JEE Main 2023 (Session 2)

April 10 Shift 2

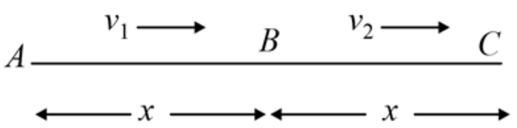
Physics

Q.1. An object moves x distance with speed v_1 and next x distance with speed v_2. The average velocity v is related to v_1 and v_2 as

A) $v = \frac{(v_1+v_2)}{2}$ B) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$ C) $v = \frac{(2v_1v_2)}{v_1+v_2}$ D) $v = \left(\frac{v_1-v_2}{2}\right)$

Answer: $v = \frac{(2v_1v_2)}{v_1+v_2}$

Solution: Let the total distance be $2x$.



For the first half distance x, let time taken be t_1.

$\because$ time $= \frac{\text{distance}}{\text{speed}}$

$\therefore t_1 = \frac{x}{v_1} \dots \text{(i)}$

For second half distance x, let time taken be t_2.

$\because$ time $= \frac{\text{distance}}{\text{speed}}$

$\therefore t_2 = \frac{x}{v_2} \quad \dots \text{(ii)}$

$\therefore$ Average speed for entire journey.

$v_{\text{av}} = \frac{\text{total distance}}{\text{total time}}$

$\Rightarrow v_{\text{av}} = \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}}$

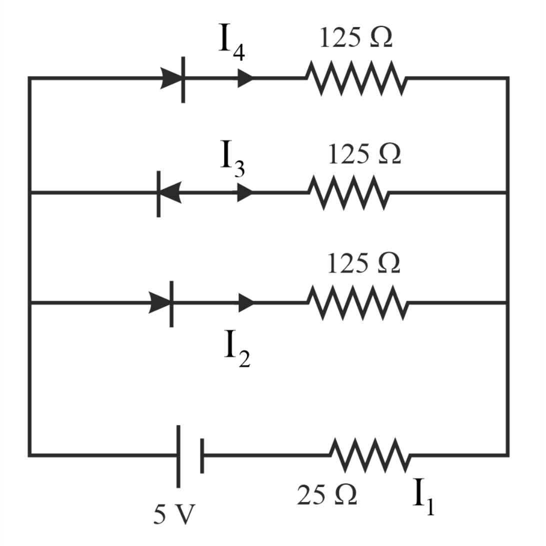
$\Rightarrow v = \frac{2x}{x\left(\frac{v_1+v_2}{v_1v_2}\right)} \quad (\because v_{\text{av}} = v)$

$\Rightarrow v = \frac{2v_1v_2}{v_1+v_2}.$

Note average speed is harmonic mean of given speeds.

Here, average speed is same as average velocity as displacement is equal to distance travelled by object.

Q.2. Following circuit contains diodes with forward bias having resistance $25\ \Omega$ and reverse bias having infinite resistance. The ratio of $\frac{I_1}{I_2}$ is equal to

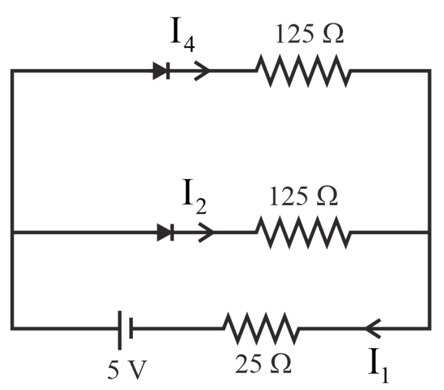


A) 1 B) 2 C) 3 D) 4

Answer: 2

Solution: Since the diode containing current I_3 is in reverse bias condition, the contribution from this diode can be neglected.

Thus, the equivalent circuit can be thought as shown below:



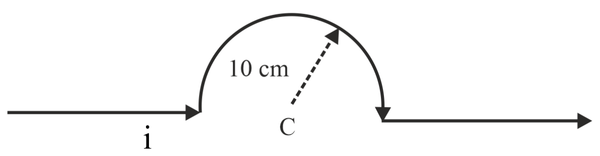
From the above diagram, it can be concluded that

$I_2 = I_4 = \frac{I_1}{2}$

Hence,

$\frac{I_1}{I_2} = 2.$

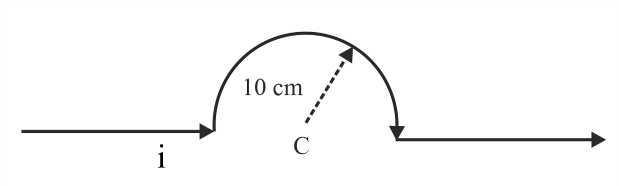
Q.3. An infinitely-long conductor has a current $14\ \mathrm{A}$ flowing as shown in the figure. Find magnetic field at centre C.



A) $88\ \mu\mathrm{T}$ B) $44\ \mu\mathrm{T}$ C) $10\ \mu\mathrm{T}$ D) $120\ \mu\mathrm{T}$

Answer: $44\ \mu\mathrm{T}$

Solution:



We know that, the magnitude of magnetic field (B) due to current carrying arc of radius(r), having a current (I) subtending an angle of θ (in **radian**) at the centre is given by,

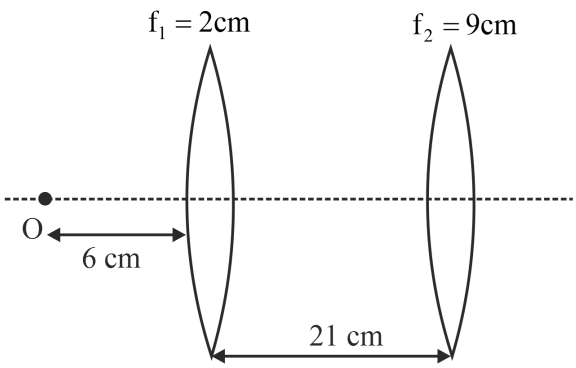
$$B = \frac{\mu_0 I}{4\pi r} \times (\theta)$$

Here, magnetic field due to two straight wires is zero as the wires are passing through C.

Now, we only need to find the magnitude of magnetic field, B due to current carrying semicircular wire of radius, $0.1\ \mathrm{m}$, having a current of $14\ \mathrm{A}$ and subtending an angle of π at centre C.

$$\begin{aligned} B &= \frac{\mu_0 I}{4\pi r} \times (\theta) \\ &= \frac{4\pi \times 10^{-7} \times 14}{4\pi \times 10^{-1}} \times \left(\pi\right) \\ &= 14\pi \times 10^{-6} \\ &\approx 44\ \mu\mathrm{T} \end{aligned}$$

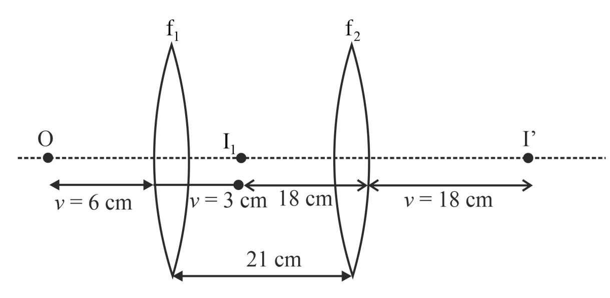
Q.4. A point object (O) is placed on the principle axis of a system of two lenses as shown. Find the distance between the image and the object.



A) $45\ \mathrm{cm}$ B) $40\ \mathrm{cm}$ C) $55\ \mathrm{cm}$ D) $50\ \mathrm{cm}$

Answer: $45\ \mathrm{cm}$

Solution: Let's consider the following diagram:



Considering the left lens at first, the image distance (v) can be calculated as follows-

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}$$
$$\Rightarrow\frac{1}{v}-\frac{1}{-6}=\frac{1}{2}$$
$$\Rightarrow\frac{1}{v}=\frac{1}{2}-\frac{1}{6}$$
$$=\frac{1}{3}$$
$$\Rightarrow v=3 \quad \ldots(1)$$

Considering the image created by the first lens as image for the second lens, the ultimate image distance (v') from the second lens can be calculated as follows-

$$\frac{1}{v'}-\frac{1}{-21-v}=\frac{1}{f_2}$$
$$\Rightarrow\frac{1}{v'}-\frac{1}{-21-(-3)}=\frac{1}{9}$$
$$\Rightarrow\frac{1}{v'}-\frac{1}{-18}=\frac{1}{9}$$
$$\Rightarrow\frac{1}{v'}=\frac{1}{9}-\frac{1}{18}$$
$$=\frac{1}{18}$$
$$\Rightarrow v'=18 \quad \ldots(2)$$

Hence, the distance (d) between the object and the image is given by

$$d=(u+21+v')\ \text{cm}$$
$$=(6+21+18)\ \text{cm}$$
$$=45\ \text{cm}$$

Q.5. If half life for a radio active decay reaction is T. Find the time after which $\left(\frac{7}{8}\right)^{th}$ of initial mass decays.

A) $3T$ B) $2T$ C) $\frac{T}{2}$ D) $4T$

Answer: $3T$

Solution: Time taken for mass of a radioactive substance to reduce to $\frac{1}{8}$th of its initial value:

The time can be calculated by setting:

$$N_t = \frac{N_0}{8}$$

in the radioactive decay equation:

$$N_t = N_0 e^{-\lambda t}$$

Therefore,

$$\frac{N_0}{8} = N_0 e^{-\lambda t}$$

Taking natural logarithm on both sides and re-arranging for t leads to

$$t = \frac{\ln\left(2^3\right)}{\lambda}$$
$$\Rightarrow t = \frac{3\ln(2)}{\lambda} \quad \left(\because t_{1/2} = \frac{\ln(2)}{\lambda}\right)$$
$$\Rightarrow t = 3t_{1/2} \quad \left(\because t_{1/2} = T\right)$$
$$\Rightarrow t = 3T$$

Q.6. Assertion (A) : Fan spins even after switch is off

Reason (R) : Fan in rotation has rotational inertia

A) A is correct and R is correct explanation of A

B) A is correct and R is incorrect explanation of A

C) A is correct and R is correct but R is not correct explanation of A

D) Both (A) and (R) are incorrect

Answer: A is correct and R is correct explanation of A

Solution: The electric fan keeps spinning after the current is cut off for a while due to rotational inertia. Any moving or rotating object possesses kinetic energy due to its motion.

Because of this stored energy, the blades will continue to rotate and work will be done by the blades in overcoming the frictional resistance acting on the blades. Thus, the kinetic energy will get consumed by doing work and when stored energy is fully exhausted, the blades will come to rest.

Hence, A is correct and R is correct explanation of A.

Q.7. When electric field is applied to the electrons in a conductor it starts

A) Moving in a straight line

B) Drifting from higher potential to lower potential

C) Drifting from lower potential to higher potential

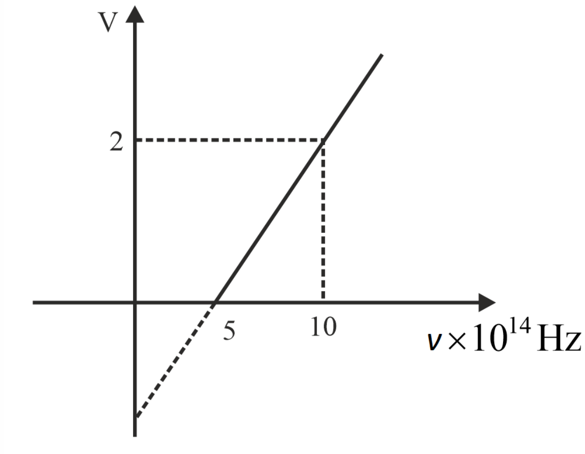
D) Moving with constant velocity

Answer: Drifting from lower potential to higher potential

Solution: We know that, $\vec{F} = q\vec{E}$.
Since charge of electrons is negative, force acting on them is opposite to the direction of electric field. Therefore, electrons move in a direction opposite to the electric field.

Now direction of electric field is from higher to lower potential therefore electrons drift from lower to higher potential.

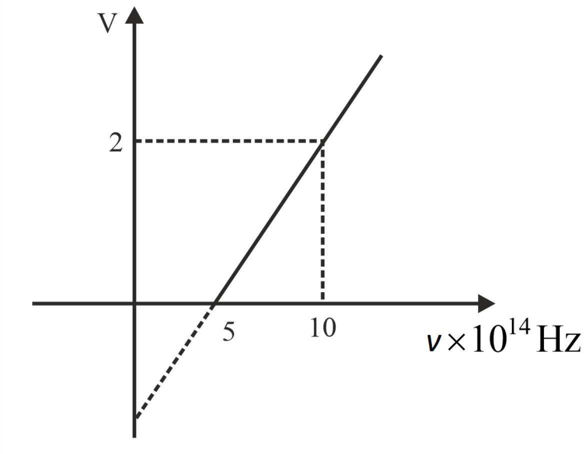
Q.8. Based on given graph between stopping potential and frequency of irradiation, work function of metal is equal to



A) 1 eV B) 3 eV C) 2 eV D) 4 eV

Answer: 2 eV

Solution:



The interception of the stopping potential versus frequency curve indicates the ratio of the stopping potential of the material to the charge of electron.

From the symmetry of the curve, it can be concluded that the extended line cuts the y-axis at -2 V.

Hence, the ratio of the work function (φ) to the electronic charge (e) can be written as

$$\frac{\varphi}{e} = 2 \text{ V}$$
$$\Rightarrow \varphi = 2 \text{ eV}$$

Q.9. Wires A and B have their Young's moduli in the ratio $1:3$, area of cross-section in the ratio of $1:2$ and lengths in ratio of $3:4$. If same force is applied on the two wires to elongate, then ratio of elongation is equal to

A) $8:1$ B) $1:12$ C) $1:8$ D) $9:2$

Answer: $9:2$

Solution: We know the formula for Young's modulus, $Y=\frac{\frac{F}{A}}{\frac{\Delta l}{l}}=\frac{Fl}{A\Delta l}$

$$\Rightarrow \Delta l=\frac{Fl}{AY}$$

Therefore, required ratio

$$\Rightarrow \frac{\Delta l_A}{\Delta l_B}=\frac{F_A}{F_B}\times\frac{l_A}{l_B}\times\frac{A_B}{A_A}\times\frac{Y_B}{Y_A}$$
$$=\frac{1}{1}\times\frac{3}{4}\times\frac{2}{1}\times\frac{3}{1}$$
$$=9:2$$

Q.10. Consider two statements:

S1: Magnetic susceptibility of diamagnetic substance is $-1\leq\chi<0$.

S2: Diamagnetic substance moves from stronger to weaker magnetic field.

A) Both statements are correct

B) Both statements are incorrect

C) S1 is correct and S2 is incorrect

D) S1 is incorrect and S2 is correct

Answer: Both statements are correct

Solution: The properties of a diamagnetic substance can be summarised as follows:

1. Atomic dipoles do not exist in diamagnetic materials since each atoms resulting magnetic moment is zero as a result of paired electrons.

2. A magnet repels materials that are diamagnetic.

3. Because the substances are only weakly attracted to the field, they tend to shift from a strong to a weak region of the external magnetic field.

4. Diamagnetic materials are characterised by constant, small negative susceptibilities, only slightly affected by changes in temperature.

Hence, both the statements are correct.

Q.11. A metallic slab of thickness $\frac{2d}{3}$ and area of surface same as that of plates of capacitor of capacitance C_1 is inserted parallel to plates of capacitor such that its new capacitance becomes equal to C_2. If d is the width between the two plates then $\frac{C_2}{C_1}$ is equal to

A) 1

B) 2

C) 3

D) 4

Answer: 3

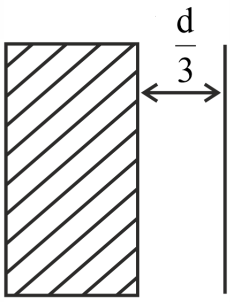
Solution: Without the metallic plate, the capacitance (C_1) of the parallel plate capacitor is given by

$$C_1=\frac{\varepsilon_0 A}{d} \quad \ldots(1)$$

As a metallic plate is inserted between the plates, the new gap (d') between the plates become

$$d'=d-\frac{2d}{3}$$
$$=\frac{d}{3}$$

It can be seen from the following diagram:



Hence, the new capacitance (C_2) is, now, given by

$$C_2=\frac{\varepsilon_0 A}{\frac{d}{3}}$$
$$=\frac{3\varepsilon_0 A}{d} \quad \ldots(2)$$

Divide equation (2) by equation (1) and simplify to obtain the required ratio.

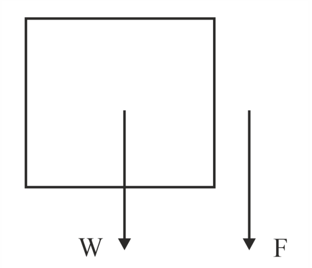
$$\frac{C_2}{C_1}=\frac{\frac{3\varepsilon_0 A}{d}}{\frac{\varepsilon_0 A}{d}}$$
$$=3$$

Q.12. Frictional force acting on the lift of mass $1400\ \mathrm{kg}$ is $2000\ \mathrm{N}$. If lift moves with constant velocity of $3\ \mathrm{m\ s}^{-1}$ in upward direction, the power (in kW) of motor is

(Take $g=10\ \mathrm{ms}^{-2}$)

Answer: 48

Solution: When the lift is moving upward, both the frictional force and the weight of the object act vertically downward, as depicted in the following figure:



Thus, the net force (F_n) on the lift when it is moving upward is given by

$$\begin{aligned} F_n &= W + F \\ &= \left(1400 \text{ kg} \times 10 \text{ m s}^{-2}\right) + 2000 \text{ N} \\ &= 16000 \text{ N} \end{aligned}$$

Hence, the required power of the motor (P) can be calculated as follows:

$$\begin{aligned} P &= F_n v \\ &= 16000 \text{ N} \times 3 \text{ m s}^{-1} \\ &= 48000 \text{ W} \times \frac{1 \text{ kW}}{1000 \text{ W}} \\ &= 48 \text{ kW} \end{aligned}$$

Chemistry

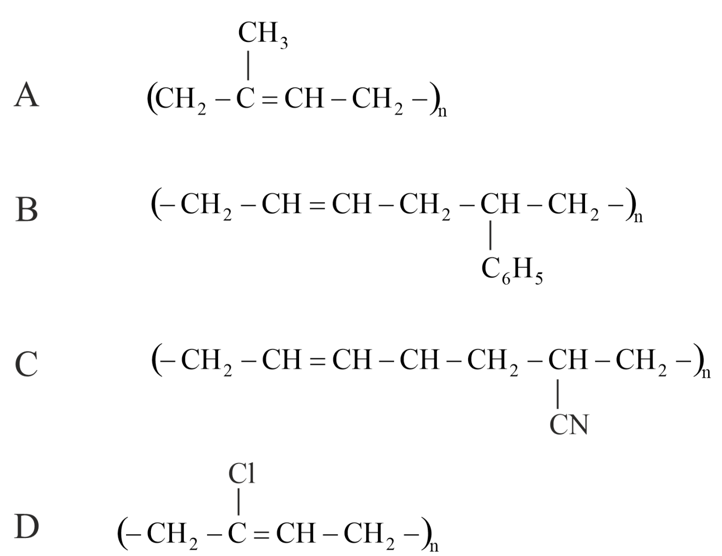
Q.13. Delicate balance of CO_2 and O_2 is not distrubed by:

A) Deforestation B) Photosynthesis C) Burning of coal D) Burning of petroleum

Answer: Photosynthesis

Solution: During photosynthesis, plants absorb CO_2 from the atmosphere and release oxygen as a byproduct. This process helps to regulate the concentration of CO_2 in the atmosphere and ensure that there is enough oxygen for organisms that require it. Deforestation, burning of coal and petroleum will increase the amount of carbondioxide so the correct option is photosynthesis.

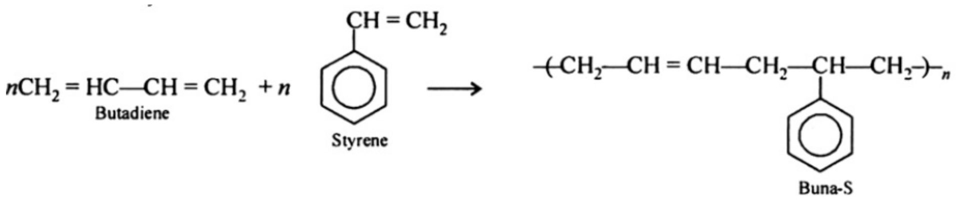
Q.14. Which of the following is correctly represents the structure of Buna-S :



A) A B) B C) C D) D

Answer: A

Solution: Buna-S is a copolymer of 1,3-butadiene and styrene. It is prepared by copolymerisation of 1,3 butadiene and styrene along with sodium. In this process peroxide is used as a catalyst at $5^{\circ}C$ therefore the formed product is also known as cold rubber.

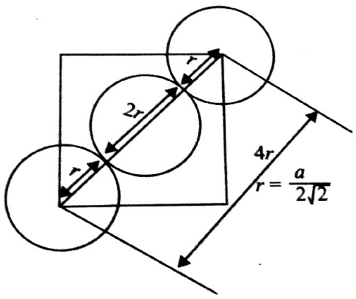


Q.15. The relationship between radius of lattice(r), edge length(a) of an FCC unit cell is :

A) $r = \frac{\sqrt{2}a}{4}$ B) $r = \frac{\sqrt{2}a}{2}$ C) $r = 2\sqrt{2}a$ D) $r = 4\sqrt{2}a$

Answer: $r = \frac{\sqrt{2}a}{4}$

Solution: In FCC unit cells, the particles are present at the corners and face centres.



The relationship between radius of lattice(r), edge length(a) of an FCC unit cell is $r = \frac{\sqrt{2}a}{4}$.

Q.16. The increasing order of metallic character is given as :

A) $Be > Ca > K$ B) $K > Ca > Be$ C) $Ca > K > Be$ D) $K > Be > Ca$

Answer: $K > Ca > Be$

Solution: Metallic character decreases along a period from left to right. Therefore, metallic character of K is greater than beryllium and calcium. Within a group, metallic character increases from top to bottom. Therefore, the metallic character of calcium is greater than beryllium.

The correct order is $K > Be > Ca$.

Q.17. During bleeding from a cut $FeCl_3$ is used to stop bleeding as:

A) Cl^{-} cause coagulation

B) Ferric ions cause coagulation

C) $FeCl_3$ dilute blood

D) Bleeding does not stop

Answer: Ferric ions cause coagulation

Solution: Blood being a colloidal solution, its coagulation by Hardy-Schulze's law states " Higher the charge on cation higher will be its efficiency to coagulate the colloidal solution". Fe^{3+} ions coagulate negatively charged blood solution. Negatively charged blood solution is coagulated byFe^{3+} ions.

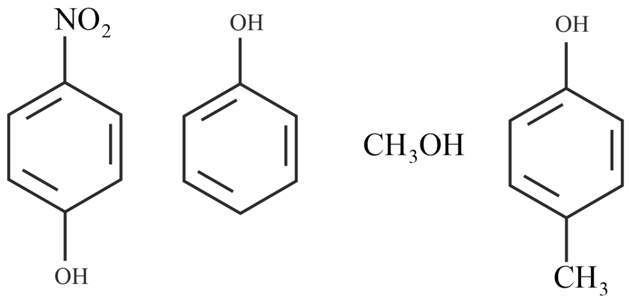
Q.18. What process is used to make soap from fat ?

A) Saponification B) Electrolysis C) Solvay process D) Haber process

Answer: Saponification

Solution: Saponification is the hydrolysis of an ester to form an alcohol and the salt of a carboxylic acid in acidic or essential conditions. Saponification is usually used to refer to the soap-forming reaction of a metallic alkali with fat or grease.

Q.19. The correct order of acidic strenght of the given compounds is:

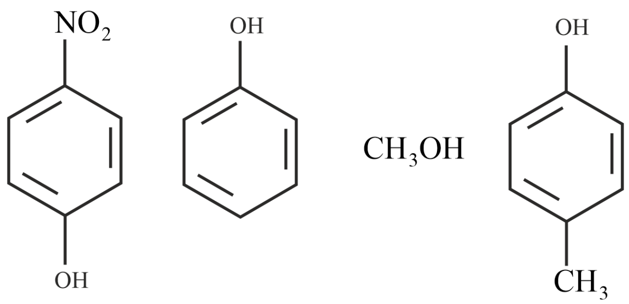


I II III IV

A) $\mathrm{I} > \mathrm{II} > \mathrm{IV} > \mathrm{III}$ B) $\mathrm{I} > \mathrm{III} > \mathrm{II} > \mathrm{IV}$ C) $\mathrm{II} > \mathrm{I} > \mathrm{IV} > \mathrm{III}$ D) $\mathrm{II} > \mathrm{III} > \mathrm{I} > \mathrm{IV}$

Answer: $\mathrm{I} > \mathrm{II} > \mathrm{IV} > \mathrm{III}$

Solution:



I II III IV

Due to the presence of nitro group along with OH group Structure I will have highest acidity. Phenol also has the highest acidity due to the presence of a hydroxyl group attached to an aromatic ring, which stabilises the negative charge that is formed when the proton is donated.

$\mathrm{p}-$cresol is a derivative of phenol with a methyl group in the para position. The electron-withdrawing effect of the methyl group reduces the electron density on the aromatic ring, making it slightly less acidic than phenol.

Methyl alcohol (methanol) has a low acidity due to the absence of any electron-withdrawing groups, which makes it difficult to donate a proton.

Q.20. Water of crystallization in soda ash and washing soda is respectively.

A) $0, 10$ B) $10, 0$ C) $0, 0$ D) $0, 1$

Answer: $0, 10$

Solution: Soda ash $(\mathrm{Na_2CO_3})$ and washing soda $(\mathrm{Na_2CO_3.10H_2O})$ both contain water of crystallization.

Soda ash does not contain any water of crystallization, as it is anhydrous.

Washing soda, on the other hand, contains 10 molecules of water of crystallization (i.e., 10 water molecules that are chemically bound to the solid salt). Its chemical formula is $\mathrm{Na_2CO_3.10H_2O}$.

Therefore, the water of crystallization in soda ash and washing soda are 0 and 10molecules, respectively.

Q.21. For a metal ion magnetic moment is calculated to be $4.9\ \mathrm{BM}$. Find the number of unpaired electron

Answer: 4

Solution: The expression for the magnetic moment is $\mu = \sqrt{\mathrm{n(n+2)}}\ \mathrm{B.M}$

Substitute values in the above expression.

$4.9 = \sqrt{\mathrm{n(n+2)}}$
$24.01 = \mathrm{n}\left(\mathrm{n}+2\right)$
$\mathrm{n} = 4$

Thus, The number of unpaired electrons is 4.

Q.22. How many electrons are gained by MnO_4^- in strongly alkaline medium.

Answer: 1

Solution: Potassium Permanganate in a strongly alkaline solution oxidises and takes up one of its electrons.

$MnO_4^- + e^- \rightarrow MnO_4^{-2}$

Change in oxidation number in the above reaction is one.
Therefore, potassium permanganate takes only one electron, thus it is a very much weaker oxidising agent in a basic medium.

Q.23. Find out the difference in oxidation state of Xe in completely hydrolysed form of XeF_4 and XeF_6.

Answer: 0

Solution: When XeF_6 is completely hydrolysed, XeO_3 (xenon trioxide) is generated.

$XeF_6 + 3H_2O \rightarrow XeO_3 + 6HF$

As xenon tetrafluoride combines with water, it produces xenon, oxygen, hydrofluoric acid, and a highly soluble xenon species. When the solution is evaporated, a white, crystal-line material known as xenon (VI) oxide, XeO_3, is formed .

$6XeF_4 + 12H_2O \rightarrow 4Xe + 2XeO_3 + 24HF + 3O_2$

In both the cases XeO_3 is formed and the oxidation state of Xe in XeO_3 is $+6$. So the difference $(+6) - (+6) = 0$.

Q.24. $NH_3, NO, N_2, F_2, CO, CO_2, H_2O$ & XeF_4

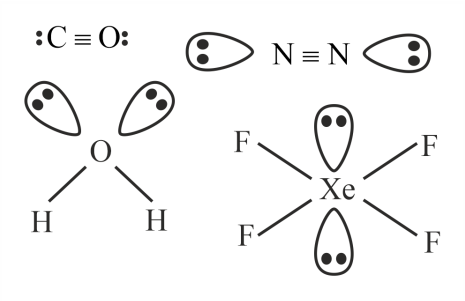
How many of above molecules are having only two lone pair of electrons.

Answer: 4

Solution: Among the given molecules four will have two lone pair of electrons.

N_2 has a triple bond between the two N atoms, it can be considered as having two lone pairs of electrons. In CO, carbon has two bonding pairs and two lone pairs of electrons, giving it a total of four electron pairs.

Water molecule and XeF_4 will also have two lone pairs.



Mathematics

Q.25. Let the circle $x^2 + y^2 = 16$ and line passing through $(1, 2)$ cuts the circle at A and B then the locus of the midpoint of AB is:

A) $x^2 + y^2 + x + y = 0$ B) $x^2 + y^2 - x + 2y = 0$ C) $x^2 + y^2 - x - 2y = 0$ D) $x^2 + y^2 + x + 2y = 0$

Answer: $x^2 + y^2 - x - 2y = 0$

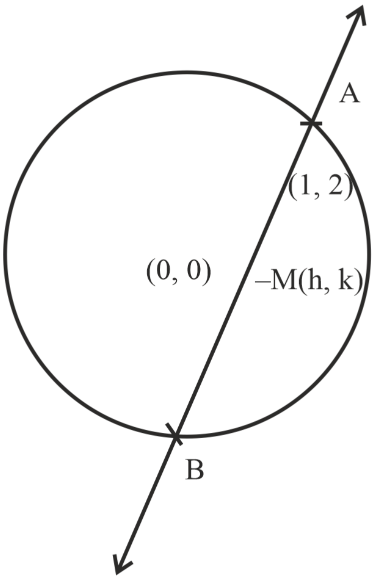
Solution: The given equation of circle is $x^2 + y^2 = 16$.

Let us check the position of the point $(1, 2)$ w.r.t circle.

$\Rightarrow S_1 = 1^2 + 2^2 - 16 < 0$.

The point lies inside the circle.

Also, the straight line cuts the circle at A and B with midpoint as $M(h, k)$.



Now Equation of AB passing through $M(h, k)$ is $T = S_1$.

$\Rightarrow h(x) + k(y) - 16 = h^2 + k^2 - 16$

But this line is also passing through $(1, 2)$.

$\Rightarrow h(1) + k(2) = h^2 + k^2$

Hence the required locus is $x^2 + y^2 - x - 2y = 0$

Q.26. Consider $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$. Domain of $f(x)$ is $[\alpha, \beta) \cup (\gamma, \delta]$, then the value of $|3\alpha + 10\beta + 5\gamma + 21\delta|$ is

A) 21

B) 25

C) 24

D) 32

Answer: 21

Solution: We know that,

Domain of $\sec^{-1} x$ is $(-\infty, -1] \cup [1, \infty)$.

Now, $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$

So,

$\frac{2x}{5x+3} \leq -1$ & $\frac{2x}{5x+3} \geq 1$

$\Rightarrow \frac{2x}{5x+3} + 1 \leq 0$ & $\frac{2x}{5x+3} - 1 \geq 0$

$\Rightarrow \frac{7x+3}{5x+3} \leq 0$ & $\frac{-3x-3}{5x+3} \geq 0$

$\Rightarrow \frac{7x+3}{5x+3} \leq 0$ & $\frac{x+1}{5x+3} \leq 0$

$\Rightarrow x \in \left[-1, -\frac{3}{5}\right) \cup \left(-\frac{3}{5}, -\frac{3}{7}\right]$

So,

$\alpha = -1,\ \beta = -\frac{3}{5},\ \gamma = -\frac{3}{5},\ \delta = -\frac{3}{7}$

Now,

$|3\alpha + 10\beta + 5\gamma + 21\delta|$
$= |-3 - 6 - 3 - 9|$
$= 21$

Q.27. In ΔABC, P is circumcentre and Q is orthocentre, then $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ is

A) $2\overrightarrow{PQ}$

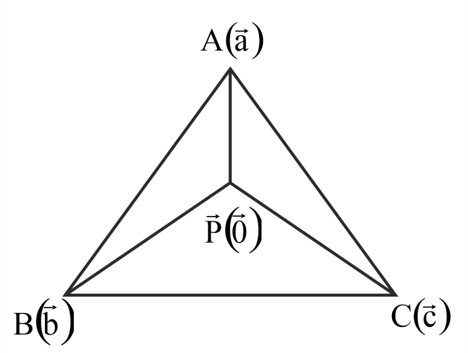
B) $\overrightarrow{PQ}$

C) $3\overrightarrow{PQ}$

D) $\frac{1}{2}\overrightarrow{PQ}$

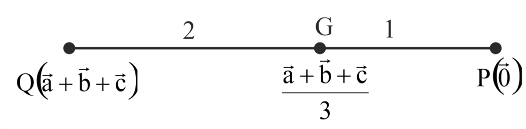
Answer: $\overrightarrow{PQ}$

Solution: Let P be origin.



Now from the above triangle we can write $\overrightarrow{PA} = \vec{a}$, $\overrightarrow{PA} = \vec{b}$, $\overrightarrow{PC} = \vec{c}$.

Also, we know that centroid G divides orthocentre Q and Circumcentre P in the ratio $2:1$.



Now $\overrightarrow{PG} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$

$\Rightarrow \overrightarrow{PQ} = 3\left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$

$\Rightarrow \overrightarrow{PQ} = \vec{a} + \vec{b} + \vec{c} = \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$.

Therefore, the required answer is $\overrightarrow{PQ}$.

Q.28. If $\frac{z+i}{4z+2i}$ is a purely real number and $z = x + iy\,(x, y \in R)$, then one of the possibility is

A) $x \neq 0,\ y \neq 1$

B) $x \neq 0,\ y = -1$

C) $x = 0,\ y \neq -\frac{1}{2}$

D) $x = 1,\ y \neq -\frac{1}{2}$

Answer: $x = 0,\ y \neq -\frac{1}{2}$

Solution: We have,

$Z = \frac{z+i}{4z+2i}$

$\Rightarrow Z = \frac{x+i(1+y)}{4x+i(4y+2)}$

$\Rightarrow Z = \frac{x+i(1+y)}{4x+i(4y+2)} \times \frac{4x-i(4y+2)}{4x-i(4y+2)}$

$\Rightarrow Z = \frac{4x^2+(4y+2)(1+y)+i[4x(1+y)-(4xy+2x)]}{16x^2+(4y+2)^2}$

Since, Z is purely real, so $\frac{4x(1+y)-(4xy+2x)}{16x^2+(4y+2)^2} = 0$

$\Rightarrow 4x + 4xy - (4xy + 2x) = 0$

$\Rightarrow 2x = 0$

$\Rightarrow x = 0$

And,

$4x + i\,(4y + 2) \neq 0$

$\Rightarrow i\,(4y + 2) \neq 0$

$\Rightarrow y \neq -\frac{1}{2}$

Q.29. If $\int_0^{t^2} \left(f(x) + x^2\right)dx = \frac{4}{3}t^3$, then $f(x)$ is

A) $2\sqrt{x} - x^2$ B) x^2 C) $x^2 + 2\sqrt{x}$ D) $-x^2$

Answer: $2\sqrt{x} - x^2$

Solution: Given:

$\int_0^{t^2} \left(f(x) + x^2\right)dx = \frac{4}{3}t^3$

Differentiating both sides w.r.t. t, we get

$\Rightarrow 2t\left[f\left(t^2\right) + t^4\right] = \frac{4}{3} \times 3t^2$

$\Rightarrow t\left[f\left(t^2\right) + t^4\right] = 2t^2$

$\Rightarrow f\left(t^2\right) + t^4 = 2t$

$\Rightarrow f\left(t^2\right) = 2t - t^4$

Put $t^2 = x$, then we get

$f(x) = 2\sqrt{x} - x^2$

Q.30. If $\int \left(\left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x}\right) \ln x\,dx = \alpha\left(\frac{x}{e}\right)^{2x} + \beta\left(\frac{e}{x}\right)^{2x} + c$ where c is constant of integration, then

A) $\alpha + \beta = 0$ B) $\alpha + \beta = 1$ C) $\alpha\beta = \frac{1}{2}$ D) $\alpha\beta = \frac{1}{4}$

Answer: $\alpha + \beta = 0$

Solution: Given,

$$\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\ln x dx=\alpha\left(\frac{x}{e}\right)^{2x}+\beta\left(\frac{e}{x}\right)^{2x}+c$$

Now let $I=\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\ln x dx$

Now let $\left(\frac{x}{e}\right)^{2x}=t$

$\Rightarrow 2x(\ln x-1)=\ln t$

$\Rightarrow \ln x dx=\frac{1}{2t}dt$

So, $I=\frac{1}{2}\int\left(t+\frac{1}{t}\right)\frac{dt}{t}$

$\Rightarrow I=\frac{1}{2}\int\left(1+\frac{1}{t^2}\right)dt$

$\Rightarrow I=\frac{1}{2}\left(t-\frac{1}{t}\right)+c$

$\Rightarrow I=\frac{1}{2}\left(\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{2x}\right)+c$

Now on comparing with $I=\alpha\left(\frac{x}{e}\right)^{2x}+\beta\left(\frac{e}{x}\right)^{2x}+c$, we get $\alpha=\frac{1}{2}$ & $\beta=\frac{-1}{2}$

Hence, $\alpha+\beta=0$

Q.31. A tangent drawn to ellipse $19x^2+15y^2=285$ is also a tangent to a circle. This circle is concentric with the conic and its radius is 4 units. The, find the angle made by the tangent with minor axis of ellipse.

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) π D) $\frac{\pi}{2}$

Answer: $\frac{\pi}{3}$

Solution: Given:

$19x^2+15y^2=285$

$\Rightarrow \frac{x^2}{15}+\frac{y^2}{19}=1 \quad ....(1)$

Let equation of tangent to ellipse be

$y=mx\pm\sqrt{15m^2+19} \quad ...(2)$

This is tangent to circle having centre $(0,0)$ and radius 4 units, so perpendicular distance from centre of the circle is equal to the radius of the circle.

$\left|\frac{\pm\sqrt{15m^2+19}}{\sqrt{1+m^2}}\right|=4$

$\Rightarrow \frac{15m^2+19}{1+m^2}=16$

$\Rightarrow 15m^2+19=16+16m^2$

$\Rightarrow m^2=3$

So, $\tan\theta=\sqrt{3}\Rightarrow\theta=\frac{\pi}{3}$

Q.32. The sum of all 4 digit numbers using the digits $1,2,2,3$ is

Answer: 26664

Solution: Total 4 digit numbers using the digits $1, 2, 2, 3$ is

$= \frac{4!}{2!} = 12$

Numbers are

$\_\ \_\ \_\ 3$
$\_\ \_\ \_\ 3$
$\_\ \_\ \_\ 3$
$\_\ \_\ \_\ 2$
$\_\ \_\ \_\ 2$
$\_\ \_\ \_\ 2$
$\_\ \_\ \_\ 2$
$\_\ \_\ \_\ 2$
$\_\ \_\ \_\ 2$
$\_\ \_\ \_\ 1$
$\_\ \_\ \_\ 1$
$\_\ \_\ \_\ 1$

Required sum is

$$= \underbrace{[3 \times 3 + 2 \times 6 + 1 \times 3]}_{\text{unit digit 's sum}} + \underbrace{[(3 \times 3 + 2 \times 6 + 1 \times 3) \times 10]}_{\text{ten 's place sum}} + \underbrace{[(3 \times 3 + 2 \times 6 + 1 \times 3) \times 100]}_{\text{hundred 's place sum}} + \underbrace{[(3 \times 3 + 2 \times 6 + 1 \times 3) \times 1000]}_{\text{thousand 's place sum}}$$

$= [24] + [24 \times 10] + [24 \times 100] + [24 \times 1000]$

$= 24 + 240 + 2400 + 24000 = 26664$

Q.33. If S_n be $4 + 11 + 21 + 34 + \ldots\ldots\ldots\ldots$, then find the value of $\frac{S_{29} - S_9}{60}$

Answer: 223

Solution: Given,

$S_n = 4 + 11 + 21 + 34 + \ldots\ldots\ldots\ldots + T_n$
$S_n = \quad 4 + 11 + 21 + 34 \ldots\ldots + T_{n-1} + T_n$

Subtracting above equations, we get

$0 = 4 + 7 + 10 + 13 + \ldots . - T_n$

$\Rightarrow T_n = 4 + 7 + 10 + 13 + \ldots$

The above series is in AP.

$\Rightarrow T_n = \frac{n}{2}(2 \times 4 + (n - 1)3)$

$\Rightarrow T_n = \frac{n}{2}(3n + 5)$

So,

$S_n = \sum \frac{3n^2 + 5n}{2}$

$\Rightarrow S_n = \frac{1}{2}\left(\frac{3n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}\right)$

$\Rightarrow S_n = \frac{n(n+1)}{2}\left(\frac{(2n+1)}{2} + \frac{5}{2}\right)$

$\Rightarrow S_{29} = \frac{29 \times 30}{2}\left(\frac{59}{2} + \frac{5}{2}\right)$

$\Rightarrow S_{29} = 29 \times 15 \times 32 = 13920$

$\Rightarrow S_9 = \frac{9 \times 10}{2}\left(\frac{19}{2} + \frac{5}{2}\right)$

$\Rightarrow S_9 = 9 \times 5 \times 12 = 540$

$\Rightarrow \frac{S_{29} - S_9}{60} = \frac{13920 - 540}{60} = 223.$

Therefore, the required answer is 223.

Q.34. If 8 persons have to travel from Point A to Point B in 3 allotted cars. If a car can carry maximum 3 persons then find the number of ways they can travel.

Answer: 1680

Solution: Let us find the different ways so that 8 persons can travel in 3 cars.

C_1	C_2	C_3
3	3	2
2	3	3
3	2	3

Hence we have 3 ways.

Now the number of ways to distribute 8 persons such that they can travel in 3 cars with any car carrying maximum of 3 persons is $\left(\frac{8!}{3!3!2!}\right) \times 3$

$= 1680$

Therefore, the required answer is 1680

Q.35. If $S = \left\{x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10\right\}$ and $\beta = \sum_{x \in S}\left(\frac{x}{3}\right)$, then the value of $\frac{1}{7}(\beta - 14)^2$ will be

Answer: 28

Solution: Given,

$9^{1-\tan^2 x} + 9^{\tan^2 x} = 10$

Now let $9^{\tan^2 x} = t$, then above equation will be,

$\frac{9}{t} + t = 10$

$\Rightarrow t^2 - 10t + 9 = 0$

$\Rightarrow t = 9 \text{ or } t = 1$

So, when $9^{\tan^2 x} = 9 \Rightarrow \tan^2 x = 1$

$\Rightarrow \tan x = \pm 1 \Rightarrow x = \pm\frac{\pi}{4}$, as given $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Now when $9^{\tan^2 x} = 1$

$\Rightarrow \tan^2 x = 0 \Rightarrow x = 0$

Hence, $\beta = \sum_{x \in S}\left(\frac{x}{3}\right) = \frac{0}{3} + \frac{\pi}{12} - \frac{\pi}{12} = 0$

So, the value of $\frac{1}{7}(\beta - 14)^2 = \frac{14^2}{7} = 28$

Q.36. The coefficient of x and x^2 in $(1+x)^p(1-x)^q$ are 4 and -5 then $2p + 3q$ is

Answer: 63

Solution: Given that The coefficient of x and x^2 in $(1+x)^p(1-x)^q$ are 4 and -5.

$\Rightarrow (1+x)^p(1-x)^q = \left(1+px+\frac{p(p-1)}{2!}x^2+\ldots\right)\left(1-qx+\frac{q(q-1)}{2!}x^2-\ldots\right)$

Now coefficient of x from the above expansion will be

$p-q$ which is equal to 4

$\Rightarrow p-q=4.$

Similarly coefficient of x^2 is -5.

$\Rightarrow \frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq=-5$

$\Rightarrow \frac{p^2-2pq+q^2}{2}-\frac{(p+q)}{2}=-5$

$\Rightarrow \frac{(p-q)^2}{2}-\frac{(p+q)}{2}=-5$

$\Rightarrow \frac{16}{2}+5=\frac{(p+q)}{2}$

$\Rightarrow p+q=26$ and $p-q=4$

On solving the above equations we get,

$p=15$ and $q=11$.

$\Rightarrow 2p+3q=2\,(15)+3\,(11)=63$

Therefore, the required answer is 63.

Q.37. Let α be the remainder when $(22)^{2022}+(2022)^{22}$ is divided by 3 and β be the remainder when the same is divided by 7, then $\alpha^2+\beta^2$ is

Answer: 5

Solution: Given that α be the remainder when $(22)^{2022}+(2022)^{22}$ is divided by 3 and β be the remainder when the same is divided by 7.

$\Rightarrow (22)^{2022}+(2022)^{22}=(21+1)^{2022}+(2022)^{22}.$

Here $(2022)^{22}$ is divisible by 3 as 2022 is divisible by 3.

So on expanding $(21+1)^{2022}$, we get

$\Rightarrow (21+1)^{2022}={}^{2022}C_0(21)^{2022}+{}^{2022}C_1(21)^{2021}+\ldots\ldots+{}^{2022}C_{2022}(1)^{2022}$

$=3\left(3^{2021}\times 7^{2022}+{}^{2022}C_1\times 3^{2020}\times 7^{2021}+\ldots\ldots\right)+1$

$=3k_1+1$

In this case the remainder is 1

Hence, $\alpha=1$

Now, $\Rightarrow (22)^{2022}+(2022)^{22}=(21+1)^{2022}+(2023-1)^{22}$

Take $(2023-1)^{22}$

$\Rightarrow (2023-1)^{22}={}^{22}C_0(2023)^{22}-{}^{22}C_1(2023)^{21}+\ldots\ldots+{}^{22}C_{22}(-1)^{22}$

$=7\left({}^{22}C_0(7)^{21}(289)^{22}-{}^{22}C_1(7)^{20}(289)^{21}+\ldots\ldots\right)+1$

$=7k_2+1$

$\Rightarrow (21+1)^{2022}+(2023-1)^{22}=7k_1+1+7k_2+1$

$=7\mu+2$

$\Rightarrow \beta=2.$

Hence, $\alpha^2+\beta^2=1^2+2^2=5.$

Therefore, the required answer is 5.