JEE Main 2023 (Session 2)

April 8 Shift 1

Physics

Q.1. For an electron and a proton $(m_p = 1847\ m_e)$ with same de-Broglie wavelength, the ratio of linear momentum is equal to:

A) $1:2$ B) $2:1847$ C) $1:1$ D) $\sqrt{1847}:1$

Answer: $1:1$

Solution: The de-Broglie's equation is given by,

$\lambda = \frac{h}{p}$.

The above equation is relating the linear momentum (p) of a particle with its de-Broglie wavelength (λ).

It is given that the de-Broglie wavelength of electron and proton is same.

Hence, the ratio of linear momentum is $1:1$.

Q.2. If the weight of an object on earth's surface is $400\ \mathrm{N}$, then weight of the same particle at a depth $\frac{R}{2}$ from surface would be (R is radius of earth)

A) $100\ \mathrm{N}$ B) $300\ \mathrm{N}$ C) $200\ \mathrm{N}$ D) $250\ \mathrm{N}$

Answer: $200\ \mathrm{N}$

Solution: The formula to calculate the variation of the acceleration due to gravity with depth can be written as

$g' = g\left(1 - \frac{d}{R}\right) \quad \ldots(1)$

Substitute the value of d into equation (1) to obtain the acceleration due to gravity at the given depth.

$$g' = g\left(1 - \frac{\frac{R}{2}}{R}\right)$$
$$= \frac{g}{2}$$

As, the acceleration due to gravity is halved, the weight of the object will also be halved.

Hence, the weight of the object at the given depth is $200\ \mathrm{N}$.

Q.3. Two forces of magnitude A and $\frac{A}{2}$ act perpendicular to each other. The magnitude of the resultant force is equal to

A) $\frac{A}{2}$ B) $\frac{\sqrt{5}A}{2}$ C) $\frac{3A}{2}$ D) $\frac{5A}{2}$

Answer: $\frac{\sqrt{5}A}{2}$

Solution: Since, the vectors act perpendicularly to each other, the angle between them is 90°.

The magnitude of the resultant (R) of the vectors, using Pythagoran theorem, be calculated as follows-

$$R^2 = A^2 + \left(\frac{A}{2}\right)^2$$
$$= \frac{5A^2}{4}$$
$$\Rightarrow R = \frac{\sqrt{5}A}{2}$$

Q.4. Two projectiles are thrown at speed $40\ \mathrm{m\ s^{-1}}$ and $60\ \mathrm{m\ s^{-1}}$ at angles 30° and 60° respectively. Find the ratio of their ranges.

A) $4:9$ B) $1:1$ C) $2:3$ D) $4:3$

Answer: $4:9$

Solution: The formula to calculate the horizontal range of a projectile is given by

$$R = \frac{u^2 \sin 2\theta}{g} \quad \ldots (1)$$

Substitute 40 m s^{-1} for u and 30° for θ into equation (1) to obtain the range (R_1) of the first particle.

$$\begin{aligned} R_1 &= \frac{40^2 \sin 60^\circ}{g} \\ &= \frac{800\sqrt{3}}{g} \quad \ldots (2) \end{aligned}$$

Similarly, substitute 60 m s^{-1} for u and 60° for θ into equation (1) to obtain the range (R_2) of the second particle.

$$\begin{aligned} R_2 &= \frac{60^2 \sin 120^\circ}{g} \\ &= \frac{1800\sqrt{3}}{g} \quad \ldots (3) \end{aligned}$$

Divide equation (2) by equation (3) to calculate the required ratio of the ranges.

$$\begin{aligned} \frac{R_1}{R_2} &= \frac{\frac{800\sqrt{3}}{g}}{\frac{1800\sqrt{3}}{g}} \\ &= \frac{4}{9} \end{aligned}$$

Hence, ratio of their ranges is $4:9$.

Q.5. An air bubble having volume 1 cm^3 at depth 40 m inside water comes to surface. What will be the volume of the bubble at the surface?

A) 5 cm^3 B) 2 cm^3 C) 4 cm^3 D) 3 cm^3

Answer: 5 cm^3

Solution: The pressure at the surface of water is the same as the atmospheric pressure. So, the pressure at the surface of water is given by

$$\begin{aligned} P_2 &= 1 \text{ atm} \\ &= 10^5 \text{ Pa} \end{aligned}$$

The pressure (P_1) at a depth of 40 m below the surface of water can be calculated as follows-

$$P_1 = P_2 + h\rho g \quad \ldots (1)$$

Substitute the values of the parameters into equation (1) to calculate the initial pressure on the bubble below the surface.

$$\begin{aligned} P_2 &= 10^5 \text{ Pa} + (40 \text{ m}) \times \left(10^3 \text{ kg m}^{-3}\right) \times \left(10 \text{ m s}^{-2}\right) \\ &= 5 \times 10^5 \text{ Pa} \end{aligned}$$

Hence, the final volume of the bubble at the surface can be calculated as follows(temperature will remain constant):

$$\begin{aligned} P_1 V_1 &= P_2 V_2 \\ \Rightarrow V_2 &= \frac{P_1 V_1}{P_2} \\ &= \frac{5 \times 10^5 \text{ Pa} \times 1 \text{ cm}^3}{10^5 \text{ Pa}} \\ &= 5 \text{ cm}^3 \end{aligned}$$

Hence, volume of the bubble at the surface is 5 cm^3.

Q.6. The height of antenna is 98 m. The radius of Earth is 6400 km. The area up to which it will transmit signal is:

A) 3642 km^2 B) 3942 km^2 C) 11200 km^2 D) 22400 km^2

Answer: 3942 km^2

Solution: Range of Line of sight (LOS) communication is given by formula,

$\text{Range, } R=\sqrt{2h_T R_e}$

where, h_T is height of the tower

and R_e is radius of the earth.

The area (A) covered by the antenna $=\pi R^2$

$$\begin{aligned} A &= 2\pi R_e h_T \\ &= 2\times 3.14\times 6400\times\left(\frac{98}{1000}\right) \\ &= 3938.8\approx 3942\ \text{km}^2 \end{aligned}$$

Q.7. If mass, radius of cross-section and height of a cylinder are (0.4 ± 0.01) g, (6 ± 0.03) m and (8 ± 0.04) m. The maximum percentage of error in the measurement of density of cylinder is

A) 1% B) 4% C) 8% D) 7%

Answer: 4%

Solution: The formula to calculate the density (ρ) of the material of a cylinder can be written as

$$\rho=\frac{m}{\pi r^2 h}\quad \ldots(1)$$

Hence, the percentage error in measuring the density of the material of the cylinder is given by

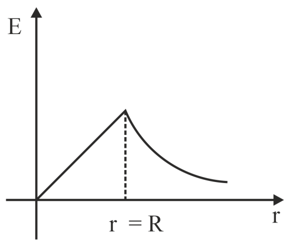
$$\frac{\Delta\rho}{\rho}\%=\left[\left(\frac{\Delta m}{m}+2\frac{\Delta r}{r}+\frac{\Delta h}{h}\right)\times 100\right]\%\quad \ldots(2)$$

Substitute the given values of the respective error in measuring individual parameters and their original values into equation (2) to calculate the percentage error in measuring the density.

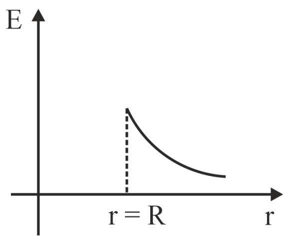
$$\begin{aligned} \frac{\Delta\rho}{\rho}\% &= \left[\left(\frac{0.01}{0.4}+2\times\frac{0.03}{6}+\frac{0.04}{8}\right)\times 100\right]\% \\ &= [(0.025+0.01+0.005)\times 100]\% \\ &= 4\% \end{aligned}$$

Q.8. The graph showing the variation of electric field (E) with the distance (r) from the centre of a conducting spherical shell is -

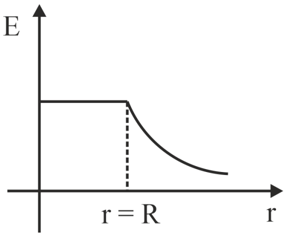
A)



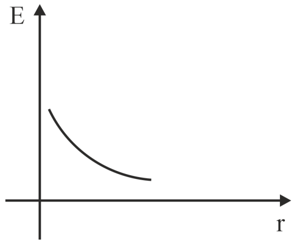
B)



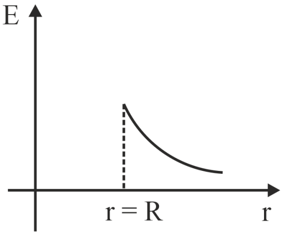
C)



D)



Answer:



Solution: For a conducting spherical shell, the total charge resides on its surface.

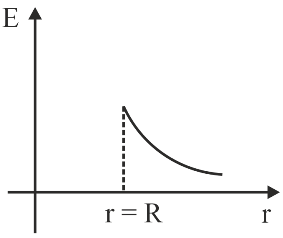
The electric field inside the shell will be zero as charge enclosed by the shell is zero. As one moves away from the surface of the spherical shell, the electric field varies according to the following formula:

$$E(r) = k\frac{Q}{r^2}$$

where, k is Coulomb's constant.

Thus, starting from the centre of the shell, the electric field remains zero up to $r = R$ and beyond this point, the electric field is inversely proportional to the square of the distance.

Hence,



Q.9. An atom of atomic mass 242, having binding energy per nucleon $8.4\ \mathrm{MeV}$, breaks into two atoms of atomic mass 121 each (with binding energy per nucleon $7.1\ \mathrm{MeV}$). Find the absolute Q value of the reaction.

A) $150\ \mathrm{MeV}$ B) $314.6\ \mathrm{MeV}$ C) $208.4\ \mathrm{MeV}$ D) $290.8\ \mathrm{MeV}$

Answer: $314.6\ \mathrm{MeV}$

Solution: The formula to calculate the $Q-$value of the given reaction can be written as

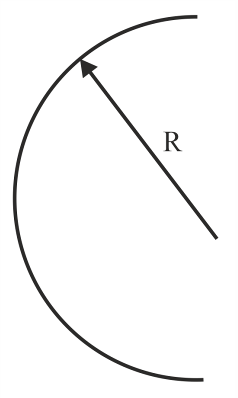
$Q = M_m E_m - n M_d E_d \quad \ldots(1)$

where, $M,\ E$ represent the atomic mass and the binding energy respectively. The subscripts $m,\ d$ represent the mother and daughter atoms respectively. Also, n represents the number of emitted daughter atoms in the reaction.

Substitute the values of the known parameters into equation (1) to calculate the required $Q-$value of the reaction.

$$Q = 242 \times 8.4 \text{ MeV} - 2 \times 121 \times 7.1 \text{ MeV}$$
$$= 314.6 \text{ MeV}$$

Q.10. The moment of inertia of semi-circular ring of mass M and radius R about an axis passing through centre and perpendicular to the plane of ring is



A) MR^2

B) $\frac{1}{2}MR^2$

C) $2MR^2$

D) $\frac{3}{4}MR^2$

Answer: MR^2

Solution: The formula to calculate the moment of inertia (I) of a ring having mass M and radius R about an axis passing through its centre perpendicularly is given by

$I = MR^2 \quad \ldots(1)$

Here, the mass of the semi-circular ring is given as M. So, the total mass of the entire ring would be $2M$.

If it were asked to find the moment of inertia of the entire ring, then the answer should be, according to equation (1), $2MR^2$.

But, as in this case, it is asked to find the moment of inertia of the semi-circular ring, then answer should be MR^2.

Q.11. The dimension of $\frac{1}{\mu_0 \varepsilon_0}$ is

A) $\mathrm{MLT^{-1}}$

B) $\mathrm{M^0LT^{-2}}$

C) $\mathrm{ML^2T^{-1}}$

D) $\mathrm{M^0L^2T^{-2}}$

Answer: $\mathrm{M^0L^2T^{-2}}$

Solution: We know the relation, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

So, $c^2 = \frac{1}{\mu_0 \varepsilon_0}$

As the unit of c is $\mathrm{m\ s^{-1}}$, so the dimensional formula for c is $\mathrm{M^0LT^{-1}}$.

Hence, the dimensional formula for c^2 is $\mathrm{M^0L^2T^{-2}}$

Q.12. In an LC oscillating circuit with $L = 75 \text{ mH}$ and $C = 30\ \mu\text{F}$, the maximum charge of capacitor is 2.7×10^{-4} C. Maximum current through the circuit will be

A) 0.18 A

B) 0.24 A

C) 0.72 A

D) 0.92 A

Answer: 0.18 A

Solution: The formula for calculating the maximum current through the LC oscillating circuit is given by, $i_{max} = q_0\omega$ ---(i)

Where, q_0 is maximum charge on the capacitor

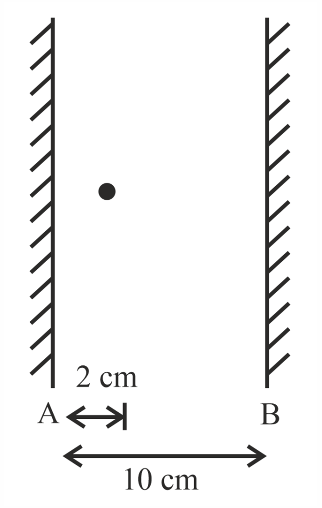
and ω is the angular frequency of oscillation.

We also, know that for LC oscillating circuit, $\omega = \frac{1}{\sqrt{LC}}$ ---(ii)

Substituting (ii) in (i), we get

$$i_{max} = \frac{q_0}{\sqrt{LC}}$$
$$= \frac{2.7\times10^{-4}}{\sqrt{75\times10^{-3}\times30\times10^{-6}}}$$
$$= 0.18 \text{ A}$$

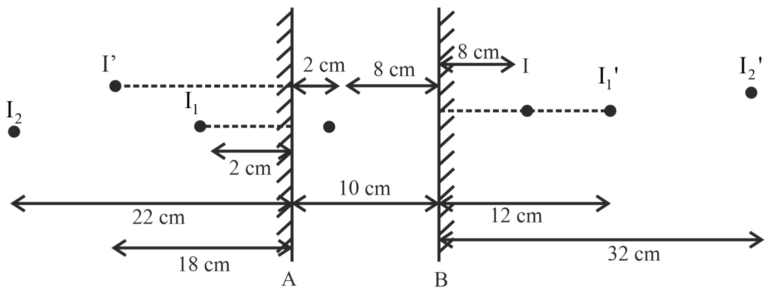
Q.13. In the given diagram, find the distance (in cm) between 2nd and 3rd image formed on the left side of mirror A.



Answer: 4

Solution: In a plane mirror, regular reflection occurs. The image formed by a plane mirror must be virtual and has the same distance from the mirror as the object does.

Let's consider the following schematic of the image formation by the two mirrors.



Considering the image formation started at mirror A at first, the first image is formed at position I_1 behind it. Considering I_1 as object and reflection at mirror B, an image is formed at I_1' behind mirror B. Similarly, the next image is formed at I_2 behind mirror A.

Now, considering the image formation started at mirror B first, the first image is formed at I behind it. Considering I as object, its image is formed at I' behind mirror A.

Hence, from the diagram, the distance (d) between the 2nd and the 3rd image formed behind mirror A can be calculated as follows-

$$d = \text{position of } I_2 - \text{position of I'}$$
$$= 22 \text{ cm} - 18 \text{ cm}$$
$$= 4 \text{ cm}$$

Chemistry

Q.14. Which of the following elements is more reactive from the given?

A) Calcium B) Magnesium C) Strontium D) Potassium

Answer: Potassium

Solution: Chemical reactivity decreases as you go left to right of the periodic table and, it increases going down the group in the case of metals.

Potassium (K) is the most reactive element among the given options $(\mathrm{Ca},\ \mathrm{Mg},\ \mathrm{Sr},\ \mathrm{K})$.

Potassium belongs to the alkali metal group and has only one valence electron in its outermost shell, making it highly reactive. It readily loses its outermost electron to form a positively charged ion, which is why it is a highly reactive metal.

On the other hand, calcium, magnesium, and strontium belong to the alkaline earth metal group, which are also reactive but less so than alkali metals. These elements have two valence electrons in their outermost shell, which they can lose to form a positively charged ion. However, the two valence electrons in their outermost shell make them less reactive than potassium, which only has one valence electron.

Q.15. The extraction of which one of the following metals involves concentration of the ore by leaching?

A) Copper B) Magnesium C) Aluminium D) Potassium

Answer: Aluminium

Solution: Bauxite is the primary source of aluminium, and it typically contains $\mathrm{SiO_2}$, iron oxides, titanium oxides as impurities. The ore is first crushed and ground to a fine powder, and then it is treated with sodium hydroxide solution (NaOH) under high temperature and pressure. This is called Bayer's process. The sodium hydroxide reacts with the aluminium oxide in the ore to form soluble sodium aluminate, while the impurities remain insoluble.

$$\mathrm{Al_2O_3(s)} + 2\,\mathrm{NaOH(aq)} + 3\mathrm{H_2O(l)} \rightarrow 2\,\mathrm{Na[Al(OH)_4](aq)}$$

Q.16. Consider the reaction:

$$\mathrm{Cu^{2+}} + \mathrm{X^-} \rightarrow \mathrm{Cu_2X_2} + \mathrm{X_2}$$

Find the product $\mathrm{X_2}$, that formed predominantly.

A) $\mathrm{Cl_2}$ B) $\mathrm{Br_2}$ C) $\mathrm{I_2}$ D) All halogens are possible

Answer: $\mathrm{I_2}$

Solution: The product $\mathrm{X_2}$ will be predominantly molecular iodine $(\mathrm{I_2})$ when $\mathrm{X^-}$ represents an iodide ion $(\mathrm{I^-})$. HI can act as a good reducing agent in the given reaction to reduce $\mathrm{Cu^{2+}}$ to $\mathrm{Cu^{+1}}$. The reduction of $\mathrm{Cu^{2+}}$ to $\mathrm{Cu^{+1}}$ involves the transfer of one electron to the $\mathrm{Cu^{2+}}$ ion, which is gained by the iodide ion $(\mathrm{I^-})$, causing it to get oxidised to molecular iodine $(\mathrm{I_2})$.

Q.17. The correct order of electronegativity of $\mathrm{Br, P, C}$ and As.

A) $\mathrm{Br > C > P > As}$ B) $\mathrm{Br > P > As > C}$ C) $\mathrm{Br > As > C > P}$ D) $\mathrm{P > Br > As > C}$

Answer: $\mathrm{Br > C > P > As}$

Solution: Electronegativity is a measure of the ability of an atom to attract electrons in a chemical bond. It generally increases across a period from left to right, and decreases down a group in the periodic table. The halogen bromine has the highest electronegativity(2.9) element among the given elements. The next highest electronegativity element is carbon(2.5) and the electronegativity values of phosphorus and arsenic are 2.19 and 2.18 respectively.

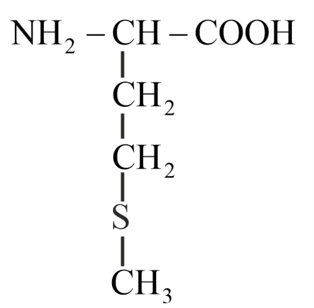
Q.18. How many of the following $\alpha-$ amino acids contain sulphur?

Lysine, Methionine, Glutamic acid, Threonine, Arginine, Cysteine, Tyrosine, Isoleucine.

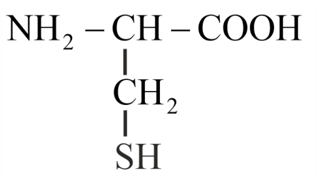
A) 4 B) 5 C) 2 D) 6

Answer: 2

Solution: There are two $\alpha-$amino acids that contain sulfur: Methionine and Cysteine. Cysteine and methionine are sulphur containing amino acids. Amino acids get linked to one another by peptide bond formation and form a polypeptide chain of proteins. Hence cysteine and methionine are found in several proteins.



methionine



cysteine

Q.19. Read the following two statements
Statement I: Ionic radius of Li^{+} is greater than Mg^{+2}
Statement II: Lithium and magnesium can't form superoxide

A) Both Statement I and Statement II are correct

B) Both Statement I and Statement II are incorrect

C) Statement I is correct but Statement II is incorrect

D) Statement I is incorrect but Statement II is correct

Answer: Both Statement I and Statement II are correct

Solution: The values of ionic radii of $\mathrm{Li}^{+}= \ 0.74$ Å and $\mathrm{Mg}^{+2}= \ 0.72$ Å respectively. Thus, the lithium ion with $+1$ charge is only marginally larger than the magnesium ion having a charge of $+2$.

The superoxide releases the most energy when formed, the superoxide is preferentially formed for the larger metals where the more complex anions are not polarised. So, $\mathrm{Li \ and \ Mg}$ do not form superoxides.

Therefore, option A is correct.

Q.20. Assertion: Butanol has highest boiling point than ethoxyethane.
Reason: Butanol exhibits intermolecular hydrogen bonding.

A) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

B) Both Assertion and Reason are true but Reason is NOT the correct explanation of Assertion.

C) Assertion is true but Reason is false.

D) Assertion is false but Reason is true.

Answer: Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Solution: The boiling point of butanol is higher than ethoxyethane due to extensive intermolecular hydrogen bonding. In case of ether there is no hydrogen bonding present. Hence, there is no molecular association. So ether has a low boiling point as compared to alcohol.

Q.21. Which cell representation is correct for the reaction given below:

$\mathrm{H_2 + 2AgCl \rightarrow 2H^{+} + \ 2\,Ag \ + \ 2\,Cl^{-}}$

A) $\mathrm{Pt\,|H_2|\ HCl\,||\,AgCl\ |\ Ag}$

B) $\mathrm{Pt\,|H_2|\ HCl\,||\,AgCl\ |\ Pt}$

C) $\mathrm{Ag\,|\,AgCl\,||\,HCl\,|H_2|\,Pt}$

D) $\mathrm{Pt\,|\,AgCl\,||\,HCl\,|H_2|\,Pt}$

Answer: $\mathrm{Pt\,|H_2|\ HCl\,||\,AgCl\ |\ Ag}$

Solution: The given reaction is :

$H_2 + 2AgCl \rightarrow 2H^+ + 2Ag + 2Cl^-$

Silver is undergoing reduction hence it will act as cathode in the given cell and hydrogen electrode acts as an anode.

The correct representation is :

$Pt|H_2|\ HCl||AgCl\ |\ Ag$

Q.22. Syn gas with Cu as catalyst produces:

A) Ethanol B) Methanal C) Methane D) Methanoic acid

Answer: Methanal

Solution: Synthesis gas (also known as syngas) is a mixture of carbon monoxide (CO) and hydrogen (H2) that is used as a fuel gas. Syn gas with Cu as catalyst produces methanal.

$CO + H_2 \xrightarrow{Cu} HCHO$

If the catalyst is $Cr_2O_3 - ZnO$ then the product is methanol.

Q.23. Which of the following is most stable, diamagnetic and octahedral in shape?

A) $K_3[Co(CN)_6]$ B) $[Co(H_2O)_6]Cl_3$ C) $Na_3[CoF_6]$ D) All have exact equal stability

Answer: $K_3[Co(CN)_6]$

Solution: The coordination compound $K_3[Co(CN)_6]$contains a cobalt(III) ion, which has a d^6 electronic configuration. The cyanide ligands in this complex are indeed strong field ligands, which means they will cause a large splitting of the d orbitals. So here cobalt is in $+3$ state having $3d^6$ configuration and also cyanide ligand which is a strong field ligand so, it is most stable and will have octahedral shape.

Q.24. Which of the following has same $d-$ electrons as chromium in chromyl chloride?

A) $Fe(III)$ B) $Ni(III)$ C) $Mn(VII)$ D) $Co(II)$

Answer: $Mn(VII)$

Solution: Chromyl chloride has the chemical formula CrO_2Cl_2, in which the chromium atom is in the $+6$ oxidation state which results in a $3d^0$electron configuration. From the given options Fe^{3+} has a d^5 electron configuration ($3d^5$), Ni^{3+} has a d^7 electron configuration ($3d^7$), and Mn^{7+} has a d^0 electron configuration ($3d^0$). Co^{2+} will have d^7 electron configuration.

So among the given options $Mn(VII)$ has same $d-$electrons as chromium in chromyl chloride.

Q.25. $XeF_4 + SbF_5 \rightarrow [XeF_m]^{+n}[SbF_p]^{-q}$

The value of $m + n + p + q =$

Answer: 11

Solution: In the given reaction, XeF_4 act as fluoride ion donor and SbF_5 act as fluoride ion acceptor.

The balanced chemical equation for the given reaction is :

$XeF_4 + SbF_5 \rightarrow [XeF_3]^+[SbF_6]^-$

A fluoride ion is transferred from xenon tetrafluoride to antimony pentafluoride.

Therefore, the value of $m + n + p + q = 11.$

Q.26. How many factors will contribute to major role in covalent character of a compound?

a. Polarising power of cation

b. Polarisability of the anion

c. Distortion caused by cation

d. Polarisability of cation

Answer: 3

Solution: Fajan's rule is stated based on the concept of polarisation which states that the covalent nature of an anion increases with an increase in its polarisation. The main factors that affect the extent of polarisation of an anion are the polarising power of the cation and the polarisability of the anion.

The power of a cation to distort the other ion is known as its polarisation power and the tendency of the anion to get polarised by the other ion is known as its polarisability. The greater the polarisation power or polarisability of an ion, the greater will be its tendency to form a covalent bond.

Therefore, The correct answer is 3.

Mathematics

Q.27. Consider the word "INDEPENDENCE". The number of words such that all the vowels are together is

A) 16800 B) 15800 C) 17900 D) 14800

Answer: 16800

Solution: There are 5 vowels in the given word which are $4E's$ & $1I$.

Since they have to always occur together we that them as a single object $E\ E\ E\ E\ I$ for the time being.

This single object together with 7 remaining object will account for 8 objects.

There 8 objects in which there are $3N's$ & $2D's$ can be arrangement in $\frac{8!}{3!2!}$ ways.

Corresponding to each of there arrangements the 5 vowels E, E, E & I can be arranged in $\frac{5!}{4!}$Hence,required number of arrangements.

$= \frac{8!}{3!2!} \times \frac{5!}{4!} = 16800$

Q.28. 7 boys and 5 girls are to be seated around a circular table such that no two girls sit together is

A) $126(5!)^2$ B) $720\,(5!)$ C) $720\,(6!)$ D) 720

Answer: $126(5!)^2$

Solution: Given,

7 boys and 5 girls are to be seated around a circular such that no two girls to be seated together,

Now we know that n objects can be arranged in a circle in $(n-1)!$ ways.

Let us first arrange 7 boys in circular arrangement in $(7-1)!$ ways.

Now there will be 7 gaps.

So let us select any 5 gaps out of 7 gaps and arrange 5 girls in the chosen gaps. This can be done in ${}^7C_5 \times 5!$ ways.

Hence, required arrangements are $6! \times {}^7C_5 \times 5!$

$= 6 \times 5! \times \frac{7\times6}{2} \times 5!$

$= 126(5!)^2$.

Therefore, required arrangements are $126(5!)^2$

Q.29. The coefficient independent of x in the expansion of $\left(3x^2 - \frac{15}{2x^5}\right)^7$ is

A) $\frac{6715}{3}$ B) $\frac{5293}{6}$ C) $\frac{5103}{4}$ D) $\frac{7193}{4}$

Answer: $\frac{5103}{4}$

Solution: The given expansion is $\left(3x^2-\frac{15}{2x^5}\right)^7$.

The general term in the binomial expansion of $(x+a)^n$ is given by $T_{r+1}={}^nC_r x^{n-r}a^r$.

$\Rightarrow T_{r+1}={}^7C_r\left(3x^2\right)^{7-r}\left(\frac{-1}{2x^5}\right)^r$

$={}^7C_r(-1)^r\left(\frac{3^{7-r}}{2^r}\right)x^{14-2r-5r}$

Now for term independent of $x \Rightarrow 14-2r-5r=0$

$\Rightarrow r=2$

Coefficient of x^0 is ${}^7C_2(-1)^2\times\left(\frac{3^{7-2}}{2^2}\right)$

$=\frac{7\times6}{2}\times\frac{3^5}{2^2}$.

$=\frac{5103}{4}$

Hence the coefficient of x^0 is $\frac{5103}{4}$.

Q.30. Shortest distance between the lines $\frac{x-5}{4}=\frac{y-3}{6}=\frac{z-2}{4}$ and $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-9}{6}$ is

A) $\frac{190}{37}$

B) $\frac{190}{\sqrt{756}}$

C) $\frac{37}{190}$

D) $\frac{756}{\sqrt{190}}$

Answer: $\frac{190}{\sqrt{756}}$

Solution: Given:

$\frac{x-5}{4}=\frac{y-3}{6}=\frac{z-2}{4}$

$\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-9}{6}$

So,

$\vec{a}_1=5\hat{i}+3\hat{j}+2\hat{k}$
$\vec{a}_2=3\hat{i}+2\hat{j}+9\hat{k}$

So,

$\vec{a}_2-\vec{a}_1=-2\hat{i}-\hat{j}+7\hat{k}$

And,

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 6 & 4\\ 7 & 5 & 6\end{vmatrix}=16\hat{i}+4\hat{j}-22\hat{k}$

$\Rightarrow\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{256+16+484}=\sqrt{756}$

Shortest distance between the lines

$=\left|\frac{\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$

$=\left|\frac{\left(-2\hat{i}-\hat{j}+7\hat{k}\right)\cdot\left(16\hat{i}+4\hat{j}-22\hat{k}\right)}{\sqrt{756}}\right|$

$=\left|\frac{32-4-154}{\sqrt{756}}\right|=\frac{190}{\sqrt{756}}\ \text{units}$

Q.31. Consider the data $x,\ y,\ 10,\ 12,\ 4,\ 6,\ 8,\ 12$. If mean of data is 9 and variance is 9.25, then the value of $3x-y$ is $(x>y)$

A) 25 B) 1 C) 24 D) 13

Answer: 25

Solution: Consider the data $x,\ y,\ 10,\ 12,\ 4,\ 6,\ 8,\ 12$.

So,

$\text{Mean}=\frac{x+y+10+12+4+6+8+12}{8}$

$\Rightarrow 9=\frac{x+y+52}{8}$

$\Rightarrow x+y+52=72$

$\Rightarrow x+y=20$

And,

$\text{Variance}=\left(\frac{\sum x_i^2}{n}\right)-\left(\frac{\sum x_i}{n}\right)^2$

$\Rightarrow 9.25=\left(\frac{x^2+y^2+100+144+16+36+64+144}{8}\right)-(9)^2$

$\Rightarrow x^2+y^2=218$

$\Rightarrow (x+y)^2-2xy=218$

$\Rightarrow 20^2-2xy=218$

$\Rightarrow 2xy=182$

$\Rightarrow xy=91$

Now solving $x+y=20$ & $xy=91$ we get,

$x=13,\ y=7\Rightarrow 3x-2y=25$

Q.32. If the coefficient of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio $1:5:20$, then the coefficient of the forth term of the expansion is:

A) 3600 B) 3658 C) 3654 D) 1000

Answer: 3654

Solution: Given,

${}^nC_{r-1}:{}^nC_r:{}^nC_{r+1}=1:5:20$

Now using the formula $\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{n-r+1}{r}$ we get,

$\therefore\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{n-r+1}{r}=\frac{5}{1}$

$\Rightarrow n-6r+1=0\ \ldots\ldots\ldots(1)$

And $\frac{{}^nC_{r+1}}{{}^nC_r}=\frac{n-r}{r+1}=\frac{20}{5}$

$\Rightarrow n-5r-4=0\ \ldots\ldots\ldots\ldots(2)$

Solving above equations we get,

$n=29$ & $r=5$

So, the coefficient of forth term will be ${}^nC_3={}^{29}C_3=\frac{29\times28\times27}{3\times2\times1}=3654$

Q.33. Dot product of two vectors is 12 and cross product is $4\hat{i}+6\hat{j}+8\hat{k}$, find product of modulus of vectors.

A) $4\sqrt{35}$ B) $2\sqrt{65}$ C) $5\sqrt{37}$ D) $6\sqrt{37}$

Answer: $2\sqrt{65}$

Solution: Let $\vec{a}\,.\,\vec{b}=12$ and $\vec{a}\times\vec{b}=4\hat{\imath}+6\hat{\jmath}+8\hat{k}$.

Let us apply Lagrange's Identity which is $\left|\vec{a}\times\vec{b}\right|^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2-\left(\vec{a}\,.\,\vec{b}\right)^2$

$\Rightarrow\left|\vec{a}\times\vec{b}\right|=\sqrt{4^2+6^2+8^2}=\sqrt{116}$

$\Rightarrow\left(\sqrt{116}\right)^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2-(12)^2$

$\Rightarrow\left|\vec{a}\right|^2\left|\vec{b}\right|^2=116+144$

$\Rightarrow\left|\vec{a}\right|^2\left|\vec{b}\right|^2=260$

$\Rightarrow\left|\vec{a}\right|\left|\vec{b}\right|=\sqrt{260}=2\sqrt{65}$ (Since Modulus is always positive)

Therefore, product of modulus of vectors is $2\sqrt{65}$.

Q.34. A bolt manufacturing company has three products $A,\ B$ and C. $50\%\ \&\ 30\%$ of the products are A and B type respectively and remaining are C type. Then, the probability that the product A is defective is 4%, that B is defective is 3% and that C is defective is 2%. A product is picked randomly and found to be defective, then the probability that it is type C is

A) $\frac{5}{33}$ B) $\frac{2}{33}$ C) $\frac{4}{33}$ D) $\frac{7}{33}$

Answer: $\frac{4}{33}$

Solution: Let $X\equiv$Event that product is defective, then

$P\left(\frac{X}{A}\right)=\frac{4}{100}$
$P\left(\frac{X}{3}\right)=\frac{3}{100}$
$P\left(\frac{X}{C}\right)=\frac{2}{100}$

Now,

$$P\left(\frac{C}{X}\right)=\frac{P(C)P\left(\frac{X}{C}\right)}{P(A)P\left(\frac{X}{A}\right)+P(B)P\left(\frac{X}{B}\right)+P(C)P\left(\frac{X}{C}\right)}$$

$$\Rightarrow P\left(\frac{C}{X}\right)=\frac{\frac{20}{100}\times\frac{2}{100}}{\frac{50}{100}\times\frac{4}{100}+\frac{30}{100}\times\frac{3}{100}+\frac{20}{100}\times\frac{2}{100}}$$

$$\Rightarrow P\left(\frac{C}{X}\right)=\frac{40}{200+90+40}$$

$$\Rightarrow P\left(\frac{C}{X}\right)=\frac{4}{33}$$

Q.35. The area under the curve of equations : $x^2\le y,\ y\le 8-x^2$ and $y\le 7$ is

A) $\frac{16}{3}$ B) 18 C) 20 D) $\frac{22}{3}$

Answer: 20

Solution: The given curves are $x^2 \le y,\ y \le 8 - x^2$ and $y \le 7$

The point of intersection of the curves $x^2 \le y$ and $y \le 8 - x^2$ is obtained by,

$\Rightarrow x^2 = 8 - x^2$

$\Rightarrow x = \pm 2$ and $y = 4$.

Hence the points are $(2, 4), (-2, 4)$.

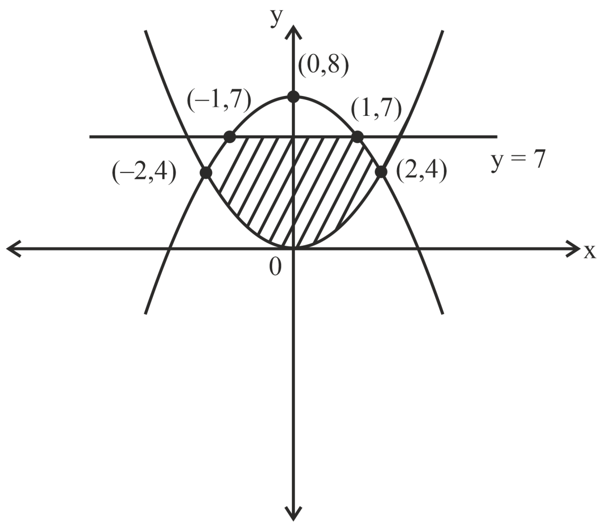
The points of intersection of the curves $y \le 8 - x^2$ and $y \le 7$ is obtained by,

$\Rightarrow 8 - x^2 = 7$

$\Rightarrow x = \pm 1$ and $y = 7$

Hence the points are $(1, 7), (-1, 7)$.

The required graph is



The required area is symmetrical about $y-$axis.

Hence required area is $A = 2\left[\int_0^4 \sqrt{y}\,\mathrm{d}y + \int_4^7 \sqrt{8-y}\,\mathrm{d}y\right]$

$$A = 2\left[\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 - \left[\frac{(8-y)^{\frac{3}{2}}}{\frac{3}{2}}\right]_4^7\right]$$

$$= \frac{4}{3}\left[(4)^{\frac{3}{2}} - (0)^{\frac{3}{2}} - \left\{(8-7)^{\frac{3}{2}} - (8-4)^{\frac{3}{2}}\right\}\right]$$

$$= \frac{4}{3}(8 - 0 - 1 + 8)$$

$= 20$ sq units.

Hence, the required area is 20 sq units.

Q.36. Check whether the function $f(x) = \frac{(1+2^x)^7}{2^x}$ is

A) Even B) Odd C) Neither even nor odd D) None of these

Answer: Neither even nor odd

Solution:

The given function is $f(x)=\frac{(1+2^x)^7}{2^x}$.

We know that if $f(-x)=-f(x)$ then $f(x)$ is odd function and if $f(-x)=f(x)$ then $f(x)$ is even function.

$\Rightarrow f(-x)=\frac{\left(1+2^{-x}\right)^7}{2^{-x}}$

$=\frac{(2^x+1)^7}{2^{7x}\times 2^{-x}}=\frac{(1+2^x)^7}{2^{6x}}$.

This is neither equal to $f(x)$ nor $-f(x)$.

Therefore, the given function is neither even nor odd.

Q.37.

If $P=\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $Q=PAP^T$, $A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then $P^TQ^{2007}P=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then find $2a+b+3c-4d$

A) 2005

B) 2006

C) 2007

D) 2008

Answer: 2005

Solution: Given:

$$P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

$$\therefore PP^{T}=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow PP^{T}=I$$

Now,

$$P^{T}\left(PAP^{T}\right)^{2007}P=P^{T}\underbrace{\left(PAP^{T}\right)\left(PAP^{T}\right)\left(PAP^{T}\right)\ldots\left(PAP^{T}\right)}_{2007\text{ times}}P$$

$$\Rightarrow P^{T}\left(PAP^{T}\right)^{2007}P=A^{2007}$$

Now,

$$A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$

$$A^{2}=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$

$$A^{3}=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$$

$$\vdots\quad\vdots\quad\vdots\ \vdots$$

$$\vdots\quad\vdots\quad\vdots\ \vdots$$

$$A^{2007}=\begin{bmatrix}1 & 2007\\ 0 & 1\end{bmatrix}$$

So,

$$\Rightarrow P^{T}\left(PAP^{T}\right)^{2007}P=\begin{bmatrix}1 & 2007\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow P^{T}QP=\begin{bmatrix}1 & 2007\\ 0 & 1\end{bmatrix}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

So,

$a=1,\ b=2007,\ c=0,\ d=1$

$2a+b+3c-4d=2+2007-4=2005$

Q.38. If $A=\begin{bmatrix}2 & 1 & 0\\ 1 & 2 & -1\\ 0 & 1 & 2\end{bmatrix}$ and $|adj\,(adj\,(adj A))|=16^{n}$ then the value of n is

Answer: 6

Solution: Given that $A=\begin{bmatrix}2 & 1 & 0\\ 1 & 2 & -1\\ 0 & 1 & 2\end{bmatrix}$ and $|adj\,(adj\,(adj A))|=16^n$.

We know that $|adj A|=|A|^{n-1}$ and $|adj\,(adj A)|=|A|^{(n-1)^2}$

Similarly, $|adj\,(adj\,(adj A))|=|A|^{(n-1)^3}$ where n is the order of the square matrix.

Let us find $|A|$.

$|A|=2\,(2\times 2-(1)\,(-1))-1\,(1\times 2-0\,(-1))+0\,(1\times 1-0\times 2)$

$=2\,(5)-(2)+0=8$

$\Rightarrow |A|^{(n-1)^3}=(8)^{(3-1)^3}=8^8$

$\Rightarrow 8^8=16^n$

$\Rightarrow \left(2^3\right)^8=\left(2^4\right)^n$

$\Rightarrow 2^{24}=2^{4n}$

$\Rightarrow 24=4n$

$\Rightarrow n=6.$

Therefore, the value of n is 6.

Q.39. Maximum value of n such that $66!$ is divisible by 3^n is

Answer: 31

Solution: We know that,

Maximum value of n for p^n where p is prime divides the number $a!$ is given by,

$\left[\frac{a}{p}\right]+\left[\frac{a}{p2}\right]+\left[\frac{a}{p3}\right]+\left[\frac{a}{p4}\right]+\ldots$

Since, 3 is a prime number, so we have

$=\left[\frac{66}{3}\right]+\left[\frac{66}{3^2}\right]+\left[\frac{66}{3^3}\right]+\left[\frac{66}{3^4}\right]+\ldots$

$=22+7+2+0$

$=31$

So, maximum value of n is 31.