

JEE Main

Shift 2



Physics

Q.1. Density (ρ) of a body depends on the force applied (F), its speed (v) and time of motion (t) by the relation $\rho = KF^a v^b t^c$, where K is a dimensionless constant. Then

- A) $a = 1, b = -4$ and $c = -2$ B) $a = 2, b = -4$ and $c = -1$ C) $a = -1, b = -4$ and $c = 2$ D) $a = 1, b = 4$ and $c = -2$

Answer: $a = 1, b = -4$ and $c = -2$

Solution: The dimensional formula for the given quantity is given by

$$\begin{aligned} [\rho] &= [F]^a [v]^b [t]^c \\ [ML^{-3}T^0] &= [MLT^{-2}]^a [LT^{-1}]^b [T]^c \\ &= [M]^a [L]^{a+b} [T]^{-2a-b+c} \dots (1) \end{aligned}$$

Equate the powers of the parameters from both sides of equation (1) to calculate the required values of the unknown parameters.

$$a = 1 \text{ and}$$

$$a + b = -3$$

$$\Rightarrow b = -3 - 1 \text{ and}$$

$$= -4$$

$$-2a - b + c = 0$$

$$\Rightarrow -2 \times 1 - (-4) + c = 0$$

$$\Rightarrow c = -2$$

Q.2. In which of the following process, the internal energy of gas remains constant?

- A) Isothermal B) Isochoric C) Isobaric D) Adiabatic

Answer: Isothermal

Solution: An isothermal process occurs at constant temperature.

Since the internal energy of a gas is only a function of its temperature, $\Delta U = 0$ for an isothermal process.

The internal energy of a system remains constant when the temperature does not change i.e., when the process is isothermal.

Q.3. A particle is projected at an angle of 30° with ground with speed 40 m s^{-1} . The speed of the particle after 2 s is (use $g = 10 \text{ m s}^{-2}$)

- A) $20\sqrt{2} \text{ m s}^{-1}$ B) $20\sqrt{3} \text{ m s}^{-1}$ C) 20 m s^{-1} D) $10\sqrt{3} \text{ m s}^{-1}$

Answer: $20\sqrt{3} \text{ m s}^{-1}$

Solution: The horizontal component of the velocity of the projectile remains the same, whereas the vertical component of the velocity changes with time.

The formula to calculate the final velocity (\vec{v}) of the projectile after time t is given by

$$\vec{v} = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j} \dots (1)$$

Substitute the values of the known parameters into equation (1) to obtain the final velocity.

$$\begin{aligned} \vec{v} &= 40 \cos 30^\circ \hat{i} - (40 \sin 30^\circ - 10 \times 2) \hat{j} \\ &= 20\sqrt{3} \hat{i} \end{aligned}$$

Hence, the magnitude of the final velocity of the projectile is $20\sqrt{3} \text{ m s}^{-1}$.

Q.4. Potential at the surface of a uniformly charged non-conducting sphere is V . Then the potential at its centre is

- A) 0 B) $\frac{V}{2}$ C) $2V$ D) $\frac{3V}{2}$

Answer: $\frac{3V}{2}$



Solution: We know that electric potential at a point ($r < R$) inside a uniformly charged non-conducting sphere is given by,

$$V = \frac{kQ}{2R^3} [3R^2 - r^2]$$

By substituting, $r = 0$, we get electric potential at centre.

So,

$$\begin{aligned} V' &= \frac{3}{2} \left[\frac{kQ}{R} \right] \\ &= \frac{3}{2} V \quad \left(\text{Since given surface potential as } V \right) \end{aligned}$$

Q.5. If $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{A} - \vec{B} = 2\hat{j}$, then find $|\vec{B}|$.

- A) 3 B) $3\sqrt{3}$ C) 2 D) $\sqrt{3}$

Answer: 3

Solution: Given,

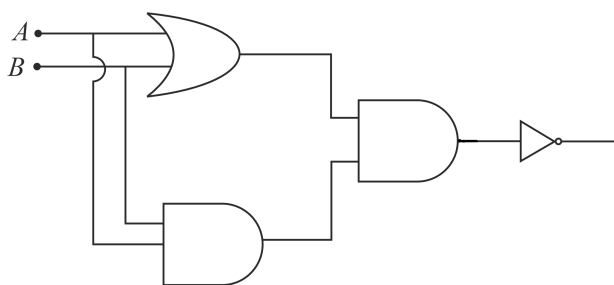
$$\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\text{and } \vec{A} - \vec{B} = 2\hat{j},$$

So,

$$\begin{aligned} \vec{B} &= \vec{A} - 2\hat{j} \\ &= (2\hat{i} + 3\hat{j} + 2\hat{k}) - 2\hat{j} \\ &= 2\hat{i} + \hat{j} + 2\hat{k} \\ \Rightarrow |\vec{B}| &= \sqrt{2^2 + 1 + 2^2} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

Q.6. The resultant gate for the following combination is

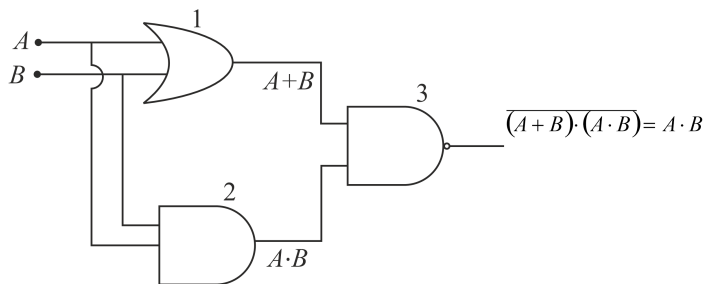


- A) NAND B) NOR C) OR D) AND

Answer: AND



Solution: Let's consider the following diagram:



As can be seen from the above diagram, gate 1 is an OR gate, gate 2 is an AND gate and gate 3 is a NAND gate.

The output from gate 1 is $(A + B)$ and that from gate 2 is $(A \cdot B)$. These two act as the input of gate 3.

Hence, the final output from gate 3 can be calculated as follows:

$$\begin{aligned}
 \overline{(A+B) \cdot (A \cdot B)} &= \overline{[(A \cdot (A \cdot B)) \cdot (B \cdot (A \cdot B))]} \\
 &= A \cdot (A \cdot B) + B \cdot (A \cdot B) \\
 &= (A \cdot A) \cdot B + A \cdot (B \cdot B) \\
 &= A \cdot B + A \cdot B \\
 &= A \cdot B
 \end{aligned}$$

Hence, the final output of the combination results an AND gate.

Q.7. If a nucleus is divided in ratio of $1 : 2^{1/3}$, then find the ratio of velocity of the parts

- A) 2 B) $2^{1/3}$ C) $2^{2/3}$ D) $2^{-1/3}$

Answer: $2^{1/3}$

Solution: Here, momentum is conserved.
As mass ratio = $1 : 2^{1/3}$,

$$\text{Velocity ratio} = 2^{1/3} : 1 \text{ [As, } mv = \text{constant]}$$

Q.8. A body is rotating with kinetic energy E . If angular velocity of body is increased to three times of initial angular velocity, the kinetic energy becomes nE . Find n .

Answer: 9

Solution: We know that the rotational kinetic energy (KE) of a body rotating with angular velocity (ω) is given by,

$$KE = \frac{1}{2} I \omega^2$$

Given,

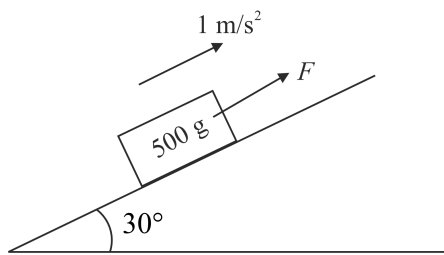
$$\omega_2 = 3\omega_1$$

$$\begin{aligned}
 KE_2 &= \frac{1}{2} I (3\omega_1)^2 \\
 &= 9 \left[\frac{1}{2} I \omega_1^2 \right] \\
 &= 9KE_1 \\
 &= 9E
 \end{aligned}$$

$$\Rightarrow n = 9$$

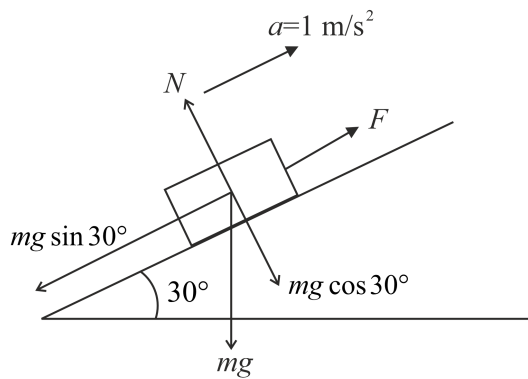


Q.9. Found the power delivered by F at $t = 10$ s. The body starts from rest.



Answer: 30

Solution: Let's consider the following free-body diagram-



From the above diagram, the force on the object can be calculated as follows:

$$F - mg \sin 30^\circ = ma$$

$$F = m(a + g \sin 30^\circ) \dots (1)$$

Substitute the values of the parameters into equation (1) to calculate the force.

$$F = [0.5 \times (1 + 10 \sin 30^\circ)] \text{ N}$$

$$= 3 \text{ N}$$

The final velocity (v) of the object can be calculated as follows:

$$v = (0 + 1 \times 10) \text{ m s}^{-1}$$

$$= 10 \text{ m s}^{-1}$$

Hence, the instantaneous power (P) delivered by the force is given by

$$P = Fv$$

$$= 3 \text{ N} \times 10 \text{ m s}^{-1}$$

$$= 30 \text{ W}$$

Q.10. Proton and electron have equal kinetic energy, the ratio of de-Broglie wavelength of proton and electron is $\frac{1}{x}$. Find x .
(Mass of proton is 1849 times mass of electron)

Answer: 43



Solution: The de-Broglie's equation is given by,

$$\lambda = \frac{h}{p} \quad \text{----(i)}$$

The above equation is relating the linear momentum (p) of a particle with its de-Broglie wavelength (λ).

We also know that, $p = \sqrt{2mKE}$ ----(ii)

Substituting (ii) in (i), we get,

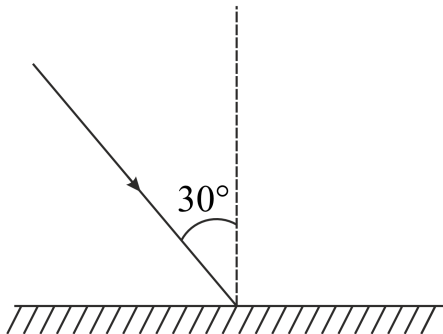
$$\lambda = \frac{h}{\sqrt{2mKE}}$$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$$

It is given that the ratio of de-Broglie wavelength of proton and electron is $\frac{1}{x}$.

$$\begin{aligned} \Rightarrow \frac{1}{x} &= \sqrt{\frac{m_e}{m_p}} \\ &= \sqrt{\frac{m_e}{1849m_e}} \\ &= \frac{1}{43} \\ \Rightarrow x &= 43 \end{aligned}$$

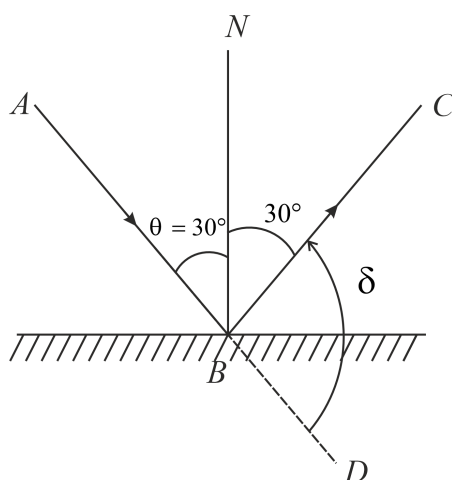
Q.11. A ray of light is incident on a plane mirror as shown in the figure. Find the deviation of the ray (in degree and clockwise direction).



Answer: 240



Solution: Let's consider the following diagram:



According to the above diagram, the angle of deviation, in the anticlockwise direction, is given by

$$\delta = 180^\circ - 2\theta \quad \dots (1)$$

Hence, the formula to calculate the angle of deviation (δ') in the clockwise direction is given by

$$\begin{aligned}\delta' &= 360^\circ - \delta \\ &= 360^\circ - (180^\circ - 2\theta) \\ &= 180^\circ + 2\theta \quad \dots (2)\end{aligned}$$

Substitute the values of the known parameters into equation (2) to calculate the required angle of deviation in the clockwise direction.

$$\begin{aligned}\delta' &= 180^\circ + 2 \times 30^\circ \\ &= 240^\circ\end{aligned}$$

Q.12. Find the average speed of N_2 at $27^\circ C$.

Answer: 476

Solution: The formula to calculate the average speed (\bar{v}) of the gas molecule at any temperature T is given by

$$\bar{v} = \sqrt{\frac{8RT}{\pi M}} \quad \dots (1)$$

Substitute the values of the known parameters into equation (1) to calculate the required average speed.

$$\begin{aligned}\bar{v} &= \sqrt{\frac{8}{22} \times \frac{25}{3} \times \frac{(27+273)}{28 \times 10^{-3}}} \text{ m s}^{-1} \\ &\approx 476 \text{ m s}^{-1}\end{aligned}$$

Chemistry

Q.13. Which alkali metal has the lowest melting point?

- A) Na B) K C) Rb D) Cs

Answer: Cs

Solution: Atomic size increases as we move down the alkali group. As a result, the binding energies of their atoms in the crystal lattice decrease. Also, the strength of metallic bonds decreases on moving down a group in the periodic table. This causes a decrease in the melting point. Among the given metals, Cs is the largest and has the least melting point.

Q.14. Statement-1: Low density polymer is formed by polymerisation of ethene in the presence of triethylaluminium and titanium tetrachloride (Ziegler-Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6 – 7 atmospheres.
Statement-2: Nylon 6 is obtained by heating caprolactum with water at 500K.

- A) Statement 1 and Statement 2 are true B) Statement 1 is false and Statement 2 is true

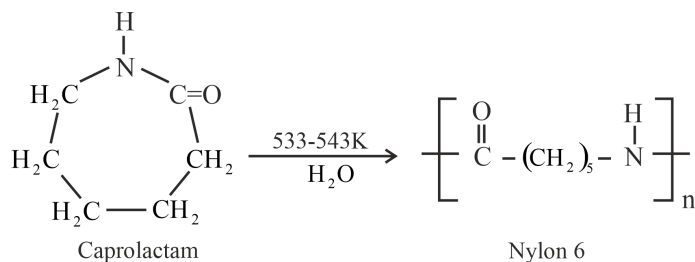


- C) Statement 1 is true and statement 2 is false D) Both statement 1 and statement 2 are wrong

Answer: Statement 1 is false and Statement 2 is true

Solution: From the given statements first statement is wrong. As High density polythene is prepared by the polymerisation of ethene in the presence of triethylaluminium and titanium tetrachloride (Ziegler-Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6 – 7 atmospheres.

Statement 2 is correct as Nylon 6 is obtained by heating caprolactam with water at 500K.



Q.15. Which of the following is least stable?

- A) HF B) BeH₂ C) LiH D) NaH

Answer: BeH₂

Solution: Beryllium hydride (BeH₂) is a covalent compound it is electron deficient and will have most covalent character. BeH₂ is known to be highly reactive and less stable due to the electron-deficient nature of beryllium.

On the other hand, HF (hydrogen fluoride), NaH (sodium hydride), and LiH (lithium hydride) are more stable compounds. HF is a polar covalent compound with a strong hydrogen bonding interaction, while NaH and LiH are ionic compounds with strong ionic bonds.

Q.16. Chemical formula of Freons :

- A) C₂F₄ B) CCl₂F₂ C) CH₂F₂ D) CH₂Cl₂

Answer: CCl₂F₂

Solution: The chlorofluorocarbon compounds of methane and ethane are collectively known as freons. For example, dichlorodifluoromethane is known as Freon-12 and its molecular formula is CCl₂F₂.

Note:- Freons are used as coolants in air conditioners and refrigerators. These are also responsible for depletion of the ozone layer also known as the ozone hole.

Q.17. Magnetic moment of [Fe(CN)₆]³⁻ and [Fe(H₂O)₆]³⁺ respectively are:

- A) 1.73 and 5.9 B) 5.9 and 1.73 C) 1.73 and 4.9 D) 1.73 and 3.9

Answer: 1.73 and 5.9

Solution: For the complex [Fe(H₂O)₆]³⁺ the ligand is weaker and it has 5 unpaired electrons in the orbitals. Hence, $\mu = \sqrt{5(5+2)} = \sqrt{35} = 5.9 \text{ BM}$

For the complex [Fe(CN)₆]³⁻ the ligand is stronger, hence the electronic configuration is $t_{2g}^5 e_g^0$ and it has one unpaired electron. Hence, $\mu = \sqrt{1(1+2)} = \sqrt{3} \text{ BM} = 1.73 \text{ BM}$.

Q.18. Statement 1: Sulphides are converted into oxide first before the reduction.

Statement 2: Because oxides can be reduced easily.

- A) Only 1 is correct B) Only 2 is correct C) Both are correct D) Both are incorrect

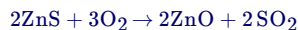
Answer: Both are correct



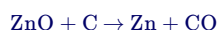
Solution: A metal oxide is generally less stable than the metal sulfide. Thus, reduction of metal oxide is easier than the reduction of metal sulfide. Hence, a sulfide ore is first converted into its oxide before reduction. This process is known as roasting.

The reaction involved in the process of roasting is represented as follows:

Conversion of Zinc sulphide to Zinc oxide in the presence of excess air:



The obtained Zinc oxide is further heated with the more reactive element Carbon, which displaces the Oxygen to form Carbon monoxide and yields the Zinc metal.



Q.19. Which of the following will give red ppt in Benedict solution?

- A) Glucose B) RNA C) DNA D) Sucrose

Answer: Glucose

Solution: Among the given compounds, glucose is the one that is most likely to give a red precipitate (ppt) in Benedict's solution.

Benedict's solution is a reagent used to test for the presence of reducing sugars, such as glucose, in a solution. When a reducing sugar is heated with Benedict's solution, it undergoes a redox reaction, where the reducing sugar reduces the copper(II) ions in Benedict's solution to form insoluble red copper(I) oxide (Cu_2O) precipitate. This is indicated by the formation of a red ppt.

Q.20. Which of the following has the maximum number of lone pairs on central atom?

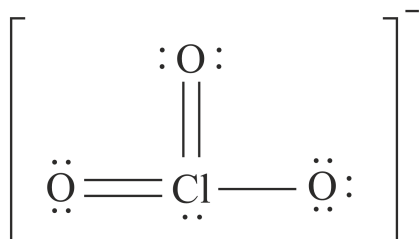
- A) ClO_3^- B) SF_4 C) XeF_4 D) I_3^-

Answer: I_3^-

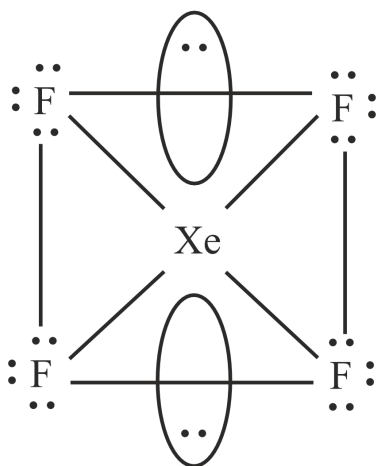


Solution:

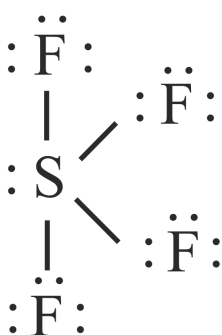
Chlorate ion is ClO_3^- and has a tetrahedral structure has one lone pair on central atom and it is shown in the figure :



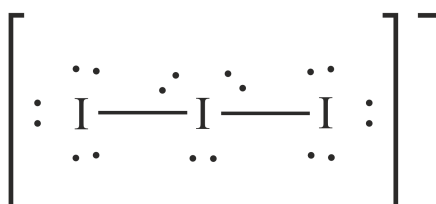
In XeF_4 , there are two lone pairings. The electron geometry of xenon is octahedral, but the molecular geometry is square planar. This is because there are six bonding electron pairs in xenon, so it has an octahedral electron geometry, but two electron pairs in the centre are unbound and lone pairs.



SF_4 has see-saw structure and has one lone pair on central atom.



The number of lone pairs on I_3^- is three and the structure is shown follow :'



Therefore, I_3^- has maximum number of lone pairs at central atom.

Q.21. 2 g of X is dissolved in 1 mol of water. Find mass percentage of X in the solution?

Answer: 10



Solution: Mass percentage can be calculated by using the formula :

$$\begin{aligned}\text{Mass percentage of X} &= \frac{\text{Mass of X}}{\text{Mass of X} + \text{Mass of water}} \times 100 \\ &= \frac{2}{2+18} \times 100 \\ &= \frac{2}{20} \times 100 \\ &= 10\end{aligned}$$

Therefore, the mass percentage of X is 10%..

Q.22. Number of intensive properties are:

E_{cell} , Molarity, Gibbs free energy, Molar mass, Mole, Molar heat capacity?

Answer: 4

Solution: E_{cell} (electric cell potential) does not depend on the size or amount of the substances in the cell and molarity is an intensive property as it does not change with the size or amount of the solution, but only depends on the ratio of solute to solution volume. Molar mass does not change with the amount of the substance, but only depends on the chemical identity of the substance. Molar heat capacity does not depend on the amount of the substance, but only on the nature of the substance itself.

These four properties, E_{cell} , Molarity, Molar mass, and Molar heat capacity, are considered intensive properties

Q.23. $\text{P}_4 + 8\text{SOCl}_2 \rightarrow 4\text{A} + 2\text{B} + \text{XSO}_2$
Sum of 4 + 2 + X is

Answer: 10

Solution: PCl_3 is obtained by the action of thionyl chloride with white phosphorus.



Therefore, sum of 4 + 2 + X = 14 + 2 + 4 = 10

Mathematics

Q.24. Using all the letters of the word "MATHS", the rank of the word "THAMS" is

A) 101 B) 102 C) 103 D) 104

Answer: 103

Solution: The given word is MATHS

Arranging the letters alphabetically, we get

AHMST

When the word starts with any of the letters A/H/M/S, the number of possibilities = $4! \times 4 = 96$

Now when the word starts with TA, then the number of possibilities = $3! = 6$

Now when the word starts with THAMS, then the number of possibilities = 1

Rank = $96 + 6 + 1 = 103$

Hence, rank of the word THAMS is 103.

Q.25. If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$ then λ and $\frac{\lambda}{3}$ are roots of:

A) $4x^2 + 24x - 27 = 0$ B) $4x^2 - 24x + 27 = 0$ C) $4x^2 - 24x - 27 = 0$ D) $4x^2 + 24x + 27 = 0$

Answer: $4x^2 - 24x + 27 = 0$



Solution:

Given that, $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$

Put $x = 0$ as $x \in R$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8}(103(0)+81)$$

On expanding the determinant we get,

$$\Rightarrow \lambda^3 = \frac{9}{8} \times 81$$

$$\Rightarrow \lambda^3 = \frac{9^3}{2^3}$$

$$\Rightarrow \lambda = \frac{9}{2} \text{ and } \frac{\lambda}{3} = \frac{9}{6}$$

Now the required quadratic equation is $x^2 - \left(\frac{9}{2} + \frac{9}{6}\right)x + \frac{9}{2} \times \frac{9}{6} = 0$

$$\Rightarrow x^2 - (6)x + \frac{27}{4} = 0$$

$$\Rightarrow 4x^2 - 24x + 27 = 0$$

Hence, the required quadratic equation is $4x^2 - 24x + 27 = 0$

Q.26.

For $\frac{dy}{dx} + \frac{5}{x(1+x^5)}y = \frac{(1+x^5)^2}{x^7}$; $y(1) = 2$, then the value of $y(2)$ is

A) $\frac{693}{128}$

B) $\frac{694}{128}$

C) $\frac{793}{128}$

D) $\frac{696}{128}$

Answer: $\frac{693}{128}$



Solution: Given:

$$\frac{dy}{dx} + \frac{5}{x(1+x^5)}y = \frac{(1+x^5)^2}{x^7}$$

This is linear differential equation.

$$\text{I. F.} = e^{\int \frac{5}{x(1+x^5)} dx}$$

$$\text{I. F.} = e^{\int \frac{5x^5}{x^6(1+x^5)} dx}$$

$$\text{I. F.} = e^{\int \frac{5}{x^6(x^{-5}+1)} dx}$$

$$\text{I. F.} = e^{-\int \frac{-5}{x^6(x^{-5}+1)} dx}$$

$$\text{I. F.} = e^{-\int \frac{d(x^{-5}+1)}{(x^{-5}+1)}}$$

$$\text{I. F.} = e^{-\log_e(x^{-5}+1)}$$

$$\text{I. F.} = (x^{-5}+1)^{-1} = \frac{x^5}{(x^5+1)}$$

So, solution is

$$y \frac{x^5}{(x^5+1)} = \int \frac{x^5}{(x^5+1)} \times \frac{(x^5+1)^2}{x^7} dx$$

$$\Rightarrow \frac{yx^5}{(x^5+1)} = \int \left(\frac{x^5+1}{x^2} \right) dx$$

$$\Rightarrow \frac{yx^5}{(x^5+1)} = \int \left(x^3 + \frac{1}{x^2} \right) dx$$

$$\Rightarrow \frac{yx^5}{(x^5+1)} = \frac{x^4}{4} - \frac{1}{x} + C$$

Since, $y(1) = 2$, so

$$\frac{2}{2} = \frac{1}{4} - 1 + C \Rightarrow C = \frac{7}{4}$$

So,

$$\frac{yx^5}{(x^5+1)} = \frac{x^4}{4} - \frac{1}{x} + \frac{7}{4}$$

Put $x = 2$, then we get

$$\frac{y(2) \times 32}{33} = \frac{16}{4} - \frac{1}{2} + \frac{7}{4}$$

$$\Rightarrow y(2) = \frac{21 \times 33}{128} = \frac{693}{128}$$

Q.27. Domain of $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ is

- A) $(-\infty, -2) \cup [6, \infty)$ B) $(-\infty, -2) \cup (6, \infty)$ C) $(-\infty, 2) \cup [6, \infty)$ D) None of these

Answer: $(-\infty, -2) \cup [6, \infty)$



Solution: Given:

$$f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$$

For $f(x)$ to be defined,

$$[x]^2 - 3[x] - 10 > 0$$

$$\Rightarrow ([x] - 5)([x] + 2) > 0$$

$$\Rightarrow [x] < -2 \text{ \& } [x] > 5$$

$$\Rightarrow x < -2 \text{ \& } x \geq 6$$

So, domain is $(-\infty, -2) \cup [6, \infty)$.

Q.28. Let mean and variance of the data 1, 2, 4, 5, x , y be 5 and 10, then the deviation of the data about mean will be

A) $\frac{8}{3}$

B) $\frac{3}{8}$

C) $\frac{16}{3}$

D) $\frac{8}{6}$

Answer: $\frac{8}{3}$

Solution: Consider the data 1, 2, 4, 5, x , y .

So,

$$\text{Mean} = \frac{1+2+4+5+x+y}{6}$$

$$\Rightarrow 5 = \frac{x+y+12}{6}$$

$$\Rightarrow x + y = 18$$

And,

$$\text{Variance} = \left(\frac{\sum x_i^2}{n} \right) - \left(\frac{\sum x_i}{n} \right)^2$$

$$\Rightarrow 10 = \left(\frac{1^2+2^2+4^2+5^2+x^2+y^2}{6} \right) - (5)^2$$

$$\Rightarrow x^2 + y^2 = 164$$

We have $x + y = 18$ and $x^2 + y^2 = 164$.

By hit and trial we get,

$$x = 10, y = 8.$$

$$\text{Mean deviation about mean is} = \frac{\sum_{i=1}^6 |x_i - \bar{x}|}{6}$$

$$= \frac{|1-5|+|2-5|+|4-5|+|5-5|+|10-5|+|8-5|}{6}$$

$$= \frac{16}{6}$$

$$= \frac{8}{3}$$

Therefore, the required answer is $\frac{8}{3}$

Q.29. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar vectors then the value of $\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$ is

A) $\begin{bmatrix} \vec{b} & \vec{d} & \vec{c} \end{bmatrix} + \begin{bmatrix} \vec{a} & \vec{d} & \vec{b} \end{bmatrix} + \begin{bmatrix} \vec{a} & \vec{d} & \vec{c} \end{bmatrix}$

B) $\begin{bmatrix} \vec{b} & \vec{d} & \vec{c} \end{bmatrix} + \begin{bmatrix} \vec{a} & \vec{b} & \vec{d} \end{bmatrix} + \begin{bmatrix} \vec{a} & \vec{d} & \vec{c} \end{bmatrix}$

C) $\begin{bmatrix} \vec{b} & \vec{c} & \vec{d} \end{bmatrix} + \begin{bmatrix} \vec{a} & \vec{b} & \vec{d} \end{bmatrix} + \begin{bmatrix} \vec{a} & \vec{d} & \vec{c} \end{bmatrix}$

D) $\begin{bmatrix} \vec{b} & \vec{c} & \vec{d} \end{bmatrix} + \begin{bmatrix} \vec{a} & \vec{d} & \vec{b} \end{bmatrix} + \begin{bmatrix} \vec{a} & \vec{d} & \vec{c} \end{bmatrix}$



Answer: $[\vec{b} \ \vec{c} \ \vec{d}] + [\vec{a} \ \vec{b} \ \vec{d}] + [\vec{a} \ \vec{d} \ \vec{c}]$

Solution: Given that $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar.

$\Rightarrow \vec{b} - \vec{a}, \vec{c} - \vec{a}, \vec{d} - \vec{a}$ are also coplanar.

$$\Rightarrow [\vec{b} - \vec{a} \ \vec{c} - \vec{a} \ \vec{d} - \vec{a}] = 0$$

$$\Rightarrow (\vec{b} - \vec{a}) \cdot ((\vec{c} - \vec{a}) \times (\vec{d} - \vec{a})) = 0$$

$$\Rightarrow (\vec{b} - \vec{a}) \cdot (\vec{c} \times \vec{d} - \vec{c} \times \vec{a} - \vec{a} \times \vec{d}) = 0 \quad \left(\text{Since } \vec{a} \times \vec{a} = 0 \right)$$

$$\Rightarrow \vec{b} \cdot (\vec{c} \times \vec{d}) - \vec{b} \cdot (\vec{c} \times \vec{a}) - \vec{b} \cdot (\vec{a} \times \vec{d}) - \vec{a} \cdot (\vec{c} \times \vec{d}) + \vec{a} \cdot (\vec{c} \times \vec{a}) + \vec{a} \cdot (\vec{a} \times \vec{d}) = 0$$

$$\Rightarrow [\vec{b} \ \vec{c} \ \vec{d}] - [\vec{b} \ \vec{c} \ \vec{a}] - [\vec{b} \ \vec{a} \ \vec{d}] - [\vec{a} \ \vec{c} \ \vec{d}] = 0$$

We know that $[\vec{x} \ \vec{y} \ \vec{z}] = [\vec{y} \ \vec{z} \ \vec{x}] = [\vec{z} \ \vec{x} \ \vec{y}]$ and $[\vec{x} \ \vec{y} \ \vec{z}] = -[\vec{x} \ \vec{z} \ \vec{y}]$

$$\Rightarrow [\vec{b} \ \vec{c} \ \vec{d}] + [\vec{a} \ \vec{b} \ \vec{d}] + [\vec{a} \ \vec{d} \ \vec{c}] = [\vec{a} \ \vec{b} \ \vec{c}]$$

Hence the required answer is $[\vec{b} \ \vec{c} \ \vec{d}] + [\vec{a} \ \vec{b} \ \vec{d}] + [\vec{a} \ \vec{d} \ \vec{c}]$

Q.30. A circle with centre at (2, 0) and maximum radius r is inscribed in the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$ then value of $12r^2$ is

A) 108

B) 172

C) 83

D) 92

Answer: 92



Solution:

Given equation of ellipse is $\frac{x^2}{36} + \frac{y^2}{9} = 1$

It is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

With $a = 6$, $b = 3$.

The centre of the circle is $C(2, 0)$.

Let $P(a \cos \theta, b \sin \theta)$ be the common point for ellipse and circle.

$$P \equiv (6 \cos \theta, 3 \sin \theta)$$

Equation of normal to the ellipse will be $\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2$.

$$\Rightarrow 6x \sec \theta - 3y \operatorname{cosec} \theta = 36 - 9$$

But the normal through P passes through the centre of the circle $C(2, 0)$.

$$\Rightarrow 6(2) \sec \theta - 3(0) \operatorname{cosec} \theta = 27$$

$$\Rightarrow \sec \theta = \frac{27}{12}$$

$$\Rightarrow \cos \theta = \frac{4}{9} \text{ and } \sin \theta = \frac{\sqrt{65}}{9}$$

$$P \equiv \left(6 \times \frac{4}{9}, 3 \times \frac{\sqrt{65}}{9} \right)$$

$$P \equiv \left(\frac{8}{3}, \frac{\sqrt{65}}{3} \right)$$

$$\text{Now } CP = r = \sqrt{\left(2 - \frac{8}{3} \right)^2 + \left(0 - \frac{\sqrt{65}}{3} \right)^2}$$

$$\Rightarrow r^2 = \frac{69}{9}$$

$$\Rightarrow 12r^2 = 12 \times \frac{69}{9} = 92$$

Therefore, the required answer is 92

Q.31. If the ratio of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$ is $1 : 3 : 5$, then sum of consecutive terms is

A) 63

B) 65

C) 60

D) 50

Answer: 63



Solution: Ratio of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$ is

$${}^{n+2}C_{r-1} : {}^{n+2}C_r : {}^{n+2}C_{r+1} :: 1 : 3 : 5$$

So,

$$\Rightarrow \frac{{}^{n+2}C_r}{{}^{n+2}C_{r-1}} = \frac{3}{1}$$

$$\Rightarrow \frac{n-r+3}{r} = 3$$

$$\Rightarrow n+3 = 4r \quad \dots (1)$$

And,

$$\frac{{}^{n+2}C_{r+1}}{{}^{n+2}C_r} = \frac{5}{3}$$

$$\Rightarrow \frac{n+2-r}{r+1} = \frac{5}{3}$$

$$\Rightarrow 3n+6-3r = 5r+5$$

$$\Rightarrow 3n+1 = 8r \quad \dots (2)$$

$$\text{So, } 3(4r-3)+1 = 8r$$

$$\Rightarrow 12r-9+1 = 8r$$

$$\Rightarrow 4r = 8$$

$$\Rightarrow r = 2$$

$$\text{So, } n = 5$$

Hence, we have

$$(1+x)^{n+2} = (1+x)^7$$

Sum of consecutive terms is

$${}^7C_1 + {}^7C_2 + {}^7C_3$$

$$= 7 + 21 + 35 = 63$$

Q.32. If $10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots \infty$, then find the real value of k

Answer: 2



Solution: Given,

$$10 = 1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + \dots \dots \dots \infty \dots \dots (1)$$

$$\frac{10}{k} = \frac{1}{k} + \frac{4}{k^2} + \frac{8}{k^3} + \frac{13}{k^4} + \dots \dots \dots \infty \dots \dots (2)$$

On subtracting above two equations we get,

$$10 - \frac{10}{k} = 1 + \frac{3}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + \dots \dots \dots \infty$$

$$\Rightarrow 10 \left(1 - \frac{1}{k}\right) = 1 + \frac{3}{k} + \frac{4}{k^2} + \frac{5}{k^3} + \frac{6}{k^4} + \dots \dots \dots \infty \dots \dots (3)$$

$$\Rightarrow 10 \left(1 - \frac{1}{k}\right) \left(\frac{1}{k}\right) = \frac{1}{k} + \frac{3}{k^2} + \frac{4}{k^3} + \frac{5}{k^4} + \frac{6}{k^5} + \dots \dots \dots \infty \dots \dots (4)$$

Again subtracting both equations we get,

$$\Rightarrow 10 \left(1 - \frac{1}{k}\right)^2 = 1 + \frac{2}{k} + \frac{1}{k^2} + \frac{1}{k^3} + \frac{1}{k^4} + \dots \dots \dots \infty$$

$$\Rightarrow 10 \left(1 - \frac{1}{k}\right)^2 = \left(1 + \frac{1}{k} + \frac{1}{k^2} + \frac{1}{k^3} + \frac{1}{k^4} + \dots \dots \dots \infty\right) + \frac{1}{k}$$

$$\Rightarrow 10 \left(1 - \frac{1}{k}\right)^2 = \left(\frac{1}{1 - \frac{1}{k}}\right) + \frac{1}{k}$$

$$\Rightarrow 10(k-1)^3 = k(k^2 + k - 1)$$

$$\Rightarrow 9k^3 - 31k^2 + 31k - 10 = 0$$

$$\Rightarrow k = 2$$

Hence, $k = 2$

Q.33. If $a + b + c + d = 11$ ($a, b, c, d > 0$), and the maximum value of $a^5 b^3 c^2 d$ is 3750β , then $\beta = \underline{\hspace{2cm}}$

Answer: 90

Solution: Given:

$$a + b + c + d = 11$$

We know that,

$$AM \geq GM$$

$$\Rightarrow \frac{\frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{a}{5} + \frac{b}{3} + \frac{b}{3} + \frac{b}{3} + \frac{c}{2} + \frac{c}{2} + \frac{d}{1}}{11} \geq \sqrt[11]{\frac{a^5 b^3 c^2 d}{5^5 3^3 2^2}}$$

$$\Rightarrow \frac{a+b+c+d}{11} \geq \sqrt[11]{\frac{a^5 b^3 c^2 d}{5^5 3^3 2^2}}$$

$$\Rightarrow \frac{11}{11} \geq \sqrt[11]{\frac{a^5 b^3 c^2 d}{5^5 3^3 2^2}}$$

$$\Rightarrow \sqrt[11]{\frac{a^5 b^3 c^2 d}{5^5 3^3 2^2}} \leq 1$$

$$\Rightarrow a^5 b^3 c^2 d \leq 5^5 3^3 2^2$$

$$\Rightarrow a^5 b^3 c^2 d \leq 90 \cdot 3750$$

So,

$$\beta = 90$$

Q.34. If $e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1 = 0$, then number of solution to the given equation will be,



Answer: 2

Solution: Given,

$$e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1 = 0$$

$$\Rightarrow e^{4x} - e^{2x} - 3 - \frac{1}{e^{2x}} + \frac{1}{e^{4x}} = 0$$

$$\Rightarrow e^{4x} + \frac{1}{e^{4x}} + 2 - \left(e^{2x} + \frac{1}{e^{2x}} \right) = 5$$

$$\Rightarrow \left(e^{2x} + \frac{1}{e^{2x}} \right)^2 - \left(e^{2x} + \frac{1}{e^{2x}} \right) = 3$$

$$\text{Now let } \left(e^{2x} + \frac{1}{e^{2x}} \right) = t$$

So, the equation becomes

$$t^2 - t - 5 = 0$$

$$\Rightarrow t = \frac{1+\sqrt{21}}{2}, \text{ ignoring negative sign as exponential function are positive,}$$

$$\text{Now } e^{2x} + \frac{1}{e^{2x}} = \frac{1+\sqrt{21}}{2}$$

$$\Rightarrow e^{4x} - \frac{1+\sqrt{21}}{2}e^{2x} + 1 = 0 \text{ which is quadratic equation in } e^{2x} \text{ with upward parabola,}$$

$$\text{Now by A.M} \geq \text{G.M we get, } e^{2x} + \frac{1}{e^{2x}} \geq 2$$

$$\text{So, } y = \frac{1+\sqrt{21}}{2} > 2, \text{ hence it will cut at two distinct point,}$$

Hence, there will be two solution.

Q.35. For a biased coin, the probability of getting head is $\frac{1}{4}$. It is tossed n times, till we get head. Given a quadratic equation $64x^2 + 2nx + 1 = 0$. If probability that the quadratic equation has no real roots is $\frac{P}{Q}$ (where, P and Q are co-prime), then the value of $Q - P$ is _____

Answer: 2187



Solution: Given a quadratic equation $64x^2 + 2nx + 1 = 0$. has no real roots, so

$$D < 0$$

$$\Rightarrow 4n^2 - 4 \cdot 64 < 0$$

$$\Rightarrow n < 8$$

$$\text{So, } n = 7$$

And, For a biased coin, the probability of getting head is $p = \frac{1}{4}$, $q = \frac{3}{4}$

So, required probability is

$$\frac{1}{4} + \frac{3}{4} \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^3 \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^4 \cdot \frac{1}{4} + \dots + \left(\frac{3}{4}\right)^6 \cdot \frac{1}{4} = \frac{P}{Q}$$

$$\Rightarrow \frac{\frac{1}{4} \left(1 - \left(\frac{3}{4}\right)^7\right)}{1 - \left(\frac{3}{4}\right)} = \frac{P}{Q}$$

$$\Rightarrow 1 - \left(\frac{3}{4}\right)^7 = \frac{P}{Q}$$

$$\Rightarrow 1 - \frac{2187}{16384} = \frac{P}{Q}$$

$$\Rightarrow \frac{P}{Q} = \frac{14197}{16384}$$

$$Q - P = 2187$$