JEE Main 2023 (Session 2)

April 11 Shift 1

Physics

Q.1. Force acting on a particle moving along $x-$axis is given by $\vec{F} = \left(2 + 3x\right)\hat{\imath}$. The work done by this force from $x = 0$ to $x = 4\ \text{m}$ is

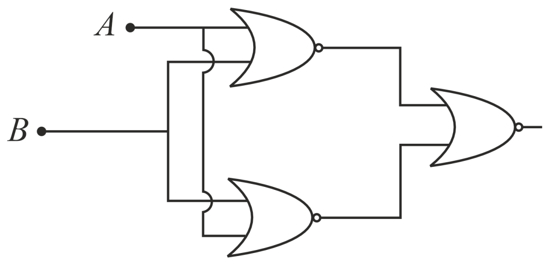
A) $16\ \text{J}$ B) $32\ \text{J}$ C) $4\ \text{J}$ D) $8\ \text{J}$

Answer: $32\ \text{J}$

Solution: We know that work done by a variable force is given by below formula,

$$W = \int_i^f F\,\text{d}x$$
$$= \int_0^4 \left(2 + 3x\right)\text{d}x$$
$$= \left[2x + \frac{3x^2}{2}\right]_0^4$$
$$= \left[8 + \left(3 \times 8\right)\right]$$
$$= 32\ \text{J}$$

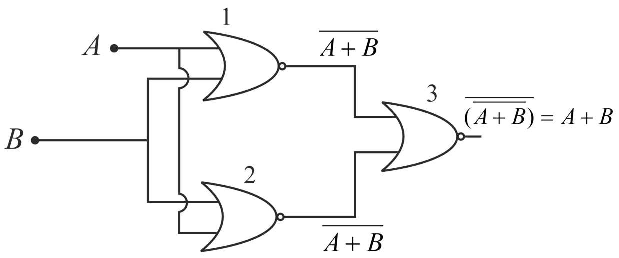
Q.2. Identify the logic operation of the following circuit:



A) AND B) OR C) NOR D) NAND

Answer: OR

Solution: The situation is depicted in the following figure:



The three gates given in the diagram represents NOR gates.

The output of the top NOR gate results $\overline{A + B}$ and the output of the bottom NOR gate also results $\overline{A + B}$.

These two outputs are used as the inputs of the third NOR gate. Hence, the result will be

$$\overline{\left(\overline{A + B}\right)} = A + B$$

Hence, the combination of the given logic gates behaves as an OR gate.

Q.3. If half life of a radio-active nuclide A is equal to average life of another radio-active nuclide B. Find the ratio of decay constant of A to that of B.

A) $\ln(2) : 1$ B) $1 : \ln(2)$ C) $2 : \ln(2)$ D) $\ln(2) : 2$

Answer: $\ln(2) : 1$

Solution: Half life of a radio-active nuclide A is given by,

$t_{1/2} = \frac{\ln(2)}{\lambda_A}$ ----(i)

Average life of radio-active nuclide B is given by,

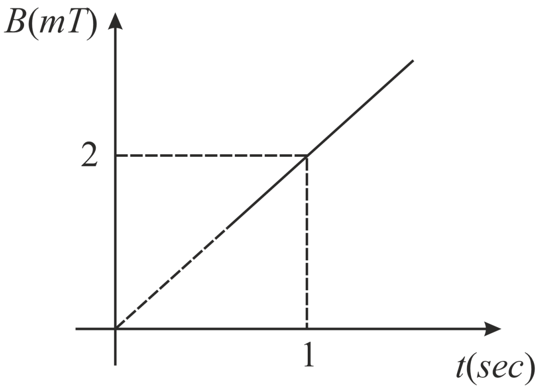
$t_{av} = \frac{1}{\lambda_B}$ ---(ii)

Given that (i) = (ii),

So, $\frac{\ln(2)}{\lambda_A} = \frac{1}{\lambda_B}$

$\Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{\ln(2)}{1}$

Q.4. Variation of magnetic field through a coil of area $4\ \mathrm{m}^2$ is shown in figure. What is the EMF induced in the coil (in mV)?



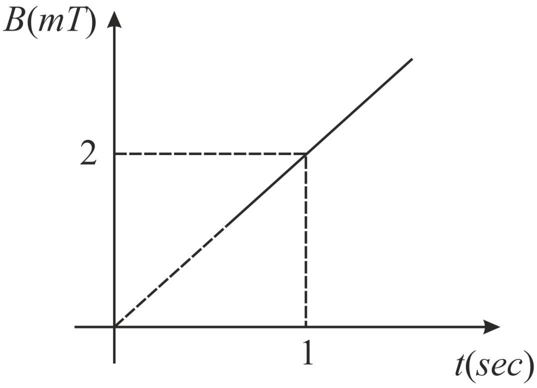
A) 8 B) 16 C) 4 D) 2

Answer: 8

Solution: The formula to calculate the induced emf (ε) is given by

$$\begin{aligned}\varepsilon &= \frac{d\varphi}{dt}\\ &= \frac{d}{dt}(BA)\\ &= A\frac{dB}{dt} \quad \ldots(1)\end{aligned}$$

The value of the rate of change of the magnetic field is the slope of the given curve between the magnetic field and time.



Substitute the values of the known parameters into equation (1) to calculate the required induced emf.

$$\begin{aligned}\varepsilon &= 4\ \mathrm{m}^2 \times \frac{2\ \mathrm{mT}}{1\ \mathrm{s}}\\ &= 8\ \mathrm{mV}\end{aligned}$$

Q.5. The characteristics of two coils are given below:

Coil A	**Coil B**
Radius $r_A = 10\ \text{cm}$	$r_B = 20\ \text{cm}$
Number of turns N_A	N_B
Current I_A	I_B

If the magnetic moments of both coils A and B are equal, choose the correct relation.

A) $2N_AI_A = N_BI_B$ B) $N_AI_A = N_BI_B$ C) $N_AI_A = 4N_BI_B$ D) $N_AI_A = 2N_BI_B$

Answer: $N_AI_A = 4N_BI_B$

Solution: The formula to calculate the magnetic moment (μ_A) of coil A is given by

$$\mu_A = N_AI_AA_a$$
$$= N_AI_A\left(\pi {r_A}^2\right)$$
$$= \pi\ N_AI_A{r_A}^2\ \ldots(1)$$

Similarly, the formula to calculate the magnetic moment (μ_B) is given by

$$\mu_B = \pi N_BI_B{r_B}^2\ \ldots(2)$$

From the given data, it can be concluded that

$$r_B = 2r_A\ \ \ldots(3)$$

Use equations (1), (2) and (3) to obtain the required relation.

$$\mu_A = \mu_B$$
$$\Rightarrow \pi N_AI_A{r_A}^2 = \pi N_BI_B{r_B}^2$$
$$= \pi N_BI_B(2r_A)^2$$
$$= 4\pi N_BI_B{r_A}^2$$
$$\Rightarrow N_AI_A = 4N_BI_B$$

Q.6. If light is passing through a medium of critical angle 45^o, then the wave speed will be

A) $\frac{3}{\sqrt{2}} \times 10^8\ \text{m s}^{-1}$ B) $3\sqrt{2} \times 10^8\ \text{m s}^{-1}$ C) $\frac{3}{2} \times 10^8\ \text{m s}^{-1}$ D) $3 \times 10^8\ \text{m s}^{-1}$

Answer: $\frac{3}{\sqrt{2}} \times 10^8\ \text{m s}^{-1}$

Solution: We know the relation, $\sin(c) = \frac{1}{\mu}$ ---(i)

where, c is critical angle of a medium of refractive index, μ

Given, $c = 45^o$

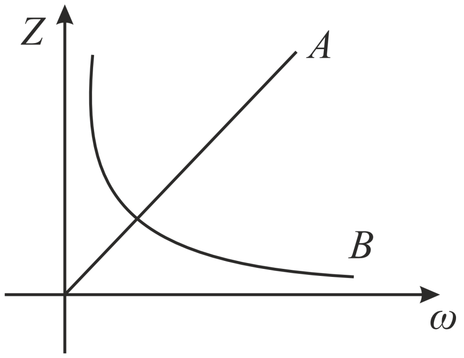
So, equation (i) becomes,

$$\frac{1}{\sqrt{2}} = \frac{1}{\mu}$$
$$\Rightarrow \mu = \sqrt{2}$$

But, we also know that, $\mu = \frac{c}{v}$

$$\Rightarrow v = \frac{c}{\mu}$$
$$\Rightarrow v = \frac{3\times10^8}{\sqrt{2}}\ \text{m s}^{-1}$$

Q.7. The variation of impedance (Z) with angular frequency (ω) for two electrical elements is shown in the graph given.



If X_L, X_C and R are inductive reactance, capacitive reactance and resistance respectively, then

A) A is resistor, B is inductor

B) A is inductor, B is capacitor

C) A is inductor, B is resistor

D) A is capacitor, B is inductor

Answer: A is inductor, B is capacitor

Solution: It is known that for an inductor, the reactance varies directly with the angular frequency, whereas for a capacitor, the reactance varies inversely with the angular frequency.

From the given variation of the reactance with the angular frequency, it can be concluded that A is inductor and B is capacitor.

Q.8. In a moving coil galvanometer, if number of turns increases by 25%, then change in voltage sensitivity is

A) 0

B) 1%

C) 25%

D) 50%

Answer: 0

Solution: The formula to calculate the voltage sensitivity of a moving coil galvanometer (v_s) is given by

$$v_s = \frac{NBA}{kR} \quad \ldots (1)$$

From the formula it can be written that

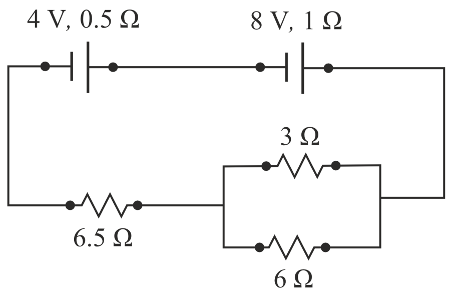
$$v_s \propto \frac{N}{R} \quad \ldots (2)$$

provided the other parameters remain constant for a galvanometer.

If the number of turns is increased, the length of the core has also to be increased accordingly. Also, the resistance of the core is directly proportional to its effective length. So, if the number of turns is increased, the increase of resistance will nullify the effect.

Hence, there will be no change in the voltage sensitivity of the galvanometer.

Q.9. Find the current flowing in $3\ \Omega$ resistor in the given circuit.



A) $0.4\ \mathrm{A}$

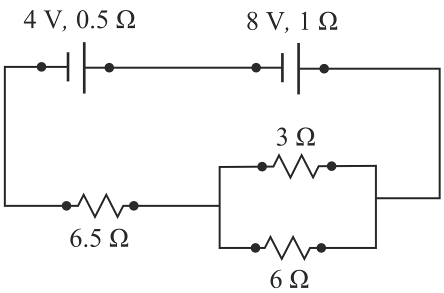
B) $0.2\ \mathrm{A}$

C) $0.8\ \mathrm{A}$

D) $0.6\ \mathrm{A}$

Answer: $0.8\ \mathrm{A}$

Solution:



The equivalent resistance (R) for $3\ \Omega$ and $6\ \Omega$ resistors, which are connected in parallel combination can be calculated as follows-

$$\frac{1}{R}=\frac{1}{3\ \Omega}+\frac{1}{6\ \Omega}$$
$$=\frac{1}{2\ \Omega}$$
$$\Rightarrow R=\ 2\ \Omega$$

The net resistance (R_n) of the entire circuit is, then, given by

$$R_n=2\ \Omega+6.5\ \Omega+0.5\ \Omega+1\ \Omega$$
$$=10\ \Omega$$

From the given figure, it is clear that both the sources of emf are connected in the same manner. So, the net voltage will be the sum of the two voltages.

Thus, the current (I) through the entire circuit can be calculated as follows-

$$I=\frac{4\ \text{V}+8\ \text{V}}{10\ \Omega}$$
$$=1.2\ \text{A}$$

So, the current (i_3) through the $3\ \Omega$ resistor is given by

$$i_3=1.2\ \text{A}\times\frac{6\ \Omega}{3\ \Omega+6\ \Omega}$$
$$=0.8\ \text{A}$$

Q.10. Two identical bulbs are first connected in series then in parallel. Find the ratio of power consumed in two cases.

A) $1:1$ B) $1:4$ C) $4:1$ D) $1:2$

Answer: $1:4$

Solution: Let a battery of voltage (ε) is connected to two identical bulbs first connected in series then in parallel.

Also, let the resistance of each bulb be r

Power consumed by any resistance R is given by

$$P=\frac{V^2}{R}$$

For series combination, $V=\varepsilon$ and $R=2r$

$$\therefore\ P_s=\frac{(\varepsilon)^2}{2r}=\frac{\varepsilon^2}{2r}$$

For parallel combination, $V=\varepsilon$ and $R=\frac{r}{2}$

$$\therefore\ P_p=\frac{\varepsilon^2}{\frac{r}{2}}=\frac{2\varepsilon^2}{r}$$

Hence, $\frac{P_s}{P_p}=\frac{\varepsilon^2}{2r}\times\frac{r}{2\varepsilon^2}=\frac{1}{4}$.

Q.11. Stopping potential of a metal when illuminated with light of wavelength λ is V_0 and that for wavelength 2λ is $\frac{V_0}{4}$. The threshold wavelength of metal is

A) λ B) 2λ C) 3λ D) 4λ

Answer: 3λ

Solution: Given,

Stopping potential is V_0 when the metal is illuminated with λ.

Stopping potential is $\frac{V_0}{4}$ when the metal is illuminated with 2λ.

In Photoelectric effect,

$eV_0 = h\nu - \phi$

$\Rightarrow eV_0 = \frac{hc}{\lambda} - \phi$-----(i)

Similarly,

$e\frac{V_0}{4} = \frac{hc}{2\lambda} - \phi$ ....(ii)

From (i) and (ii)

$\frac{hc}{\lambda} - \phi = 4\left(\frac{hc}{2\lambda} - \phi\right)$

$\Rightarrow \frac{hc}{\lambda} - \phi = \frac{2hc}{\lambda} - 4\phi$

$\Rightarrow 3\phi = \frac{hc}{\lambda}$

$\Rightarrow \phi = \frac{hc}{3\lambda}$

Therefore,

$\frac{hc}{\lambda max} = \frac{hc}{3\lambda} \Rightarrow \lambda max =$ threshold wavelength $= 3\lambda$

Q.12. Equation of progressive wave is $y = A\sin(160t - 0.5x)$. Let the speed of the wave be $10x$, then find x.

Answer: 32

Solution: Here, the direction of wave movement is along $+X$ axis.

The standard equation of a wave moving in $+X$ direction is given by,

$y = A\sin(\omega t - kx)$

where, the speed of wave is given by, $\text{speed} = \frac{\omega}{k}$

Upon substitution, we get

$\text{speed} = \frac{160}{0.5}$

$= 320 \text{ m s}^{-1}$

But given that the speed of the wave is $10x$.

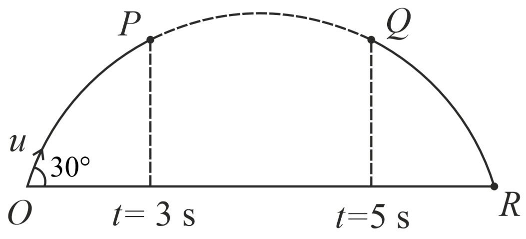
$\Rightarrow 10x = 320$

$\Rightarrow x = 32$

Q.13. A particle is projected at an angle of 30° and the horizontal height of the particle at 3 s and 5 s are the same. Find the speed of the projectile in m s^{-1}. $\left(g = 10 \text{ m s}^{-2}\right)$

Answer: 80

Solution: Let's consider the following diagram for the path followed by the projectile:



From the diagram, it is clear that the particle needs $3\ \text{s}$ to travel the distance OP and $5\ \text{s}$ to travel the distance OQ.

Since, projectile motion is a symmetric motion, from the above diagram, it can be concluded that the particle requires $3\ \text{s}$ to travel the distance QR.

Hence, the time of flight (T) of the projectile is given by

$$T = 3\ \text{s} + 5\ \text{s}$$
$$= 8\ \text{s}$$

The formula to calculate the time of flight of a projectile is given by

$$T = \frac{2u \sin\theta}{g} \quad \ldots(1)$$

Substitute the values of the known parameters into equation (1) and solve to calculate the required speed of the projectile.

$$8\ \text{s} = \frac{2u \sin 30^\circ}{10\ \text{m s}^{-2}}$$
$$\Rightarrow u = 80\ \text{m s}^{-1}$$

Chemistry

Q.14. Find the spin only magnetic momentum ratio for complexes $[Cr(CN)_6]^{-3}$ & $[Cr(H_2O)_6]^{+3}$.

A) $1:2$ B) $1:1$ C) $3:4$ D) $2:1$

Answer: $1:1$

Solution: The oxidation state of chromium in the complexes $[Cr(CN)_6]^{-3}$ & $[Cr(H_2O)_6]^{+3}$ is $+3$. The outer electronic configuration of Cr^{+3} is $3d^3$. Hence, the hybridisation of both the complexes is d^2sp^3 irrespective of ligand strength. The number of unpaired electrons are same. Therefore, the spin magnetic moment ratio is $1:1$

Q.15. In a container at a constant temperature, arrange the RMS velocoty order of the following

$Ne,\ Cl_2,\ UF_6$

A) $Ne > Cl_2 > UF_6$ B) $UF_6 > Cl_2 > Ne$ C) $Cl_2 > UF_6 > Ne$ D) $Ne > UF_6 > Cl_2$

Answer: $Ne > Cl_2 > UF_6$

Solution: We know that RMS velocity can be calculated by using the formula :

$$U_{RMS} = \sqrt{\frac{3RT}{M}}$$

Here, M is molecular weight

RMS velocity is inversely proportional to the molecular weight. The molecular weight order for the given molecules is $UF_6 > Cl_2 > Ne$.

Therefore, RMS velocity order is $Ne > Cl_2 > UF_6$

Q.16. Correct order of first Ionisation energy for Li, Be, C, B, N, O, F.

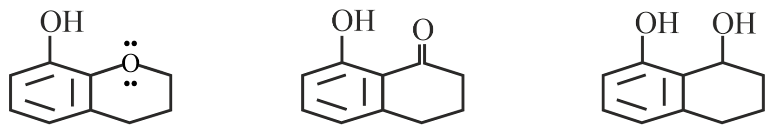
A) $F > N > O > C > Be > B > Li$ B) $F > B > C > O > N > Be > Li$

C) $O > N > F > C > B > Be > Li$ D) $B > N > O > C > F > Be > Li$

Answer: $F > N > O > C > Be > B > Li$

Solution: All the elements belong to the same period i.e second. When we are moving in a period the electrons to be added in the same shell but nuclear charge and number of electrons will differ.
So according to the above definition, across a period the Ionisation potential increases from left to right due to increase in net nuclear charge of elements. But there is an exception in the case of Be and B the ionisation potential of Be is higher than that of B. So the correct order is $\mathrm{F} > \mathrm{N} > \mathrm{O} > \mathrm{C} > \mathrm{Be} > \mathrm{B} > \mathrm{Li}$.

Q.17. Increasing order of the following for the electrophilic substitution reaction as:



(a) (b) (c)

A) $a > b > c$

B) $\mathrm{b > c > a}$

C) $\mathrm{b < c < a}$

D) $\mathrm{c < b < a}$

Answer: $\mathrm{b < c < a}$

Solution: In electrophilic aromatic substitution, an atom attached to an aromatic ring is substituted by an electrophile. In electrophilic aromatic substitution, benzene acts as a nucleophile. Thus, the benzene ring donates electrons from inside its ring.

When electron withdrawing groups are present on the benzene ring the ring is less reactive towards electrophilic substitution reaction. And when electron donating groups are present on the benzene ring the ring is more reactive towards electrophilic substitution reaction.

In compound (a) oxygen shows $+\mathrm{R}$ effect with benzene ring. In compound (b) carbonyl group acts as electron withdrawing group and compound (c) shows $-\mathrm{I}$-effect due to the presence of $-\mathrm{OH}$ group.

Therefore, the increasing order of electrophilic substitution reaction will be :

$\mathrm{b < c < a}$

Q.18. Match the following :

(a) $\mathrm{ClO_2}^-$	(p) Linear
(b) $\mathrm{N_3}^-$	(q) Tetrahedral
(c) $\mathrm{NH_4}^+$	(r) Bent
(d) $\mathrm{SF_4}$	(s) See-saw

A) a-s, b-p. c-q, d-r

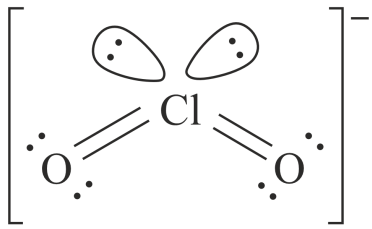
B) a-r, b-p, c-q, d-s

C) a-p, b-r, c-q, d-s

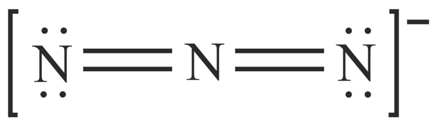
D) a-q, b-r, c-p, d-s

Answer: a-r, b-p, c-q, d-s

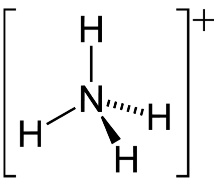
Solution: The molecular geometry of ClO_2^- is bent with two bond pairs and two lone pairs as shown below.



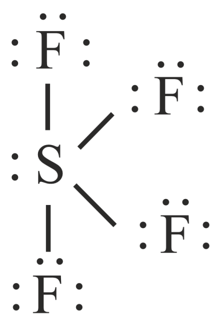
$N_3^-$is known as azide ion. It has a linear structure.



Ammonium ion has tetrahedral structure



and SF_4 has see-saw structure.



Q.19. Which of the following is not ambidentate ligand

A) $C_2O_4^{2-}$, H_2O B) $EDTA^{-4}$, NO_2^- C) NO_2^-, SCN^- D) SCN^-, CN^-

Answer: $C_2O_4^{2-}$, H_2O

Solution: The ambidentate ligands are those that can bond through different atoms or donor sites. Among the given options $C_2O_4^{2-}$ is a bidentate ligand and H_2O is a weak field ligand apart from this all the given options are ambidentate ligands.

Q.20. Which of the following can be represented as a meridional isomer ?

A) $[Pt(NH_3)_3Cl_3]^+$ B) $[Pt(en)_3]^{+4}$ C) $[Pt(en)_2Cl_2]^{+2}$ D) $[Pt(en)_2(NH_3)_2]^{+4}$

Answer: $[Pt(NH_3)_3Cl_3]^+$

Solution: The type of geometrical isomerism occurs in an octahedral complex of the type $[Ma_3b_3]$, in which three donor atoms of the same ligand occupy positions around the meridian of the octahedron, then it called a meridional isomer.

In the given molecules, $[Pt(NH_3)_3Cl_3]^+$ can show Facial and meridional isomerism as it is an octahedral complex with $[Ma_3b_3]$ type.

Q.21. Identify the correct statement about the compound $GaAlCl_4$.

A) Chlorine atom is bonded to both Ga and Al.

B) Ga is cationic part and less electronegative than Al

C) Chlorine atom forms co-ordinate bond with Ga.

D) Chlorine atom is bonded to Al.

Answer: Chlorine atom is bonded to Al.

Solution: The compound $GaAlCl_4$ is a complex compound and can be represented as $Ga[AlCl_4]$. The complex contain Ga^+ & $[AlCl_4]^-$ ions. In the anion all chlorine given in the molecular formula are connected to aluminium ion in the complex.

Q.22. Statement I: A water sample having B.O.D.= 4 ppm is of good quality.

Statement II: If concentration of Zn and NO_3^- each is 5 ppm, then water is of good quality.

A) Both statement I and statement II are corect

B) Statement I is incorrect and statement II is correct

C) Statement I is correct and stateent II is incorrect.

D) Neither statement I and nor statement II is correct.

Answer: Both statement I and statement II are corect

Solution: Clean water would have BOD value of less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more. A water sample having 4 ppm is of good quality. So the statement I is correct. Similarly, the concentration of Zn and NO_3^- in water is an important parameter to monitor in water quality assessment, as their elevated concentrations can indicate pollution. In second statement the concentration is 5 ppm which indicates good quality of water. The permissible concentration of zinc is 5ppm and nitrate ion is 50 ppm.

Q.23. Statement I: CH_4 and H_2O in presence of Ni catalyst produces H_2 gas.

Statement II: Sodium nitrite reacts with NH_4Cl gives H_2, N_2 and H_2O.

A) Both statement I and statement II are corect

B) Statement I is incorrect and statement II is correct

C) Statement I is correct and stateent II is incorrect.

D) Neither statement I and nor statement II is correct.

Answer: Statement I is correct and stateent II is incorrect.

Solution: Statement I is correct as methane reacts with water in presence of Nickel catalyst will produce hydrogen gas. The reaction is as follows:

$$CH_4(g) + H_2O(g) \xrightarrow[Ni]{1270K} CO(g) + H_2(g)$$

Statement II is incorrect as In the preperation of dinitrogen in laboratory, aqueous solution of ammonium chloride is treated with sodium nitrite which results in water and nitrogen and a salt. Here hydrogen is not formed. So statement II is wrong.

$$NH_4Cl(aq) + NaNO_2(aq) \rightarrow N_2(g) + 2H_2O(l) + NaCl(aq)$$

Q.24. To $25\,ml$ of $1.05M$ KI , $1\ M$ $AgNO_3$ is added drop wise. In the colloidal solution formed, fixed and diffused layer consists of respectively:($AgNO_3$ is in excess)

A) I^- and NO_3^-

B) Ag^+ and NO_3^-

C) Ag^+ and K^+

D) K^+ and Ag^+

Answer: Ag^+ and NO_3^-

Solution: The process involves the formation of a colloidal solution, where Ag^+ ions from $AgNO_3$ react with KI to form AgI, which precipitates as a colloidal solution due to its low solubility. The Ag^+ ions, being positively charged, will form the fixed layer around the colloidal particles. This layer provides stability to the colloidal particles by preventing them from coagulating or settling.

On the other hand, the NO_3^- ions, being negatively charged, will form the diffused layer (also known as the inner or counter-ion layer) around the colloidal particles. This layer helps to maintain the charge balance and stability of the colloidal particles by neutralizing the positive charge of Ag^+ ions in the fixed layer.

So, in the given colloidal solution, the fixed layer consists of Ag^+ ions, and the diffused layer consists of NO_3^- ions.

Q.25. 25% of $250g$ sugar solution and 40% of $500g$ sugar solution are mixed then find out the mass percentage in the solution.

Answer: 35

Solution: Let's calculate the total mass of sugar in the mixed solution:

Mass of 25% sugar solution = 250g

Mass of 40% sugar solution = 500g

Sugar content in 25% sugar solution = $25\% \text{ of } 250\text{g} = 0.25 \times 250\text{g} = 62.5\text{g}$

Sugar content in 40% sugar solution = $40\% \text{ of } 500\text{g} = 0.40 \times 500\text{g} = 200\text{g}$

Total mass of sugar in the mixed solution = Sugar content in 25% sugar solution + Sugar content in 40% sugar solution $= 62.5\text{g} + 200\text{g} = 262.5\text{g}$

Total mass of the mixed solution = Mass of 25% sugar solution + Mass of 40% sugar solution = $250\text{g} + 500\text{g} = 750\text{g}$

Now, let's calculate the mass percentage of sugar in the mixed solution:

Mass percentage of sugar in the mixed solution = $\frac{\text{Total mass of sugar in the mixed solution}}{\text{Total mass of the mixed solution}} \times 100$

$= \frac{262.5\text{g}}{750\text{g}} \times 100 = 35\%$

So, the mass percentage of sugar in the mixed solution is 35%.

Q.26. Find the number of atoms per unit cell if edge length is 300 pm, density$= 3 \text{ g/cm}^3$, molecular mass $= 40 \text{ g}$ (nearest integer)

Answer: 1

Solution: Thee the number of atoms per unit cell can be calculated from the following equation :

$$\text{d} = \frac{\text{Z} \times \text{M}}{\text{N}_\text{A} \times \text{a}^3}$$

Given, $\text{d} = 3 \text{ g/cm}^3$, $\text{M} = 40 \text{ g}$,

$\text{a} = 408 \text{ pm} = 408 \times 10^{-8} \text{ cm}$

$$\text{Z} = \frac{6 \times 10^{23} \times \left(300 \times 10^{-8}\right)^3 \times 3}{40}$$

$\text{Z} = 1.2$

Therefore, nearest integer is 1.

Mathematics

Q.27. If $x + y + z = 17$ and x, y, z are non-negative integers, then find the number of integral solutions.

A) 136 B) 171 C) 90 D) 130

Answer: 171

Solution: The given equation is $x + y + z = 17$ where x, y, z are non-negative integers.

The number of non-negative integral solutions of the equation $x_1 + x_2 + x_3 + \ldots + x_r = n$ is ${}^{n+r-1}C_{r-1}$.

In the given equation $n = 17$, $r = 3$

Required number of non-negative integral solutions is $= {}^{17+3-1}C_{3-1} = {}^{19}C_2 = 171$

Hence, the required answer is 171.

Q.28. Let $M = \left[a_{ij}\right]_{2\times2}$, $0 \le i,\ j \le 2$ where $a_{ij} \in \{0, 1, 2\}$ and A be the event such that M is invertible then $P(A)$ is

A) $\frac{49}{81}$ B) $\frac{16}{27}$ C) $\frac{47}{81}$ D) $\frac{46}{81}$

Answer: $\frac{16}{27}$

Solution: Given that M is invertible means M is a non-singular matrix.

That means $|M| \neq 0$.

Also, given that $M = \left[a_{ij}\right]_{2\times2}$, $0 \leq i,\ j \leq 2$

Let A be the event that M is invertible.

$\Rightarrow P(A) = \frac{n(A)}{n(S)}$

$n(S)$ is the total number of cases.

$\Rightarrow n(S) = 3 \times 3 \times 3 \times 3 = 81$ (Each entry of the matrix can be $\{0, 1, 2\}$).

For the favourable cases, let us find the number of matrices for which the determinant is non zero.

$\Rightarrow M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow |M| = ad - bc \neq 0$

$\Rightarrow ad \neq bc$

Let us assume that $a = 0$ then d can take values $0, 1, 2$ and b and c can take $1, 2$ such that $ad \neq bc$.

Number of cases with $a = 0,\ ad \neq bc$ will be $2 \times 2 \times 3 = 12$ cases.

Similarly b, c, d can be zero.

So Total number of cases such that $ad \neq bc$ will be $n(A) = 12 \times 4 = 48$.

Hence, $P(A) = \frac{48}{81} = \frac{16}{27}$.

Q.29. Consider the function $f(x) = \left[x^2 - x\right] + \{x\}$, where, $[\cdot] \rightarrow \text{G.I.F}$ and $\{\cdot\}$ is fractional part function.

A) $f(x)$ is continuous at $x = 0, 1$

B) $f(x)$ is continuous and differentiable at $x = 0, 1$

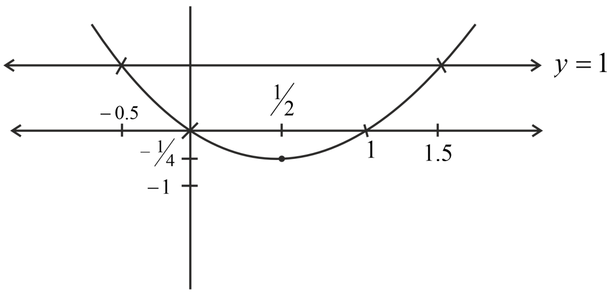
C) $f(x)$ is continuous but not differentiable at $x = 0, 1$

D) $f(x)$ is continuous at $x = 1$ but discontinuous at $x = 0$

Answer: $f(x)$ is continuous at $x = 1$ but discontinuous at $x = 0$

Solution: Given:

$f(x)=\left[x^2-x\right]+\{x\}$



So, we have

$$f(x)=\begin{cases} x+1; & -0.5<x<0 \\ 0; & x=0 \\ x-1; & 0<x<1 \\ x-1; & 1\le x<1.5 \end{cases}$$

Since,

$\left[f\left(0^-\right)=1\right]\neq\left[f\left(0^+\right)=-1\right]\neq\left[f(0)=0\right]$

So, $f(x)$ is discontinuous at $x=0$, hence non-differentiable at $x=1$.

Since, $\left[f\left(1^-\right)=0\right]=\left[f\left(1^+\right)=0\right]=\left[f(1)=0\right]$, hence $f(x)$ is continuous at $x=1$.

Q.30. Two complex numbers w_1 & w_2, given by $w_1=3+5i$ and $w_2=5+4i$ are both rotated by 90° with respect to origin anticlock-wise and clockwise respectively to get the new complex numbers w_3 & w_4. Then principal argument of w_3-w_4 is

A) $-\pi-\tan^{-1}\frac{8}{9}$ B) $-\pi-\tan^{-1}\frac{33}{5}$ C) $\pi-\tan^{-1}\frac{8}{9}$ D) $\pi-\tan^{-1}\frac{33}{5}$

Answer: $\pi-\tan^{-1}\frac{8}{9}$

Solution: Given,

Two complex numbers w_1 & w_2

$w_1=3+5i$ and $w_2=5+4i$ are both rotated by 90° with respect to origin anticlock-wise and clockwise respectively,

So by concept of rotation we get,

$w_3=iw_1=i(3+5i)=-5+3i$

And $w_4=-iw_2=-i(5+4i)=4-5i$

Now principal argument of $w_3-w_4=-9+8i$ will be $\pi-\tan^{-1}\frac{8}{9}$ {as complex number is in second quadrant so principal argument is given by $\pi-\tan^{-1}\left|\frac{y}{x}\right|$}

Q.31. Let a and b are roots of $x^2-7x-1=0$, then the value of $\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}$ is

A) 29 B) 49 C) 53 D) 51

Answer: 51

Solution: Since, a and b are roots of $x^2-7x-1=0$, so

$a^2-7a-1=0$

$\Rightarrow a^2-1=7a$

$\Rightarrow a^4+1-2a^2=49a^2$

$\Rightarrow a^4+1=51a^2 \quad \ldots(1)$

Now,

$\frac{a^{21}+b^{21}+a^{17}+b^{17}}{a^{19}+b^{19}}$

$=\frac{a^{17}\left(a^4+1\right)+b^{17}\left(b^4+1\right)}{a^{19}+b^{19}}$

$=\frac{51a^{19}+51b^{19}}{a^{19}+b^{19}}$

$=51$

Q.32. Find the number of solutions of $\cos^4\theta-2\cos^2\theta+\sin^2\theta+1=0$ where $\theta\in[0,2\pi]$.

A) 1 B) 2 C) 3 D) 4

Answer: 3

Solution: Given that $\cos^4\theta-2\cos^2\theta+\sin^2\theta+1=0$

$\Rightarrow\left(1-2\cos^2\theta+\cos^4\theta\right)+\sin^2\theta=0$

$\Rightarrow\left(1-\cos^2\theta\right)^2+\sin^2\theta=0$

$\Rightarrow\left(\sin^2\theta\right)^2+\sin^2\theta=0$

$\Rightarrow\sin^2\theta\left(\sin^2\theta+1\right)=0$

$\Rightarrow\sin^2\theta=0$

$\Rightarrow\theta=0,\pi,2\pi$. for $\theta\in[0,2\pi]$

Therefore, the number of solutions are 3.

Q.33. A rectangle is drawn by lines $x=0,\ x=2,\ y=0\ \&\ y=5$. Point A and B lie on coordinate axes. If the line AB divides the area of rectangle in $4:1$, then the locus of the midpoint of AB is :

A) Circle B) Hyperbola C) Ellipse D) Straight line

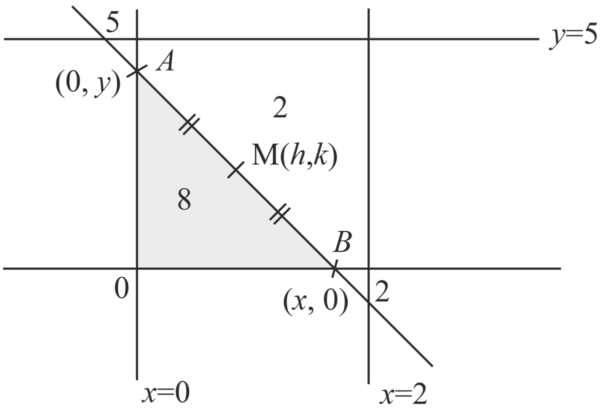
Answer: Hyperbola

Solution: Given,

A rectangle is drawn by lines $x=0,\ x=2,\ y=0$ & $y=5$.

Point A and B lie on coordinate axes. If the line AB divides the area of rectangle in $4:1$

Now plotting the diagram of the above data we get,



Now let midpoint be $M(h,k)$ of the line AB,

So coordinate of $A(0,y)\equiv(0,2k)$ and $B(x,0)\equiv B(2h,0)$

Now area of the rectangle is $5\times2=10$, so area of triangle OAB will be 8 as line divides the area in $4:1$,

So, area of triangle $OAB=\frac{1}{2}\times2h\times2k=8$

$\Rightarrow hk=4$

$\Rightarrow xy=4$ which is the equation of hyperbola

Q.34. Find the area bounded by $\begin{cases}x^2+(y-2)^2\le4\\ y^2\le2x\end{cases}$

A) $\pi+\frac{4}{3}$

B) $\pi-\frac{4}{3}$

C) $2\pi+\frac{8}{3}$

D) $2\pi-\frac{8}{3}$

Answer: $\pi-\frac{4}{3}$

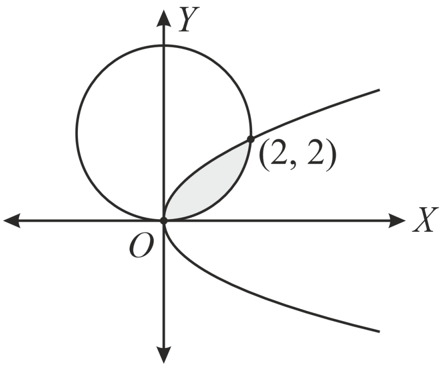
Solution: Given curves are

$x^2+(y-2)^2 \leq 2^2$. This is a circle with centre $C(0,2)$ and radius $r=2$.

$y^2 \leq 2x$ this is a parabola.

The point of intersection of these two curves will be $(0,0)$ and $(2,2)$.

Hence the required diagram will be



Let us integrate the area w.r.t $y-$axis.

Required area $=\int_0^2\left[\left(\sqrt{4-(y-2)^2}\right)-\frac{y^2}{2}\right]\mathrm{d}y$

We know that $\int\sqrt{a^2-x^2}dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

$=\left[\frac{y}{2}\sqrt{4-(y-2)^2}+\frac{4}{2}\sin^{-1}\left(\frac{y-2}{2}\right)-\frac{y^3}{6}\right]_0^2$

$=-\frac{8}{6}-\left(2\times\left(\frac{-\pi}{2}\right)\right)$

$=\pi-\frac{4}{3}$

Therefore the required area is $\pi-\frac{4}{3}$.

Q.35. Consider the plane $2x+y-3z=6$. If (α,β,γ) is the image point of $(2,3,5)$ in the given plane, then $\alpha+\beta+\gamma$ is

Answer: 10

Solution: Given plane is $2x+y-3z=6$.

Since, (α,β,γ) is the image point of $(2,3,5)$ in the given plane, so

$\frac{\alpha-2}{2}=\frac{\beta-3}{1}=\frac{\gamma-5}{-3}=-\frac{2(4+3-15-6)}{4+1+9}$

$\Rightarrow\frac{\alpha-2}{2}=\frac{\beta-3}{1}=\frac{\gamma-5}{-3}=2$

$\Rightarrow\frac{\alpha-2}{2}=2;\ \frac{\beta-3}{1}=2;\ \frac{\gamma-5}{-3}=2$

$\therefore\alpha=6,\ \beta=5,\ \gamma=-1$

Hence, $\alpha+\beta+\gamma=6+5-1=10$

Q.36. If solution of differential equation $\left(1-x^2y^2\right)dx=xdy+ydx$ is $y(x)$ and $y(2)=4$, then $\frac{5y(5)+1}{5y(5)-1}$ is

A) $\frac{9}{7}e^6$ B) $\frac{81}{49}e^2$ C) e^2 D) $\frac{9}{7}e^2$

Answer: $\frac{9}{7}e^6$

Solution: Given:

$\left(1-x^2y^2\right)dx = xdy + ydx$

$\Rightarrow dx = \frac{d(xy)}{1-(xy)^2}$

$\Rightarrow \int dx = \int \frac{d(xy)}{1-(xy)^2}$

$\Rightarrow x = \frac{1}{2}\log\left|\frac{1+xy}{1-xy}\right| + C$

$\Rightarrow 2x = \log\left|\frac{1+xy}{1-xy}\right| + C_1$

Since, $y(2) = 4$, so

$4 = \log\left|\frac{1+8}{1-8}\right| + C_1 \Rightarrow C_1 = 4 - \log\left(\frac{9}{7}\right)$

So,

$2x = \log\left|\frac{1-xy}{1+xy}\right| + 4 - \log\left(\frac{9}{7}\right)$

Put $x = 5$ we get,

$10 = \log\left|\frac{1+5y(5)}{1-5y(5)}\right| + 4 - \log\left(\frac{9}{7}\right)$

$\Rightarrow \log\left|\frac{1+5y(5)}{1-5y(5)}\right| = 6 + \log\left(\frac{9}{7}\right)$

$\Rightarrow \left|\frac{1+5y(5)}{1-5y(5)}\right| = e^{\left(6+\log\left(\frac{9}{7}\right)\right)}$

$\Rightarrow \frac{5y(5)+1}{5y(5)-1} = \frac{9}{7}e^6$

Q.37. 5 boys with allotted roll numbers and seat numbers are seated in such a way that no one sits on the allotted seats. The number of ways of such seating arrangement is

Answer: 44

Solution: 5 boys with allotted roll numbers and seat numbers are seated in such a way that no one sits on the allotted seats. The number of ways of such seating arrangement is equal to the derangement of 5 boys i.e.,

$= 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$

$= 5! - 5! + 60 - 20 + 5 - 1$

$= 44$

Q.38. The number of rational terms in the expansion of $\left(3^{\frac{3}{4}} + 5^{\frac{3}{2}}\right)^{60}$ are

Answer: 16

Solution: Given expansion is $\left(3^{\frac{3}{4}}+5^{\frac{3}{2}}\right)^{60}$.

The general term in the expansion $(x+a)^n$ is $T_{r+1}={}^nC_r\cdot x^{n-r}\cdot a^r$

$\Rightarrow T_{r+1}={}^{60}C_r\left(3^{\frac{3}{4}}\right)^{60-r}\left(5^{\frac{3}{2}}\right)^r,\ 0\le r\le 60$

$={}^{60}C_r(3)^{\frac{3(60-r)}{4}}(5)^{\frac{3r}{2}}$

For $(3)^{\frac{3(60-r)}{4}}(5)^{\frac{3r}{2}}$ to be rational r should be a multiple of 4.

$\Rightarrow r=0,4,8,12,.....60$

$\Rightarrow a_n=a+(n-1)d$

$\Rightarrow 60=0+(n-1)4$

$\Rightarrow n=16$

That means r can take 16 values.

Hence, the required answer is 16.

Q.39. The mean of coefficients of $x,\ x^2,\ x^3,\ ....,\ x^7$ in the binomial expansion of $(2+x)^9$ is,

Answer: 2736

Solution: We know that.

Binomial expansion of $(2+x)^9$ is given by,

$(2+x)^9={}^9C_0x^02^9+{}^9C_1x^12^8+..........{}^9C_9x^92^0$

Now put $x=1$ we get,

$(2+1)^9={}^9C_02^9+{}^9C_12^8++{}^9C_22^7+..........{}^9C_92^0$

$\Rightarrow 3^9={}^9C_02^9+{}^9C_12^8++{}^9C_22^7+..........{}^9C_92^0$

So, sum of coefficient of $x,\ x^2,\ x^3,\ ....,\ x^7$ will be,

$\Rightarrow {}^9C_12^8+{}^9C_22^7+..........{}^9C_72^2=3^9-{}^9C_02^9-{}^9C_82^1-{}^9C_92^0$

$\Rightarrow {}^9C_12^8+{}^9C_22^7+..........{}^9C_72^2=3^9-2^9-18-1$

$\Rightarrow {}^9C_12^8+{}^9C_22^7+..........{}^9C_72^2=19683-572-19$

$\Rightarrow {}^9C_12^8+{}^9C_22^7+..........{}^9C_72^2=19152$

Now mean is given by $\frac{19152}{7}=2736$