JEE Main 2023 (Session 2)

April 6 Shift 2

Physics

Q.1. An object starts moving with an initial speed $10 \mathrm{~m~s}^{-1}$ and acceleration $2 \mathrm{~m~s}^{-2}$ along positive x-direction. The time taken to attain $60 \mathrm{~m~s}^{-1}$ speed is

A) $25 \mathrm{~s}$ B) $20 \mathrm{~s}$ C) $30 \mathrm{~s}$ D) $15 \mathrm{~s}$

Answer: $25 \mathrm{~s}$

Solution: The equation of motion for the object is given by

$v=u+at \quad \ldots(1)$

Substitute the values of the known parameters into equation (1) and solve to calculate the required time.

$$60 = 10+2\times t$$
$$\Rightarrow t = \frac{60-10}{2}$$
$$= 25$$

Hence, the required time is $25 \mathrm{~s}$

Q.2. Potential energy of an electron is defined as $U=\frac{1}{2}m\omega^2x^2$ and follows Bohr's law. Radius of orbit as function of n depends on

A) n^2 B) $\frac{1}{\sqrt{n}}$ C) $\sqrt{n}$ D) $n^{2/3}$

Answer: $\sqrt{n}$

Solution: Here, in the question, x is the radius of orbit.

So, we need to find x as a function of n.

i.e., $x=f(n)$

The angular momentum of an electron in a Bohr orbit is given as below,

$mvx=\frac{nh}{2\pi}$-----(i)

But, $v=x\omega$-----(ii)

Substituting (ii) in (i), we get

$m\omega x^2=\frac{nh}{2\pi}$

Hence, $x \propto \sqrt{n}$

Q.3. If W is the weight on the surface of Earth then weight of same body at a height $\frac{Re}{4}$ above the surface of Earth is equal to

($R_e=$ Radius of Earth)

A) $\frac{4}{5}W$ B) $\frac{16}{25}W$ C) $\frac{25}{16}W$ D) $\frac{5}{4}W$

Answer: $\frac{16}{25}W$

Solution: The variation of acceleration due to gravity with height from the surface of the Earth can be written as

$$g' = \frac{GMe}{(Re+h)^2} \quad \ldots (1)$$

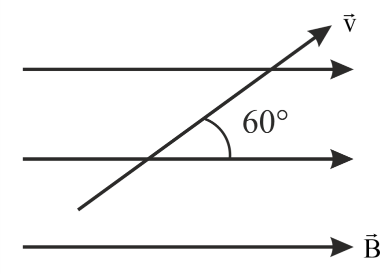
Substitute $\frac{Re}{4}$ for h into equation (1) and simplify to obtain the new value of the acceleration due to gravity at the given height.

$$g' = \frac{GMe}{\left(Re+\frac{Re}{4}\right)^2}$$
$$= \frac{16GMe}{25Re^2}$$
$$= \frac{16}{25}g \quad \ldots (2)$$

Multiply m on both sides of equation (2) to obtain the new weight of the object.

$$mg' = \frac{16}{25}mg$$
$$\Rightarrow W' = \frac{16}{25}W$$

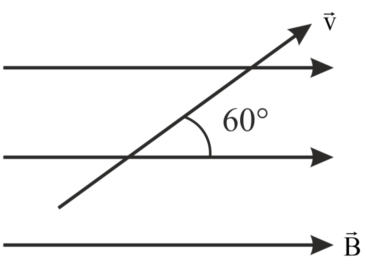
Q.4. A proton is projected with speed v in magnetic field B of magnitude $1\ \mathrm{T}$. If angle between velocity and magnetic field is 60^o as shown below. Kinetic energy of proton is $2\ \mathrm{eV}$ (mass of proton $= 1.67 \times 10^{-27}\ \mathrm{kg}$, $e = 1.6 \times 10^{-19}\ \mathrm{C}$). The pitch of the path of proton is approximately



A) $6.28 \times 10^{-2}\ \mathrm{m}$ B) $6.28 \times 10^{-4}\ \mathrm{m}$ C) $3.14 \times 10^{-2}\ \mathrm{m}$ D) $3.14 \times 10^{-4}\ \mathrm{m}$

Answer: $6.28 \times 10^{-4}\ \mathrm{m}$

Solution:



$$\text{Pitch } = v\cos(60^o) \times T$$
$$= \sqrt{\frac{2KE}{m}} \times \frac{1}{2} \times \frac{2\pi m}{eB}$$
$$= \sqrt{2mKE} \times \frac{\pi}{e}$$
$$= \sqrt{2 \times 1.67 \times 10^{-27} \times \left(2 \times 1.6 \times 10^{-19}\right)} \times \frac{3.14}{1.6\times10^{-19}}$$
$$= \sqrt{10.688 \times 10^{-46}} \times 1.9625 \times 10^{19}$$
$$= 3.269 \times 10^{-23} \times 1.9625 \times 10^{19}$$
$$= 6.415 \times 10^{-4} \approx 6.28 \times 10^{-4}\ \mathrm{m}$$

Q.5. Find the ratio of root-mean-square speed of oxygen gas molecules to that of hydrogen gas molecules, if temperature of both the gases are same.

A) $\frac{1}{4}$ B) $\frac{1}{16}$ C) $\frac{1}{32}$ D) $\frac{1}{8}$

Answer: $\frac{1}{4}$

Solution: The formula to calculate the rms speed for oxygen gas is given by

$$v_O = \sqrt{\frac{3RT}{M_O}} \quad \ldots (1)$$

The formula to calculate the rms speed for hydrogen gas is given by

$$v_h = \sqrt{\frac{3RT}{M_h}} \quad \ldots (2)$$

Divide the equations to obtain the ratio of the rms speeds.

$$\frac{v_O}{v_h} = \frac{\sqrt{3RT/M_O}}{\sqrt{3RT/M_h}}$$
$$= \sqrt{\frac{M_h}{M_O}} \quad \ldots (3)$$

Substitute the values of the molecular masses of the consecutive gases into equation (3) to calculate the required ratio,

$$\Rightarrow \frac{v_O}{v_h} = \sqrt{\frac{2}{32}}$$
$$= \frac{1}{4}$$

Q.6. In amplitude modulation with carrier frequency (A_c) and modulating frequency (A_m), modulation index is 60%. If $A_c - A_m = 3\ \mathrm{V}$, then $A_c + A_m$ is equal to

A) $6\ \mathrm{V}$ B) $12\ \mathrm{V}$ C) $4\ \mathrm{V}$ D) $15\ \mathrm{V}$

Answer: $12\ \mathrm{V}$

Solution: The formula to calculate the modulation index (m) is given by

$$m = \frac{A_m}{A_c} \quad \ldots\ldots (1)$$

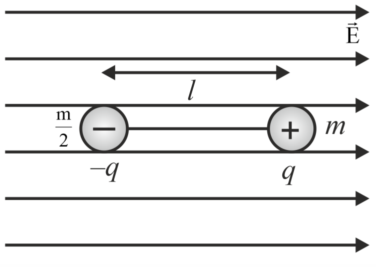
Use the method of Componendo Dividendo on both sides of equation (1) and simplify to obtain the required sum.

$$\frac{m+1}{m-1} = \frac{A_m + A_c}{A_m - A_c}$$
$$\Rightarrow A_m + A_c = \frac{m+1}{m-1}(A_m - A_c) \quad \ldots (2)$$

Substitute the values of the known parameters into equation (2) to calculate the required value.

$$A_m + A_c = \frac{0.6+1}{0.6-1} \times (-3\ \mathrm{V})$$
$$= 12\ \mathrm{V}$$

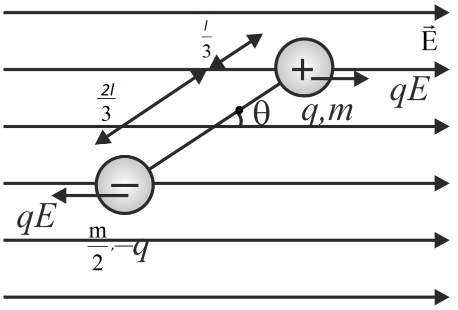
Q.7. An electric dipole is shown in the figure. If it is displaced angularly by a small angle with respect to electric field, then angular frequency of oscillation is given by



A) $\sqrt{\frac{6qE}{ml}}$ B) $\sqrt{\frac{3qE}{ml}}$ C) $\sqrt{\frac{2qE}{ml}}$ D) $\sqrt{\frac{qE}{ml}}$

Answer: $\sqrt{\frac{3qE}{ml}}$

Solution:



Here, dipole oscillates about centre of mass (COM).

Also, COM is towards larger mass and its location is $\frac{l}{3}$ from m.

$\tau = -qlE\sin(\theta) = I\alpha$

For small angles, $\sin(\theta) \approx \theta$,

So, $\tau = -qlE\theta = I\alpha$---(i)

Calculation of I:

$I = \frac{m}{2}\left(\frac{2l}{3}\right)^2 + m\left(\frac{l}{3}\right)^2 = \frac{ml^2}{3}$---(ii)

Substituting (ii) in (i), we get,

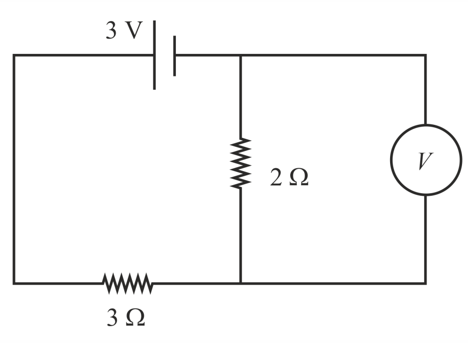
$\frac{-3qlE\theta}{ml^2} = \alpha$

On comparing above equation with,

$\alpha = -\omega^2\theta$

We get, $\omega = \sqrt{\frac{3qE}{ml}}$

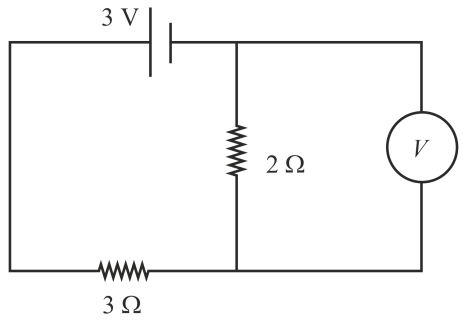
Q.8. In the circuit shown reading of the ideal voltmeter used is equal to _____ volts.



A) $3\ \text{V}$ B) $1.8\ \text{V}$ C) $1.2\ \text{V}$ D) $Zero$

Answer: $1.2\ \text{V}$

Solution:



The equivalent resistance (R) of the circuit can be calculated as follows

$$R = 3\ \Omega + 2\ \Omega$$
$$= 5\ \Omega$$

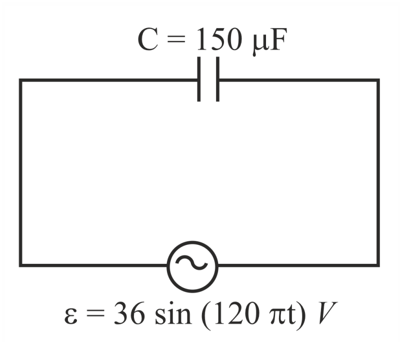
The current (i) in the circuit can be calculated as follows

$$i = \frac{3\ \text{V}}{5\ \Omega}$$
$$= 0.6\ \text{A}$$

Hence, the reading in the voltmeter is given by

$$v = 0.6\ \text{A} \times 2\ \Omega$$
$$= 1.2\ \text{V}$$

Q.9. In the given AC circuit, find maximum current through the capacitor



A) 0.65π A B) 0.35π A C) 0.2π A D) 0.8π A

Answer: 0.65π A

Solution: The formula to calculate the instantaneous charge (q) on the capacitor is given by

$$q = C\mathscr{E}$$
$$= 36C\sin(120\pi t) \quad \ldots(1)$$

The value of the instantaneous current (i) through the circuit is can be calculated as follows

$$i = \frac{dq}{dt}$$
$$= \frac{d}{dt}(36C\sin(120\pi t))$$
$$= 4320\pi C\cos(120\pi t) \quad \ldots.(2)$$

The value of the maximum current is, then, given by

$$i_m = 4320\pi \times 150 \times 10^{-6}\ \text{A}$$
$$\approx 0.65\pi\ \text{A}$$

Q.10. Radius of first orbit in $H-atom$ is a_0. Then de-Broglie wavelength of electron in the third orbit is

A) $3\pi a_0$ B) $6\pi a_0$ C) $9\pi a_0$ D) $12\pi a_0$

Answer: $6\pi a_0$

Solution: Radius of nth orbit is given by,

$r = a_0 \frac{n^2}{Z} = a_0 n^2 \left(for\ H - atom,\ Z = 1 \right)$ ---(i)

The de-Broglie wavelength of electron is given by,

$\lambda = \frac{h}{mv}$ ---(ii)

We also know by Bohr's law, that

$mvr = \frac{nh}{2\pi}$

$\Rightarrow mv = \frac{nh}{2\pi r}$ ---(iii)

Substituting, (iii) in (ii), we get,

$\lambda = \frac{2\pi r}{n}$ ---(iv)

Substituting (i) in (iv), we get,

$\lambda = 2\pi a_0 n$

Hence, de-Broglie wavelength of electron in the third orbit is $6\pi a_0$.

Q.11. Choose the incorrect statement from the given statements

A) Planets revolve around the sun with constant linear speed.

B) Energy of planet in elliptical orbit is constant.

C) Satellite in circular motion have constant energy.

D) Body falling towards the earth results in negligible displacement of earth

Answer: Planets revolve around the sun with constant linear speed.

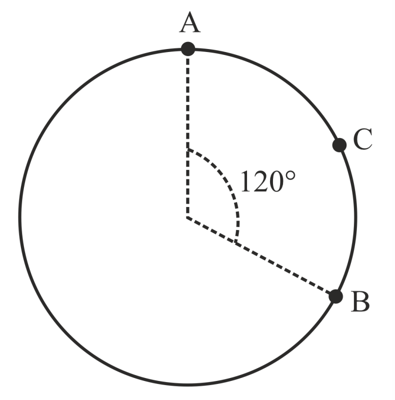
Solution: From Kepler's second law of planetary motion, the linear speed of a planet is maximum when its distance from the sun is least. So it changes depending on the position of the planet.

Planets follow two conservation laws: total energy stays constant & angular momentum stays constant throughout the elliptical orbital motion.

In circular motion about the Earth, a satellite remains at a fixed distance from the surface of the Earth at all the time, therefore Satellite in circular motion have constant energy.

As mass of earth is very large compared to any object, during free fall of the object, centre of mass of the system remains constant but due to large mass of the earth very little displacement of earth happens.

Q.12. A particle moves from A to B via C with uniform speed $\pi\ \text{m s}^{-1}$. Average velocity during the journey is equal to



A) $\sqrt{3}\ \text{m s}^{-1}$

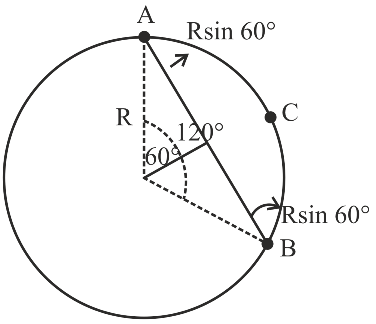
B) $\frac{\sqrt{3}}{2}\ \text{m s}^{-1}$

C) $\frac{3\sqrt{3}}{2}\ \text{m s}^{-1}$

D) $2\ \text{m s}^{-1}$

Answer: $\frac{3\sqrt{3}}{2}\ \text{m s}^{-1}$

Solution: Let's consider the following figure-



From the figure, the displacement of the particle on moving from point A to B via C can be calculated as

$$AB = R\sin 60^\circ + R\sin 60^\circ$$
$$= 2R\sin 60^\circ \quad \ldots.(1)$$

Also, the time taken (t) by the particle to move from A to B is given by

$$t = \frac{AC}{v}$$
$$= \frac{\frac{2\pi R}{3}}{\pi} \text{ s}$$
$$= \frac{2R}{3} \text{ s} \quad \ldots.(2)$$

Hence, the average velocity of the particle (v_a) can be calculated as follows

$$v_a = \frac{AB}{t}$$
$$= \frac{2R\sin 60^\circ \text{ m}}{2\frac{R}{3} \text{ s}}$$
$$= \frac{3\sqrt{3}}{2} \text{ m s}^{-1}$$

Q.13. A solid sphere and a ring have equal masses and equal radius of gyration. If sphere is rotating about its diameter and ring about an axis passing through the centre and perpendicular to its plane, then the ratio of radius is $\sqrt{\frac{x}{2}}$, then find the value of x.

Answer: 5

Solution: The formula to calculate the moment of inertia of the sphere is given by

$$I_s = mK^2 = \frac{2}{5}mR_s{}^2 \quad \ldots(1)$$

The formula to calculate the moment of inertia of the ring is given by

$$I_r = mK^2 = mR_r{}^2 \quad \ldots(2)$$

Equate equations (1) and (2) and solve to obtain the required ratio.

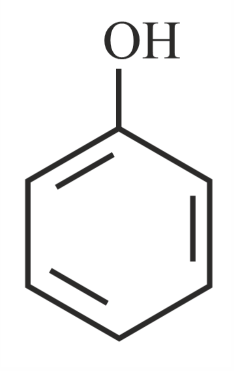
$$\frac{2}{5}mR_s{}^2 = mR_r{}^2$$
$$\Rightarrow \frac{R_s{}^2}{R_r{}^2} = \frac{5}{2}$$
$$\Rightarrow \frac{R_s}{R_r} = \sqrt{\frac{5}{2}}$$

Comparing the above equation with the given expression it can be concluded that $x = 5$.

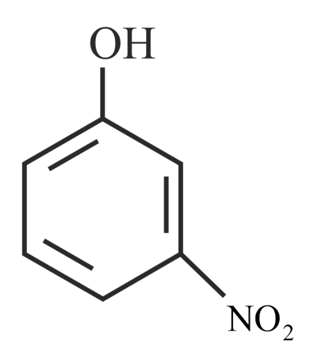
Chemistry

Q.14. Which of the following compound is most acidic?

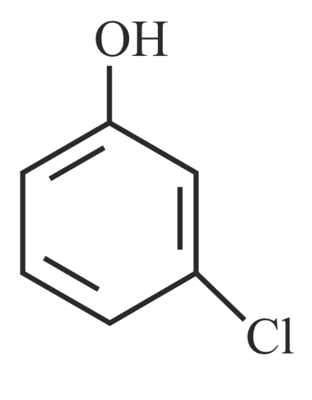
A)



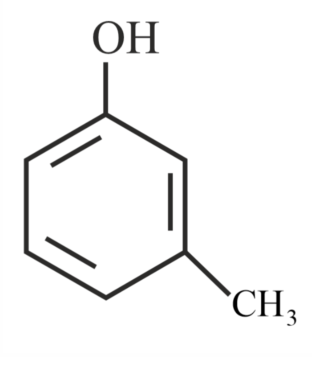
B)



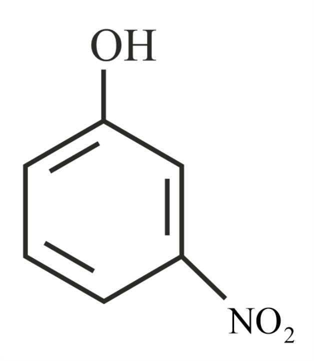
C)



D)

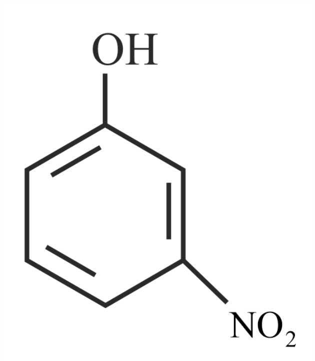


Answer:



Solution: The acidity of a compound is directly proportional to the stability of its conjugate base. When an acidic compound donates a proton, it forms a conjugate base. The stability of the conjugate base determines the strength of the acid. Here conjugate base is stable when we have electron withdrawing group (NO_2) by $-I$ effect. The electron-withdrawing effect of the nitro group in meta-nitrophenol increases its acidity by enhancing the ability of the hydroxyl group to donate a proton.

So Compound



is stable among the given.

Q.15. Nessler's reagent does not have :

A) K B) Hg C) N D) I

Answer: N

Solution: An alkaline solution of K_2HgI_4 is called Nessler's reagent. Nessler's reagent is used for the qualitative analysis of ammonia. A brown colour precipitate is formed when Nessler's reagent is added to concentrated ammonium salts.

Nessler's reagent does not contain nitrogen, therefore, option C is correct.

Q.16. Which of the following is not obtained on electrolysis of brine solution ?

A) $NaOH$ B) H_2 gas C) Cl_2 gas D) Na

Answer: Na

Solution: Sodium chloride and water ionizes as:

$$NaCl(aq) \rightarrow Na^{+}(aq) + Cl^{-}(aq) \text{ (almost completely ionized)}$$

$$H_2O(l) \rightleftharpoons H^{+}(aq) + OH^{-}(aq) \text{ (only slightly ionized)}$$

At cathode: Both Na^{+}(aq) and H^{+}(aq) are present near the cathode. Since the discharge potential of H^{+} ions is lower than that of Na^{+} ions, therefore, H^{+} ions are discharged in preference to Na^{+} ions. Hence, H_2 gas is evolved at the cathode while Na^{+} ions remain in the solution.

$$2H^{+} + 2e^{-} \rightarrow H_2(g)$$

At anode: Both Cl^{-} and OH^{-} ions are present near the anode. Since the discharge potential of Cl^{-} ions is lower than that of OH^{-} ions, therefore, Cl^{-} ions are discharged in preference to OH^{-} ions. Hence, Cl_2 gas is evolved at the anode while OH^{-} ions remain in the solution.

$$2\,Cl^{-} \rightarrow Cl_2(g) + 2e^{-}$$

Therefore, the products formed are $H_2,\ Cl_2 \text{ and } NaOH.$

Therefore, the correct option is D.

Q.17. Statement-1: Morphine and many of its homologues, when administered in medicinal doses, relieve pain and produce sleep
Statement-2: Morphine narcotics are sometimes referred to as opiates, since they are obtained from the opium poppy.

A) Both Statement- 1 and Statement- 2 are correct
B) Both Statement- 1 and Statement- 2 are incorrect
C) Statement- 1 is correct and Statement- 2 is incorrect
D) Statement- 1 is incorrect and Statement- 2 is correct

Answer: Both Statement- 1 and Statement- 2 are correct

Solution: Morphine and many of its homologues, when administered in medicinal doses, relieve pain and produce sleep. In poisonous doses, these produce stupor, coma, convulsions and ultimately death. Morphine narcotics are sometimes referred to as opiates, since they are obtained from the opium poppy.
These analgesics are chiefly used for the relief of postoperative pain, cardiac pain and pains of terminal cancer, and in childbirth.

Q.18. Oxidation state of Mn in $KMnO_4$ changes by three units in which medium ?

A) Strongly acidic
B) Strongly Basic
C) Aqueous neutral
D) Weakly acidic

Answer: Aqueous neutral

Solution: $KMnO_4$ acts as an oxidising agent in the neutral medium and gets reduced to MnO_2, in acidic medium it changes to Mn^{2+} and in strongly basic medium it changes to MnO_4^{-}.

The oxidation state of Mn in $KMnO_4$ is $+7$

The oxidation state of Mn in MnO_2 is $+4$

Here the change in oxidation state is 3.

Therefore, option C is correct.

Q.19. Which of the following is most basic?

A) Tl_2O_3
B) Tl_2O
C) Cr_2O_3
D) B_2O_3

Answer: Tl_2O

Solution: Lower the oxidation state, more basic is the oxide. In Tl_2O_3 the oxidation state of Tl is $+3$. In Tl_2O the oxidation state is $+1$. Here Tl has least oxidation state and it will be more basic oxide. Cr_2O_3 is amphoteric in nature. B_2O_3 is an acidic oxide.

Q.20. Which of the following has highest hydration energy?

A) Be^{2+}
B) Mg^{2+}
C) Ca^{2+}
D) Ba^{2+}

Answer: Be^{2+}

Solution: Be^{2+} ion has small size and high charge. Hence, it has high polarising power and can attract several water molecules. Thus, it has the highest hydration energy among the given ions. Smaller sized and highly charged metal ions have higher hydration energy. In the alkaline earth metals the hydration energy for charged ions is greater than the large-sized charged ions. Therefore, order of hydration energy in this group is

$Be^{2+} > Mg^{2+} > Ca^{2+} > Sr^{2+} > Ba^{2+}$.

Q.21. IUPAC name of the compound $K_3[Co(C_2O_4)_3]$ is:

A) Potassium trioxalatocobalt(III)

B) Potassium trioxalatocobaltate (III)

C) Potassium cobalttrioxalate (II)

D) Potassium oxalatocobaltate (III)

Answer: Potassium trioxalatocobaltate (III)

Solution: Ligands that include a numerical prefix in the name use the prefixes bis for 2, tris for 3, or tetrakis for 4 to indicate their number.

If the complex ion is an anion, we drop the ending of the metal name and add –ate.

In the given coordination compound, Co is the cobalt metal, $C_2O_4^{2-}$ are the ligands named as oxalato.

The charge on $[Co(C_2O_4)_3]$ is -3 and oxalate is having an oxidation state of -2.

$X + 3(-2) = -3$
$X = +3$

Hence, the IUPAC name of the compound $K_3[Co(C_2O_4)_3]$ is Potassium trioxalatocobaltate(III).

Q.22. Which one doesn't gives Pb^{+2} test?

A) Iodite

B) Chromate

C) Sulfate

D) Nitrate

Answer: Nitrate

Solution: Pb^{+2} test is used to identify the presence of lead ions in a solution. Lead ions react with many anions to form insoluble salts, which are usually colored and can be easily identified.

Iodite ions and chromate ions both react with lead ions to form insoluble lead salts, so they would give a positive Pb^{+2} test.

Sulfate ions also react with lead ions to form a white precipitate of lead sulfate ($PbSO_4$), indicating the presence of lead ions in the solution.

However, nitrate ions do not react with lead ions to form an insoluble salt. Nitrate ions are usually soluble in water and do not form precipitates with most cations, including lead ions. Therefore, nitrate ions would not give a positive Pb^{+2} test.

Q.23. Consider the following

(i). D.D.T (ii) Aldrin (iii) Sodium arsinate (iv) Sodium chlorate

How many of these are pesticides?

A) 1

B) 2

C) 3

D) 4

Answer: 2

Solution: Dichlorodiphenyltrichloroethane (DDT) and Aldrin are indeed pesticides. DDT was widely used as an insecticide, aldrin is also a pesticide that was commonly used to control insects.

Sodium arsenate and sodium chlorate are indeed herbicides.Sodium arsenate was once used to control weeds.

Q.24. How many of the following have same Relative lowering vapour pressure?

(A) $1M\ NaCl$

(B) $1.5\ M\ AlCl_3$

(C) $1M\ urea$

(D) $2M\ Na_2SO_4$

Answer: 2

Solution: Relative lowering vapour pressure can be calculated by using the formula :

$\frac{\Delta P}{P^\circ}=i.\ X_{Solute}\equiv i.M$

i = Van't Hoff's factor

X = mole fraction

M = Molarity

(A) $\frac{\Delta P}{P^\circ}=1\times 2$

(B) $\frac{\Delta P}{P^\circ}=1\times 1=1$

(C) $\frac{\Delta P}{P^\circ}=4\times 1.5=6$

(D) $\frac{\Delta P}{P^\circ}=3\times 2=6$

Therefore, C and D have same Relative lowering vapour pressure.

Q.25. Types of unit cells are cubic, tetragonal, orthorhombic, hexagonal, monoclinic, triclinic, and rhombohedral.

How many of them can have BCC unit cell ?

Answer: 3

Solution: Cubic system shows three types of Bravais lattices - Primitive, base centered and face centered.

Tetragonal system shows two types of Bravais lattices - Primitive, body centered.

Orthorhombic system shows four types of Bravais lattices - Primitive, body centered, base centered and face centered.

Hexagonal system shows one type of Bravais lattice which is Primitive. Rhombohedral system shows one type of Bravais lattice which is Primitive. Monoclinic system shows two types of Bravais lattices - Primitive, base centered. Triclinic system shows one type of Bravais lattice which is Primitive.

Therefore, the correct answer is 3.

Mathematics

Q.26. If the coefficient of x^7 in $\left(\alpha x^2+\frac{1}{2\beta x}\right)^{11}$ and x^{-7} in $\left(x+\frac{1}{3\beta x^2}\right)^{11}$ are equal, then

A) $\alpha^6\beta=\frac{2^5}{3^6}$

B) $\alpha^6\beta=\frac{2^6}{3^5}$

C) $\alpha\beta^6=\frac{2^5}{3^6}$

D) $\alpha\beta^6=\frac{2^6}{3^5}$

Answer: $\alpha^6\beta=\frac{2^5}{3^6}$

Solution: Given,

The coefficient of x^7 in $\left(\alpha x^2+\frac{1}{2\beta x}\right)^{11}$ and x^{-7} in $\left(x+\frac{1}{3\beta x^2}\right)^{11}$ are equal,

Now finding the coefficient of x^7 in $\left(\alpha x^2+\frac{1}{2\beta x}\right)^{11}$ by using general term of binomial we get,

$T_{r+1}={}^{11}C_r\left(\alpha x^2\right)^{11-r}\left(\frac{1}{2\beta x}\right)^r$

$\Rightarrow T_{r+1}={}^{11}C_r(\alpha)^{11-r}\left(\frac{1}{2\beta}\right)^r x^{22-2r-r}$

Now solving $22-3r=7 \Rightarrow r=5$

Hence, $T_6={}^{11}C_5(\alpha)^6\left(\frac{1}{2\beta}\right)^5 x^7$

Now finding coefficient of x^{-7} in $\left(x+\frac{1}{3\beta x^2}\right)^{11}$ by using general term of binomial expansion we get,

$T_{r+1}={}^{11}C_r(x)^{11-r}\left(\frac{1}{3\beta x^2}\right)^r$

$\Rightarrow T_{r+1}={}^{11}C_r(x)^{11-3r}\left(\frac{1}{3\beta}\right)^r$

Now equating $11-3r=-7 \Rightarrow r=6$

So, $T_7={}^{11}C_6(x)^{-7}\left(\frac{1}{3\beta}\right)^6$

Now equating coefficient of x^7 & x^{-7} we get,

${}^{11}C_5(\alpha)^6\left(\frac{1}{2\beta}\right)^5={}^{11}C_6\left(\frac{1}{3\beta}\right)^6$

$\Rightarrow (\alpha)^6\left(\frac{1}{2\beta}\right)^5=\left(\frac{1}{3\beta}\right)^6$

$\Rightarrow \alpha^6\beta=\frac{2^5}{3^6}$

Q.27. The system of equations

$x+y+z=6$

$x+2y+\alpha z=5$

$x+2y+6z=\beta$

A) Infinitely many solutions for $\alpha=6,\ \beta=3$

B) Infinitely many solutions for $\alpha=6,\ \beta=5$

C) Unique solutions for $\alpha=6,\ \beta=5$

D) No solution for $\alpha=6,\ \beta=5$

Answer: Infinitely many solutions for $\alpha=6,\ \beta=5$

Solution: Given,

$x+y+z=6$

$x+2y+\alpha z=5$

$x+2y+6z=\beta$

The system of equation can be written as

$\Delta=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & \alpha\\ 1 & 2 & 6\end{bmatrix}$

For Unique solution, $\Delta \neq 0$

$\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & \alpha\\ 1 & 2 & 6\end{vmatrix}=(12-2\alpha)-(6-\alpha)+(2-2)$

$=6-\alpha$

That means for $\alpha=6$, $\Delta=0$

Now when $\alpha=6$

$\Delta_1=\begin{vmatrix}6 & 1 & 1\\ 5 & 2 & 6\\ \beta & 2 & 6\end{vmatrix}=0-(30-6\beta)+(10-2\beta)$

$=4(\beta-5)$

$\Delta_2=\begin{vmatrix}1 & 6 & 1\\ 1 & 5 & 6\\ 1 & \beta & 6\end{vmatrix}=(30-6\beta)-0+(\beta-5)$

$=25-5\beta$

$\Delta_3=\begin{vmatrix}1 & 1 & 6\\ 1 & 2 & 5\\ 1 & 2 & \beta\end{vmatrix}=(2\beta-10)-(\beta-5)+0$

$=\beta-5$

Clearly, at $\beta=5,\ \ \Delta_{\mathrm{i}}=0\ \text{ for }\ \mathrm{i}=1,2,3$

Therefore, at $\alpha=6, \beta=5$ system has infinitely many solution.

Q.28. If $f(x)+f(\pi-\mathrm{x})=\pi^2$, then $\int_0^\pi f(x)\sin x dx=$

A) π^2

B) $\frac{\pi^2}{2}$

C) $\frac{\pi^2}{4}$

D) $\frac{\pi^2}{3}$

Answer: π^2

Solution: Let

$I=\int_0^{\pi} f(x)\sin x dx \quad \ldots(1)$

Now using the property $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$ we get,

$\Rightarrow I=\int_0^{\pi} f(\pi-x)\sin(\pi-\mathrm{x})dx$

$\Rightarrow I=\int_0^{\pi} f(\pi-x)\sin\mathrm{x}\, dx \quad \ldots(2)$

Adding (1) & (2), we get

$2I=\int_0^{\pi}[f(x)+f(\pi-x)]\sin\mathrm{x}\, dx$

$\Rightarrow 2I=\pi^2\int_0^{\pi}\sin\mathrm{x}\, dx$

$\Rightarrow 2I=\pi^2[-\cos\mathrm{x}]_0^{\pi}$

$\Rightarrow 2I=2\pi^2$

$\Rightarrow I=\pi^2$

Q.29. Area (in sq. units) included between $y=f(x)=|x-1|+|x-2|$ and $y=3$ is

A) 4 B) 8 C) 2 D) None of these

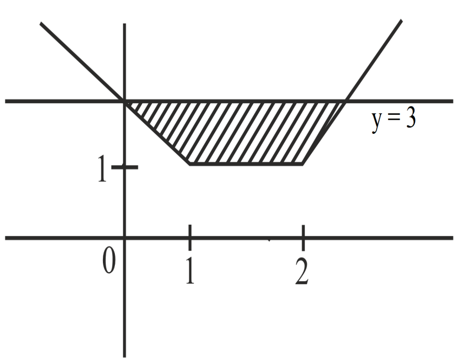
Answer: 4

Solution: Given:

$y=f(x)=|x-1|+|x-2|$

$$\Rightarrow f(x)=\begin{cases}3-2x: & x<1\\ 1; & 1\le x<2\\ 2x-3; & x\ge 2\end{cases}$$

Let us draw diagram of $y=f(x)$ & $y=3$ we get,



Required area is the area of trapezium which is

$=\frac{1}{2}(1+3)\times 2$

$=4$ sq. units

Q.30. Sum of all values of α for which $\hat{\imath}-2\hat{\jmath}+3\hat{k},\ 2\hat{\imath}-3\hat{\jmath}+4\hat{k},\ (\alpha+1)\hat{\imath}+2\hat{k}$ and $9\hat{\imath}+(\alpha-8)\hat{\jmath}+6\hat{k}$ are coplanar.

A) 6 B) 2 C) -2 D) 4

Answer: 2

Solution: Let the given vectors be $\vec{A}=\hat{\mathrm{i}}-2\hat{\mathrm{j}}+3\hat{\mathrm{k}}$, $\vec{B}=2\hat{\mathrm{i}}-3\hat{\mathrm{j}}+4\hat{\mathrm{k}}$, $\vec{C}=(\alpha+1)\hat{\mathrm{i}}+2\hat{\mathrm{k}}$ and $\vec{D}=9\hat{\mathrm{i}}+(\alpha-8)\hat{\mathrm{j}}+6\hat{\mathrm{k}}$

We know that if the vectors are coplanar then $\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right]=0$

$\Rightarrow \overrightarrow{AB}=\left(2\hat{\mathrm{i}}-3\hat{\mathrm{j}}+4\hat{\mathrm{k}}\right)-\left(\hat{\mathrm{i}}-2\hat{\mathrm{j}}+3\hat{\mathrm{k}}\right)=\hat{i}-\hat{j}+\hat{k}$

$\Rightarrow \overrightarrow{AC}=\left((\alpha+1)\hat{\mathrm{i}}+2\hat{\mathrm{k}}\right)-\left(\hat{\mathrm{i}}-2\hat{\mathrm{j}}+3\hat{\mathrm{k}}\right)=\alpha\hat{i}+2\hat{j}-\hat{k}$

$\Rightarrow \overrightarrow{AD}=\left(9\hat{\mathrm{i}}+(\alpha-8)\hat{\mathrm{j}}+6\hat{\mathrm{k}}\right)-\left(\hat{\mathrm{i}}-2\hat{\mathrm{j}}+3\hat{\mathrm{k}}\right)=8\hat{i}+(\alpha-6)\hat{j}+3k$

Now,

$$\Rightarrow \begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & \alpha-6 & 3 \end{vmatrix}=0$$

$\Rightarrow 1(6+\alpha-6)+(3\alpha+8)+\left(\alpha^2-6\alpha-16\right)=0$

On Simplifying we get,

$\Rightarrow \alpha^2-2\alpha-8=0$

Therefore, sum of the roots is $-(-2)=2$.

Q.31. If V is volume of parallelepiped whose edges determined by vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, then volume of parallelepiped whose edges determined by vectors $\vec{a}$, $\vec{a}+\vec{b}$, $\vec{a}+2\vec{b}+3\vec{c}$ is

A) $6V$ B) V C) $2V$ D) $3V$

Answer: $3V$

Solution: We know that,

Volume of parallelepiped whose edges determined by vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ is $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$.

Now it is given that V is volume of parallelepiped $\vec{a}$, $\vec{b}$, $\vec{c}$, so required volume of parallelepiped with new edges will be

$=\left[\vec{a}\ \ \vec{a}+\vec{b}\ \ \vec{a}+2\vec{b}+3\vec{c}\right]$

$$=\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix}\left[\vec{a}\ \vec{b}\ \vec{c}\right]$$

$=3\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$=3V$

Q.32. Consider

$S_1:(p\Rightarrow q)\vee(\sim p\wedge q)$ is a tautology

$S_2:(q\Rightarrow p)\vee(\sim p\wedge q)$ is a contradiction

A) S_1 is true and S_2 is false

B) S_1 is false and S_2 is true

C) Both S_1 and S_2 are false

D) Both S_1 and S_2 are true

Answer: Both S_1 and S_2 are false

Solution:

p	q	$\sim p$	$\sim p\wedge q$	$p\Rightarrow q$	$q\Rightarrow p$	$(p\Rightarrow q)\vee(\sim p\wedge q)$	$(q\Rightarrow p)\vee(\sim p\wedge q)$
T	T	F	F	T	T	T	T
T	F	F	F	F	T	F	T
F	T	T	T	T	F	T	T
F	F	T	F	T	T	T	T

Hence, $(q\Rightarrow p)\vee(\sim p\wedge q)$ is a tautology, so both statements are false.

Q.33. If $(21)^{18}+20(21)^{17}+(20)^2(21)^{16}+\ldots\ldots+20^{18}=k\left(21^{19}-20^{19}\right)$ then $k=$

A) $\frac{19}{20}$ B) 1 C) $\frac{21}{20}$ D) $\frac{20}{21}$

Answer: 1

Solution: The given series is $(21)^{18}+20(21)^{17}+(20)^2(21)^{16}+\ldots\ldots+20^{18}=k\left(21^{19}-20^{19}\right)$

The given series is in Geometric Progression.

The total number of terms are $n=19$.

The first term is $a=21^{18}$.

And common ratio is $r=\frac{a_2}{a_1}=\frac{20(21)^{17}}{21^{18}}=\frac{20}{21}$

We know that the sum of n terms in GP is given by $S_n=\frac{a(1-r^n)}{1-r}$ when $|r|<1$

$\Rightarrow S_{19}=\frac{21^{18}\left(1-\left(\frac{20}{21}\right)^{19}\right)}{1-\left(\frac{20}{21}\right)}$

$=\frac{21^{18}\left(21^{19}-20^{19}\right)}{21^{19}}\times 21$

$=1\left(21^{19}-20^{19}\right)$

On comparing this with $k\left(21^{19}-20^{19}\right)$, we get $k=1$.

Hence, the value of $k=1$.

Q.34. If $1^2-2^2+3^2-4^2+\ldots.-(2022)^2+(2023)^2=m^2n$ where $m,n\in N$ and $m>19$ then $n-m^2$ is

A) 615 B) 562 C) 812 D) 264

Answer: 615

Solution: Let

$S=1^2-2^2+3^2-4^2+\ldots.+(2021)^2-(2022)^2+(2023)^2$

$\Rightarrow S=(1-2)(1+2)+(3-4)(3+4)+\ldots.+(2021-2022)(2021+2022)+(2023)^2$

$\Rightarrow S=-[3+7+11+15+\ldots.+4043]+(2023)^2$

$\Rightarrow S=-\frac{1011}{2}(6+1010\times 4)+(2023)^2$

$\Rightarrow S=-1011\times 2023+(2023)^2$

$\Rightarrow S=2023\times 1012$

$\Rightarrow S=34^2\times 1771$

So,

$m=34,\ n=1771$

Hence,

$n-m^2=615$

Q.35. The rank of the word "PUBLIC" is

Answer: 582

Solution: The given word is PUBLIC

Arranging the letters alphabetically, we get

BCILPU

When the word starts with any of the letters B/C/I/L, the number of possibilities$= 5! \times 4 = 480$

Now when the word starts with PB, then the number of possibilities$= 4! = 24$

Now when the word starts with PC, then the number of possibilities $= 4! = 24$

Now when the word starts with PI, then the number of possibilities $= 4! = 24$

Now when the word starts with PL, then the number of possibilities $= 4! = 24$

Now when the word starts with PUBC, then the number of possibilities$= 2! = 2$

Now when the word starts with PUBI, then the number of possibilities $= 2! = 2$

Now when the word starts with PUBLC, then the number of possibilities$= 1$

Now when the word starts with PUBLIC, then the number of possibilities $= 1$

Rank $= 480 + 24 \times 4 + 2 \times 2 + 1 \times 2 = 582$

Hence, rank of the work PUBLIC is 582.

Q.36. Find all the 4 letter words with 2 vowels and 2 consonants from the word UNIVERSE.

Answer: 504

Solution: Given word is UNIVERSE.

The vowels are I,U,E,E.

The consonants are N,V,R,S.

Let us take 2 cases to solve the given problem.

Case $1:$ When two vowels are same and two consonants are chosen.

The required number of ways of choosing two same vowels is ${}^2C_2 = 1$ and

The number of ways of choosing two consonants from four consonants is ${}^4C_2 = \frac{4\times3}{2!} = 6$.

Now the number of permutations of four letters in the above case 1 will be$= \frac{{}^2C_2 \times {}^4C_2}{2!} \times 4! = \frac{1\times6}{2} \times 24 = 72$

Case $2:$ When two vowels are different and two consonants are chosen.

The required number of ways of choosing two different vowels(U,I,E) is ${}^3C_2 = \frac{3\times2}{2!} = 3$.

Now the number of permutations of four letters in the above case 2 will be $= {}^3C_2 \times {}^4C_2 \times 4! = 3 \times 6 \times 24 = 432$.

Therefore, the total number of arrangements are $432 + 72 = 504$.

Q.37. Three dice are thrown. The probability that no outcomes are similar is $\frac{p}{q}$. What is $q - p$? (p and q are co-primes).

Answer: 4

Solution: Given that three dice are thrown.

The total number of outcomes when three dice are thrown together is $6^3 = 6 \times 6 \times 6$.

The number of outcomes such that all the outcomes are different is $= 6 \times 5 \times 4$.

For ex: If the outcome in dice 1 is $''6''$ then the number of outcomes for dice 2 should be $5\ (1, 2, 3, 4, 5)$ which excludes the outcome $''6''$.

Hence, the required probability is $= \frac{6 \times 5 \times 4}{6 \times 6 \times 6}$

$= \frac{20}{36} = \frac{5}{9} = \frac{p}{q}$

$\Rightarrow q - p = 9 - 5 = 4$

Therefore, the required answer is 4.