JEE Main 2023 (Session 2)

April 13 Shift 2

Physics

Q.1. What should be the minimum size of antenna required for successful transmission of wave having wavelength λ.

A) 2λ B) $\frac{\lambda}{4}$ C) $\frac{\lambda}{2}$ D) λ

Answer: $\frac{\lambda}{4}$

Solution: The transmitting and receiving antennas' heights should be comparable to one-quarter of the wavelength for effective radiation and reception. Therefore, the minimum length of the antenna should be $\frac{\lambda}{4}$ in order to transmit electromagnetic wave signals effectively.

Q.2. A $10\ \mu\mathrm{C}$ charge is divided into two equal parts and kept at $1\ \mathrm{cm}$ distance. Find repulsion between charges?

A) $225\ \mathrm{N}$ B) $450\ \mathrm{N}$ C) $2250\ \mathrm{N}$ D) $4500\ \mathrm{N}$

Answer: $2250\ \mathrm{N}$

Solution: As the charge is divided into two equal parts, each charge becomes $5\ \mu\mathrm{C}$.

The formula to calculate the repulsive force between the charges is given by

$$F=k\frac{q^2}{r^2}\quad\ldots\left(1\right)$$

Substitute the values of the known parameters into equation (1) to calculate the required force.

$$F=9\times10^9\times\frac{\left(5\times10^{-6}\right)^2}{(0.01)^2}\ \mathrm{N}$$
$$=2250\ \mathrm{N}$$

Q.3. Two identical trains cross each other moving on parallel tracks, opposite in direction. Speed of one of the train is $70\ \mathrm{km\ h^{-1}}$ and second train has a speed of $110\ \mathrm{km\ h^{-1}}$. If it takes $8\ \mathrm{s}$ for two trains to cross each other, then length of trains is equal to

A) $100\ \mathrm{m}$ B) $200\ \mathrm{m}$ C) $300\ \mathrm{m}$ D) $400\ \mathrm{m}$

Answer: $200\ \mathrm{m}$

Solution: Since the trains are coming from opposite direction, the total relative displacement of each train is twice the length of the train.

The formula to calculate the time taken by each train to overcome the relative displacement is given by

$$t=\frac{2l}{v_1+v_2}\quad\ldots\left(1\right)$$

where, l is the length of each train.

Substitute the values of the known parameters into equation (1) and solve to calculate the required length of the trains.

$$8\ \mathrm{s}=\frac{2l}{110\times\frac{5}{18}\ \mathrm{m\,s^{-1}}+70\times\frac{5}{18}\ \mathrm{m\,s^{-1}}}$$
$$\Rightarrow 2l=8\ \mathrm{s}\times50\ \mathrm{m\ s^{-1}}$$
$$\Rightarrow l=200\ \mathrm{m}$$

Q.4. A particle is performing S.H.M whose distance from mean position varies as $x=A\sin(\omega t)$. Find the position of particle from mean position, where kinetic energy and potential energy is equal.

A) $\frac{A}{2}$ B) $\frac{A}{\sqrt{2}}$ C) $\frac{A}{2\sqrt{2}}$ D) $\frac{A}{4}$

Answer: $\frac{A}{\sqrt{2}}$

Solution: The formula to calculate the kinetic energy of a particle executing SHM is given by

$K=\frac{1}{2}m\omega^2\left(A^2-x^2\right) \quad \ldots(1)$

The formula to calculate the potential energy of the particle is given by

$U=\frac{1}{2}m\omega^2x^2 \quad \ldots(2)$

Equate equations (1) and (2) and solve to obtain the required position of the particle.

$$\frac{1}{2}m\omega^2\left(A^2-x^2\right)=\frac{1}{2}m\omega^2x^2$$
$$\Rightarrow A^2-x^2=x^2$$
$$\Rightarrow 2x^2=A^2$$
$$\Rightarrow x=\frac{A}{\sqrt{2}}$$

Q.5. An electron is moving along positive x direction in xy plane, magnetic field points in negative z direction, then the force due to magnetic field on electron points in the direction

A) $\hat{j}$ B) $-\hat{j}$ C) $\hat{k}$ D) $-\hat{k}$

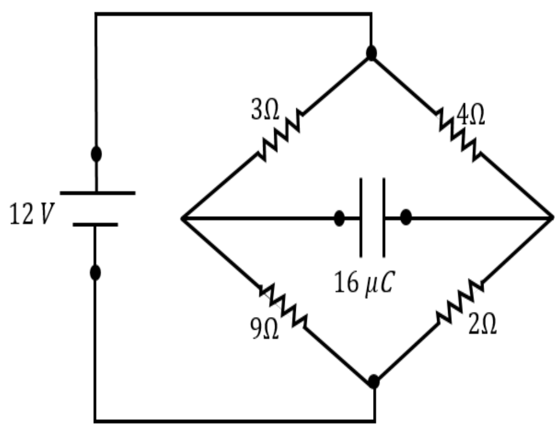
Answer: $-\hat{j}$

Solution: We know that, $\vec{F}=q\left(\vec{v}\times\vec{B}\right)$

So, for electron, the direction of force acting on it is given by $-\left(\hat{i}\times-\hat{k}\right)=-\hat{j}$

We take negative sign for velocity as charge of electron is negative.

Q.6. Find the energy stored in the capacitor in the given circuit.



A) $0.2\ \text{mJ}$ B) $0.4\ \text{mJ}$ C) $0.6\ \text{mJ}$ D) $0.8\ \text{mJ}$

Answer: $0.2\ \text{mJ}$

Solution: There will be no current through the capacitor.

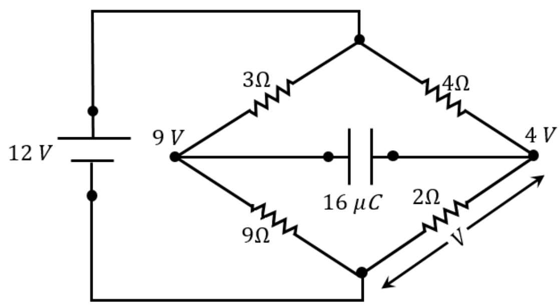
The potential difference across the $2\ \Omega$ resistor can be calculated as follows:

$$V_2 = \frac{2\ \Omega}{2\ \Omega + 4\ \Omega} \times 12\ \text{V}$$
$$= 4\ \text{V} \quad \ldots(1)$$

Similarly, the potential difference across the $9\ \Omega$ resistor can be calculated as follows:

$$V_9 = \frac{9\ \Omega}{9\ \Omega + 3\ \Omega} \times 12\ \text{V}$$
$$= 9\ \text{V} \quad \ldots(2)$$

The scenario is shown in the following figure:



The formula to calculate the energy stored in the capacitor is given by

$$U = \frac{1}{2}C(V_9 - V_2)^2 \quad \ldots(3)$$

Substitute the values of the known parameters into equation (3) to calculate the required energy stored.

$$U = \frac{1}{2} \times 16 \times 10^{-6} \times (9-4)^2\ \text{J}$$
$$= 2 \times 10^{-3}\ \text{J} \times \frac{1000\ \text{mJ}}{1\ \text{J}}$$
$$= 0.2\ \text{mJ}$$

Q.7. A mixture of gases with adiabatic coefficient equal to $\frac{3}{2}$ is compressed from initial state $(P_0,\ V_0)$ to one fourth volume adiabatically. Its final pressure will be equal to

A) P_0 B) $2P_0$ C) $4P_0$ D) $8P_0$

Answer: $8P_0$

Solution: The relation between the pressure and the volume of a gas for an adiabatic compression is given by

$$P_i V_i^{\gamma} = P_f V_f^{\gamma} \quad \ldots(1)$$

Substitute the known values of the parameters into equation (1) and simplify to obtain the final pressure.

$$P_0 V_0^{\frac{3}{2}} = P_f\left(\frac{V_0}{4}\right)^{\frac{3}{2}}$$
$$\Rightarrow P_f = (4)^{\frac{3}{2}} P_0$$
$$= 8P_0$$

Q.8. A bi-convex lens of focal length $10\ \text{cm}$ is cut perpendicularly to principal axis. Find power (in D) of new lens.

Answer: 5

Solution: Calculation of the focal length of the biconvex lens:

Let the focal length of lens be f before cutting.

$$\Rightarrow \frac{1}{f}=\left(\mu-1\right)\left(\frac{1}{R}-\frac{1}{-R}\right)$$
$$\Rightarrow \frac{1}{f}=\left(\mu-1\right)\left(\frac{1}{R}+\frac{1}{R}\right)$$
$$\Rightarrow \frac{1}{f}=\left(\mu-1\right)\left(\frac{2}{R}\right)$$

But, $P=\frac{1}{\frac{10}{100}}=10\ \mathrm{D}$

Calculation of the focal length of the lens after it's been cut:

Let the focal length of the lens after it's been cut be f_1

For the plano-convex lens,

$$\Rightarrow \frac{1}{f_1}=\left(\mu-1\right)\left(\frac{1}{R}-\frac{1}{\infty}\right)$$
$$\Rightarrow \frac{1}{f_1}=\left(\mu-1\right)\left(\frac{1}{R}\right)$$
$$\Rightarrow \frac{1}{f_1}=\frac{1}{2f}$$
$$\Rightarrow P_1=0.5\times P=0.5\times 10=5\ \mathrm{D}$$

Q.9. Body accelerates from rest to $u\ \mathrm{m\ s^{-1}}$, energy is E. If it accelerates from rest to $2u\ \mathrm{m\ s^{-1}}$, then energy is nE. Find n.

Answer: 4

Solution: Case 1:

We know that, $E=\frac{1}{2}mu^2-0$

$$\Rightarrow E=\frac{1}{2}mu^2$$

Case 2:

$$E'=\frac{1}{2}m(2u)^2-0=4\times\left(\frac{1}{2}mu^2\right)=4E$$

$$\Rightarrow n=4$$

Q.10. If a substance absorbs radiation of wavelength $500\ \mathrm{nm}$ and emits radiation of wavelength $600\ \mathrm{nm}$, then the net change in energy is $x\times 10^{-2}\ \mathrm{eV}$. Find the value of x to the nearest integer.

Answer: 41

Solution: The formula to calculate the energy of absorbed photon is given by

$$E_a = \frac{hc}{\lambda_a} \quad \ldots (1)$$

and, the energy of the emitted photon is given by

$$E_e = \frac{hc}{\lambda_e} \quad \ldots (2)$$

Hence, the net change of energy can be calculated as follows:

$$\begin{aligned}\Delta E &= E_a - E_e \\ &= \frac{hc}{\lambda_a} - \frac{hc}{\lambda_e} \\ &= hc\left(\frac{1}{\lambda_a} - \frac{1}{\lambda_e}\right) \quad \ldots (3)\end{aligned}$$

Substitute the values of the known parameters into equation (3) to calculate the net change in energy.

$$\begin{aligned}\Delta E &= 1242 \text{ eV} - \text{nm} \times \left(\frac{1}{500 \text{ nm}} - \frac{1}{600 \text{ nm}}\right) \\ &\approx 41 \times 10^{-2} \text{ eV}\end{aligned}$$

Comparing the calculated value with the given expression, it can be concluded that $x = 41$.

Q.11. A car of mass 200 kg is revolving in a circular track of radius 70 m with angular velocity 0.2 rad s^{-1}, then find the centripetal force in Newton.

Answer: 560

Solution: Centripetal force is given by formula,

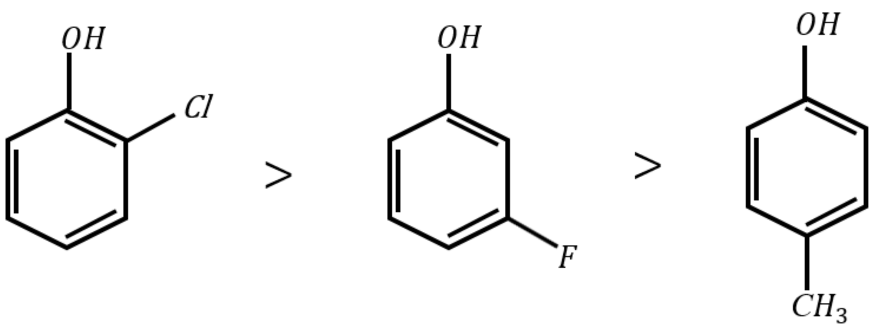
$$F = m\omega^2 r$$

On substituting the values in above equation, we get,

$$\begin{aligned}F &= 200 \times (0.2)^2 \times 70 \\ &= 14000 \times 0.04 \\ &= 560 \text{ N}\end{aligned}$$

Chemistry

Q.12. Assertion(A): Acidic nature follows the order



Reason: F is better electron withdrawing group than Cl.

A) (A) and (R) both are correct, and (R) is the correct explanation of (A)

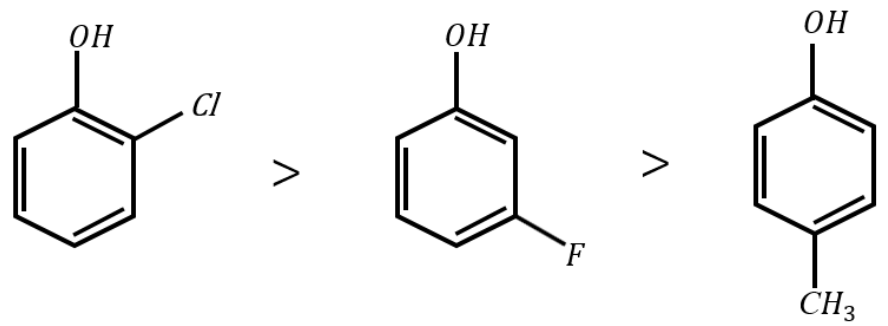
B) (A) and (R) both are correct, and (R) is not the correct explanation of (A)

C) (A) is correct but (R) is not correct

D) (A) is incorrect but (R) is correct

Answer: (A) and (R) both are correct, and (R) is not the correct explanation of (A)

Solution: Electron withdrawing groups increases the acidic nature of the phenol. Amongst the given molecules' cresol is the least acidic due to the presence of methyl group(electron releasing group). Fluorine is stronger electron withdrawing group than the chlorine. But fluorine shows only $-I$ effect at the meta position. But chlorine at ortho position shows $-I$ effect as well as vacant d-orbital effect which increases the acidic nature more than the fluoro phenol.



Q.13. The correct increasing order of the magnitude of enthalpies of formation for alkali metal halides is:

A) $\mathrm{NaI} < \mathrm{NaF} < \mathrm{NaBr} < \mathrm{NaCl}$ B) $\mathrm{NaF} > \mathrm{NaCl} > \mathrm{NaBr} > \mathrm{NaI}$ C) $\mathrm{NaF} < \mathrm{NaBr} < \mathrm{NaCl} < \mathrm{NaI}$ D) $\mathrm{NaCl} < \mathrm{NaBr} < \mathrm{NaF} < \mathrm{NaI}$

Answer: $\mathrm{NaF} > \mathrm{NaCl} > \mathrm{NaBr} > \mathrm{NaI}$

Solution: Alkali metal halides can be prepared by the reaction of the appropriate oxide,hydroxide or carbonate with aqueous hydrohalic acid(HX). All of these halides have high negative enthalpies of formation. Order of enthalpy of formation: $\mathrm{Fluoride} > \mathrm{chloride} > \mathrm{bromide} > \mathrm{iodide}$.

This fluoride are the most stable halides. So the correct order of magnitude of enthalpies of group 1 halides is: $\mathrm{NaF} > \mathrm{NaCl} > \mathrm{NaBr} > \mathrm{NaI}$.

Q.14. Statement 1: SO_2 and H_2O both have bent structures.

Statement 2: SO_2 have less angle than H_2O.

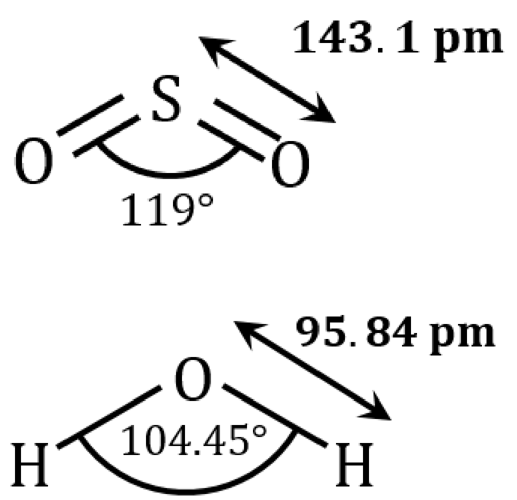
A) Statement 1 is true and Statement 2 is false. B) Statement 1 is fasle and Statement 2 is true.

C) Both the statements are true. D) Both the statements are false.

Answer: Statement 1 is true and Statement 2 is false.

Solution: Statement 1 is correct. Both SO_2 (sulfur dioxide) and H_2O (water) have bent or V-shaped molecular geometries. This is due to the presence of lone pairs of electrons on the central atom, which cause repulsion and result in a bent molecular shape.

Statement 2 is incorrect. In reality, SO_2 has a larger bond angle than H_2O. The bond angle in SO_2 is approximately 120^o, while the bond angle in H_2O is approximately 104.5^o.



Q.15. Which of the following is oxidised by oxygen in acidic medium?

A) Cl^-, Br^- B) Br^-, I^- C) Br^- D) I^-

Answer: Br^-, I^-

Solution: The oxidation of a species by oxygen in acidic medium can be explained based on the standard reduction potentials (E°) of the species involved. Based on the standard reduction potentials, the species that can be oxidised by oxygen in acidic medium is bromide (Br^-) ions and Iodide ions(I^-).

The standard reduction potentials are

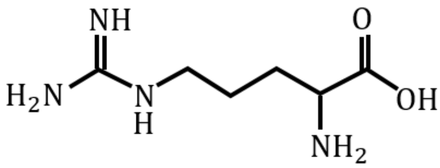
$Cl_2 + 2e^- \rightarrow 2Cl^-$ $E^\circ = +1.36$ V
$Br_2 + 2e^- \rightarrow 2Br^-$ $E^\circ = +1.07$ V
$I_2 + 2e^- \rightarrow 2I^-$ $E^\circ = +0.54$ V

Q.16. A naturally occurring amino acid that contains only one basic functional group.

A) Arginine B) Lysine C) Histidine D) Isoleucine

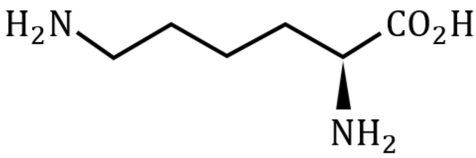
Answer: Isoleucine

Solution: The structure of arginine is as follows :



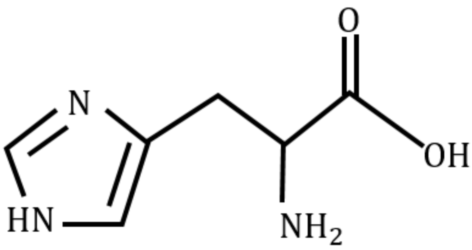
It is two basic functional groups, amine and guanidine.

The structure of lysine is as follows :



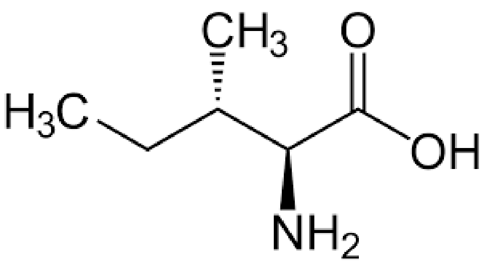
It contain two basic amino groups.

The structure of histidine is as follows :



It contain more than one basic groups.

The structure of isoleucine is as follows :



It is a monobasic in nature.

Therefore, correct option is D.

Q.17. Match the given in column I with column II:

Column I	Column II
(a) Nylon - 6,6	(p) Thermosetting
(b) Nylon - 6	(q) Polyester
(c) Phenol formaldehyde resin	(r) Homopolymer
(d) Dacron	(s) Polyamides

A) a-p, b-q, c-s, d-r B) a-q, b-p, c-r, d-s C) a-pq, b-rs, c-q, d-p D) a-s, b-rs, c-p, d-q

Answer: a-s, b-rs, c-p, d-q

Solution: A polyamide is a synthetic polymer made by the linkage of an amino group of one molecule and a carboxylic acid group of another, including many synthetic fibres such as nylon. Therefore, nylon – 6,6 and nylon - 6 are polyamides.

Nylon 6 is produced by ring-opening chain growth polymerisation of caprolactam in the presence of water vapour and an acid catalyst at the melt. Therefore, it is a homo polymer.

Phenol formaldehyde resins are thermosetting polymers. For example Bakelite is thermosetting polymer.

Dacron is also called as Terylene which is obtained from the polymerisation of Terephthalic acid ethylene glycol. Dacron consists of ester groups, it is a polyester.

Q.18. If the formula of borax is $Na_2B_4O_x(OH)_y.zH_2O$, Find the value of $x+y+z$?

Answer: 17

Solution: The molecular formula of borax is $Na_2B_4O_7.10H_2O$. The structural formula of borax,$Na_2[B_4O_5(OH)_4]\cdot 8H_2O$.

In this formula, $x,\ y,\ \text{and}\ z$ represent the stoichiometric coefficients or the number of molecules of water, hydroxide, and borax respectively.

Based on the correct formula, the values of $x,\ y,\ \text{and}\ z$ are as follows: $5+4+8=17$.

Q.19. Number of greenhouse gases are:

Water vapours, ozone, molecular hydrogen, I_2.

Answer: 2

Solution: Besides carbon dioxide, other green house gases are methane, water vapour, nitrous oxide, Chloro fluoro carbons and ozone. Methane is produced naturally when vegetation is burnt. So from the given options' water vapour and ozone are green house gases.

Q.20. Consider the reaction

$Cr_2O_7^{2-}+xH^++Fe^{2+}\rightarrow yFe^{3+}+2Cr^{3+}+zH_2O$

Sum of $x, y\ \text{and}\ z$ are

Answer: 27

Solution: $Cr_2O_7^{2-}+xH^++Fe^{2+}\rightarrow yFe^{3+}+2Cr^{3+}+zH_2O$

In the above reaction, the change oxidation state per molecule of dichromate ion is 6 and change in oxidation state per one Fe^{2+} ion is one.

Based on this, we can balance equation is as follows,

$Cr_2O_7^{2-}+14H^++6Fe^{2+}\rightarrow 6Fe^{3+}+2Cr^{3+}+7H_2O$

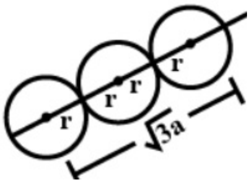
The value of $x=14,\ y=6\ \text{and}\ z=7$

Hence, the value of $x+y+z$ is 27.

Q.21. Given length of body diagonal of unit cell is 4Å . The radius of sodium atom forming BCC lattice is $x\times 10^{-1}$. The value of x is (The nearest integer)

Answer: 17

Solution: The Body centred cubic unit cell with its body diagonal is shown below.



The relation between edge length and radius of atom is $\sqrt{3}\mathrm{a}=4\mathrm{r}$

Now, radius of sodium $\mathrm{r}=\frac{\sqrt{3}\times 4}{4}=1.732\text{Å}$

The radius value can be represented as follows,

$\mathrm{r}=17.32\times 10^{-1}\text{Å}$

Q.22. The orbital angular momentum of electron in $3\mathrm{s}$ is $\frac{\mathrm{xh}}{2\pi}$. Then the value of x is :

Answer: 0

Solution: The orbital angular momentum of an electron $=\frac{\mathrm{h}}{2\pi}\sqrt{\mathrm{l}(\mathrm{l}+1)}$.

where $\mathrm{l}=$ Azimuthal quantum number.

For $3\mathrm{s}$ orbital, the value of $\mathrm{l}=0$.

Thus, the orbital angular momentum = $\frac{\mathrm{h}}{2\pi}\sqrt{0\,(0+1)}=0$ (zero).

Mathematics

Q.23. Coefficient of x^4 in $\left(2x^3-\frac{1}{3x^8}\right)^5$ is

A) $-\frac{80}{3}$ B) $\frac{80}{3}$ C) $-\frac{70}{3}$ D) $\frac{70}{3}$

Answer: $-\frac{80}{3}$

Solution: General term of the binomial expansion of $\left(2x^3-\frac{1}{3x^8}\right)^5$ is

$T_{r+1}={}^5C_r\left(2x^3\right)^{5-r}\left(-\frac{1}{3x^8}\right)^r$

$\Rightarrow T_{r+1}={}^5C_r(2)^{5-r}\left(-\frac{1}{3}\right)^r x^{15-11r}$

For coefficient of x^4, we must have

$15-11r=4\Rightarrow r=1$

So, required coefficient is

$={}^5C_1(2)^4\left(-\frac{1}{3}\right)$

$=-5\times 16\times\frac{1}{3}$

$=-\frac{80}{3}$

Q.24. If $\sin^{-1}x=2\tan^{-1}x$, then number of integral values of x is equal to:

A) 0 B) 1 C) 2 D) More than 2

Answer: More than 2

Solution: Given that $\sin^{-1}x = 2\tan^{-1}x$

$\Rightarrow \sin^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$\Rightarrow x = \left(\frac{2x}{1+x^2}\right)$

$\Rightarrow x + x^3 = 2x$

$\Rightarrow x^3 - x = 0$

$\Rightarrow x\left(x^2 - 1\right) = 0$

$\Rightarrow x = 0, -1, 1.$

Therefore x has more than 2 roots.

Q.25. If $x^2 - \sqrt{2}x + 2 = 0$ has roots $\alpha^{14} + \beta^{14}$ is:

A) -256 B) -128 C) $-128\sqrt{2}$ D) $-256\sqrt{2}$

Answer: -128

Solution: Given,

$x^2 - \sqrt{2}x + 2 = 0$

$\Rightarrow x = \frac{\sqrt{2}\pm\sqrt{6}i}{2}$

$\Rightarrow x = \sqrt{2}\left(\frac{1\pm\sqrt{3}i}{2}\right)$

$\Rightarrow x = \sqrt{2}\left(\frac{1}{2} \pm \frac{\sqrt{3}i}{2}\right)$

$\Rightarrow x = \sqrt{2}\left(\cos\frac{\pi}{3} \pm i\sin\frac{\pi}{3}\right)$

So, $\alpha = \sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = \sqrt{2}e^{\frac{i\pi}{3}}$

And $\beta = \sqrt{2}\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) = \sqrt{2}e^{\frac{-i\pi}{3}}$

Now $\alpha^{14} + \beta^{14} = \left(\sqrt{2}e^{\frac{i\pi}{3}}\right)^{14} + \left(\sqrt{2}e^{\frac{-i\pi}{3}}\right)^{14}$

$\Rightarrow \alpha^{14} + \beta^{14} = \left(\sqrt{2}\right)^{14}\left(e^{\frac{i14\pi}{3}} + e^{\frac{-i14\pi}{3}}\right)$

$\Rightarrow \alpha^{14} + \beta^{14} = \left(\sqrt{2}\right)^{14}\left(\cos\frac{14\pi}{3} + i\sin\frac{14\pi}{3} + \cos\frac{14\pi}{3} - i\sin\frac{14\pi}{3}\right)$

$\Rightarrow \alpha^{14} + \beta^{14} = 2^7\left(2\cos\frac{14\pi}{3}\right)$

$\Rightarrow \alpha^{14} + \beta^{14} = 2^8\left(\cos\left(4\pi + \frac{2\pi}{3}\right)\right)$

$\Rightarrow \alpha^{14} + \beta^{14} = 2^8\left(\cos\frac{2\pi}{3}\right)$

$\Rightarrow \alpha^{14} + \beta^{14} = -2^8 \times \frac{1}{2} = -128$

Q.26. $\int_0^{\frac{\pi}{4}}\left(\frac{\tan^{50}x}{\tan^{51}x+\tan^{49}x}\right)dx =$

A) $\frac{1}{4}$ B) $\frac{1}{8}$ C) $\frac{1}{2}$ D) $\frac{1}{16}$

Answer: $\frac{1}{4}$

Solution: Let

$I=\int_0^{\frac{\pi}{4}}\left(\frac{\tan^{50}x}{\tan^{51}x+\tan^{49}x}\right)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\left(\frac{1}{\tan x+\cot x}\right)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\left(\frac{1}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}\right)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}\left(\frac{\sin x\cos x}{\sin^2x+\cos^2x}\right)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}(\sin x\cos x)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{4}}(\sin x)d(\sin x)$

$\Rightarrow I=\left[\frac{\sin^2x}{2}\right]_0^{\frac{\pi}{4}}$

$\Rightarrow I=\frac{1}{4}$

Q.27. The range of $\frac{4+(\sin x)^4}{1+x^2}$ is

A) $(0,1]$ B) $(0,4]$ C) $(0,3]$ D) none of these

Answer: $(0,4]$

Solution: The given function is $\frac{4+(\sin x)^4}{1+x^2}$

The function is maximum when the denominator is minimum.

The graph of $y=1+x^2$ is an upward parabola with vertex at $(0,1)$.

That means the function $1+x^2$ is minimum when $x=0$

The maximum value of the given function $\frac{4+(\sin x)^4}{1+x^2}$ is at $x=0$.

$=\frac{4+0}{1+0}=4$

And the minimum value of the function is when $x\to\infty$.

$\Rightarrow \text{As }\ x\to\infty,\ \frac{4+(\text{sinx})^4}{1+\text{x}^2}\to 0$

Therefore the range of the given function is $(0,4]$.

Q.28. The number of six digit number formed by using the digits $\{1,2,3,4,5,6\}$ which are divisible by 6 (repetition is not allowed)

A) 120 B) 360 C) 240 D) 720

Answer: 360

Solution: Given,

Six-digit number be formed from $\{1, 2, 3, 4, 5, 6\}$

Now addition of $1 + 2 + 3 + 4 + 5 + 6 = 21$ which is divisible by 3

Now number to be divisible by 6, it must be divisible by 2 & 3,

So number which are divisible by 2 will have last place as $2,\ 4$ & 6 which can be done by 3 ways,

Now first 5 places can be filled by $5! = 120$ ways,

So, total number which are divisible by 6 will be $120 \times 3 = 360$ ways.

Q.29. If matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ \alpha & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ and $|A| = 2$ then value of $|\alpha adj(\alpha adj(\alpha A))|$ is

A) 2^{25} B) 2^{24} C) 2^{20} D) 2^{16}

Answer: 2^{25}

Solution:

Given that $A = \begin{bmatrix} 1 & 2 & 1 \\ \alpha & 3 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ and $|A| = 2$

$\Rightarrow |A| = 1(3 - 2) - 2(\alpha - 6) + 1(\alpha - 9)$

$\Rightarrow 1 - 2\alpha + 12 + \alpha - 9 = 2$

$\Rightarrow \alpha = 2$

We know that $|adjA| = |A|^{n-1} |adj(adjA)| = |A|^{(n-1)^2}$ and $adj(kA) = k^{n-1} adj(A)$

$\Rightarrow |\alpha adj(\alpha adj(\alpha A))| = |2adj(2adj(2A))|$

$= 2^3 \left|adj\left(2 \times 2^{3-1} adj(A)\right)\right|$

$= 2^3 \left|adj\left(2^3 adj(A)\right)\right|$

$= 2^3 \left|\left(2^3\right)^2 adj(adj(A))\right|$

$= 2^3 \left|2^6 adj(adj(A))\right|$

$= 2^3 \times \left(2^6\right)^3 |adj(adj(A))|$

$= 2^{21} |A|^{(3-1)^2}$

$= 2^{21} |2|^4$

$= 2^{25}$

Therefore, the required answer is 2^{25}.

Q.30. In a given data set, mean of 40 observations is 50 and standard deviation is 12. Two readings which were 20 & 25 mistakenly taken as 40 and 45. Find correct variance of data set.

A) 169 B) 150 C) 178 D) 180

Answer: 178

Solution: Let the observations be $x_1,\ x_2,\ x_3,\ \ldots,\ x_{38},\ 40,\ 45$,

Mean$=50$

$\Rightarrow \frac{x_1+x_2+x_3+\ldots+x_{38}+40+45}{40}=50$

$\Rightarrow x_1+x_2+x_3+\ldots+x_{38}=1915 \quad \ldots.(\mathrm{i})$

Hence, new mean

$(\overline{X})_{\text{new}}=\frac{1915+20+25}{40}=49$

Now,

$$\mathrm{S.D}=\sqrt{\frac{\sum_{\mathrm{i}=1}^{38}\mathrm{x}_{\mathrm{i}}^2+40^2+45^2}{40}-(50)^2}$$

$$\Rightarrow 12=\sqrt{\frac{\sum_{\mathrm{i}=1}^{38}\mathrm{x}_{\mathrm{i}}^2+3625}{40}-2500}$$

$\Rightarrow \sum_{\mathrm{i}=1}^{38}\mathrm{x}_{\mathrm{i}}^2=102135$

Now,

$$(\text{Variance})_{\text{new}}=\frac{\sum_{\mathrm{i}=1}^{38}\mathrm{x}_{\mathrm{i}}^2+20^2+25^2}{40}-(49)^2$$

$$\Rightarrow (\text{Variance})_{\text{new}}=\frac{102135+20^2+25^2}{40}-(49)^2$$

$\Rightarrow (\text{Variance})_{\text{new}}=2579-2401$

$\Rightarrow (\text{Variance})_{\text{new}}=178$

Q.31. Given $\frac{x+3}{-3}=\frac{y-2}{2}=\frac{z-5}{5}$. Which of the following lines in options is coplanar with the given line?

A) $\frac{x+1}{-1}=\frac{y-1}{1}=\frac{z-5}{5}$ B) $\frac{x+1}{1}=\frac{y+1}{-1}=\frac{z-5}{5}$ C) $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-5}{5}$ D) $\frac{x-1}{-1}=\frac{y+2}{-2}=\frac{z-5}{4}$

Answer: $\frac{x+1}{-1}=\frac{y-1}{1}=\frac{z-5}{5}$

Solution:

We know that two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are coplanar if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$

Let us verify this option.

The given lines are $\frac{x+3}{-3}=\frac{y-2}{2}=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-1}{1}=\frac{z-5}{5}$.

$$=\begin{vmatrix} -1-(-3) & 1-2 & 5-5 \\ -3 & 2 & 5 \\ -1 & 1 & 5 \end{vmatrix}$$

$$=\begin{vmatrix} 2 & -1 & 0 \\ -3 & 2 & 5 \\ -1 & 1 & 5 \end{vmatrix}$$

$=2(10-5)-(-1)(-15-(-5))+0$

$=2(5)+(-10)=0$

Therefore, this is the correct option.

Q.32. The statement $(\sim p\wedge q)\vee(p\wedge \sim q)\vee(\sim p\wedge \sim q)$ is equivalent to

A) tautology B) fallacy C) $(p \vee q)$ D) $\sim(p \wedge q)$

Answer: $\sim(p \wedge q)$

Solution:

p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$(\sim p \wedge q)$	$(p \wedge \sim q)$	$(\sim p \wedge \sim q)$	$(\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$
T	T	F	F	F	F	F	F	F
T	F	F	T	T	F	T	F	T
F	T	T	F	T	T	F	F	T
F	F	T	T	T	F	F	T	T

Therefore, $(\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) \equiv \sim(p \wedge q)$.

Q.33. The rank of the word "MONDAY" is

Answer: 327

Solution: The given word is MONDAY

Arranging the letters alphabetically, we get

ADMNOY

When the word starts with any of the letters A/D, the number of possibilities$= 5! \times 2 = 240$

Now when the word starts with MA, then the number of possibilities$= 4! = 24$

Now when the word starts with MD, then the number of possibilities $= 4! = 24$

Now when the word starts with MN, then the number of possibilities $= 4! = 24$

Now when the word starts with MOA, then the number of possibilities$= 3! = 6$

Now when the word starts with MOD, then the number of possibilities $= 3! = 6$

Now when the word starts with MONA, then the number of possibilities $= 2! = 2$

Now when the word starts with MONDAY, then the number of possibilities$= 1$

Rank $= 240 + 24 \times 3 + 6 \times 2 + 2 + 1 = 327$

Hence, rank of the word MONDAY is 327.