently hardened.

Therefore, option A is correct.

Q.19. Which of the reaction is correct among the following with appropriate enzyme?

- A) Sucrose  $\rightarrow$  Glucose + Fructose : Enzyme – Invertase                      B) Glucose  $\rightarrow$  CO<sub>2</sub> + Ethanol : Enzyme – Maltase



- C) Protein  $\rightarrow$  Amino acid : Enzyme – Zymase                      D) Starch  $\rightarrow$  Maltose : Enzyme – Pepsin

**Answer:** Sucrose  $\rightarrow$  Glucose + Fructose : Enzyme – Invertase

**Solution:** Hydrolysis of sucrose (in the presence of either dilute acid, invertase or sucrase) results in the cleavage of glycosidic bond to give glucose and fructose. The enzyme invertase catalyzes this reaction by breaking the glycosidic bond that links glucose and fructose in the sucrose molecule, releasing them as separate molecules.

Q.20. Y form FCC lattice. X occupied  $\frac{1}{3}$  of tetrahedral voids. The formula of the compound is

- A)  $X_3Y_2$                       B)  $XY_3$                       C)  $X_2Y_3$                       D)  $X_3Y$

**Answer:**  $X_2Y_3$

**Solution:** Number of atoms of element Y in the FCC unit cell = 4.

Number of tetrahedral voids formed =  $2 \times$  Number of atoms of element Y.

Number of tetrahedral voids formed =  $2 \times 4 = 8$ .

Number of tetrahedral voids occupied by atoms of X =  $\frac{1}{3} \times 8 = \frac{8}{3}$ .

Ratio of number of atoms of X and Y =  $\frac{8}{3} : 4 = 2 : 3$

So the formula of compound is  $X_2Y_3$ .

Q.21. Photochemical smog is maximum in

- A) Himalayan Region                      B) Warm moist climate                      C) Marshy Lands                      D) Sunny desert areas

**Answer:** Sunny desert areas

**Solution:** Photochemical smog occurs in warm, dry and sunny climate. The main components of the photochemical smog result from the action of sunlight on unsaturated hydrocarbons and nitrogen oxides produced by automobiles and factories. Photochemical smog has high concentration of oxidising agents. It tends to occur more often in summer, because that is when we have the most sunlight.

Therefore, photochemical smog is maximum in Sunny desert areas.

Q.22. Match the following

Name of the Reaction	Reagents
(a) Etard reaction	(p) NaOCl
(b) Iodoform	(q) CO / HCl, Anhyd. $AlCl_3$
(c) Gattermann	(r) $CrO_2Cl_2$ , $CS_2$ , $H_3O^+$
(d) HVZ	(s) $X_2$ / Red P, $H_2O$

- A) a-q, b-r, c-p, d-s                      B) a-r, b-p, c-q, d-s                      C) a-r, b-p, c-p, d-s                      D) a-s, b-r, c-p, d-q

**Answer:** a-r, b-p, c-q, d-s

**Solution:** Iodoform test is used to check the presence of carbonyl compounds with the structure  $R - CO - CH_3$  or alcohols with the structure  $R - CH(OH) - CH_3$  in a given unknown substance. The reagent used in this reaction is NaOCl.

The Étard reaction is a chemical reaction that involves the direct oxidation of an aromatic or heterocyclic bound methyl group to an aldehyde using chromyl chloride. The reagents used in this reaction is  $CrO_2Cl_2$ ,  $CS_2$ ,  $H_3O^+$ .

When benzene or its derivative is treated with carbon monoxide and hydrogen chloride in the presence of anhydrous aluminium chloride or cuprous chloride, it gives benzaldehyde or substituted benzaldehyde. This reaction is known as Gattermann Koch reaction.

Hell Volhard Zelinsky (HVZ) reaction involves alpha bromination of carboxylic acids. The reagent used for this reaction is  $X_2$ / Red P,  $H_2O$ .

Q.23. We are given with the reaction



- A) This reaction can also take place in acetic acid.  
 B) This reaction is called Swarts reaction.  
 C) This reaction shifts in forward direction using the principle of Le-Chatelier's principle.



D) This reaction will take place even if Br is replaced with F.

**Answer:** This reaction shifts in forward direction using the principle of Le-Chatelier's principle.

**Solution:** Finkelstein reaction is used to prepare alkyl iodides starting from alkyl chlorides and alkyl bromides. In this reaction alkyl chlorides or bromides are treated with NaI in presence of acetone to form alkyl iodides. In acetone, NaI is soluble, as its lattice energy is lesser than the solvation energy of acetone. But NaCl or NaBr is insoluble in acetone, hence it gets precipitated. So stress is exerted on the reaction equilibrium due to deposition of NaCl. According to Le-chateliers principle, in order to minimise the exerted stress, reaction tends to go for the completion. Hence, this reaction shifts in forward direction using this principle.

Q.24. Some amount of urea is added to 1000 gm of H<sub>2</sub>O due to which the vapour pressure decreases by the 25% of the original vapour pressure. Find out the mass of urea added

- A) 833.3 gm                      B) 800.02 gm                      C) 673.22 gm                      D) 786.43 gm

**Answer:** 833.3 gm

**Solution:** The vapour pressure of the solution decreases by 25% of the original vapour pressure, which means that the new vapour pressure is 75% of the original vapour pressure.

From Relative lowering of vapour pressure for very dilute solutions

$$\frac{P^0 - P}{P^0} = \frac{n_{\text{solute}}}{n_{\text{solvent}}}$$

Here P is the vapour pressure of solvent in solution.

$$\frac{100 - 75}{100} = \frac{\frac{W}{60}}{\frac{1000}{18}}$$

$$\frac{25}{100} \times \frac{1000}{18} \times 60 = W$$

$$W = 833.3 \text{ gm}$$

Q.25. Oxidation state of Mo in Ammonium Phosphomolybdate is:

- A) +5                      B) +6                      C) +8                      D) +3

**Answer:** +6

**Solution:** The molecular formula of Ammonium Phosphomolybdate is: (NH<sub>4</sub>)<sub>3</sub>PO<sub>4</sub> · 12 MoO<sub>3</sub>. It is a polyatomic anion composed of 12 molybdenum atoms.

The sum of the oxidation states of all the atoms in a neutral molecule is equal to zero. The oxidation state of hydrogen is +1, oxygen is -2, nitrogen is -3, Phosphorus is +5 and Molybdenum is X.

Now, the charge on ammonium ion is +1 and phosphate is -3.

$$3(+1) + (-3) + 12X + 36(-2) = 0$$

$$3 - 3 + 12X - 72 = 0$$

$$12X = +72$$

$$X = +\frac{72}{12}$$

$$X = +6$$

Q.26. If radius of ground state hydrogen is 51 pm, find out the radius of 5<sup>th</sup> orbit of Li<sup>2+</sup> ions in pm.

**Answer:** 425

**Solution:** For Hydrogen-like particles, radius of the n<sup>th</sup> orbit is given by the expression

$$r = \frac{r_0 n^2}{Z}$$

Here, r<sub>0</sub> = radius of the first orbit of the hydrogen atom = 51 pm

Z is the atomic number of Hydrogen-like species.

$$r_{\text{Li}^{2+}} = \frac{5^2}{3} \times 51$$

$$r_{\text{Li}^{2+}} = 425 \text{ pm}$$



## Mathematics

Q.27. The number of ways to distribute 20 chocolates among three students such that each student will get atleast one chocolate is

- A)  ${}^{22}C_2$                       B)  ${}^{19}C_2$                       C)  ${}^{19}C_3$                       D)  ${}^{22}C_3$

**Answer:**  ${}^{19}C_2$

**Solution:** Let the number of chocolates given to distributed to three students be  $x_1, x_2, x_3$  then we have,

$$x_1 + x_2 + x_3 = 20 \quad \dots (1)$$

such that

$$x_1 \geq 1 \Rightarrow x_1 - 1 \geq 0$$

$$x_2 \geq 1 \Rightarrow x_2 - 1 \geq 0$$

$$x_3 \geq 1 \Rightarrow x_3 - 1 \geq 0$$

Hence, by (1) we have

$$(x_1 - 1) + (x_2 - 1) + (x_3 - 1) = 17$$

Required number of ways

$$\begin{aligned} &= {}^{17+3-1}C_{3-1} \\ &= {}^{19}C_2 \end{aligned}$$

Q.28. The coefficient of  $x^{18}$  in the expansion of  $\left(x^4 - \frac{1}{x^4}\right)^{15}$ .

- A)  ${}^{14}C_7$                       B)  ${}^{15}C_8$                       C)  ${}^{15}C_6$                       D)  ${}^{14}C_8$

**Answer:**  ${}^{15}C_6$

**Solution:** The given expansion is  $\left(x^4 - \frac{1}{x^4}\right)^{15}$ .

The general term in the binomial expansion of  $(x + a)^n$  is given by  $T_{r+1} = {}^nC_r x^{n-r} a^r$ .

$$\Rightarrow T_{r+1} = {}^{15}C_r (x^4)^{15-r} \left(\frac{-1}{x^4}\right)^r$$

$$= {}^{15}C_r (-1)^r x^{60-4r-3r}$$

$$\text{Now } 60 - 4r - 3r = 18$$

$$\Rightarrow r = 6$$

$$\text{Coefficient of } x^{18} \text{ is } {}^{15}C_6 (-1)^6$$

$$= {}^{15}C_6.$$

Hence the coefficient of  $x^{18}$  is  ${}^{15}C_6$ .

Q.29. Sum of first 20 terms of the series 5, 11, 19, 29, 41 ..... is

- A) 3130                      B) 3520                      C) 2790                      D) 1880

**Answer:** 3520



**Solution:** Let the given series be  $S_n$ , then we can write as

$$S_n = 5 + 11 + 19 + 29 + 41 + \dots + T_n$$

$$S_n = 5 + 11 + 19 + 29 + 41 + \dots + T_{n-1} + T_n$$

Subtracting above equations, we get

$$0 = 5 + 6 + 8 + 10 + \dots - T_n$$

$$\Rightarrow T_n = 5 + \left[ \frac{(n-1)}{2} (12 + (n-2)2) \right]$$

$$\Rightarrow T_n = 5 + (n-1)(n+4)$$

$$\Rightarrow T_n = n^2 + 3n + 1$$

So,

$$S_{20} = \sum_{n=1}^{20} T_n$$

$$\Rightarrow S_{20} = \sum_{n=1}^{20} (n^2 + 3n + 1)$$

$$\Rightarrow S_{20} = \frac{20 \times 21 \times 41}{6} + \frac{3 \times 20 \times 21}{2} + 20$$

$$\Rightarrow S_{20} = 3520$$

Q.30. If  $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$ , then  $18 \int_1^2 f(x) dx$  is

A)  $10 \log 2 + 6$

B)  $10 \log 2 - 6$

C)  $5 \log 2 + 6$

D)  $5 \log 2 - 6$

**Answer:**  $10 \log 2 - 6$

**Solution:** Given:

$$5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3 \quad \dots (1)$$

$$\text{Replace } x \rightarrow \frac{1}{x}$$

$$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3 \quad \dots (2)$$

Eliminating  $f\left(\frac{1}{x}\right)$  from (1) & (2), we get

$$9f(x) = \frac{5}{x} + 15 - 4x - 12$$

$$\Rightarrow 9f(x) = \frac{5}{x} - 4x + 3$$

$$\Rightarrow 18f(x) = \frac{10}{x} - 8x + 6$$

$$\Rightarrow 18 \int_1^2 f(x) dx = \int_1^2 \left( \frac{10}{x} - 8x + 6 \right) dx$$

$$\Rightarrow 18 \int_1^2 f(x) dx = \left[ 10 \log x - 4x^2 + 6x \right]_1^2$$

$$\Rightarrow 18 \int_1^2 f(x) dx = [(10 \log 2 - 4) - (10 \log 1 + 2)]$$

$$\Rightarrow 18 \int_1^2 f(x) dx = 10 \log 2 - 6$$

Q.31. The sum of roots of  $|x^2 - 8x + 15| - 2x + 7 = 0$  is

A)  $11 + \sqrt{3}$

B)  $11 - \sqrt{3}$

C)  $9 + \sqrt{3}$

D)  $9 - \sqrt{3}$

**Answer:**  $9 + \sqrt{3}$





**Solution:** Given:

$$|x^2 - 8x + 15| - 2x + 7 = 0$$

$$\Rightarrow |(x-3)(x-5)| - 2x + 7 = 0 \quad \dots (1)$$

Case 1 : When  $x \in (-\infty, 3] \cup [5, \infty)$ , then we have

$$x^2 - 8x + 15 - 2x + 7 = 0$$

$$\Rightarrow x^2 - 10x + 22 = 0$$

$$\Rightarrow x = \frac{10 \pm \sqrt{12}}{2}$$

$$\Rightarrow x = 5 + \sqrt{3}, 5 - \sqrt{3}$$

But  $(5 - \sqrt{3}) \notin (-\infty, 3] \cup [5, \infty)$ , so we have one root  $5 + \sqrt{3}$ .

Case 2 : When  $x \in (3, 5)$ , then we have

$$-(x^2 - 8x + 15) - 2x + 7 = 0$$

$$\Rightarrow x^2 - 6x + 8 = 0$$

$$\Rightarrow x = \frac{6 \pm \sqrt{4}}{2}$$

$$\Rightarrow x = 4, 2$$

But  $2 \notin (3, 5)$ , so we have one root 4.

Required sum is  $5 + \sqrt{3} + 4 = 9 + \sqrt{3}$ .

Q.32. If the image of the point  $P(1, 2, 3)$  about the plane  $2x - y + 3z = 2$  is  $Q$ , then the area of the triangle  $PQR$ , where co-ordinates of  $R$  is  $(4, 10, 12)$

A)  $\frac{\sqrt{1531}}{2}$

B)  $\frac{\sqrt{1675}}{2}$

C)  $\frac{\sqrt{2443}}{2}$

D)  $\frac{\sqrt{1784}}{2}$

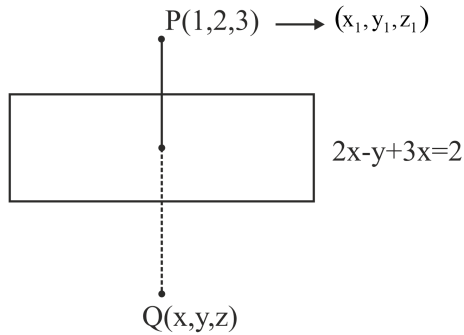
**Answer:**  $\frac{\sqrt{1531}}{2}$



**Solution:** Given co-ordinates of  $P(1, 2, 3)$  and  $R(4, 10, 12)$ .

We know that the image of a point  $A(x_1, y_1, z_1)$  w.r.t the plane  $ax + by + cz + d = 0$  is  $Q(x, y, z)$  and can be evaluated using the formula

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$



The image of the point  $P(1, 2, 3)$  w.r.t the plane  $2x - y + 3z - 2 = 0$  is  $Q(x, y, z)$ .

$$\Rightarrow \frac{x-1}{2} = \frac{y-2}{(-1)} = \frac{z-3}{3} = \frac{-2(2(1)+(-1)(2)+3(3)-2)}{2^2+(-1)^2+3^2}$$

On simplifying we get,

$$\Rightarrow \frac{x-1}{2} = \frac{y-2}{(-1)} = \frac{z-3}{3} = -1$$

Therefore, the image is  $Q(-1, 3, 0)$ .

Now the area of the  $\Delta PQR$  is  $= \frac{1}{2} |\vec{PQ} \times \vec{PR}|$ .

$$\vec{PQ} = (-1-1)\hat{i} + (3-2)\hat{j} + (0-3)\hat{k} = -2\hat{i} + \hat{j} - 3\hat{k} \text{ and}$$

$$\vec{PR} = (4-1)\hat{i} + (10-2)\hat{j} + (12-3)\hat{k} = 3\hat{i} + 8\hat{j} + 9\hat{k}$$

Now let us evaluate using determinants.

$$\Rightarrow \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -3 \\ 3 & 8 & 9 \end{vmatrix}$$

$$= \hat{i}(9 - (-24)) - \hat{j}(-18 - (-9)) + \hat{k}(-16 - 3)$$

$$= 33\hat{i} + 9\hat{j} - 19\hat{k}$$

$$\text{Area of the required triangle is } = \frac{1}{2} |33\hat{i} + 9\hat{j} - 19\hat{k}|$$

$$= \frac{\sqrt{33^2+9^2+(-19)^2}}{2} = \frac{\sqrt{1531}}{2}.$$

Therefore, the area of the triangle is  $\frac{\sqrt{1531}}{2}$  sq units.

Q.33.  $(P \Rightarrow Q) \vee (R \Rightarrow Q)$  is equivalent to

- A)  $(P \wedge R) \Rightarrow Q$       B)  $(P \vee R) \Rightarrow Q$       C)  $(Q \Rightarrow R) \vee (P \Rightarrow R)$       D)  $(R \Rightarrow P) \wedge (Q \Rightarrow R)$

**Answer:**  $(P \wedge R) \Rightarrow Q$



**Solution:** We have been given  $(P \Rightarrow Q) \vee (R \Rightarrow Q)$

We know that  $a \Rightarrow b \equiv \neg a \vee b$ .

Now  $P \Rightarrow Q \equiv \neg P \vee Q$  and

$R \Rightarrow Q \equiv \neg R \vee Q$

$(P \Rightarrow Q) \vee (R \Rightarrow Q) \equiv (\neg P \vee Q) \vee (\neg R \vee Q)$

$\equiv (\neg P \vee \neg R) \vee Q$

$\equiv \neg (P \wedge R) \vee Q$  ( By Applying De-Morgans Law)

$\equiv (P \wedge R) \Rightarrow Q$ .

Hence,  $(P \Rightarrow Q) \vee (R \Rightarrow Q) \equiv (P \wedge R) \Rightarrow Q$ .

Q.34. Let  $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ ,  $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$ ,  $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$  and  $\vec{d}$  is a vector perpendicular to both  $\vec{b}$  and  $\vec{c}$  and  $\vec{a} \cdot \vec{d} = 18$ , then  $|\vec{a} \times \vec{d}|^2$  is

A) 720

B) 640

C) 680

D) 760

**Answer:** 720

**Solution:** Let  $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ ,  $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$ ,  $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$  and

Since,  $\vec{d}$  is a vector perpendicular to both  $\vec{b}$  and  $\vec{c}$ , so

$$\vec{d} = \lambda (\vec{b} \times \vec{c})$$

$$\Rightarrow \vec{d} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix}$$

$$\Rightarrow \vec{d} = \lambda (2\hat{i} - \hat{j} + 2\hat{k})$$

$$\Rightarrow \vec{a} \cdot \vec{d} = \lambda (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (2\hat{i} - \hat{j} + 2\hat{k})$$

$$\Rightarrow 18 = \lambda (4 - 3 + 8)$$

$$\Rightarrow \lambda = 2$$

So,

$$\vec{d} = 4\hat{i} - 2\hat{j} + 4\hat{k}$$

Now,

$$|\vec{a} \times \vec{d}|^2 = |\vec{a}|^2 |\vec{d}|^2 - (\vec{a} \cdot \vec{d})^2$$

$$\Rightarrow |\vec{a} \times \vec{d}|^2 = (4 + 9 + 16)(16 + 4 + 16) - 18^2$$

$$\Rightarrow |\vec{a} \times \vec{d}|^2 = 720$$

Q.35. The integration  $\int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$  is

A)  $\frac{x}{x \tan x + 1} + \log |x \sin x + \cos x| + c$

B)  $\frac{x}{x \tan x + 1} - \log |x \sin x + \cos x| + c$

C)  $\frac{-x^2}{x \tan x + 1} + 2 \log |x \sin x + \cos x| + c$

D)  $\frac{x^2}{x \tan x + 1} + 2 \log |x \sin x + \cos x| + c$

**Answer:**  $\frac{-x^2}{x \tan x + 1} + 2 \log |x \sin x + \cos x| + c$



**Solution:**

We need to integrate  $\int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$ .

Let us apply integration by parts.

Let the first function be  $f(x) = x^2$  and the second function be  $g(x) = \frac{(x \sec^2 x + \tan x)}{(x \tan x + 1)^2}$ .

Now we know that  $\int f(x)g(x)dx = f(x) \int g(x)dx - \int f'(x) \int (g(x)dx)dx$

$$\Rightarrow \int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx = x^2 \int g(x) dx - \int 2x \int (g(x)dx) dx$$

Now let us evaluate  $\int g(x)dx = \int \frac{(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$

Let us substitute  $x \tan x + 1 = t$

$$\Rightarrow \tan x + x \sec^2 x = \frac{dt}{dx}$$

$$\Rightarrow \int \frac{(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx = \int \frac{dt}{t^2}$$

$$= \frac{-1}{t} + c$$

$$= \frac{-1}{x \tan x + 1} + c$$

Now we get,

$$\int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx = x^2 \left( \frac{-1}{x \tan x + 1} \right) - \int 2x \left( \frac{-1}{x \tan x + 1} \right) dx$$

$$= \left( \frac{-x^2}{x \tan x + 1} \right) + 2 \int \left( \frac{x \cos x}{x \sin x + \cos x} \right) dx + c_1$$

Let  $x \sin x + \cos x = p$

$$\Rightarrow \sin x + x \cos x - \sin x = \frac{dp}{dx}$$

So, the integral  $\int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx$  will be,

$$= \left( \frac{-x^2}{x \tan x + 1} \right) + 2 \int \left( \frac{dp}{p} \right) + c_1$$

$$= \left( \frac{-x^2}{x \tan x + 1} \right) + 2 \log |p| + C$$

$$= \left( \frac{-x^2}{x \tan x + 1} \right) + 2 \log |x \sin x + \cos x| + C$$

$$\text{Therefore, } \int \frac{x^2(x \sec^2 x + \tan x)}{(x \tan x + 1)^2} dx = \left( \frac{-x^2}{x \tan x + 1} \right) + 2 \log |x \sin x + \cos x| + C$$

Q.36. Let  $a_1, a_2, \dots, a_n$  are in arithmetic progression having common difference  $d$ , then the value of

$$\lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left( \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \right) \text{ is}$$

**Answer:** 1



**Solution:** Given:

$$\begin{aligned}
 & \lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left( \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \right) \\
 &= \lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left( \frac{\sqrt{a_2} - \sqrt{a_1}}{a_2 - a_1} + \frac{\sqrt{a_3} - \sqrt{a_2}}{a_3 - a_2} + \dots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}} \right) \\
 &= \lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left( \frac{\sqrt{a_2} - \sqrt{a_1}}{d} + \frac{\sqrt{a_3} - \sqrt{a_2}}{d} + \dots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{d} \right) \\
 &= \lim_{n \rightarrow \infty} \sqrt{\frac{d}{n}} \left( \frac{\sqrt{a_n} - \sqrt{a_1}}{d} \right) \\
 &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{d}} \left( \frac{\sqrt{a_n} - \sqrt{a_1}}{\sqrt{n}} \right) \\
 &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{d}} \left( \frac{\sqrt{a_1 + (n-1)d} - \sqrt{a_1}}{\sqrt{n}} \right) \\
 &= \lim_{n \rightarrow \infty} \frac{1}{\sqrt{d}} \left( \sqrt{\frac{a_1}{n} + \left(1 - \frac{1}{n}\right)d} - \sqrt{\frac{a_1}{n}} \right) \\
 &= \lim_{n \rightarrow \infty} \frac{\sqrt{d}}{\sqrt{d}} = 1
 \end{aligned}$$

Q.37. If  ${}^{2n}C_3 : {}^nC_3 = 10$  then  $\frac{n^2+3n}{n^2-3n+4}$  is equal to

**Answer:** 2

**Solution:** We have been given that  ${}^{2n}C_3 : {}^nC_3 = 10$

We know that  ${}^nC_r = \frac{n!}{r!(n-r)!}$

$$\begin{aligned}
 & \Rightarrow \frac{\frac{(2n)!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = 10 \\
 & \Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = 10 \\
 & \Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 10 \\
 & \Rightarrow \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = 10 \\
 & \Rightarrow \frac{(4)(2n-1)}{(n-2)} = 10 \\
 & \Rightarrow 8n - 4 = 10n - 20 \\
 & \Rightarrow n = 8
 \end{aligned}$$

Hence, the value of  $\frac{n^2+3n}{n^2-3n+4}$  is  $= \frac{8^2+3(8)}{8^2-3(8)+4} = \frac{88}{44} = 2$

Therefore, the value of  $\frac{n^2+3n}{n^2-3n+4}$  is 2.

Q.38. The ratio of 5<sup>th</sup> term from the beginning and 5<sup>th</sup> term from the end is  $\sqrt{6} : 1$  in  $\left( \frac{1}{2^{\frac{1}{4}}} + 3^{-\frac{1}{4}} \right)^n$ , then the value of  $n$  is



**Answer:** 10

**Solution:** We know that  $5^{\text{th}}$  term from the end is equal to  $(n - 5 + 2)^{\text{th}}$  from the beginning, so

$$\begin{aligned} \frac{T_{4+1}}{T_{n-4+1}} &= \frac{\sqrt{6}}{1} \\ \Rightarrow \frac{{}^nC_4 \left(\frac{1}{24}\right)^{n-4} \left(3\frac{1}{4}\right)^4}{{}^nC_{n-4} \left(\frac{1}{24}\right)^4 \left(3\frac{1}{4}\right)^{n-4}} &= \sqrt{6} \\ \Rightarrow \left(\frac{1}{24}\right)^{n-8} \left(3\frac{1}{4}\right)^{n-8} &= 6^{\frac{1}{2}} \\ \Rightarrow \left(\frac{1}{24}\right)^{n-8} \left(3\frac{1}{4}\right)^{n-8} &= 2^{\frac{1}{2}} \times 3^{\frac{1}{2}} \end{aligned}$$

So,

$$\frac{n-8}{4} = \frac{1}{2} \Rightarrow n = 10$$

Q.39. Matrix  $A$  is  $2 \times 2$  matrix and  $A^2 = I$ , no elements of the matrix is zero. Let sum of the diagonal elements be  $a$  and  $\det(A) = b$ , then the value of  $3a^2 + b^2$  is

**Answer:** 1



**Solution:** We have been given that order of  $A$  is  $2 \times 2$  and also no elements of the matrix is zero.

$$\text{Let } A = \begin{bmatrix} u & v \\ w & x \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} u & v \\ w & x \end{bmatrix} \times \begin{bmatrix} u & v \\ w & x \end{bmatrix}$$

$$= \begin{bmatrix} u^2 + vw & uv + vx \\ uw + xw & vw + x^2 \end{bmatrix}$$

$$\text{But this is equal to } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u^2 + vw & uv + vx \\ uw + xw & vw + x^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By comparing the corresponding elements of the above two matrices and equating them we get.

$$\Rightarrow uv + vx = 0$$

$$\Rightarrow v(u + x) = 0$$

But  $v \neq 0$

$$\Rightarrow u + x = 0$$

$$\Rightarrow \text{Tr}(A) = 0$$

We have been given that the sum of the diagonal elements is  $a$

$$\Rightarrow a = 0 \dots (1)$$

$$\text{Also, } A^2 = I$$

$$\Rightarrow |A|^2 = |I| = 1$$

$$\Rightarrow |A| = \pm 1$$

$$\Rightarrow b = \pm 1$$

$$\Rightarrow b^2 = 1$$

$$\text{Hence, } 3a^2 + b^2 = 3(0)^2 + 1 = 1.$$

Therefore, the value of  $3a^2 + b^2$  is 1