JEE Main

Shift 1

Physics

Q.1. If a planet has mass equal to 16 times the mass of earth, and radius equal to 4 times that of earth. The ratio of escape speed of planet to that of earth is

A) $2:1$ B) $1:2$ C) $\sqrt{2}:1$ D) $4:1$

Answer: $2:1$

Solution: Escape velocity formula is given by,

$$v=\sqrt{\frac{2GM}{R}}$$

Given,

Mass of planet $(M_p)=16M_e$

Radius of planet $(R_p)=4R_e$

So,

$$\frac{vp}{ve}=\sqrt{\frac{Mp}{Me}\times\frac{Re}{Rp}}$$
$$=\sqrt{\frac{16}{4}}$$
$$=2:1$$

Q.2. Find ratio of de-Broglie wavelength of a proton and an $\alpha-$particle, when accelerated through a potential difference of $2\ \mathrm{V}$ and $4\ \mathrm{V}$ respectively.

A) $4:1$ B) $2:1$ C) $1:8$ D) $16:1$

Answer: $4:1$

Solution: Using energy conservation, $KE=PE$ or $\frac{p^2}{2m}=qV$

$\Rightarrow p=\sqrt{2mqV}$

Where, $p=$ momentum of particle,

$m=$ mass of particle,

$q=$ charge of particle, and

$V=$ potential difference.

We know that de-Broglie wavelength is given by,

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mqV}}$$

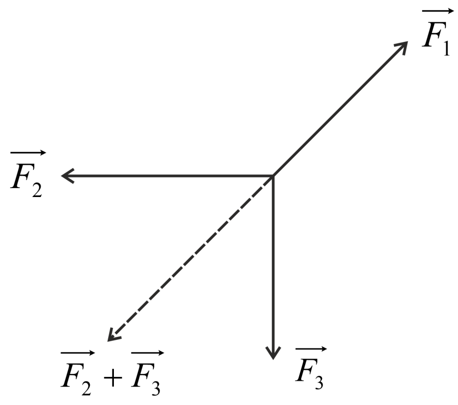
$$\Rightarrow\frac{\lambda p}{\lambda\alpha}=\sqrt{\frac{m\alpha}{mp}\times\frac{q\alpha}{qp}\times\frac{V\alpha}{Vp}}$$
$$=\sqrt{\frac{4}{1}\times\frac{2}{1}\times\frac{4}{2}}$$
$$=4:1$$

Q.3. A body of mass $5\ \mathrm{kg}$ is in equilibrium due to forces $F_1,\ F_2$ and F_3. F_2 and F_3 are perpendicular to each other. If F_1 is removed then find the acceleration of body. Given $F_2=\ 6\ \mathrm{N}$ and $F_3=8\ \mathrm{N}$.

A) $2\ \mathrm{m\ s^{-2}}$ B) $3\ \mathrm{m\ s^{-2}}$ C) $4\ \mathrm{m\ s^{-2}}$ D) $5\ \mathrm{m\ s^{-2}}$

Answer: $2\ \mathrm{m\ s^{-2}}$

Solution: Since, the forces F_2, F_3 are mutually perpendicular and all the three forces keep the object in equilibrium, the force F_1 must act along the opposite direction to the resultant fo the forces F_2 and F_3, as depicted in the following figure:



The magnitude of force F_1 is, then, given by

$$\left|\vec{F}_1\right|^2 = \left|\vec{F}_2\right|^2 + \left|\vec{F}_3\right|^2$$

$$\Rightarrow \left|\vec{F}_1\right| = \sqrt{\left|\vec{F}_2\right|^2 + \left|\vec{F}_3\right|^2}$$

$$= \sqrt{(6\ \text{N})^2 + (8\ \text{N})^2}$$

$$= 10\ \text{N}$$

Hence, the acceleration (a) of the particle, in absence of force F_1, will be directed along the direction of the resultant of the other two forces and the magnitude of the acceleration is given by

$$a = \frac{F_1}{m}$$

$$= \frac{10\ \text{N}}{5\ \text{kg}}$$

$$= 2\ \text{m s}^{-2}$$

Q.4. Ratio between rms speed of Ar to the most probable speed of O_2 at $27^\circ\ \text{C}$ is

A) $\sqrt{\frac{8}{\pi}}$ B) $\sqrt{\frac{6}{5}}$ C) $\sqrt{\frac{4}{\pi}}$ D) $\sqrt{\frac{4}{3}}$

Answer: $\sqrt{\frac{6}{5}}$

Solution: The rms speed of Ar is given by

$$v_{rms} = \sqrt{\frac{3RT}{M_{Ar}}} \quad \ldots (1)$$

The most probable speed of O_2 is given by

$$v_{mp} = \sqrt{\frac{2RT}{M_{O_2}}} \quad \ldots (2)$$

Divide equation (1) by equation (2) and simplify to obtain the ratio of the two speeds.

$$\frac{v_{rms}}{v_{mp}} = \frac{\sqrt{\frac{3RT}{M_{Ar}}}}{\sqrt{\frac{2RT}{M_{O_2}}}}$$

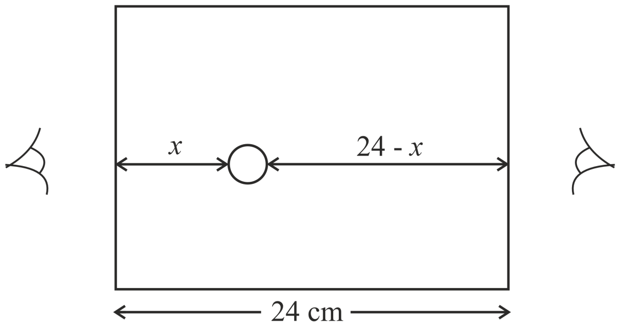
$$= \sqrt{\frac{3M_{O_2}}{2M_{Ar}}} \quad \ldots (3)$$

Substitute the values of the known parameters into equation (3) to calculate the required ratio.

$$\frac{v_{rms}}{v_{mp}} = \sqrt{\frac{3\times 32}{2\times 40}}$$

$$= \sqrt{\frac{6}{5}}$$

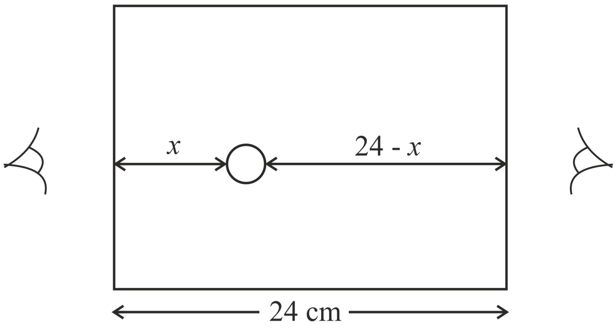
Q.5. In an ice cube of thickness $24\ \text{cm}$, a bubble is trapped as shown in figure. If apparent depths are $12\ \text{cm}$ and $4\ \text{cm}$ from side 1 and side 2 respectively, then refractive index of ice cube is



A) $\frac{4}{3}$ B) 2 C) $\frac{3}{2}$ D) 2.4

Answer: $\frac{3}{2}$

Solution: Let's consider that the bubble is situated at a distance x from the left edge of the ice cube, as depicted by the following figure.



From the figure, considering the observer to be on the left side of the cube, it can be written that,

$$\frac{\text{Apparent depth}}{\text{Real depth}} = \frac{1}{\mu}$$
$$\frac{12}{x} = \frac{1}{\mu} \quad \ldots(1)$$

where, μ is the refractive index of ice cube.

Also, considering the observer on the right side of the cube, it can be written that,

$$\frac{\text{Apparent depth}}{\text{Real depth}} = \frac{1}{\mu}$$
$$\frac{4}{24-x} = \frac{1}{\mu} \quad \ldots(2)$$

Equate equations (1) and (2) and solve to calculate the value of x.

$$\frac{12}{x} = \frac{4}{24-x}$$
$$\Rightarrow 288 - 12x = 4x$$
$$\Rightarrow 16x = 288$$
$$\Rightarrow x = 18$$

Substitute the value of x into equation (1) and solve to calculate the required refractive index of ice cube.

$$\frac{12}{18} = \frac{1}{\mu}$$
$$\Rightarrow \mu = \frac{18}{12}$$
$$= \frac{3}{2}$$

Q.6. A dipole having dipole moment $\overrightarrow{M}$ is placed in two magnetic fields of strengths B_1 and B_2 respectively. If dipole oscillates 60 times in 20 seconds in B_1 magnetic field and 60 oscillations in 30 seconds in B_2 magnetic field. Then find $\left(\frac{B_1}{B_2}\right)$

A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) $\frac{4}{9}$ D) $\frac{9}{4}$

Answer: $\frac{9}{4}$

Solution: The time period of oscillation of a magnetic dipole is given by,

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Where, I is the moment of Inertia, M is the magnetic dipole moment and B is the magnetic field.

$$\Rightarrow T \propto \frac{1}{\sqrt{B}}$$

$$\Rightarrow \frac{B1}{B2} = \left(\frac{T2}{T1}\right)^2$$

$$= \left(\frac{30}{60} \times \frac{60}{20}\right)^2$$

$$= \frac{9}{4}$$

Q.7. The length of a conductor having resistance $160\ \Omega$, is compressed to 25% of its initial value. The new resistance will be

A) $10\ \Omega$ B) $20\ \Omega$ C) $15\ \Omega$ D) $17\ \Omega$

Answer: $10\ \Omega$

Solution: Here, as length (l) is changed, cross-sectional area (A) will also change. But, overall volume (V) remains unchanged.

So, in $R = \rho\frac{l}{A}$ substitute $A = \frac{V}{l}$

$$\Rightarrow R = \rho\frac{l^2}{V}$$

But ρ and V are constant here.

So, $R \propto l^2 \Rightarrow$ If length is compressed to 25%,
The new length $l' = 25\%$ of l

$$= \frac{25}{100} \times l = \frac{l}{4}$$

$$\Rightarrow \frac{R'}{R} = \left(\frac{l'}{l}\right)^2$$

$$= \left(\frac{l}{4l}\right)^2$$

$$= \frac{1}{16}$$

$$\Rightarrow R' = \frac{R}{16} = \frac{160}{16} = 10\ \Omega$$

Q.8. Statement (1) : In LCR circuit, by increasing frequency, current increases first then decreases.

Statement (2) : Power factor of LCR circuit is one at resonance.

A) Statement 1 is correct and Statement 2 is incorrect

B) Statement 1 is incorrect and Statement 2 is correct

C) Both Statement 1 and Statement 2 are correct

D) Both Statement 1 and Statement 2 are incorrect

Answer: Both Statement 1 and Statement 2 are correct

Solution: The frequency response curve of a series LCR circuit shows that the magnitude of the current amplitude is a function of angular frequency and plotting this onto a graph shows us that the response starts at near to zero, reaches maximum value at the resonance frequency and then drops again to nearly zero as ω becomes infinite.

Power factor for series LCR circuit is given by formula,

$$\cos(\phi) = \frac{R}{Z}.$$

At resonance, for series LCR circuit, $X_L = X_C \Rightarrow R = Z$

So, power factor is one for series LCR circuit at resonance.

Q.9. Match the physical quantity in column 1 with the respective dimensions in column 2 and choose the correct option.

Column 1	Column 2
(I) Spring constant	(P) $\left[ML^2T^0\right]$
(II) Moment of inertia	(Q) $\left[M^0L^0T^{-1}\right]$
(III) Angular momentum	(R) $\left[ML^0T^{-2}\right]$
(IV) Angular speed	(S) $\left[ML^2T^{-1}\right]$

A) $I \rightarrow P,\ II \rightarrow Q,\ III \rightarrow R,\ IV \rightarrow S$

B) $I \rightarrow R,\ II \rightarrow P,\ III \rightarrow Q,\ IV \rightarrow S$

C) $I \rightarrow R,\ II \rightarrow S,\ III \rightarrow Q,\ IV \rightarrow P$

D) $I \rightarrow R,\ II \rightarrow P,\ III \rightarrow S,\ IV \rightarrow Q$

Answer: $I \rightarrow R,\ II \rightarrow P,\ III \rightarrow S,\ IV \rightarrow Q$

Solution: The dimensional formula can be matched as follows:

$$[F] = [kx]$$
$$\Rightarrow \left[MLT^{-2}\right] = [k]\,[L]$$
$$\Rightarrow [k] = \left[ML^0T^{-2}\right]$$

$$[I] = \left[MR^2\right]$$
$$= \left[ML^2T^0\right]$$

$$[L] = [mvr]$$
$$= [M]\left[LT^{-1}\right][L]$$
$$= \left[ML^2T^{-1}\right]$$

Now, $[\omega] = \left[\frac{2\pi}{T}\right] = \left[M^0L^0T^{-1}\right]$

Hence, the correct option is $I \rightarrow R,\ II \rightarrow P,\ III \rightarrow S,\ IV \rightarrow Q$.

Q.10. A circular ring is placed in magnetic field of $0.4\ \mathrm{T}$. Suddenly, its radius starts shrinking at the rate of $1\ \mathrm{mm\ s^{-1}}$. Find the induced emf in the ring at $r = 2\ \mathrm{cm}$.

A) $16\pi\ \mu\mathrm{V}$

B) $8\pi\ \mu\mathrm{V}$

C) $16\pi\ \mathrm{mV}$

D) $8\pi\ \mathrm{mV}$

Answer: $16\pi\ \mu\mathrm{V}$

Solution: The formula to calculate the induced emf (ε) in the circular ring can be obtained as follows:

$$\varepsilon = -\frac{d\varphi}{dt}$$
$$= -\frac{d}{dt}(BA)$$
$$= -B\frac{d}{dt}\left(\pi r^2\right)$$
$$= -\pi B(2r)\frac{dr}{dt}$$
$$= -2\pi Br\frac{dr}{dt} \quad \ldots(1)$$

Substitute the values of the known parameters into equation (1) to calculate the required induced emf in the ring.

$$\varepsilon = -2\pi \times 0.4\ \mathrm{T} \times 0.02\ \mathrm{m} \times \left(-0.001\ \mathrm{m\ s^{-1}}\right)$$
$$= 0.000016\pi\ \mathrm{V} \times \frac{10^6\ \mu\mathrm{V}}{1\ \mathrm{V}}$$
$$= 16\pi\ \mu\mathrm{V}$$

Q.11. A particle is thrown vertically upward with initial velocity of $150\ \mathrm{m\ s^{-1}}$. Find the ratio of its speed at $t = 3\ \mathrm{s}$ and $t = 5\ \mathrm{s}$. (Take $g = 10\ \mathrm{m\ s^{-2}}$)

Answer: 1.20

Solution: The velocity(v_3) of the particle after $t = 3\ \text{s}$ is given by

$$v_3 = [u - gt]$$
$$= (150 - 10 \times 3)\ \text{m s}^{-1}$$
$$= 120\ \text{m s}^{-1} \quad \dots (1)$$

The velocity (v_5) of the particle after $t = 5\ \text{s}$ is given by

$$v_5 = [u - gt]$$
$$= (150 - 10 \times 5)\ \text{m s}^{-1}$$
$$= 100\ \text{m s}^{-1} \quad \dots (2)$$

Divide equation (1) by equation (2) to obtain the required ratio.

$$\frac{v_3}{v_5} = \frac{120\ \text{m s}^{-1}}{100\ \text{m s}^{-1}}$$
$$= 1.20$$

Q.12. 64 identical balls made of conducting material each having potential of $10\ \text{mV}$ are joined to form a bigger ball. The potential of bigger ball is ______ V.

Answer: 0.16

Solution: We know that for a conducting sphere, capacitance is given by, $C = 4\pi\varepsilon_0 R$

So, potential of small sphere, $V = \frac{Q}{C} = \frac{q}{4\pi\varepsilon_0 r} = \frac{kq}{r}$

Given,

64 balls of radius r forms single bigger ball of radius R.

$$\Rightarrow 64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$
$$\Rightarrow 64 r^3 = R^3$$
$$\Rightarrow R = 4r$$

Also, charge on bigger ball becomes $Q = 64q$

The potential on bigger ball is given by,

$$V' = \frac{kQ}{R}$$
$$= \frac{k \times 64q}{4r}$$
$$= 16 \times \frac{kq}{r}$$
$$= 16V$$
$$= 16 \times 10 \times 10^{-3}\ \text{V}$$
$$= 0.16\ \text{V}$$

Q.13. A photon of energy $12.75\ \text{eV}$ falls on a H-atom. Find out the numbers of spectral lines observed.

Answer: 6

Solution: The first few values of the binding energies of H-atom are as follows:

$n=1 \rightarrow E_1 = -13.6 \text{ eV}$
$n=2 \rightarrow E_2 = -3.4 \text{ eV}$
$n=3 \rightarrow E_3 = -1.5 \text{ eV}$
$n=4 \rightarrow E_4 = -0.85 \text{ eV}$
$n=5 \rightarrow E_5 = -0.55 \text{ eV}$

The difference in energy levels between $n=1$ and $n=4$ is given by

$$\Delta E = -13.6 \text{ eV} - (-0.85 \text{ eV}) \\ = -12.75 \text{ eV}$$

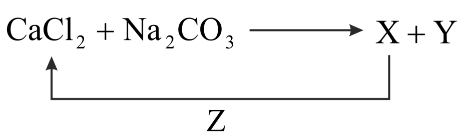
The above calculation shows that the incident photon transfers one electron from $n=1$ to $n=4$ state.

Hence, the number of possible transitions (N) can be calculated as follows:

$$N = {}^4C_2 = \frac{4!}{2!(4-2)!} = 6$$

Chemistry

Q.14. Consider the following reaction:



A) $X: CaCO_3, Y: NaCl, Z: HCl$

B) $X: CaO, Y: NaCl + CO_2, Z: KCl$

C) $X: CaO, Y: NaCl + CO_2, Z: NaCl$

D) $X: CaCO_3, Y: NaCl, Z: KCl$

Answer: $X: CaCO_3, Y: NaCl, Z: HCl$

Solution: When $CaCl_2$ and Na_2CO_3 react, a double displacement reaction or precipitation reaction is likely to occur.

So, the balanced chemical equation for the reaction would be:

$CaCl_2 + Na_2CO_3 \rightarrow CaCO_3 + 2NaCl$

So the products X and Y are $CaCO_3$ and $NaCl$.

Then X on reaction with HCl gives Calcium chloride, water and carbon dioxide. This is an acid-base reaction (neutralization)

$CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$

Q.15. Match the column I with column II

Column I	Column II
A. Biodegradable	P. Polyacrylonitrile
B. Synthetic	Q.PHBV
C. Natural	R. Dacron
D. Polyester	S. Rubber

A) $A-Q, B-P, C-S, D-R$

B) $A-P, B-R, C-Q, D-S$

C) $A-R, B-P, C-Q, D-S$

D) $A-R, B-R, C-S, D-Q$

Answer: $A-Q, B-P, C-S, D-R$

Solution: PHBV(Poly(3-hydroxybutyrate-co-3-hydroxyvalerate)) is known for its biodegradability, which means it can be broken down by natural processes into smaller molecules by the action of microorganisms, enzymes, or other biological mechanisms.

Polyacrylonitrile (PAN) is a synthetic polymer that belongs to the family of acrylic polymers.

Natural rubber is a type of natural polymer. It is derived from the sap or latex of the rubber tree (Hevea brasiliensis) and is a naturally occurring polymer.

Dacron is a type of polyester fabric made from polyethylene terephthalate (PET). It is a thermoplastic polymer that is commonly used in clothing, bedding, and as a fibrefill material in various products.

Q.16. Statement 1: Boron is hard as it has high lattice energy.

Statement 2: Boron has high melting and boiling points as compared to other group elements.

A) Statement 1 is correct and Statement 2 is incorrect

B) Statement 1 is incorrect and Statement 2 is correct

C) Both the statements are correct

D) Both the statements are incorrect

Answer: Both the statements are correct

Solution: Boron is hard as it has high lattice energy. Due to very strong crystall lattice boron has unusually highest melting point. Rest of the members are soft metals with low meling point and high conductivity. Boron forms strong covalent bonds with the neighbouring atoms. Thus boron atom are closely packed in its solid state, so a large amount of heat is needed to break the bonds between atoms.

Q.17. The bond order and magnetic property of acetylide ion is same that of

A) NO^+ B) ${N_2}^+$ C) ${O_2}^-$ D) ${O_2}^+$

Answer: NO^+

Solution: Acetanylide has bond order 3.

NO -Bond order $= \frac{10-5}{2} = 2.5$

From NO the NO^+ should be formed and the bond order increases to 3.0.

For ${N_2}^+$ it is 2.5

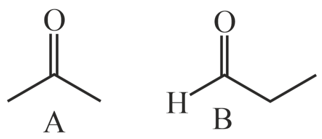
For ${O_2}^-$ it is 1.5

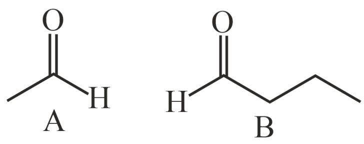
For ${O_2}^+$ it is 2.5

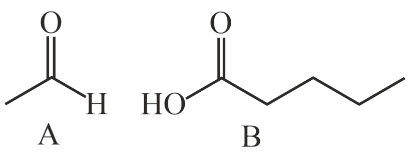
So the answer is option A.

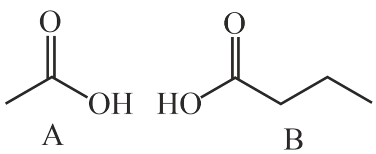
Q.18. Find the product A and B in the given reaction :

Hex-2-ene$\xrightarrow[H_2O_2]{O_3}$ $A + B$

a


b


c


d


A) a B) b C) c D) d

Answer: b

Solution: This reaction is known as ozonolysis where hex-2-ene reacts with O_3 to give ozonide which on reacts with hydrogen peroxide gives butanal and ethanal as major products.

$$CH_2 - CH = CH - CH_2 - CH_2 - CH_3 \xrightarrow[H_2O_2]{O_3} CH_3CHO + CH_3 - CH_2 - CH_2 - CHO$$

Q.19. Match the following

Column I Type of Hydride	Column II Formula
(A) Electro deficient	(1) MgH_2
(B) Electron precise	(2) HF
(C) Electron rich	(3) CH_4
(D) Saline hydride	(4) B_2H_6

A) A-4,B-3, C-1,D-2 B) A-4,B-1,C-3, D-2 C) A-4,B-3,C-2, D-1 D) A-1,B-2,C-3, D-4

Answer: A-4,B-3,C-2, D-1

Solution: In diborane, 8 covalent bonds and 12 electrons are present. But it requires a total 14 electrons to form bonding. It lacks 2 electron pairs. Thus B_2H_6 is electron deficient.

The type of hydrides that have the exact number of electrons to form a covalent bond is called electron precise. These types of compounds are usually formed by group 14 elements. Therefore, CH_4 electron precise species.

HF is electron rich hydride as it is having extra electrons present as lone pair on fluorine.

Saline hydrides are compounds form between hydrogen and the most active metals, especially with the alkali and alkaline-earth metals of group one and two elements.Therefore, MgH_2 is saline hydride.

Q.20. Statement-1 : $SbCl_5$ is more covalent than $SbCl_3$.

Statement-2 : Higher oxidation state halides of group 15 are more stable.

A) Both the statements are correct B) Both the statements are wrong

C) Statement 1 is correct and statement 2 is wrong D) Statement 2 is correct and statement 1 is wrong

Answer: Statement 1 is correct and statement 2 is wrong

Solution: The oxidation state of $+5$ in pentahalides is more covalent as compared to the $+3$ oxidation state in trihalides. Due to the higher positive oxidation state of the central atom in pentahalide state, these atoms will have larger polarising power than the halogen atom attached to them since the polarising power is directly proportional to the charge. More is the polarisation, larger will be the covalent character of the bond. Hence, due to larger polarisation of bond in pentahalide state as compared to trihalide state, the $SbCl_5$ is more covalent than $SbCl_3$.

As we move down in Group 15, we can see the inert pair effect. This effect impacts the penetration effect of s orbitals. These orbitals penetrate and come closer to the nucleus and enjoy greater nuclear attraction. Also, these orbitals, then, don't participate in bonding. This is the reason why the stability of $+3$ oxidation state increases in heavier elements of Group 15 in comparison with $+5$ oxidation state.

Q.21. Density of Group-1 metals follow the order:

A) $Li > Na > K > Rb$ B) $Li > K > Na > Rb$ C) $Rb > K > Na > Li$ D) $Rb > Na > K > Li$

Answer: $Rb > Na > K > Li$

Solution: Due to having the lowest atomic weight and the largest atomic radius of all the elements in their periods, the alkali metals are the least dense metals in the periodic table with Lithium, Sodium, and Potassium being the only three metals in the periodic table that are less dense than water. Atomic radius, which increases going down the group; thus, the volume of an alkali metal atom increases going down the group. The mass of an alkali metal atom also increases going down the group.

Correct order of density of alkali metals is $Rb > Na > K > Li$

Q.22. Critical temperature (T_C) of gases A, B, C and D are given as $5.3,\ 20.3,\ 128.5,\ 166.5$. Find the correct order of adsorption of gases from the options given below.

A) $A > B > C > D$ B) $D > C > B > A$ C) $C > D > A > B$ D) $B > A > C > D$

Answer: $D > C > B > A$

Solution: Critical temperature of a gas is the temperature at or above which vapor of the gas cannot be liquefied, no matter how much pressure is applied. The extent of adsorption is directly proportional to the critical temperature of the gas. Higher is the critical temperature of the gas, greater is the extent of adsorption.

Therefore, the correct option is B.

Q.23. Select the correct statement about the lead storage battery.

A) $PbSO_4$ converts into PbO_2 at anode during discharging.

B) $PbSO_4$ converts into PbO_2 at cathode during discharge.

C) 38% H_2SO_4 solution is taken as the electrolyte.

D) H_2SO_4 is produced during discharging.

Answer: 38% H_2SO_4 solution is taken as the electrolyte.

Solution: Lead storage battery consists of a lead anode and a grid of lead packed with lead dioxide as cathode. A 38% solution of sulphuric acid is used as an electrolyte. First statement is wrong as lead sulphate is formed at anode and also at cathode lead dioxide reacts. So among the given options Option C is correct.

Q.24. Statement 1: In Elingham diagram, the change in slope for Mg to MgO reaction occurs at 1120^oC.

Statement 2: Sudden change in entropy also occurs at 1120^oC.

A) Both statements are correct.

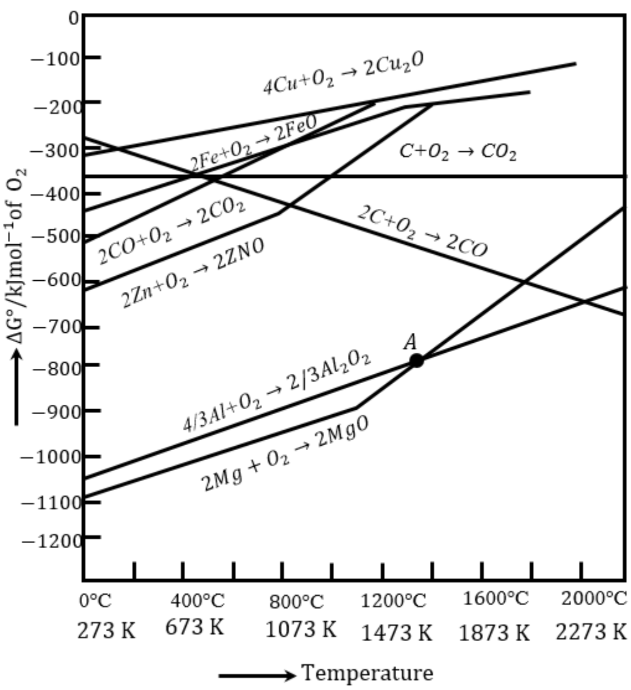
B) Both statements are incorrect.

C) Statement 1 is correct but Statement 2 is incorrect.

D) Statement 1 is incorrect but Statement 2 is correct.

Answer: Both statements are correct.

Solution: The Elingham diagram is a graphical representation of the thermodynamic stability of metal oxides. It shows the relationship between the free energy change (ΔG) of a metal reacting with oxygen gas (O_2) to form its oxide (MO) at different temperatures.



From the graph each metal oxide line started to move upward towards positive value of ΔG. The metal oxide formation is not possible if it is unstable. So every metal oxide is unstable at a certain point. From the diagram it is clear that both the statements are correct.

Q.25. pH of 1 litre of HCl solution is 1. How much water (in litres) is added to make $pH = 2$?

Answer: 9

Solution:

$pH = 1$
$[H^+] = 10^{-1} = 0.1M$
$pH = 2$
$[H^+] = 10^{-2} = 0.01M$

If volume $V_1 = 1$ lit, then molarity, $M_1 = 10^{-1}$

Given, $M_2 = 10^{-2}$

We know the formula,

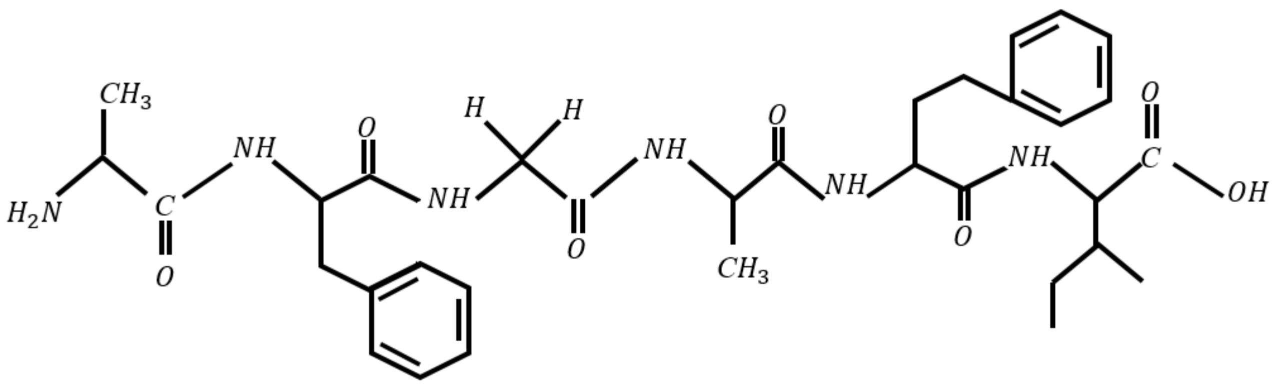
$M_1V_1 = M_2V_2$

$10^{-1} \times 1 = 10^{-2} \times V_2$
$V_2 = 10$ L

The volume of water added, $V_2 - V_1 = 10 - 1 = 9$ L

Q.26. The number of sp^2 hybridised carbon atoms in the following peptide is :

Ala-Phe-Gly-Ala-Phe-Leu



Answer: 18

Solution: In the given structure, there are two benzene rings and each benzene ring have six sp^2 hybridised carbon atoms. There are six $-C=O$ which participate in sp^2 hybridisation. Therefore, the number of sp^2 hybridised carbon atoms in the give peptide is 18.

Mathematics

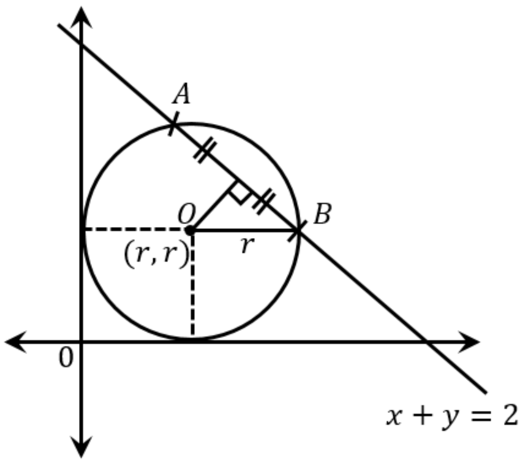
Q.27. Two circles having radius r_1 & r_2 touch both the coordinate axes, if line $x + y = 2$ makes the intercept 2 on both the circles then the value of ${r_1}^2 + {r_2}^2 - r_1r_2$ will be

A) $\frac{9}{2}$ B) 6 C) 7 D) 8

Answer: 7

Solution: Assuming a circle of radius r which touches both the axis is given by, $(x-r)^2+(y-r)^2=r^2$

Now given line $x+y=2$ makes the intercept 2 the circle, so plotting the diagram we get,



Now from diagram,

$AC=BC=1,\ OC=\sqrt{r^2-1}$

Now using the formula of perpendicular distance of point from the line we get,

$OC=\left|\frac{2r-2}{\sqrt{2}}\right|=\sqrt{r^2-1}$

$\Rightarrow 2(r-1)^2=r^2-1$

$\Rightarrow r^2-4r+3=0$

$\Rightarrow r_1=1,\ r_2=3$

Hence, $r_1^2+r_2^2-r_1r_2=1+9-3=7$

Q.28. If $\left(1+x^2\right)dy=y\,(y-x)dx$ and $y\,(1)=1$ then the value of $y\left(\sqrt{2}\right)$ will be

A) $\frac{4}{\sqrt{2}}$ B) $\frac{3}{\sqrt{2}}$ C) $\frac{1}{\sqrt{2}}$ D) $\sqrt{2}$

Answer: $\frac{1}{\sqrt{2}}$

Solution: Given,

$\left(1+x^2\right)dy=y\,(y-x)dx$ and $y\,(1)=1$

Now rewriting the above equation we get,

$\frac{dy}{dx}+\frac{x}{\left(1+x^2\right)}\cdot y=\frac{y^2}{\left(1+x^2\right)}$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}+\frac{x}{\left(1+x^2\right)}\cdot\frac{1}{y}=\frac{1}{\left(1+x^2\right)}$

Now let $\frac{1}{y}=t\Rightarrow\frac{-1}{y^2}\cdot\frac{dy}{dx}=\frac{dt}{dx}$

So, the equation becomes,

$-\frac{dt}{dx}+\frac{x}{\left(1+x^2\right)}\cdot t=\frac{1}{\left(1+x^2\right)}$

$\Rightarrow\frac{dt}{dx}-\frac{x}{\left(1+x^2\right)}\cdot t=\frac{-1}{\left(1+x^2\right)}$

Now integrating factor will be $IF = e^{-\int \frac{x}{1+x^2}dx} = e^{\frac{-1}{2}\ln\left(1+x^2\right)} = \frac{1}{\sqrt{1+x^2}}$

Now solution of differential equation is given by,

$t \times \frac{1}{\sqrt{1+x^2}} = -\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2}}dx$

Now solving integral $I = \int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2}}dx$

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$ we get,

$I = \int \frac{\sec^2\theta}{\sec^2\theta \cdot \sec\theta}d\theta = \int \cos\theta d\theta = \sin\theta + c$

$\Rightarrow I = \frac{x}{\sqrt{1+x^2}} \left\{\text{as x} = \tan\theta \Rightarrow \frac{\text{x}}{\sqrt{1+\text{x}^2}} = \sin\theta\right\}$

So, $t \times \frac{1}{\sqrt{1+x^2}} = -\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2}}dx$

$\Rightarrow t \times \frac{1}{\sqrt{1+x^2}} = \frac{-x}{\sqrt{1+x^2}} + c$

$\Rightarrow \frac{1}{y} \times \frac{1}{\sqrt{1+x^2}} = \frac{-x}{\sqrt{1+x^2}} + c$

Now using $y(1) = 1$ we get, $c = \sqrt{2}$

$\frac{1}{y} \times \frac{1}{\sqrt{1+x^2}} = \frac{-x}{\sqrt{1+x^2}} + c$

$\Rightarrow \frac{1}{y} \times \frac{1}{\sqrt{1+x^2}} = \frac{-x}{\sqrt{1+x^2}} + \sqrt{2}$

Now $y\left(2\sqrt{2}\right)$ will be,

$\frac{1}{y} \times \frac{1}{\sqrt{1+8}} = \frac{-2\sqrt{2}}{\sqrt{1+8}} + \sqrt{2}$

$\Rightarrow \frac{1}{y} \times \frac{1}{3} = \frac{-2\sqrt{2}}{3} + \sqrt{2}$

$\Rightarrow \frac{1}{y} = \sqrt{2}$

$\Rightarrow y = \frac{1}{\sqrt{2}}$

Q.29. For the expression $(1-x)^{100}$. Then sum of coefficients of first 50 terms is:

A) ${}^{99}C_{49}$

B) $-\frac{{}^{100}C_{50}}{2}$

C) $-{}^{99}C_{49}$

D) $-{}^{101}C_{50}$

Answer: $-\frac{{}^{100}C_{50}}{2}$

Solution: We know that $(1-x)^{100} = {}^{100}C_0 - {}^{100}C_1 \cdot x + {}^{100}C_2 \cdot x^2 - \ldots\ldots - {}^{100}C_{49} \cdot x^{49} + \ldots\ldots + {}^{100}C_{100} \cdot x^{100}$

The sum of first fifty coefficients be S.

$\Rightarrow S = {}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - \ldots\ldots - {}^{100}C_{49}$

Let us substitute $x = 1$ in the expansion.

$\Rightarrow (1-1)^{100} = {}^{100}C_0 - {}^{100}C_1 \cdot 1 + {}^{100}C_2 \cdot 1^2 - \ldots\ldots - {}^{100}C_{49} \cdot 1^{49} + \ldots\ldots + {}^{100}C_{100} \cdot 1^{100}$

$\Rightarrow 0 = {}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - \ldots\ldots - {}^{100}C_{49} + {}^{100}C_{50} - {}^{100}C_{51} + {}^{100}C_{52} \cdot \ldots\ldots + {}^{100}C_{100}$

There are total of 101 terms and the middle term is $T_{51} = {}^{100}C_{50}$.

As ${}^nC_r = {}^nC_{n-r}$

$\Rightarrow 0 = {}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - \ldots\ldots - {}^{100}C_{49} + {}^{100}C_{50} - {}^{100}C_{49} + {}^{100}C_{48} \cdot \ldots\ldots + {}^{100}C_0$

$\Rightarrow 0 = {}^{100}C_{50} + {}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - \ldots\ldots - {}^{100}C_{49} - {}^{100}C_{49} + {}^{100}C_{48} \cdot \ldots\ldots + {}^{100}C_0$

$\Rightarrow 0 = {}^{100}C_{50} + 2\left({}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - \ldots\ldots - {}^{100}C_{49}\right)$

$\Rightarrow -\frac{{}^{100}C_{50}}{2} = \left({}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - \ldots\ldots - {}^{100}C_{49}\right)$

$\Rightarrow {}^{100}C_0 - {}^{100}C_1 + {}^{100}C_2 - \ldots\ldots - {}^{100}C_{49} = -\frac{{}^{100}C_{50}}{2}$

Therefore, the required answer is $-\frac{{}^{100}C_{50}}{2}$.

Q.30. If $\Delta(k) = \begin{vmatrix} 1 & 2k-1 & 2k \\ n & n^2 & n(n+1) \\ \cos^2 n & \cos^2(n+1) & (n+2) \end{vmatrix}$, then find the value of $\sum_{k=1}^{n} \Delta(k) =$

A) n B) 1 C) $\frac{n^2}{2}$ D) 0

Answer: 0

Solution: Given,

$$\Delta(k) = \begin{vmatrix} 1 & 2k-1 & 2k \\ n & n^2 & n(n+1) \\ \cos^2 n & \cos^2(n+1) & (n+2) \end{vmatrix}$$

Now finding $\sum_{k=1}^{n} \Delta(k)$ we get,

$$\sum_{k=1}^{n} \Delta(k) = \begin{vmatrix} \sum_{k=1}^{n} 1 & \sum_{k=1}^{n} (2k-1) & \sum_{k=1}^{n} (2k) \\ n & n^2 & n(n+1) \\ \cos^2 n & \cos^2(n+1) & (n+2) \end{vmatrix}$$

$$\Rightarrow \sum_{k=1}^{n} \Delta(k) = \begin{vmatrix} n & n^2 & n(n+1) \\ n & n^2 & n(n+1) \\ \cos^2 n & \cos^2(n+1) & (n+2) \end{vmatrix}$$

$\Rightarrow \sum_{k=1}^{n} \Delta(k) = 0$

Q.31. Area of region enclosed by curve $y = x^3$ and its tangent at $(-1, -1)$.

A) 4 B) $\frac{27}{4}$ C) $\frac{4}{27}$ D) 27

Answer: $\frac{27}{4}$

Solution: Given curve is $y=x^3$.

Slope of tangent at $y=x^3$ is $y'=3x^2$.

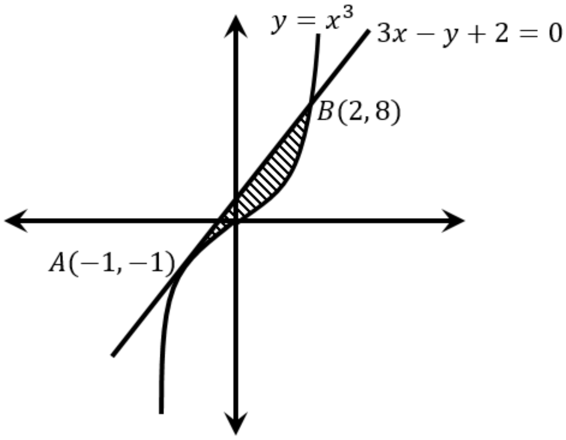
$\Rightarrow y'|_{x=-1}=3(-1)^2=3$.

Equation of tangent at (x_1,y_1) on a curve with slope m is $y-y_1=m(x-x_1)$.

$\Rightarrow y-(-1)=3(x-(-1))$

$\Rightarrow y=3x+2$.

The Point of intersection of the tangent and the curve will be $(2,8)$.



Required area is $=\int_{-1}^{2}\left(3x+2-x^3\right)\mathrm{d}x$

$=\left(\frac{3x^2}{2}+2x-\frac{x^4}{4}\right)_{-1}^{2}$

$=\left(\frac{3(2)^2}{2}+2(2)-\frac{(2)^4}{4}\right)-\left(\frac{3(-1)^2}{2}+2(-1)-\frac{(-1)^4}{4}\right)$

$=(3\times 2+4-4)-\left(\frac{3}{2}-2-\frac{1}{4}\right)$

$=8-\frac{3}{2}+\frac{1}{4}=\frac{32-6+1}{4}=\frac{27}{4}$sq units.

Therefore, the required area is $\frac{27}{4}$ sq units.

Q.32. Given A,B,C represents angles of ΔABC and $\cos A+2\cos B+\cos C=2$ and $AB=3,\ BC=7$ then $\cos A-\cos C$ is

A) $\frac{-10}{7}$ B) $\frac{10}{7}$ C) $\frac{5}{7}$ D) $\frac{-5}{7}$

Answer: $\frac{-10}{7}$

Solution: Given,

$AB = 3,\ BC = 7.$

$\Rightarrow c = 3,\ a = 7.$

Apply cosine rule in the given equation $\cos A + 2\cos B + \cos C = 2$, we get

$\Rightarrow \left(\frac{b^2+c^2-a^2}{2bc}\right) + 2\left(\frac{c^2+a^2-b^2}{2ac}\right) + \left(\frac{a^2+b^2-c^2}{2ab}\right) = 2$

$\Rightarrow \left(\frac{b^2+9-49}{6b}\right) + \left(\frac{9+49-b^2}{21}\right) + \left(\frac{49+b^2-9}{14b}\right) = 2$

$\Rightarrow \left(\frac{b^2-40}{6b}\right) + \left(\frac{58-b^2}{21}\right) + \left(\frac{40+b^2}{14b}\right) = 2$

$\Rightarrow b^3 - 5b^2 - 16b + 80 = 0$

$\Rightarrow (b-4)(b+4)(b-5) = 0$

$\Rightarrow b = -4, 4, 5.$

But $b \neq -4.$

Also, if $b = 4$ triangle cannot be constructed.

That means $b = 5.$

$\Rightarrow \cos A - \cos C = \left(\frac{b^2+c^2-a^2}{2bc}\right) - \left(\frac{a^2+b^2-c^2}{2ab}\right)$

$= \left(\frac{5^2+3^2-7^2}{2(5\times3)}\right) - \left(\frac{7^2+5^2-3^2}{2(7\times5)}\right)$

On simplifying we get, $= \frac{-10}{7}.$

Therefore, the required answer is $\frac{-10}{7}$

Q.33. Let $x^2 + 6\sqrt{x} + 4 = 0$ be any quadratic equation and α, β are roots of that equation then, $\frac{\alpha^{34}\beta^{24}+\alpha^{32}\beta^{26}+2\alpha^{33}\beta^{25}}{\alpha^{31}\beta^{20}+\alpha^{28}\beta^{23}+3\alpha^{30}\beta^{21}+3\alpha^{29}\beta^{22}} =$

A) $-\frac{2^7}{3}\sqrt{6}$

B) $\frac{2^7}{3}\sqrt{6}$

C) $-\frac{2^8}{3}\sqrt{6}$

D) $\frac{2^8}{3}\sqrt{6}$

Answer: $-\frac{2^7}{3}\sqrt{6}$

Solution: Given,

$x^2+\sqrt{6}x+4=0$ quadratic equation has roots α, β therefore

Sum of roots will be $\alpha+\beta=-\sqrt{6}$

And product of roots will be $\alpha\beta=4$

Now,

$$\frac{\alpha^{34}\beta^{24}+\alpha^{32}\beta^{26}+2\alpha^{33}\beta^{25}}{\alpha^{31}\beta^{20}+\alpha^{28}\beta^{23}+3\alpha^{30}\beta^{21}+3\alpha^{29}\beta^{22}}$$

$$=\frac{\alpha^{32}\beta^{24}\left(\alpha^2+\beta^2+2\alpha\beta\right)}{\alpha^{28}\beta^{20}\left(\alpha^3+\beta^3+3\alpha^2\beta+3\alpha\beta^2\right)}$$

$$=\frac{\alpha^4\beta^4(\alpha+\beta)^2}{(\alpha+\beta)^3}$$

$$=\frac{\alpha^4\beta^4}{(\alpha+\beta)}=\frac{4^4}{-\sqrt{6}}=-\frac{2^7}{3}\sqrt{6}$$

Q.34. If $a\hat{i}+\hat{j}+\hat{k},\ \hat{i}+b\hat{j}+\hat{k},\ \ \hat{i}+\hat{j}+c\hat{k}$ are coplanar then the value $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is

Answer: 1

Solution: Given that $a\hat{i}+\hat{j}+\hat{k},\ \hat{i}+b\hat{j}+\hat{k},\ \ \hat{i}+\hat{j}+c\hat{k}$ are coplanar.

$$\Rightarrow\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}=0$$

Apply row transformations, $R_1\rightarrow R_1-R_2$ and $R_2\rightarrow R_2-R_3$.

$$\Rightarrow\begin{vmatrix} a-1 & 1-b & 1-1 \\ 1-1 & b-1 & 1-c \\ 1 & 1 & c \end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix} a-1 & 1-b & 0 \\ 0 & b-1 & 1-c \\ 1 & 1 & c \end{vmatrix}=0.$$

Expand the determinant along R_3.

$\Rightarrow 1\left[(1-b)(1-c)-0\right]-\left[(a-1)(1-c)-0\right]+c\left[(a-1)(b-1)-0\right]=0$

Multiply and Divide with $(1-a)(1-b)(1-c)$.

$$\Rightarrow\frac{(1-b)(1-c)}{(1-a)(1-b)(1-c)}+\frac{(1-a)(1-c)}{(1-a)(1-b)(1-c)}+\frac{c(1-a)(1-b)}{(1-a)(1-b)(1-c)}=0$$

$$\Rightarrow\frac{1}{(1-a)}+\frac{1}{(1-b)}+\frac{c}{(1-c)}=0$$

$$\Rightarrow\frac{1}{(1-a)}+\frac{1}{(1-b)}+\frac{c}{(1-c)}+\frac{1}{1-c}-\frac{1}{1-c}=0$$

$$\Rightarrow\frac{1}{(1-a)}+\frac{1}{(1-b)}-\frac{(1-c)}{(1-c)}+\frac{1}{1-c}=0$$

$$\Rightarrow\frac{1}{(1-a)}+\frac{1}{(1-b)}+\frac{1}{1-c}=1.$$

Therefore, the required answer is 1.

Q.35. If $\frac{{}^nC_n}{n+1}+\frac{{}^nC_{n-1}}{n}+\ldots\ldots+\frac{1}{2}{}^nC_1+{}^nC_0=\frac{255}{8}$, then value of n is

Answer: 7

Solution: Given that $\frac{{}^nC_n}{n+1}+\frac{{}^nC_{n-1}}{n}+\ldots\ldots+\frac{1}{2}{}^nC_1+{}^nC_0=\frac{255}{8}$

Let us take $\int_0^1(1+x)^n\,dx=\int_0^1\left({}^nC_0+{}^nC_1x+{}^nC_2x^2+\ldots\ldots+{}^nC_nx^n\right)dx$

$$\Rightarrow\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1=\left[{}^nC_0x+{}^nC_1\frac{x^2}{2}+{}^nC_2\frac{x^3}{3}+\ldots\ldots+{}^nC_n\frac{x^{n+1}}{n+1}\right]_0^1$$

$$\Rightarrow\left[\frac{(1+1)^{n+1}}{n+1}-\frac{1^{n+1}}{n+1}\right]=\left[{}^nC_0(1)+{}^nC_1\frac{1}{2}+{}^nC_2\frac{1}{3}+\ldots\ldots+{}^nC_n\frac{(1)^{n+1}}{n+1}-0\right]$$

$$\Rightarrow\frac{2^{n+1}-1}{n+1}=\left[{}^nC_0+{}^nC_1\frac{1}{2}+{}^nC_2\frac{1}{3}+\ldots\ldots+\frac{{}^nC_n}{n+1}\right]$$

$$\Rightarrow\frac{2^{n+1}-1}{n+1}=\left[\frac{255}{8}\right]$$

$$\Rightarrow\frac{2^{n+1}-1}{n+1}=\left[\frac{2^{7+1}-1}{7+1}\right]$$

On comparing we get, $n=7$.

Therefore, the value of n is 7.

Q.36. If the value of $\int_{-0.15}^{0.15}\left|100x^2-1\right|dx=\frac{k}{3000}$, then the value of k will be

Answer: 575

Solution: Given,

$\int_{-0.15}^{0.15}\left|100x^2-1\right|dx=\frac{k}{3000}$

Now we know that, $\int_{-a}^{a}f(x)\,dx=2\int_0^a f(x)\,dx$ if $f(x)$ is even

So, $\int_{-0.15}^{0.15}\left|100x^2-1\right|dx=2\int_0^{0.15}\left|100x^2-1\right|dx$

Now solving, $2\int_0^{0.15}\left|100x^2-1\right|dx=2\left(\int_0^{0.1}-\left(100x^2-1\right)dx+\int_{0.1}^{0.15}100x^2-1\,dx\right)$

$$\Rightarrow 2\int_0^{0.15}\left|100x^2-1\right|dx=2\left(\left[x-\frac{100x^3}{3}\right]_0^{0.1}+\left[\frac{100x^3}{3}-x\right]_{0.1}^{0.15}\right)$$

$$\Rightarrow 2\int_0^{0.15}\left|100x^2-1\right|dx=2\left(\left(\frac{1}{10}-\frac{1}{30}\right)+\left(\frac{100\times15^3}{3\times100^3}-\frac{15}{100}\right)-\left(\frac{1}{30}-\frac{1}{10}\right)\right)$$

$$\Rightarrow 2\int_0^{0.15}\left|100x^2-1\right|dx=2\left(\frac{20}{30}+\left(\frac{225}{2000}-\frac{15}{100}\right)+\frac{20}{30}\right)$$

$$\Rightarrow 2\int_0^{0.15}\left|100x^2-1\right|dx=\frac{575}{3000}$$

Now comparing with $\int_{-0.15}^{0.15}\left|100x^2-1\right|dx=\frac{k}{3000}$, we get $k=575$

Q.37. $N>40000$, where N is divisible by 5. How many such 5 digit numbers can be formed using $0,1,3,5,7,9$ without repition.

Answer: 120

Solution: We need to find number of 5 digit numbers which are divisible by 5 which are greater than 40000.

Case 1 : Let the number begin with 5 and end with 0 as $\underline{5}\ \_\ \_\ \_\ \underline{0}$ and in the middle 3 places remaining 4 digits can be filled in $4\times3\times2=24$ ways.

Case 2 :Let the number begin with 7 and end with 0 as $\underline{7}\ \_\ \_\ \_\ \underline{0}$ and in the middle 3 places remaining 4 digits can be filled in $4\times3\times2=24$ ways.

Case 3 :Let the number begin with 7 and end with 5 as $\underline{7}\ \_\ \_\ \_\ \underline{5}$ and in the middle 3 places remaining 4 digits can be filled in $4\times3\times2=24$ ways.

Similarly when the number starts with 9 there are 48 ways.

Therefore, the total number of ways are $24+24+24+48=120$ ways.

Q.38. 5 positive numbers $a_1,\ a_2, \ldots,\ a_5$ are in geometric progression. Their mean and variance are $\frac{31}{10}$ and $\frac{m}{n}$. The mean of the reciprocals is $\frac{31}{40}$, then $m+n$ is ______

Answer: 211

Solution: 5 positive numbers $a_1,\ a_2, \ldots,\ a_5$ are in geometric progression. So, let the numbers be $\frac{a}{r^2},\ \frac{a}{r},\ a,\ ar,\ ar^2$

According to question,

$$\frac{\frac{a}{r^2}+\frac{a}{r}+a+ar+ar^2}{5}=\frac{31}{10} \quad \ldots(1)$$

And,

$$\frac{\frac{r^2}{a}+\frac{r}{a}+\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}}{5}=\frac{31}{40}$$

$$\Rightarrow \frac{\frac{1}{a}\left(r^2+r+1+\frac{1}{r}+\frac{1}{r^2}\right)}{5}=\frac{31}{40} \quad \ldots(2)$$

By (1) & (2), we get

$a^2=4\ \Rightarrow a=2$

So,

$$\left(r^2+r+1+\frac{1}{r}+\frac{1}{r^2}\right)=\frac{31}{4}$$

$$\Rightarrow \left(r+\frac{1}{r}\right)^2+r+\frac{1}{r}=\frac{27}{4}+2$$

$$\Rightarrow t^2+t=\frac{35}{4}$$

where, $t=r+\frac{1}{r}$

$4t^2+4t-35=0$

$\Rightarrow t=\frac{5}{2} \Rightarrow r=2$

So, numbers are $\frac{1}{2},\ 1,\ 2,\ 4,\ 8$

Variance is

$$=\frac{\frac{1}{4}+1+4+16+64}{5}-\left(\frac{\frac{1}{2}+1+2+4+8}{5}\right)^2$$

$$=\frac{186}{25}=\frac{m}{n}$$

$m+n=211$

Q.39. Three numbers $a,\ b,\ c$ are in A.P. and they are used to make $9-$ digit number using each digit thrice such that atleast three consecutive digits are in A.P., then number of such numbers is _______

Answer: 1260

Solution: Three numbers $a,\ b,\ c$ are in A.P. ad they are used to making $9-$ digit number using each digit thrice such that atleast three consecutive digits are in A.P., So, c, b, a will also be in A.P.

Now, we have 7 position to put (a, b, c) or (c, b, a) as $\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_}_{7 \text{ positions}}\ \_\ \_$.

Number of such numbers is

$= {}^{2}C_{1} \cdot {}^{7}C_{1} \cdot \frac{6!}{2! \cdot 2! \cdot 2!}$

$= 1260$