JEE Main 2023 (Session 2)

April 13 Shift 1

Physics

Q.1. If the height of a tower used for LOS communication is increased by 21%, then percentage change in range is

A) 10% B) 21% C) 19% D) 42%

Answer: 10%

Solution: Range of Line of sight (LOS) communication is given by formula,

$\text{Range, } R=\sqrt{2h_T R_e}$

where, h_T is height of the tower

and R_e is radius of the earth

Let R' be the final range of LOS after 21% increase in the height of the tower.

Then, the final height of tower will be $1.21h_T$.

So, $\text{Final range, } R' = \sqrt{2(1.21h_T)R_e}$

$\Rightarrow R' = 1.1R$

Percentage change in R is given by $\frac{\Delta R}{R}\times 100=\frac{R'-R}{R}\times 100=\frac{0.1R}{R}\times 100=10.$

Therefore, the $\%$ change in range is 10%.

Q.2. A dipole of charge $0.01\ \mathrm{C}$ and separation $0.4\ \mathrm{mm}$, is placed in an electric field of strength $10\ \mathrm{dyn\ C^{-1}}$. Find the maximum torque exerted on the dipole in the field

A) $4\times 10^{-9}\ \mathrm{N\ m}$ B) $2\times 10^{-10}\ \mathrm{N\ m}$ C) $4\times 10^{-10}\ \mathrm{N\ m}$ D) $2\times 10^{-9}\ \mathrm{N\ m}$

Answer: $4\times 10^{-10}\ \mathrm{N\ m}$

Solution: The formula to calculate the maximum torque on the dipole is given by

$\tau = qlE \quad \ldots(1)$

Substitute the values of the known parameters into equation (1) to calculate the maximum torque on the dipole.

$\tau=0.01\ \mathrm{C}\times 0.4\ \mathrm{mm}\times\frac{1\ \mathrm{m}}{1000\ \mathrm{mm}}\times 10\ \mathrm{dyn\ C^{-1}}\times\frac{1\ \mathrm{N}}{10^5\ \mathrm{dyn}}$

$=4\times 10^{-10}\ \mathrm{N\ m}$

Q.3. Two bodies having same linear momentum have ratio of kinetic energy as $16:9$. Find the ratio of masses of these bodies.

A) $\frac{9}{16}$ B) $\frac{4}{3}$ C) $\frac{3}{4}$ D) $\frac{16}{9}$

Answer: $\frac{9}{16}$

Solution: We know that kinetic energy (KE) of an object of mass (m) is related to its linear momentum (p) as, $KE=\frac{p^2}{2m}$

Given that two bodies have same linear momentum.

So, $m\propto\frac{1}{KE}$

Therefore, the ratio of masses of these two bodies is $\frac{9}{16}$.

Q.4. What is centre of gravity of semi-circular disc of radius (R)?

A) $\frac{2R}{\pi}$ B) $\frac{4R}{3\pi}$ C) $\frac{R}{2}$ D) $\frac{3R}{8}$

Answer: $\frac{4R}{3\pi}$

Solution: $\sigma = mass\ per\ unit\ area$

Consider an elemental semi-circular ring of radius r.

Total mass $\left(m\right) = \int \sigma dA = \int_0^R \sigma\left(\pi r dr\right) = \left[\frac{\pi\sigma}{2}\left(R^2\right)\right]$

Mass of ring of radius (r) and thickness (dr)

is $dm = [\sigma \times \pi r dr]$ and its centre of mass is at $\left(\frac{2r}{\pi}\right)$

$$Ycm = \frac{1}{m}\int ydm = \frac{1}{m}\int_0^r \frac{2r}{\pi} \times \left(\sigma\pi r dr\right) = \frac{2\sigma}{m}\int_0^r r^2 dr$$

$$\Rightarrow Ycm = \frac{2\sigma}{m}\left[\frac{r^3}{3}\right]_0^R = \frac{2\sigma\left(R^3\right)}{3\left[\frac{\pi\sigma}{2}\left(R^2\right)\right]}$$

$$\Rightarrow Ycm = \frac{4R}{3\pi}$$

Q.5. Pressure for polytropic process P varies with volume V as $P = aV^{-3}$, Find out the bulk modulus.

A) $3V$ B) $3P$ C) P D) V

Answer: $3P$

Solution: The formula to calculate the bulk modulus is given by

$$B = -\frac{dP}{\frac{dV}{V}}$$
$$= -V\frac{dP}{dV} \quad \ldots(1)$$

Given that the pressure varies with volume as

$$P = aV^{-3} \quad \ldots(2)$$

Substitute the expression from equation (2) into equation (1) and simplify to obtain the bulk modulus.

$$B = -V\frac{d}{dV}\left(aV^{-3}\right)$$
$$= -aV\left(-3V^{-4}\right)$$
$$= 3aV^{-3}$$
$$= 3P$$

Q.6. The work function for two metals are $9\ \text{eV}$ and $4.5\ \text{eV}$. Find the approximate difference between their threshold wavelength. (use $hc = 1240\ \text{eV} - \text{nm}$)

A) $138\ \text{nm}$ B) $130\ \text{nm}$ C) $112\ \text{nm}$ D) $145\ \text{nm}$

Answer: $138\ \text{nm}$

Solution: The threshold wavelength and the work function of a metal are related by the formula

$$\lambda_{th} = \frac{hc}{\varphi} \quad \ldots(1)$$

Use equation (1) to obtain the expression for the difference in the threshold wavelengths of the two metals:

$$\lambda_{th}^{(2)} - \lambda_{th}^{(1)} = \frac{hc}{\varphi_2} - \frac{hc}{\varphi_1}$$
$$= hc\left(\frac{1}{\varphi_2} - \frac{1}{\varphi_1}\right) \quad \ldots(2)$$

Substitute the values of the known parameters into equation (2) to calculate the required difference in the threshold wavelengths.

$$\lambda_{th}^{(2)} - \lambda_{th}^{(1)} = 1240\ \text{eV} - \text{nm} \times \left(\frac{1}{4.5\ \text{eV}} - \frac{1}{9\ \text{eV}}\right)$$
$$\approx 138\ \text{nm}$$

Q.7. For the given radioactive decay ${}^{298}_{94}X \rightarrow {}^{294}_{92}Y + {}^{4}_{2}\alpha + Q - \text{value}$, binding energy per nucleon of X, Y and α are $a,\ b$ and c. The $Q - \text{value}$ is equal to

A) $294b + 4c - 298a$ B) $92b + 2c - 94a$ C) $294b + 4c + 298a$ D) $92b + 2c + 94a$

Answer: $294b + 4c - 298a$

Solution: The "$Q - \text{value}$" of the decay, is the difference of the mass of the parent and the combined mass of the daughter and the α -particle, multiplied by c^2.

It is also equal to the difference between the sum of the binding energies of the daughter and the α-particles and that of the parent nucleus.

Therefore, $Q - \text{value} = 294b + 4c - 298a$

Q.8. The energy of He^{+} in 2nd orbit is $-13.6\ \text{eV}$ then energy of Be^{+++} in $n = 4$ is

A) $-3.4\ \text{eV}$ B) $-27.2\ \text{eV}$ C) $-13.6\ \text{eV}$ D) $-54.4\ \text{eV}$

Answer: $-13.6\ \text{eV}$

Solution: Energy of an electron in n^{th} orbit can be written as

$$E_n = -13.6\frac{Z^2}{n^2}\ \text{eV} \quad \ldots (1)$$

Substitute the values of the known parameters for Beryllium ion into equation (1) to calculate the required energy of the ion.

$$E_4 = -13.6 \times \frac{4^2}{4^2}\ \text{eV}$$
$$= -13.6\ \text{eV}$$

Q.9. Which of the following shows time varying magnetic field?

A) Linearly varying electric field B) Permanent magnet

C) Antenna signal D) Constant electric field

Answer: Antenna signal

Solution: Time varying current in the antenna produces a magnetic field by Ampere's law. Similarly, antenna signal produces a time varying magnetic field.

Q.10. If a wire of resistance R is connected across V_0, then power is P_0. The wire is cut into two equal parts and connected with V_0 individually, then sum of power dissipated is P_1, then $\frac{P_0}{P_1}$ is $\frac{1}{x}$. Find the value of x.

Answer: 4

Solution: We know clearly that, $P_0 = \frac{V_0^2}{R}$

Since, the wire is cut into two equal parts, the resistance is divided equally leading to resistance of $\frac{R}{2}$ for each wire.

So,

$$P_1 = \frac{V_0^2}{\frac{R}{2}} + \frac{V_0^2}{\frac{R}{2}} = 4\frac{V_0^2}{R} = 4P_0$$
$$\Rightarrow \frac{P_0}{P_1} = \frac{1}{4}$$

Hence, $x = 4$.

Q.11. A particle is performing SHM having position $x = A\cos\ 30^{\circ}$ and $A = 40\ \text{cm}$. If its kinetic energy at this position is $200\ \text{J}$, the value of force constant in $\left(\frac{\text{kN}}{\text{m}}\right)$ is

Answer: 10

Solution: The formula to calculate the kinetic energy of a particle executing SHM is given by

$$K=\frac{1}{2}k\left(A^2-x^2\right)\quad\ldots(1)$$

where, K is the kinetic energy, k is the force constant, A is the amplitude of vibration and x is its instantaneous position.

Simplify equation (1) to obtain the expression for the force constant.

$$\begin{aligned}2K&=k\left(A^2-A^2\cos^2 30^\circ\right)\\&=kA^2\left(1-\cos^2 30^\circ\right)\\&=kA^2\sin^2 30^\circ\\\Rightarrow k&=\frac{2K}{A^2\sin^2 30^\circ}\quad\ldots(2)\end{aligned}$$

Substitute the values of the known parameters into equation (2) to calculate the required value of the force constant.

$$\begin{aligned}k&=\frac{2\times 200\ \mathrm{J}}{(0.4\ \mathrm{m})^2\times\sin^2 30^\circ}\\&=10000\ \mathrm{N\ m^{-1}}\times\frac{1\ \mathrm{kN}}{1000\ \mathrm{N}}\\&=10\ \frac{\mathrm{kN}}{\mathrm{m}}\end{aligned}$$

Q.12. A solid sphere rolls on a horizontal plane. The ratio of angular momentum about COM to total energy is $\frac{\pi}{22}$. Find the angular speed of the sphere.

Answer: 4

Solution: The formula to calculate the moment of inertia of a solid sphere is given by

$$I=\frac{2}{5}MR^2\quad\ldots(1)$$

The angular momentum of the sphere about its centre of mass can be written as

$$\begin{aligned}L&=I\omega\\&=\frac{2}{5}MR^2\omega\quad\ldots(2)\end{aligned}$$

The kinetic energy of the sphere can be considered in two ways-

i) the sum of the rotational kinetic energy of the centre of mass and the translational kinetic energy of its centre of mass,

ii) the pure rotational energy of the sphere about the point of contact of the sphere with the ground.

Considering the second form of kinetic energy, the expression for it can be written as

$$\begin{aligned}K&=\frac{1}{2}\left(\frac{7}{5}MR^2\right)\omega^2\\&=\frac{7}{10}MR^2\omega^2\quad\ldots(3)\end{aligned}$$

Divide equation (2) by equation (3) and simplify to obtain the required angular speed of the sphere.

$$\begin{aligned}\frac{L}{K}&=\frac{\frac{2}{5}MR^2\omega}{\frac{7}{10}MR^2\omega^2}\\&=\frac{4}{7\omega}\\\Rightarrow\frac{\pi}{22}&=\frac{4}{7\omega}\\\Rightarrow\omega&=\frac{4\times 22}{7\pi}\\&=\frac{4\times 22}{7\times\frac{22}{7}}\\&=4\end{aligned}$$

Q.13. If $m=5\pm 0.2$ and $v=20\pm 0.4$, calculate percentage error in measurement of KE.

Answer: 8

Solution: As, $KE = \frac{1}{2}mv^2$

Hence, percentage error in calculation of KE will be:

$$\left[\frac{\Delta KE}{KE}\right] \times 100 = \left[\left[\frac{\Delta m}{m}\right] + \left[2\frac{\Delta v}{v}\right]\right] \times 100$$
$$= \left[\frac{0.2}{5} + 2 \times \frac{0.4}{20}\right] \times 100$$
$$= 8\%$$

Chemistry

Q.14. Which of the following free radical helps in depletion of ozone layer?

A) $NO\cdot$ B) $Cl\cdot$ C) $OH\cdot$ D) $CH_3\cdot$

Answer: $Cl\cdot$

Solution: The free radical that helps in the depletion of the ozone layer is Cl (chlorine). Chlorine radicals are released into the stratosphere from human-made chlorofluorocarbons (CFCs) and other halogen-containing compounds. Once in the stratosphere, these chlorine radicals can react with ozone (O_3) molecules, leading to a chemical reaction that results in the depletion of ozone.

Q.15. What happens when lyophilic sol is added to lyophobic sol?

A) Prevention from coagulation B) Precipitation

C) Emulsion D) Electrophoresis

Answer: Prevention from coagulation

Solution: When a lyophilic sol is added to the lyophobic sol, the lyophilic particles form a layer around lyophobic particles and thus protect the layer from electrolytes. On adding lyophilic sol to lyophobic sol lyophobic sol is protected and a film of lyophilic sol is formed over lyophobic sol.

Q.16. In which of the following options the species changes from paramagnetic to diamagnetic and bond order increases.

A) $N_2 \to N_2^+$ B) $O_2 \to O_2^{-2}$ C) $NO \to NO^+$ D) $O_2 \to O_2+$

Answer: $NO \to NO^+$

Solution: The mathematical formula to calculate bond order is as follows :

Bond order
$= \frac{1}{2} \times \left(\text{number of electrons in the bonding orbitals}\right) - \left(\text{number of electrons in the antibonding orbitals}\right)$

Bond order of $NO = \frac{1}{2}(10 - 5) = 2.5$

Bond order of $NO^+ = \frac{1}{2}(10 - 4) = 3$

Nitric oxide is paramagnetic because of the presence of an odd electron (unpaired electron). When this electron is lost, NO^+ is formed which is diamagnetic.

Q.17. Which of the following is incorrect about Borazine?

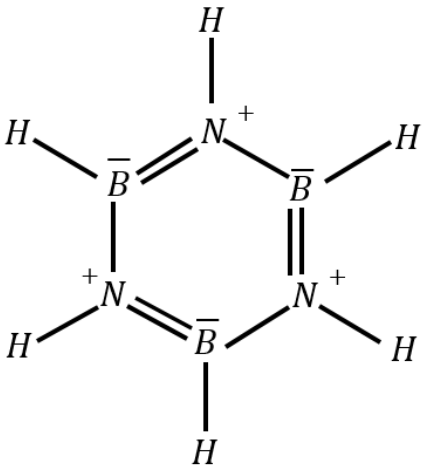
A) It reacts with water. B) It resembles with benzene.

C) It contains banana bond. D) It contains coordinate bond.

Answer: It contains banana bond.

Solution: The statement "It contains a banana bond" is incorrect about Borazine.

Borazine, also known as inorganic benzene or borazole, is a chemical compound with the formula $B_3N_3H_6$. It is an inorganic analog of benzene, with boron (B) atoms replacing the carbon (C) atoms and nitrogen (N) atoms replacing the hydrogen (H) atoms in benzene. Borazine is a hexagonal planar molecule with alternating boron and nitrogen atoms, and it exhibits some structural and chemical similarities to benzene.



Borazine

Q.18. Which of the following is incorrect match ?

A) Zinc-Liquation B) Cu-Electrolysis C) Ni-Mond's Process D) Ti-Van Arkel method

Answer: Zinc-Liquation

Solution: Mond's process involves the combination of carbon monoxide with nickel readily and reversibly and gives nickel tetracarbonyl. This process is used to extract and purify nickel.

The Van Arkel technique is used to refine Zirconium or Titanium. Impurities found in Zirconium or Titanium in the form of nitrogen and oxygen are eliminated using this technique.

Copper is purified by electrolysis. Electricity is passed through solutions containing copper compounds, such as copper sulphate. The anode (positive electrode) is made from impure copper and the cathode (negative electrode) is made from pure copper.

Mercury and zinc are purified by distillation method. Liquation is a metallurgical method to remove lead containing silver from copper and remove antimony minerals from ore.

Q.19. Statement-1: Permutit process is more efficient than synthetic resin method.

Statement-2: In the synthetic resin method, soluble salt of sodium is formed.

A) Statement 1 is true and Statement 2 is false. B) Statement 1 is false and Statement 2 is true.

C) Both statements are true. D) Both statements are false.

Answer: Both statements are false.

Solution: Statement 1: Nowadays hard water is softened by synthetic cationic exchangers. This method is more efficient than zeolite process/permutit process. So the given statement is wrong.

Statement 2: In synthetic resin method, sodium ions exchanges with calcium and magnesium ions present in water to make the water soft. So this statement is also wrong.

$$2\,RNa(s) + M^{2+}(aq) \rightarrow R_2M(s) + 2\,Na^{+}(aq)$$

Q.20. An organic compound on combustion gives $0.22\ g\ of\ CO_2,\ and\ 0.126\ g\ of\ H_2O$. If the percentage of C in given organic compound is 40%, the percentage of hydrogen will be?

A) 8.55 B) 9.33 C) 10 D) 60

Answer: 9.33

Solution: Percentage of carbon, $\%\mathrm{C}=\frac{12}{44}\times\frac{0.22}{\text{Weight of sample}}\times 100$

$$40=\frac{12}{44}\times\frac{0.22}{\text{weight of sample}}\times 100$$

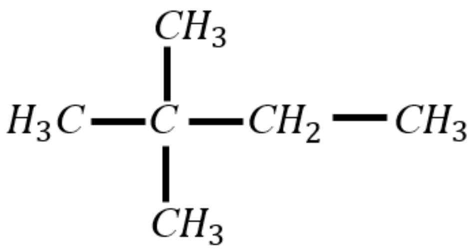
Weight of sample$=0.15\ \mathrm{g}$

The percentage of hydrogen, $\%\mathrm{H}=\frac{2}{18}\times\frac{0.126}{0.15}\times 100=9.33\%$

Q.21. What is the major product formed in the following reaction.

$$\mathrm{CH_3}-\left(\mathrm{CH_2}\right)_4-\mathrm{CH_3}\xrightarrow[\text{HCl, heat}]{\text{Anhy.AlCl}_3}\text{Major product}$$

A)



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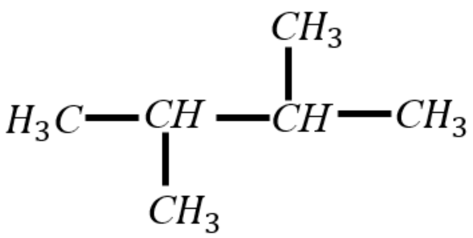
B)



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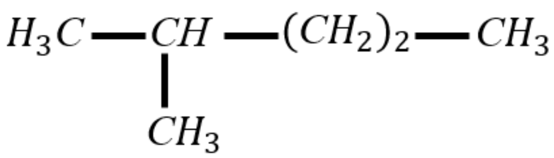
C)



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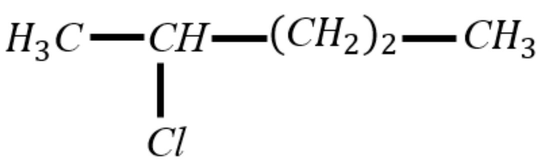
D)



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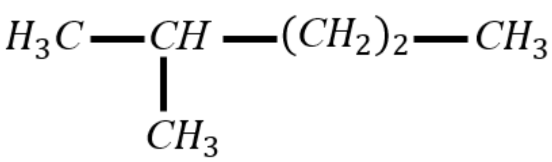
Answer:



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Solution: When n-hexane is heated with anhydrous $AlCl_3$ and HCl

$$\underset{n-\text{hexane}}{CH_3(CH_2)_4CH_3} \xrightarrow[\text{Heat}]{AlCl_3\ ,HCl} \underset{2-\text{methyl pentane}}{CH_3-\overset{\overset{\displaystyle CH_3}{|}}{CH}\,CH_2CH_2CH_3}$$

$$+\underset{3-\text{methyl pantane}}{H_3CCH_2-\underset{\underset{\displaystyle CH_3}{|}}{CH}CH_2CH_3}$$

gas in the presence of high temperature, it isomerises to give a mixture of branched chain alkanes. The presence of anhydrous aluminum chloride and hydrogen chloride gas as catalysts at high temperatures promotes the isomerization of n-hexane by facilitating the rearrangement of the carbon atoms in the hydrocarbon chain

Q.22. ClF_5 exist in which state at room temperature?

A) Gaseous state and square pyramidal, colorless

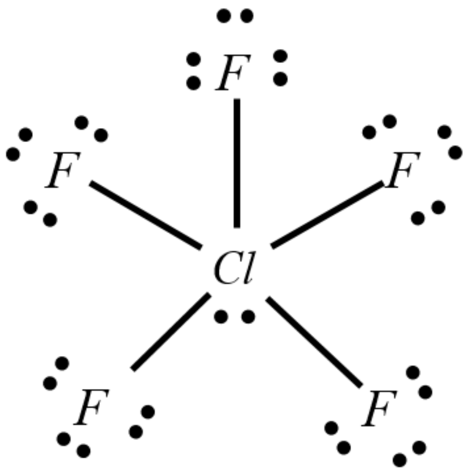
B) Liquid state and trigonal bipyramidal, colorless

C) Gaseous state and trigonal bipyramidal, colorless

D) Liquid state and square pyramidal, colorless

Answer: Liquid state and square pyramidal, colorless

Solution: Chlorine pentafluoride (ClF_5) is a Colourless gas, made by reacting chlorine trifluoride with fluorine at high temperatures and high pressures. This exists in liquid state and the shape is square pyramidal.



Q.23. The pair of lanthanides with exceptionally high 3^{rd} ionisation enthalpy than neighbouring elements:

A) Lu and Yb

B) Eu and Gb

C) Eu and Yb

D) Dy and Yb

Answer: Eu and Yb

Solution: Europium has an atomic number of 63 and a stable electronic configuration of $[Xe]4f^76s^2$. When it loses three electrons to form a cation (Eu^{3+}), it results in the loss of one electron from the 4f subshell, resulting in a highly stable $4f^6$ electron configuration, resulting in a higher ionisation enthalpy.

Ytterbium, on the other hand, has an atomic number of 70 and a stable electronic configuration of $[Xe]4f^{14}6s^2$. When it loses three electrons to form a cation (Yb^{3+}), it results in the loss of three electrons from the $4f$ subshell, resulting in a stable $4f^{13}$ electron configuration, resulting in higher ionisation enthalpy.

Q.24. Match the following :

Column I	Column II
(A) Nylon-6	(1) Caprolactum
(B) Natural rubber	(2)Chloroprene
(C) Vulcanised rubber	(3) Isoprene
(D) Neoprene	(4) Sulphur containg rubber

A) A-1, B-3, C-4, D-2 B) A-1,B-2, C-4, D-3 C) A-4, B-3, C-1, D–2 D) A-2, B-3, C-4, D-1

Answer: A-1, B-3, C-4, D-2

Solution: Nylon 6 is produced by ring-opening chain growth polymerisation of caprolactam in the presence of water vapour and an acid catalyst at the melt.

The monomer of natural rubber is isoprene. Isoprene is produced by many plants.

Vulcanisation is a chemical process in which the rubber is heated with sulphur, accelerator and activator at $140-160^{\circ}\mathrm{C}$.

Neoprene is a family of synthetic rubbers formed by chloroprene polymerisation also known as polychloroprene.

Q.25. Radius of second orbit of He^{+} is $r_{\circ}$. Radius of fourth orbit of Be^{3+} is $xr_{\circ}$. Find x ?

Answer: 2

Solution: The formula used to calculate radius of nth orbit is $r_n = 0.529 \times \frac{n^2}{Z}$

Here, $n = \text{Principal quantum number}, Z = \text{Atomic number}$

Radius of second orbit of He^{+}, $r_{\circ} = 0.529 \times \frac{4}{2}$

$r_{\circ} = 0.529 \times 2$

Radius of fourth orbit of Be^{+3}, $r_{\mathrm{Be}^{+3}} = 0.529 \times \frac{4^2}{4}$

$r_{\mathrm{Be}^{+3}} = 0.529 \times 4 = 2r_{\circ}$

Therefore, the value of $x = 2$

Q.26. For the first order reactions, the ratio of $t_{50\%}$ and $t_{87.5\%}$ will be $1 : x$. Find the value of x ?

Answer: 3

Solution: The time for completion of the reaction and concentration of reactant relation is as follows,

$t_{x\%} = \frac{2.303}{k}\log\frac{100}{100-x}$

$t_{50\%} = \frac{2.303}{k}\log\frac{100}{50}$

$t_{87.5\%} = \frac{2.303}{k}\log\frac{100}{12.5}$

$t_{50\%} : t_{87.5\%} = \log 2 : \log 8 = 1:3$

Therefore, for the first order reactions, the ratio of $t_{50\%}$ and $t_{87.5\%}$ will be $1:3$

Mathematics

Q.27. Find the sum of series $2 \times 2^2 - 2 \times 3^2 + 2 \times 4^2 + \ldots\ldots (20 \text{ terms})$

A) 462 B) -462 C) 460 D) -460

Answer: -460

Solution: Let $S = 2 \times 2^2 - 2 \times 3^2 + 2 \times 4^2 + \ldots\ldots - 2 \times 21^2$

$= 2\left(2^2 - 3^2 + 4^2 + \ldots\ldots - 21^2\right)$

$= 2\left[\left(2^2 + 4^2 + 6^2 + 8^2 + \ldots + 20^2\right) - \left(3^2 + 5^2 + 7^2 + \ldots\ldots + 21^2\right)\right]$

$= 2\left[2\left(2^2 + 4^2 + 6^2 + 8^2 + \ldots + 20^2\right) - \left(2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + \ldots\ldots + 20^2 + 21^2\right)\right]$

$= 2\left[2\left(2^2 + 4^2 + 6^2 + 8^2 + \ldots + 20^2\right) - \left(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + \ldots\ldots + 20^2 + 21^2\right) + 1^2\right]$

$= 2\left[2^3\left(1^2 + 2^2 + 3^2 + 4^2 + \ldots + 10^2\right) - \left(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + \ldots\ldots + 20^2 + 21^2\right) + 1^2\right]$

We know that $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$= 2\left[2^3\left(\frac{10(10+1)(2\times10+1)}{6}\right) - \left(\frac{21(21+1)(2\times21+1)}{6}\right) + 1^2\right]$

$= 2\left[8 \times \frac{10\times11\times21}{6} - \frac{21\times22\times43}{6} + 1\right]$

$= 2\left[3080 - 3311 + 1\right]$

$= 2\left[-230\right] = -460$

Therefore, the sum of the required series is -460

Q.28. The number of seven digit numbers using $1, 2, 3, 4$ whose sum of digits is 12 is

A) 413 B) 311 C) 308 D) 393

Answer: 413

Solution: The given numbers are $1, 2, 3, 4$ and sum of digits should be 12.

Case 1: $\underline{4}\ \underline{3}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}$,Here the sum of the digits is 12.

The number of ways arranging them is $= \frac{7!}{5!} = 42.$

Case 2: $\underline{4}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}$.

The number of ways arranging them is $= \frac{7!}{2!4!} = 105$

Case 3: $\underline{3}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1}\ \underline{1}$.

The number of ways arranging them is $= \frac{7!}{3!3!} = 140$

Case 4: $\underline{3}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}$.

The number of ways arranging them is $= \frac{7!}{2!4!} = 105$

Case 4: $\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1}$

The number of ways arranging them is $= \frac{7!}{5!2!} = 21$

The total number of ways are $= 42 + 105 + 140 + 105 + 21$

$= 413$

Therefore, the required answer is 413.

Q.29. If $\frac{\mathrm{d}y}{\mathrm{d}x} = y + 7$ and $y(0) = 0$, then the value of $y(1)$ is

A) $7(e-1)$ B) $2(e-1)$ C) $7e$ D) None of these

Answer: $7(e-1)$

Solution: Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = y + 7$

$\Rightarrow \frac{dy}{y+7} = dx$

Now integrate on both sides of the equation.

$\Rightarrow \int \frac{dy}{y+7} = \int dx$

$\Rightarrow \ln|y+7| = x + c$

But $y(0) = 0$

$\Rightarrow \ln|0+7| = 0 + c$

Let us find $y(1)$.

$\Rightarrow \ln|y+7| = 1 + \ln|7|$

$\Rightarrow \ln|y+7| = \ln|7e|$

$\Rightarrow y + 7 = 7e$

$\Rightarrow y(1) = 7(e-1)$

Therefore, the required answer is $7(e-1)$

Q.30. Consider:

Statement 1 : $\lim_{n\to\infty} \frac{1}{n^2}(1+2+3+\ldots+n) = 1$

Statement 2 : $\lim_{n\to\infty} \frac{1}{n^{16}}\left(1^{15}+2^{15}+3^{15}+\ldots+n^{15}\right) = \frac{1}{16}$

A) Both statements 1 and 2 are true.

B) Both statements 1 and 2 are false.

C) Statement 1 is true and 2 is false

D) Statement 1 is false and 2 is true

Answer: Statement 1 is false and 2 is true

Solution: Statement 1 :

$\lim_{n\to\infty} \frac{1}{n^2}(1+2+3+\ldots+n)$

$= \lim_{n\to\infty} \left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{n}{n}\right)\frac{1}{n}$

$= \lim_{n\to\infty} \left(\sum_{r=1}^{n}\left(\frac{r}{n}\right)\right)\frac{1}{n}$

$= \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$

Statement 2 :

$\lim_{n\to\infty} \frac{1}{n^{16}}\left(1^{15}+2^{15}+3^{15}+\ldots+n^{15}\right)$

$= \lim_{n\to\infty} \left(\left(\frac{1}{n}\right)^{15}+\left(\frac{2}{n}\right)^{15}+\left(\frac{3}{n}\right)^{15}+\ldots+\left(\frac{n}{n}\right)^{15}\right)\frac{1}{n}$

$= \lim_{n\to\infty} \left(\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{15}\right)\frac{1}{n}$

$= \int_0^1 x^{15}dx = \left[\frac{x^{16}}{16}\right]_0^1 = \frac{1}{16}$

Q.31. If $3f(x) + 2f\left(\frac{1}{x}\right) = \frac{1}{x} - 10$, then $f(3) =$

A) 2

B) -3

C) -4

D) None of these

Answer: -3

Solution: Given:

$3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10 \quad \ldots.(1)$

Replace $x \rightarrow \frac{1}{x}$

$3f\left(\frac{1}{x}\right)+2f(x)=x-10 \quad \ldots.(2)$

Eliminating $f\left(\frac{1}{x}\right)$ from (1) & (2), we get

$\Rightarrow 5f(x)=\frac{3}{x}-2x-10$

Let us replace x with 3

$\Rightarrow 5f(3)=\frac{3}{3}-2(3)-10$

$\Rightarrow f(3)=-3$

Therefore, the required answer is -3

Q.32. Maximum value of $f(x)=x-\sin 2x+\frac{1}{3}\sin 3x$ in $[0,\pi]$ is

A) $\frac{5\pi}{6}+\frac{\sqrt{3}}{2}+\frac{1}{3}$ B) 0 C) $\frac{5\pi}{6}-\frac{\sqrt{3}}{2}+\frac{1}{3}$ D) $\frac{5\pi}{6}+\frac{\sqrt{3}}{2}-\frac{1}{3}$

Answer: $\frac{5\pi}{6}+\frac{\sqrt{3}}{2}+\frac{1}{3}$

Solution: Given:

$$f(x) = x - \sin 2x + \frac{1}{3}\sin 3x$$

$$\Rightarrow f'(x) = 1 - 2\cos 2x + \cos 3x$$

$$\Rightarrow f'(x) = 1 - 2\left(2\cos^2 x - 1\right) + 4\cos^3 x - 3\cos x$$

$$\Rightarrow f'(x) = 3 - 4\cos^2 x + 4\cos^3 x - 3\cos x$$

$$\Rightarrow f'(x) = (\cos x - 1)\left(4\cos^2 x - 3\right)$$

For critical points,

$$f'(x) = 0$$

$$\Rightarrow (\cos x - 1)\left(4\cos^2 x - 3\right) = 0$$

$$\Rightarrow \cos x = 1, \ \pm\frac{\sqrt{3}}{2}$$

$$\Rightarrow x = 0, \ \frac{\pi}{6}, \ \frac{5\pi}{6}$$

And,

$$f''(x) = 4\sin 2x - 12\cos^2 x \sin x + 3\sin x$$

Now,

$$f''(0) = 0$$
$$f''\left(\frac{\pi}{6}\right) > 0$$
$$f''\left(\frac{5\pi}{6}\right) < 0\,(\text{maxima})$$

So, maximum value of function is

$$f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} - \sin\left(\frac{5\pi}{3}\right) + \frac{1}{3}\sin\left(\frac{5\pi}{2}\right)$$

$$\Rightarrow f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + \sin\left(\frac{\pi}{3}\right) + \frac{1}{3}\sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3}$$

Q.33. If $\frac{dy}{dx} = 6e^x + e^{2x} + e^{3x}$, then $y(2) - y(0)$ is

A) $6e^2 + \frac{1}{2}e^4 + \frac{1}{3}e^6 - \frac{41}{6}$ B) $6e^2 + \frac{1}{2}e^4 + \frac{1}{3}e^6 + \frac{41}{6}$ C) $6e^2 + \frac{1}{2}e^4 + \frac{1}{3}e^6 - \frac{51}{6}$ D) $6e^2 + \frac{1}{2}e^4 + \frac{1}{3}e^6 + \frac{51}{6}$

Answer: $6e^2 + \frac{1}{2}e^4 + \frac{1}{3}e^6 - \frac{41}{6}$

Solution: Given:

$$\frac{dy}{dx} = 6e^x + e^{2x} + e^{3x}$$

$$\Rightarrow dy = \left(6e^x + e^{2x} + e^{3x}\right)dx$$

$$\Rightarrow \int dy = \int\left(6e^x + e^{2x} + e^{3x}\right)dx$$

$$\Rightarrow y = 6e^x + \frac{1}{2}e^{2x} + \frac{1}{3}e^{3x} + C$$

So,

$$y(2) - y(0) = 6e^2 + \frac{1}{2}e^4 + \frac{1}{3}e^6 + C - \left(6 + \frac{1}{2} + \frac{1}{3} + C\right)$$

$$\Rightarrow y(2) - y(0) = 6e^2 + \frac{1}{2}e^4 + \frac{1}{3}e^6 - \frac{41}{6}$$

Q.34. If $\vec{a}=2\hat{i}+3\hat{j}+5\hat{k},\ \vec{b}=3\hat{i}+3\hat{j}+7\hat{k},\ \vec{c}=7\hat{i}+8\hat{j}+9\hat{k}$ and $\vec{a}\times\vec{b}=\vec{c}+\vec{d}$ then $|\vec{d}|$ is

A) $\sqrt{194}$ B) $8\sqrt{2}$ C) $\sqrt{168}$ D) $5\sqrt{5}$

Answer: $\sqrt{194}$

Solution: Given that $\vec{a}=2\hat{i}+3\hat{j}+5\hat{k},\ \vec{b}=3\hat{i}+3\hat{j}+7\hat{k},\ \vec{c}=7\hat{i}+8\hat{j}+9\hat{k}$

Let us find $\vec{a}\times\vec{b}$.

$$\Rightarrow \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 5\\ 3 & 3 & 7\end{vmatrix}$$

$$=(21-15)\hat{i}-(14-15)\hat{j}+(6-9)\hat{k}$$

$$=6\hat{i}+\hat{j}-3\hat{k}$$

But $\vec{a}\times\vec{b}=\vec{c}+\vec{d}$

$$\Rightarrow 6\hat{i}+\hat{j}-3\hat{k}=7\hat{i}+8\hat{j}+9\hat{k}+\vec{d}$$

$$\Rightarrow \vec{d}=-\hat{i}-7\hat{j}-12\hat{k}$$

$$\Rightarrow |\vec{d}|=\sqrt{(-1)^2+(-7)^2+(-12)^2}=\sqrt{194}$$

Therefore, the required answer is $\sqrt{194}$

Q.35. The integral $\int_0^\infty \frac{6}{e^{3x}+6e^{2x}+11e^x+6}dx$ equals

A) $\log_e\left(\frac{32}{27}\right)$ B) $\log_e\left(\frac{32}{9}\right)$ C) $\log_e\left(\frac{16}{27}\right)$ D) $\log_e\left(\frac{32}{3}\right)$

Answer: $\log_e\left(\frac{32}{27}\right)$

Solution: Let

$$I=\int_0^\infty\left(\frac{6}{e^{3x}+6e^{2x}+11e^x+6}\right)dx$$

$$\Rightarrow I=\int_0^\infty\frac{6}{(e^x+1)(e^x+2)(e^x+3)}dx$$

Using partial fraction, we can write

$$\Rightarrow I=6\int_0^\infty\left[\frac{1}{2(e^x+1)}-\frac{1}{(e^x+2)}+\frac{1}{2(e^x+3)}\right]dx$$

$$\Rightarrow I=6\int_0^\infty\left[\frac{1}{2(e^{-x}+1)}-\frac{1}{(1+2e^{-x})}+\frac{1}{2(1+3e^{-x})}\right]e^{-x}dx$$

$$\Rightarrow I=6\left[-\frac{1}{2}\log_e\left(e^{-x}+1\right)+\frac{1}{2}\log_e\left(e^{-x}+2\right)-\frac{1}{6}\log_e\left(e^{-x}+3\right)\right]_0^\infty$$

$$\Rightarrow I=6\left[0-\left\{-\frac{1}{2}\log_e 2+\frac{1}{2}\log_e 3-\frac{1}{6}\log_e 4\right\}\right]$$

$$\Rightarrow I=3\log_e 2-3\log_e 3+\log_e 4$$

$$\Rightarrow I=5\log_e 2-3\log_e 3$$

$$\Rightarrow I=\log_e\left(\frac{32}{27}\right)$$

Q.36. Remainder when 2^{2022} is divided by 15 is equal to

Answer: 4

Solution: We know that,

$(1+x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + \dots\dots\dots {}^nC_n x^n$

Now rewriting 2^{2022} as $2^{2022} = \left(2^8\right)^{252} \times 2^6 = (255+1)^{252} \times 2^6$

Now $(1+255)^{252} = 1 + {}^{252}C_1 255 + {}^{252}C_2 255^2 \dots\dots + {}^{252}C_{252} 255^{252}$

So, when $(1+255)^{252}$ is divided by 15 remainder will be 1 as 255 is divisible by 15,

Now $(255+1)^{252} \times 2^6$ when divided by 15 will give the remainder as $2^6 = 64$, now again 64 divided by 15 will give the remainder as 4,

Hence, 2^{2022} when divided by 15 gives the remainder as 4

Q.37. Let the number of matrices of order 3×3 are possible using the digits $\{0, 2, 3, \dots, 10\}$ is m^n, then $m + n$ is

Answer: 20

Solution: For 3×3 matrices, we must have matrix of the form

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Now each element of matrices can be filled with 11 numbers from 0 to 10,

So, Number of matrices$= 11^9$

So, $m = 11,\ n = 9$

$m + n = 20$

Q.38. If $g(x) = \sqrt{x+1}$ and $f(g(x)) = 3 - \sqrt{x+1}$ then $f(0) =$

Answer: 3

Solution: Given,

$g(x) = \sqrt{x+1}$ and $f(g(x)) = 3 - \sqrt{x+1}$.

$\Rightarrow g(-1) = \sqrt{-1+1} = 0$

Now $f(g(-1)) = 3 - \sqrt{-1+1}$

$\Rightarrow f(0) = 3$

Therefore, the value of $f(0)$ is 3.

Q.39. Let there be 10 A.P's whose first term are $(1, 2, 3. \dots 10)$ respectively and common differences are $(1, 3, 5, \dots)$ respectively and S_i denotes the sum of 10 terms of i^{th} A.P, then $\sum_{i=1}^{10} S_i$ is

Answer: 5050

Solution: Given,

First term of A.P are $1, 2, 3 \ldots . 10$, so general term of the first term will be i

And common difference are $1, 3, 5, \ldots .$, so general term of common difference is given by $2i - 1$

Now sum of the A.P is given by,

$S_i = \frac{10}{2}\left[2 \times i + (10 - 1)(2i - 1)\right]$

$\Rightarrow S_i = 5\left[2 \times i + 9(2i - 1)\right]$

$\Rightarrow S_i = 100i - 45$

So, $\sum_{i=1}^{10} S_i = \sum_{i=1}^{10} 100i - 45 \sum_{i=1}^{10} 1$

$\Rightarrow \sum_{i=1}^{10} S_i = 100\left(\frac{10 \times 11}{2}\right) - 45 \times 10$

$\Rightarrow \sum_{i=1}^{10} S_i = 100(55) - 45 \times 10$

$\Rightarrow \sum_{i=1}^{10} S_i = 5050$