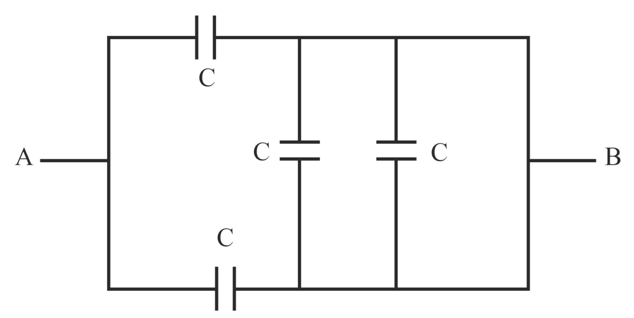
JEE Main 2023 (Session 2)

April 10 Shift 1

Physics

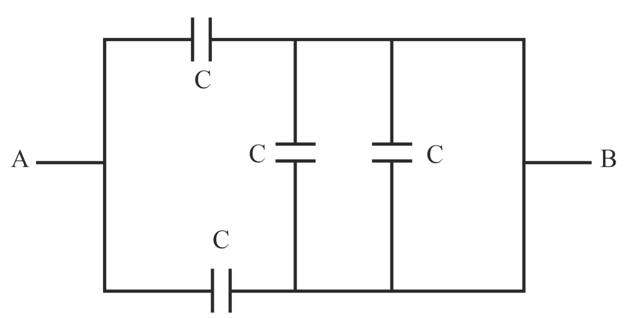
Q.1. Find the equivalent capacitance across points A and B in the given electrical circuit.



A) $\frac{C}{2}$ B) $2C$ C) $\frac{5C}{3}$ D) $\frac{3C}{4}$

Answer: $2C$

Solution:



From the diagram it is clear that the two middle capacitors are made short-circuit with respect to points A and B. Hence, while calculating the equivalent capacitance, these two middle capacitors can be neglected.

Hence, the equivalent capacitance (C_{eq}) for the given circuit can be calculated as follows(remaining two capacitors are parallel combination):

$$C_{eq} = C + C$$
$$= 2C$$

Q.2. A particle of mass m moving with a velocity v collides with a particle of mass $2m$ at rest and sticks to it. Velocity of combined mass is equal to

A) v B) $\frac{v}{2}$ C) $\frac{v}{3}$ D) $\frac{v}{4}$

Answer: $\frac{v}{3}$

Solution: Since there is no external force acting on the combined system, linear momentum is conserved.

So, initial momentum is equal to final momentum.

Let v' be the final velocity of combination.

Then, by law of conservation of linear momentum, we have below expression.

$$(m \times v) + (2m \times 0) = (m + 2m) \times (v')$$
$$\Rightarrow mv = 3mv'$$
$$\Rightarrow v' = \frac{v}{3}$$

Q.3. An object weighs 200 N at the surface of Earth. Find the weight at a depth of $\frac{R}{2}$, where R is the radius of Earth.

A) 100 N B) 300 N C) 50 N D) 150 N

Answer: 100 N

Solution: The variation of acceleration due to gravity with depth can be expressed as follows:

$g' = g\left(1 - \frac{d}{R}\right) \quad \ldots (1)$

Substitute $\frac{R}{2}$ for d into equation (1) to obtain the new acceleration due to gravity at the given depth.

$$g' = g\left(1 - \frac{\frac{R}{2}}{R}\right)$$

$$= \frac{g}{2}$$

Since, the acceleration due to gravity is halved, the weight of the object will also be halved at the given depth.

Hence, the weight of the object at the given depth is $100\ \mathrm{N}$.

Q.4. For an object radiating heat at $300\ \mathrm{K}$, the wavelength corresponding to maximum intensity is λ. If the temperature of body is increased by $300\ \mathrm{K}$, the new wavelength corresponding to maximum intensity will be

A) $\frac{\lambda}{2}$ B) 2λ C) λ D) $\frac{5\lambda}{2}$

Answer: $\frac{\lambda}{2}$

Solution: According to Wien's displacement law,

$\lambda_m T = \text{constant}$

where, λ_m is wavelength of maximum intensity and T is absolute temperature.

Let λ_m' be the required wavelength at $600\ \mathrm{K}$ (Since temperature is increased by $300\ \mathrm{K}$)

$\Rightarrow \lambda_m' \times 600 = \lambda \times 300$

$\Rightarrow \lambda_m' = \frac{\lambda}{2}$

Q.5. A particle, when projected at 15^o with horizontal, has a range of $50\ \mathrm{m}$. Find the range when projected at 45^o with horizontal.

A) $50\ \mathrm{m}$ B) $100\ \mathrm{m}$ C) $80\ \mathrm{m}$ D) $120\ \mathrm{m}$

Answer: $100\ \mathrm{m}$

Solution: The formula for horizontal range of a projectile is given by, $R = \frac{u^2 \sin(2\theta)}{g}$

For angle of projection, $\theta = 15^o$, range is given by,

$$R = \frac{u^2 \sin(30^o)}{g}$$

$$= \frac{u^2}{2g} = 50\ \mathrm{m}\ \left(\text{Given}\right)$$

For angle of projection, $\theta = 45^o$, range is given by,

$$R' = \frac{u^2 \sin(90^o)}{g}$$

$$= \frac{u^2}{g} = 100\ \mathrm{m}$$

Q.6. Statement (1) : An LCR circuit connected to an AC source has maximum average power at resonance.

Statement (2) : A resistor only circuit with zero phase difference has maximum average power.

A) (1) and (2) both are correct
B) (1) is correct but (2) is incorrect
C) (1) is incorrect but (2) is correct
D) (1) and (2) both are incorrect

Answer: (1) and (2) both are correct

Solution: We know that at resonance, an LCR circuit connected to an AC source has current amplitude at its maximum value.

In addition, average power reaches a maximum when impedance (Z), which depends on the frequency, is a minimum, that is, when $X_C = X_L$ and $Z = R$. Thus, at resonance, the average power output of the source in an LCR series circuit is a maximum.

Thus, both the statements are correct.

Q.7. A monoatomic gas initially at pressure P and volume V is compressed to $\frac{1}{8}^{\text{th}}$ of its volume adiabatically. Final pressure of the gas is equal to

A) $4P$ B) $8P$ C) $16P$ D) $32P$

Answer: $32P$

Solution: The initial and final parameters of the gas are related by the equation

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$P_f = P_i \left(\frac{V_i}{V_f}\right)^{\gamma} \quad \ldots (1)$$

Substitute the values of the known parameters into equation (1) to obtain the final pressure of the gas.

$$P_f = P\left(\frac{V_i}{\frac{V_i}{8}}\right)^{\frac{5}{3}}$$

$$= 32P$$

Q.8. What is the maximum percentage error in the measurement of quantity l, if it is given by $l = \frac{a^2 b^3}{c\sqrt{d}}$? Given the percentage error in the calculation of $a,\ b,\ c$ and d are $1\%,\ 2\%,\ 3\%$ and 4% respectively.

A) 4% B) 12% C) 9% D) 13%

Answer: 13%

Solution: The formula to calculate the percentage error in measuring the quantity l can be expressed as

$$\frac{\Delta l}{l}\% = \left[\left(2\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}\right) \times 100\right]\% \quad \ldots (1)$$

Substitute the values of the known parameters into equation (1) to calculate the required percentage error to measure the given quantity.

$$\frac{\Delta l}{l}\% = \left[\left(2 \times 0.01 + 3 \times 0.02 + 0.03 + \frac{1}{2} \times 0.04\right) \times 100\right]\%$$

$$= 13\%$$

Q.9. The equation of progressive wave is $y = 5\sin(6t + 0.03x)$. Find the speed of wave.

Answer: 200

Solution: Here, the direction of wave movement is along $-X$ axis.

The standard equation of a wave moving in $-X$ direction is given by,

$y = A\sin(\omega t + kx)$

where, the speed of wave is given by, $\text{speed} = \frac{\omega}{k}$

Upon substitution, we get

$$\text{speed} = \frac{6}{0.03}$$

$$= 200 \text{ m s}^{-1}$$

Q.10. Earth shrinks to $\frac{1}{64}$ times of its initial volume. Time period of Earth rotation is found to be $\frac{24}{x}$ hr. Find the value of x.

Answer: 16

Solution: As the radius of Earth is decreased, its moment of inertia will also change.

The time period (T) and the angular frequency (ω) are related by the relation

$$\omega = \frac{2\pi}{T} \quad \ldots(1)$$

Considering Earth as a perfect sphere of radius R, its initial angular momentum (L) is given by

$$L = \left(\frac{2}{5}MR^2\right)\omega \quad \ldots(2)$$

From equations (1) and (2), it implies that

$$L = \frac{2}{5}MR^2 \times \frac{2\pi}{T}$$
$$\Rightarrow T = \frac{4\pi}{5L}MR^2 \quad \ldots(3)$$

The radius and volume of a spere is related by the relation

$$R \propto V^{\frac{1}{3}} \quad \ldots\left(4\right)$$

Using equation (4), the new radius of the Earth (R') is given by

$$\frac{R'}{R} = \left(\frac{\frac{V}{64}}{V}\right)^{\frac{1}{3}}$$
$$\Rightarrow R' = \frac{R}{4} \quad \ldots(5)$$

Substitute the expression for the new radius into equation (3) to obtain the new time period (T') when the volume is decreased.

$$T' = \frac{4\pi}{5L}M\left(\frac{R}{4}\right)^2$$
$$= \frac{T}{16} \quad \ldots(6)$$

Comparing equation (6) with the given expression for the new time period, it can be concluded that $x = 16$.

Q.11. A conducting rod of length $1\ \mathrm{m}$ is moved across a magnetic field of $0.15\ \mathrm{T}$, with constant speed of $4\ \mathrm{m\ s^{-1}}$. Find force (in N) on rod.

Answer: 0

Solution: The formula to calculate the emf (ε) induced in the moving rod of length l, moving with a speed of v under a magnetic field B can be expressed as(in this case length of the rod is perpendicular to both magnetic field and velocity of the rod)

$$\varepsilon = Blv \quad \ldots(1)$$

Substitute the values of the known parameters into equation (1) to calculate the required induced emf.

$$\varepsilon = 0.15\ \mathrm{T} \times 1\ \mathrm{m} \times 4\ \mathrm{m\ s^{-1}}$$
$$= 0.6\ \mathrm{V}$$

Now as the rod is not connected to any circuit, there will not be any current flowing & therefore no magnetic force on the rod.

Hence, the required value of the force is 0.

Q.12. 10 resistors each of $10\ \Omega$ resistance when connected together give minimum equivalent resistance R_1 and maximum equivalent resistance R_2 among various possible combinations. So $\frac{R_2}{R_1}$ is equal to

Answer: 100

Solution: A set of resistors connected in a circuit produces maximum value of equivalent resistance when they are connected in series combination. On the other hand, the equivalent resistance is minimum when the set of resistors are connected in parallel combination.

When ten resistors of resistance $10\ \Omega$ each are connected in series combination, the equivalent resistance (R_2) is given by

$$R_2 = 10\ \Omega + 10\ \Omega + \ldots\ldots 10\ \text{times}$$
$$= 100\ \Omega \quad \ldots(1)$$

When these are connected in parallel combination, the equivalent resistance (R_1) of the circuit is given by

$$\frac{1}{R_1} = \frac{1}{10\,\Omega} + \frac{1}{10\,\Omega} + \ldots\ldots\ldots\ 10\ \text{times}$$
$$\Rightarrow R_1 = 1\ \Omega \quad \ldots(2)$$

Divide equation (1) by equation (2) to calculate the required ratio.

$$\frac{R_2}{R_1} = \frac{100\,\Omega}{1\,\Omega}$$
$$= 100$$

Q.13. In an AM wave, amplitude of modulating wave $= 3$ units and amplitude of carrier wave $= 15$ units. Find the ratio of maximum to minimum intensity $\frac{Imax}{Imin}$.

Answer: 2.25

Solution: Given,

Amplitude of modulating wave, $(A_M) = 3$ units

Amplitude of carrier wave, $(A_C) = 15$ units

As both modulating and carrier waves are superimposed, $A_{max} = A_C + A_M$

and $A_{min} = A_C - A_M$

Since intensity is proportional to the square of amplitude.

$$\frac{Imax}{Imin} = \left(\frac{A_C + A_M}{A_C - A_M}\right)^2$$
$$= \left(\frac{15+3}{15-3}\right)^2$$
$$= \left(\frac{18}{12}\right)^2$$
$$= 1.5^2 = 2.25$$

Chemistry

Q.14. Stabiliser used for concentration of sulphide ore is

A) Fatty acids B) Pine iol C) Cresols D) Xanthates

Answer: Cresols

Solution: Froth flotation method is used to remove gangue from the sulphide ores by the formation of a powdered ores' suspension in water with the use of collectors and stabilisers. The role of the stabiliser in the froth flotation process is used to stabilise the froth such as cresols and aniline. It increases the non-wettability of the mineral particles.

Q.15. Which of the following complex compound is diamagnetic and low spin?

A) $[Co(NH_3)_6]^{+3}$ B) $[CoCl_6]^{-3}$ C) $[CoF_6]^{-3}$ D) $[Fe(H_2O)_6]^{+3}$

Answer: $[Co(NH_3)_6]^{+3}$

Solution: In $[Co(NH_3)_6]^{+3}$ the oxidation state of cobalt is $+3$. Ammonia is a strong field ligand due to that pairing of electrons takes place. The hybridisation of central metal atom is d^2sp^3. Since it has no unpaired electrons, it is diamagnetic.

Strong field ligands cause larger splitting of d orbitals and pairing of electrons is favoured. Hence, it is a low spin complex.

Q.16. One which does not stabilise secondary and tertiary protein.

A) $O-O$ linkage B) $S-S$ linkage C) Vanderwalls force D) Hydrogen bonding

Answer: $O-O$ linkage

Solution: In primary amines, peptide linkages (also known as peptide bonds) are formed between the amine group of one amino acid and the carboxyl group of another amino acid, resulting in the formation of a peptide chain.

In secondary structure of polypeptides, the peptide chains are stabilised by hydrogen bonding between the amino and carboxyl groups of adjacent amino acids.

In tertiary structure, the protein chain is folded into a three-dimensional conformation, and the stabilised by various types of bonds, including disulphide bonds, ionic bonds, hydrogen bonds, and hydrophobic interactions, among others.

So among the given options $O-O$ linkage doesn't stabilise secondary and tertiary protein.

Q.17. Which compound does not exist from the following?

A) $BeCl_2$ B) NaO_2 C) $PbEt_4$ D) $(NH_4)_2BeF_4$

Answer: NaO_2

Solution: Among the alkali metals, sodium is known to form peroxides, whereas the other alkali metals (such as potassium, rubidium, and cesium) are known to form superoxides. Sodium reacts with oxygen to form sodium oxide (Na_2O) as well as with excess oxygen to form sodium peroxide (Na_2O_2). However, sodium does not form the superoxide ion (O_2^-) under normal conditions. Therefore, the compound that does not exist from the given options is NaO_2.

Q.18. Prolongated heating of ferrous ammonium sulphate is avoided to prevent

A) Oxidation B) Reduction C) hydrolysis D) Breaking

Answer: Oxidation

Solution: On heating, ferrous sulphate crystals lose water and anhydrous ferrous sulphate ($FeSO_4$) is formed. On prolonged heating ferrous ion oxidised to ferric ions.

The reaction is as follows,

$2FeSO_4 \rightarrow Fe_2O_3 + SO_2 + SO_3$

Q.19. Enthalpy of adsorption and enthalpy of formation of micelle are respectively

A) Positive, Positive B) Positive, Negative C) Negative, Positive D) Negative, Negative

Answer: Negative, Positive

Solution: The amount of heat evolved when one mole of an adsorbate (gas or liquid) is adsorbed on the surface of an adsorbent is called enthalpy of adsorption. In the vast majority of cases, adsorption is an exothermic in nature. Therefore, enthalpy of adsorption is negative.

Micelle formation decreases the stability of the colloidal solution so energy of the mixture should increase which means $\Delta H > 0$. Therefore, the enthalpy of formation of micelle is positive.

Q.20. Read the following two statements.
Statement I: Potassium dichromate is used in volumetric analysis.
Statement II: $K_2Cr_2O_7$ is more soluble in water than $Na_2Cr_2O_7$

A) Both statements I and II are correct. B) Both statements I and II are incorrect.

C) Statement I is correct and II is incorrect. D) Statement I is incorrect and II is correct.

Answer: Statement I is correct and II is incorrect.

Solution: Potassium dichromate is preferred over sodium dichromate in volumetric analysis, because sodium dichromate is hygroscopic in nature and, therefore, accurate weighing is not possible in normal atmospheric conditions.

$Na_2Cr_2O_7$is hygroscopic in nature and is more soluble in water than $K_2Cr_2O_7$.

Therefore, option C is correct.

Q.21. Match the following

Column I	Column II
(i). Cotton mills	(a). Biodegradable waste
(ii). Paper mills	(b). Gypsum
(iii). Fertilizers	(c). Non-biodegradable waste
(iv). Thermal power plant	(d). Fly ash

A) $\mathrm{i-c, ii-a, b; iii-c, iv-b}$

B) $\mathrm{i-a; ii-a, iii-b, iv-d}$

C) $\mathrm{i-a, c; ii-b, iii-b, iv-a}$

D) $\mathrm{i-c, ii-b, c; iii-b, c, iv-a}$

Answer: $\mathrm{i-a; ii-a, iii-b, iv-d}$

Solution: Cotton mills generate biodegradable waste such as cotton fiber, cottonseed, and other organic materials. Paper mills produce biodegradable waste such as pulp sludge, bark, and wood chips. Fertilizers are made from raw materials such as phosphate rock, sulfur, and potassium salts. During their production, gypsum is generated as a waste product. Thermal power plants generate non-biodegradable waste such as fly ash, bottom ash, and slag as by-products of coal combustion.

Q.22. Match the column I with column II

Column I	Column II
A. Dacron	P. Thermosetting
B. Urea and formaldehyde resin	Q. Biodegradable
C. Nylon-2, Nylon-6	R. Polyester
D. Nylon$-6, 6$	S. Uses for making bristles of brushes

A) $\mathrm{A-R, B-P, C-S, D-Q}$ B) $\mathrm{A-P, B-R, C-Q, D-S}$ C) $\mathrm{A-R, B-P, C-Q, D-S}$ D) $\mathrm{A-R, B-R, C-S, D-Q}$

Answer: $\mathrm{A-R, B-P, C-Q, D-S}$

Solution: Dacron is a type of polyester fabric made from polyethylene terephthalate (PET). It is a thermoplastic polymer that is commonly used in clothing, bedding, and as a fibrefill material in various products.

Urea formaldehyde resin is a type of thermosetting plastic that is commonly used in the manufacture of adhesives, coatings, and molded products.

Nylon-2 and Nylon-6 are types of biodegradable nylon that can break down naturally in the environment over time. These types of nylon are typically used in applications such as packaging, textiles, and medical products.

Nylon$-6, 6$ is a type of nylon that is known for its high durability and abrasion resistance. It is commonly used in the manufacture of bristles for brushes, as well as in other applications such as automotive parts, fishing line, and clothing.

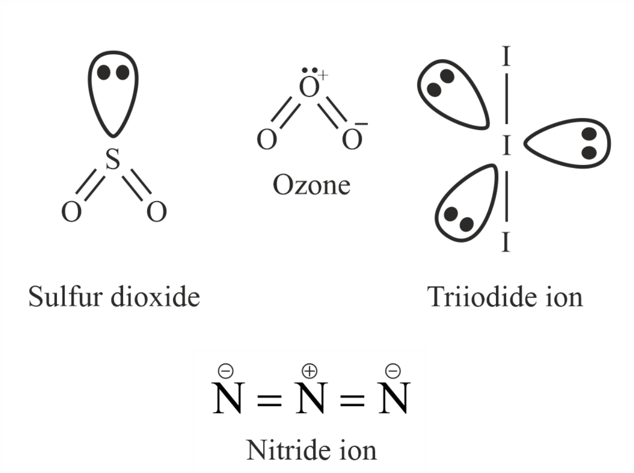
Q.23. How many of the following are bent in shape?

$\mathrm{SO_2, O_3, I_3^-, N_3^-}$.

Answer: 2

Solution: SO_2 has a bent molecular geometry due to the presence of a lone pair of electrons on sulfur atom, which distorts the molecular shape. In O_3 central oxygen atom is surrounded by two unshared electrons (one lone pair) and six bonding electrons (three bonds). So it is bent in shape.

I_3^- and N_3^- are linear.



Q.24. The sum of number of lone pairs in central atom in IF_5 and IF_7 is:

Answer: 1

Solution: In IF_5, the central atom is iodine, which has 7 valence electrons. Iodine in this molecule has 5 bonding pairs and 1 lone pair. Therefore, the number of lone pairs on the central atom of IF_5 is 1.

In IF_7, the central atom is also iodine, which has 7 valence electrons. Iodine in this molecule has 7 bond pairs and 0 lone pairs. Therefore, the number of lone pairs on the central atom of IF_7 is 0.

Therefore, the sum of the number of lone pairs in the central atom of IF_5 and IF_7 is $1 + 0 = 1$..

Q.25. The degree of dissociation of monobasic acid is 0.3, By what percent is the observed depression in freezing point greater than the calculated depression in freezing point?

Answer: 30

Solution: $\underset{1-\alpha}{HA} \leftrightharpoons \underset{\alpha}{H^+} + \underset{\alpha}{A^-}$

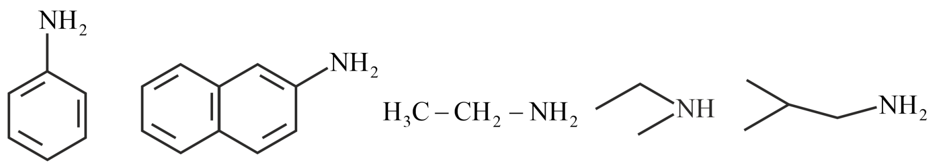
Van't Hoff factor, $i = 1 - \alpha + \alpha + \alpha = 1 + \alpha$

Given, degree of dissociation, $\alpha = 0.3$

Therefore, $i = 1 + 0.3 = 1.3$

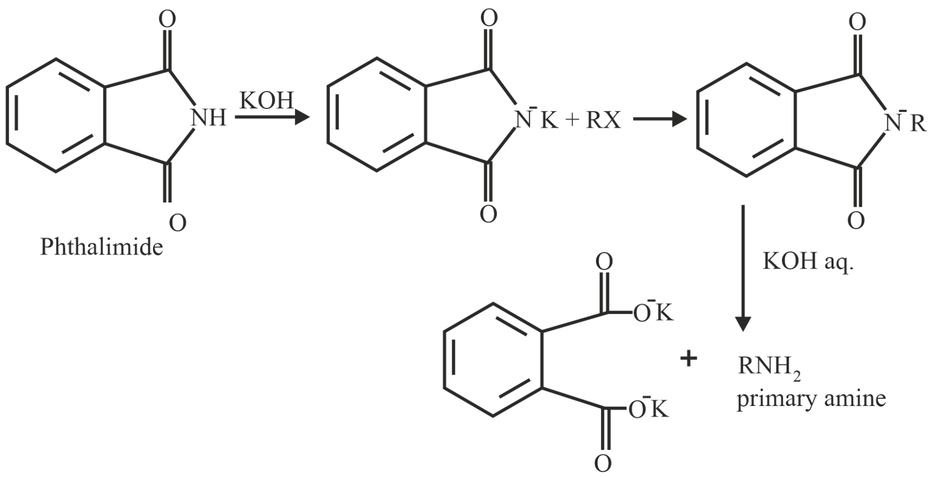
The observed depression in freezing point is 1.3 times of the calculated depression in freezing point. Therefore, the difference is 0.3 and the percentage is 30.

Q.26. How many compounds can be easily prepared by Gabriel Phthalimide synthesis, which on reaction with Hinsberg reagent produces a compound which is soluble in KOH?



Answer: 2

Solution: Gabriel's Phthalimide synthesis is used for the conversion of primary alkyl halide into a primary amine.
In Gabriel phthalimide synthesis, a base abstract proton from phthalimide gives a nucleophile phthalimide ion which attacks on the unhindered primary alkyl halide. The base hydrolysis of alkylated phthalimide gives the primary unhindered amine and phthalimide ion.



Mathematics

Q.27. The negation of the statement $(p \vee q) \wedge \sim r$ is

A) $(\sim p \wedge \sim q) \wedge r$ B) $(\sim p \wedge \sim q) \vee r$ C) $(\sim p \wedge q) \vee r$ D) $(p \wedge \sim q) \vee r$

Answer: $(\sim p \wedge \sim q) \vee r$

Solution: The negation of the statement $(p \vee q) \wedge \sim r$ is

$\sim [(p \vee q) \wedge \sim r]$

$\equiv \sim (p \vee q) \vee r$

$[\because \sim (A \wedge B) \equiv \sim A \vee \sim B]$

$\equiv (\sim p \wedge \sim q) \vee r$

$[\because \sim (A \vee B) \equiv \sim A \wedge \sim B]$

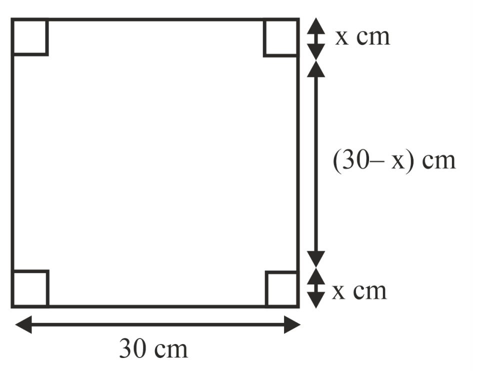
Q.28. From a square of side 30 cm, the squares of side $x \text{ cm}$ is cut off to make cuboid of maximum volume. The surface area of cuboid with open top is

A) 400 cm^2 B) 464 cm^2 C) 800 cm^2 D) 900 cm^2

Answer: 800 cm^2

Solution: Given that, the side of square is $30\ \mathrm{cm}$ and $x\ \mathrm{cm}$ squares are cut off.

The required diagram is



Now the dimensions of the cuboid formed will be

$l(x) = 30 - 2x$, $b(x) = 30 - 2x$ and $h(x) = x$.

The Volume of the cuboid will be $V(x) = (30 - 2x)^2 (x)$

Now to get Maximum value,

$\Rightarrow \frac{\mathrm{d}V(x)}{\mathrm{d}x} = 0$

$\Rightarrow 2(30 - 2x)(-2)x + (30 - 2x)^2(1) = 0$

$\Rightarrow (30 - 2x)(-4x + 30 - 2x) = 0$

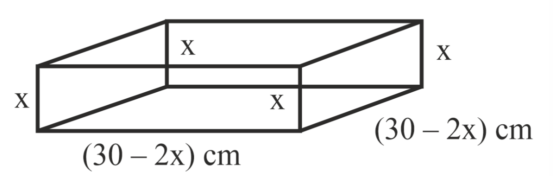
On simplifying we get,

$\Rightarrow \mathrm{x} = 15\ \mathrm{cm},\ 5\ \mathrm{cm}$

But x cannot be $15\ \mathrm{cm}$ as the volume becomes zero.

Hence $\mathrm{x} = 5\ \mathrm{cm}$.

Now to find the surface area of the cuboid,



Surface area will be $= (30 - 2x) \times x \times 4 + (30 - 2x)^2$

$= (30 - 2 \times 5) \times 5 \times 4 + (30 - 2 \times 5)^2$

$= 800\ \mathrm{cm}^2$.

Therefore, the required surface area will be $800\ \mathrm{cm}^2$.

Q.29. Using the number $1, 2, 3, \ldots, 7$, total numbers of 7 digit number which does not contain string 154 or 2367 is, (repetition is not allowed)

A) 4897 B) 4898 C) 4896 D) 4899

Answer: 4898

Solution: Given,

The number $1, 2, 3, \ldots., 7$

Now total numbers of 7 digit number will be $7!$ without repetition,

Now total numbers which contain string 154 will be $5!$

Total numbers which contain string 2367 will be $4!$

And number which contain both string 2367 & 154 will be 2

So, total number which does not contain string 154 or 2367 will be $7! - (5! + 4! - 2)$

$= 5040 - (120 + 24 - 2)$

$= 5040 - 142 = 4898$

Q.30. Find the value of $96 \cdot \cos\frac{\pi}{33} \cdot \cos\frac{2\pi}{33} \cdot \cos\frac{4\pi}{33} \ldots\ldots\ldots \cos\frac{16\pi}{33}$

A) 0 B) 1 C) 2 D) 3

Answer: 3

Solution: Given,

Expression $96 \cdot \cos\frac{\pi}{33} \cdot \cos\frac{2\pi}{33} \cdot \cos\frac{4\pi}{33} \ldots\ldots\ldots \cos\frac{16\pi}{33}$

Now we know that,

$\cos A \cdot \cos 2A \cdot \cos 2^2 A \cdot \cos 2^3 A \ldots\ldots \cdot \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$

Now using the above formula in given expression we get,

$96 \cdot \cos\frac{\pi}{33} \cdot \cos\frac{2\pi}{33} \cdot \cos\frac{4\pi}{33} \ldots\ldots\ldots \cos\frac{16\pi}{33}$

$= 96 \times \frac{\sin\frac{32\pi}{33}}{2^5 \sin\frac{\pi}{33}}$

$= 96 \times \frac{\sin\left(\pi - \frac{\pi}{33}\right)}{2^5 \sin\frac{\pi}{33}}$

$= 96 \times \frac{\sin\left(\frac{\pi}{33}\right)}{2^5 \sin\frac{\pi}{33}}$ $\{\text{as } \sin(\pi - \alpha) = \sin\alpha\}$

$= 96 \times \frac{1}{32} = 3$

Q.31. Slope of tangent to a curve at a variable point is $\frac{x^2+y^2}{2xy}$ and $y(2) = 0$, then $y(8)$ is

A) $\sqrt{2}$ B) $\sqrt{5}$ C) $4\sqrt{3}$ D) None of these

Answer: $4\sqrt{3}$

Solution: Given:

$\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$

Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$v+x\frac{dv}{dx}=\frac{1+v^2}{2v}$

$\Rightarrow x\frac{dv}{dx}=\frac{1-v^2}{2v}$

$\Rightarrow\int\left(\frac{2v}{v^2-1}\right)dv=-\int\frac{dx}{x}$

$\Rightarrow\log_e\left|v^2-1\right|=\log_e\left(\frac{C}{x}\right)$

$\Rightarrow\frac{y^2-x^2}{x^2}=\frac{C}{x}$

$\Rightarrow y^2-x^2=Cx$

Put $x=2$ and $y=0$ we get,

$0-2^2=2C\Rightarrow C=-2$

$\Rightarrow y^2=x^2-2x$

$\Rightarrow y(8)=\sqrt{8^2-16}$

$\Rightarrow y(8)=\sqrt{48}=4\sqrt{3}$

Q.32. If the order of the matrix A is 3×3 and $|A|=2$, then the value of $\left|3adj\left(|3A|A^2\right)\right|$ is

A) $2^{21}\cdot3^{10}$ B) $2^{10}\cdot3^{21}$ C) $2^{12}\cdot3^{15}$ D) $2^{15}\cdot3^{12}$

Answer: $2^{10}\cdot3^{21}$

Solution: We need to find the value of $\left|3adj\left(|3A|A^2\right)\right|$

We know that $|kA|=k^n|A|$ where n is the order of the matrix and k is a constant.

$\Rightarrow|3A|=3^3(2)$

$\Rightarrow\left|3adj\left(|3A|A^2\right)\right|=3^3\left|adj\left(\left(3^3\cdot2\right)A^2\right)\right|$

We know that $adj(kA)=k^{n-1}adj(A)$

$=3^3\left|\left(3^3\cdot2\right)^2adj\left(A^2\right)\right|$

Now we know that $|adjA|=|A|^{n-1}$

$=3^3\left(\left(3^3\cdot2\right)^2\right)^3\left|adj\left(A^2\right)\right|$

$=3^3\left(3^3\cdot2\right)^6\left|A^2\right|^{3-1}$

$=3^3\left(3^3\cdot2\right)^6(2)^4$

$=2^{10}\cdot3^{21}$

Hence, the required answer is $2^{10}\cdot3^{21}$

Q.33. Find the number of integral values of x which satisfy the inequality $x^2-10x+19<6$

A) 7 B) 10 C) 6 D) 8

Answer: 7

Solution: Given:

$x^2 - 10x + 19 < 6$

$\Rightarrow x^2 - 10x + 13 < 0$

Now,

$x^2 - 10x + 13 = 0$

$\Rightarrow x = \frac{10 \pm \sqrt{48}}{2}$

$\Rightarrow x = 5 \pm 2\sqrt{3}$

So,

$x^2 - 10x + 13 < 0$

$\Rightarrow \left(x - \left(5 + 2\sqrt{3}\right)\right)\left(x - \left(5 - 2\sqrt{3}\right)\right) < 0$

$\Rightarrow x \in \left(5 - 2\sqrt{3}, 5 + 2\sqrt{3}\right)$

$\Rightarrow x \in (1.5, 8.4)$

So, integral values are $x = 2, 3, 4, 5, 6, 7, 8$ i.e., 7 values.

Q.34. The coefficient of x^7 in $\left(1 - 2x + x^3\right)^{10}$ is

A) 5410 B) 2080 C) 4080 D) 6234

Answer: 4080

Solution: We need to find the coefficient of x^7 in $\left(1 - 2x + x^3\right)^{10}$.

We know that for $(x + y + z)^n$,

$Tn = \frac{n!}{a!b!c!}(x)^a(y)^b(z)^c$ such that $a + b + c = n$

Now for $\left(1 - 2x + x^3\right)^{10}$

$Tn = \frac{10!}{a!b!c!}(1)^a(-2x)^b\left(x^3\right)^c$

$= \frac{10!}{a!b!c!} - 2^b x^{b+3c}$. with $a + b + c = 10$

Here we need coefficient of x^7.

Hence the combinations would be

a	b	c
3	7	0
5	4	1
7	1	2

which satisfies both $a + b + c = 10$ and $b + 3c = 7$.

Hence, coefficient of x^7 is $= \frac{10!}{3!7!0!}(-2)^7 + \frac{10!}{5!4!1!}(-2)^4 + \frac{10!}{7!1!2!}(-2)^1$

$= 120 \times (-128) + 20160 + (-720)$

$= 4080$

Therefore, the required value is 4080

Q.35. If $a^2 + (ar)^2 + \left(ar^2\right)^2 = 33033, (a, r \in N)$ then the value of $a + ar + ar^2$ is

A) 148 B) 249 C) 230 D) 231

Answer: 231

Solution: Given that $a^2+(ar)^2+\left(ar^2\right)^2=33033,(a, r \in N)$

$\Rightarrow a^2\left(1+r^2+r^4\right)=11^2 \times 273$

On comparing both sides of the equation we get,

$a=11$ and $1+r^2+r^4=273$

$\Rightarrow r^2+r^4=272$

$\Rightarrow r^2\left(1+r^2\right)=16 \times 17$

$\Rightarrow r=4$

Now $a+ar+ar^2=11+11 \times 4+11 \times 16$

$=11+44+176=231.$

Therefore, the required value is 231

Q.36. Shortest distance between the lines $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ is

A) $\sqrt{29}$ units B) $2\sqrt{29}$ units C) $3\sqrt{29}$ units D) $5\sqrt{29}$ units

Answer: $2\sqrt{29}$ units

Solution: Given:

$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$

$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

So,

$\vec{a}_1=-\hat{i}-\hat{j}-\hat{k}$

$\vec{a}_2=3\hat{i}+5\hat{j}+7\hat{k}$

So,

$\vec{a}_2-\vec{a}_1=4\hat{i}+6\hat{j}+8\hat{k}$

And,

$\vec{b}_1 \times \vec{b}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}=-4\hat{i}-6\hat{j}-8\hat{k}$

$\Rightarrow\left|\vec{b}_1 \times \vec{b}_2\right|=\sqrt{16+36+64}=\sqrt{116}$

Shortest distance between the lines

$=\left|\frac{\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$

$=\left|\frac{\left(4\hat{i}+6\hat{j}+8\hat{k}\right)\cdot\left(-4\hat{i}-6\hat{j}-8\hat{k}\right)}{\sqrt{116}}\right|$

$=\left|\frac{-16-36-64}{\sqrt{116}}\right|=\sqrt{116}=2\sqrt{29}$ units

Q.37. If $3,8,13,.....373$ is in AP then sum of terms which is not divisible by 3 is

Answer: 9525

Solution: Given sequence is $3, 8, 13, \ldots \ldots 373$.

Here $a = 3,\ d = 5\ \text{and}\ a_n = 373$

Now $a_n = a + (n - 1)d$

$\Rightarrow 373 = 3 + (n - 1)5$

$\Rightarrow n = 75$

We know that $S_n = \frac{n}{2}(a + a_n)$

$\Rightarrow S_{75} = \frac{75}{2}(3 + 373)$

$\Rightarrow S_{75} = 14100$

Now let us write the sequence of terms which are divisible by 3.

We get, $3, 18, 33, \ldots \ldots 363$

$\Rightarrow 363 = 3 + (n - 1)15$

$\Rightarrow n = 25$

Now let us find the sum of terms divisible by 3.

$\Rightarrow S_{div\ by\ 3} = \frac{25}{2}(3 + 363) = 4575$

Required sum $=$Sum of 75 terms$-$Sum of terms divisible by 3

$= 14100 - 4575$

$= 9525.$

Therefore, the required sum is 9525

Q.38. If the coefficient of x^7 in the expansion of $\left(ax - \frac{1}{bx^2}\right)^{13}$ is equal to the coefficient of x^{-5} in the expansion of $\left(ax + \frac{1}{bx^2}\right)^{13}$, then a^4b^4 is

Answer: 22

Solution: The coefficient of x^7 in the expansion of $\left(ax^2-\frac{1}{bx}\right)^{13}$ is equal to the coefficient of x^{-5} in $\left(ax+\frac{1}{bx^2}\right)^{13}$.

We know that, the general term T_{r+1} in the expansion $(a+b)^n$ is

$T_{r+1}={}^nC_r a^{n-r} b^r$

Applying to $\left(ax-\frac{1}{bx^2}\right)^{13}$, we get

$T_{r+1}={}^{13}C_r(ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$

$\Rightarrow T_{r+1}=(-1)^r\times{}^{13}C_r(a)^{13-r}(x)^{13-3r}(b)^{-r}$

Therefore, $13-3r=7\Rightarrow r=2$ for coefficient of x^7.

Thus,

$T_3={}^{13}C_2\left(\frac{a^{11}}{b^2}\right)$

Similarly, applying to $\left(ax+\frac{1}{bx^2}\right)^{13}$, we get

$T_{r+1}={}^{13}C_r(ax)^{13-r}\left(\frac{1}{bx^2}\right)^r$

$\Rightarrow T_{r+1}={}^{13}C_r(a)^{13-r}(x)^{13-3r}(b)^{-r}$

Therefore, $13-3r=-5$ for coefficient of x^{-5}

$\Rightarrow r=6$

So,

$T_7={}^{13}C_6(a)^7(b)^{-6}$

Hence, applying the given condition we get

${}^{13}C_2\left(\frac{a^{11}}{b^2}\right)={}^{13}C_6(a)^7(b)^{-6}$

$\Rightarrow a^4b^4=\frac{{}^{13}C_6}{{}^{13}C_2}$

$\Rightarrow a^4b^4=\frac{13!}{7!\cdot 6!}\times\frac{2!\cdot 11!}{13!}$

$\Rightarrow a^4b^4=\frac{11\times10\times9\times8}{6\times5\times4\times3}$

$\Rightarrow a^4b^4=22$

Q.39. Two dice are rolled and sum of two digits is N then probability that $2^N<N!$ is $\frac{m}{n}$, where m and n are coprime, then $11m-3n$ is

Answer: 85

Solution: Given that $2^N < N!$

N is the sum of numbers of two dice.

$\Rightarrow 2 \leq N \leq 12$

Let us check the given condition $2^N < N!$

This condition is only true for $N \geq 4$

We need to find the probability for $N \geq 4$.

$\Rightarrow P(N \geq 4) = 1 - P(N = 2) - P(N = 3)$

$= 1 - \frac{1}{36} - \frac{2}{36}$

$= \frac{11}{12} = \frac{m}{n}$

Now let us find $11m - 3n$.

$= 11 \times 11 - 3 \times 12 = 85.$

Hence, the requried answer is 85.