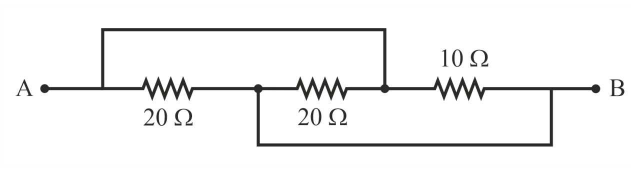
JEE Main 2023 (Session 2)

April 8 Shift 2

Physics

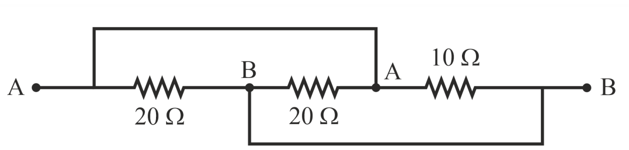
Q.1. Find the equivalent resistance between points A and B for the following figure.



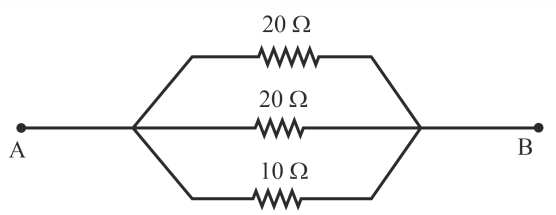
A) $5\ \Omega$ B) $10\ \Omega$ C) $20\ \Omega$ D) $40\ \Omega$

Answer: $5\ \Omega$

Solution:



The equivalent circuit for the given figure is shown below:

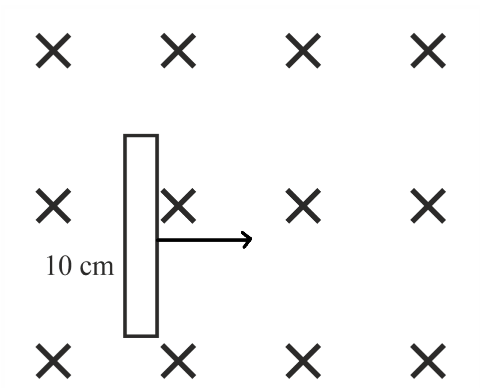


The equivalent resistance (R_{eq}) between points A and B can be calculated as follows:

$$\frac{1}{Req}=\frac{1}{20\ \Omega}+\frac{1}{20\ \Omega}+\frac{1}{10\ \Omega}$$
$$=\frac{1}{10\ \Omega}+\frac{1}{10\ \Omega}$$
$$=\frac{2}{10\ \Omega}$$
$$\Rightarrow Req=\ 5\ \Omega$$

Q.2. Direction of magnetic field is inside the plane of the paper as shown in the figure. A moving rod of length $10\ \text{cm}$ has induced EMF of $0.08\ \text{V}$. Find the velocity of the rod.

(Given $B=0.4\ \text{T}$)



A) $1\ \text{m s}^{-1}$ B) $2\ \text{m s}^{-1}$ C) $3\ \text{m s}^{-1}$ D) $4\ \text{m s}^{-1}$

Answer: $2\ \text{m s}^{-1}$

Solution: The formula to calculate the motional emf (ε) induced in a conductor when it is moving through a magnetic field is given by

$\varepsilon = Blv \quad \ldots(1)$

Substitute the values of the known parameters into equation (1) and solve to calculate the speed of the conductor.

$$0.08 = 0.4 \times 0.1 \times v$$
$$\Rightarrow v = \frac{0.08}{0.4 \times 0.1}$$
$$= 2$$

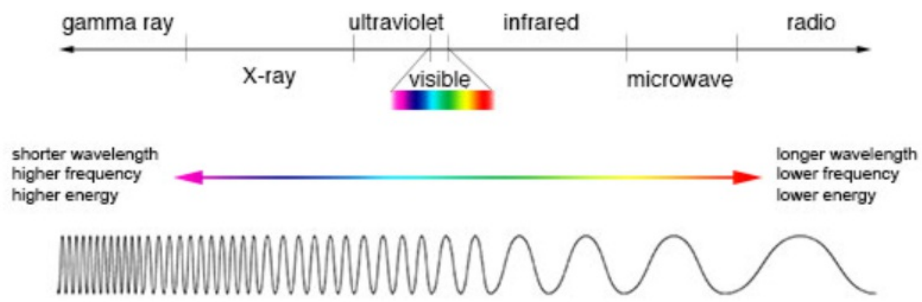
Hence, the speed of the conductor is 2 m s^{-1}.

Q.3. Which of the following is the highest energy electromagnetic wave?

A) X-ray B) Infrared C) Microwaves D) Radiowave

Answer: X-ray

Solution:



As we can see from above diagram, among the options given, X-ray has the smallest wavelength.

Now, energy of electromagnetic wave is given by,

$E = \frac{hc}{\lambda}$.

Therefore, X-ray will have the highest energy compared to Infrared, Microwaves & Radiowaves.

Q.4. Which of the following expression gives the value of acceleration due to gravity g' at the altitude h above the surface of Earth.

(R =radius of Earth, g =acceleration due to gravity at surface of Earth)

A) $g' = g\frac{h^2}{R^2}$ B) $g' = g\frac{R^2}{(R+h)^2}$ C) $g' = g\left(1 - \frac{h}{R}\right)$ D) $g' = g\left(1 - \frac{h^2}{R^2}\right)$

Answer: $g' = g\frac{R^2}{(R+h)^2}$

Solution: Acceleration due to gravity at the surface of the earth is given by, $g = \frac{GM}{R^2} \quad \ldots Eq(1)$.

At height h, it will become $g' = \frac{GM}{(R+h)^2} \quad \ldots Eq(2)$

From both equations, we can write

$$\frac{g'}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}$$
$$\Rightarrow g' = g\frac{R^2}{(R+h)^2}$$

Q.5. Find the distance from a point charge of magnitude 5×10^{-9} C, where electric potential is 50 V

A) 90 cm B) 70 cm C) 60 cm D) 50 cm

Answer: 90 cm

Solution: Electric potential due to point charge is given by,

$$V=\frac{kQ}{R}.$$

Therefore,

$$R=\frac{kQ}{V}=\frac{\left(9\times10^{9}\right)\times\left(5\times10^{-9}\right)}{50}=0.9\ \text{m}=90\ \text{cm}$$

Q.6. A Carnot engine working between $27^{\circ}\ \text{C}$ and $127^{\circ}\ \text{C}$ performs $2\ \text{kJ}$ of work. The amount of heat rejected is equal to

A) $4\ \text{kJ}$

B) $6\ \text{kJ}$

C) $8\ \text{kJ}$

D) $12\ \text{kJ}$

Answer: $6\ \text{kJ}$

Solution: The formula to calculate the efficiency (η) of a Carnot engine can be written as

$$\eta=1-\frac{Q_1}{Q_2}=1-\frac{T_1}{T_2}\quad\ldots\left(1\right)$$

Substitute the values of the known parameters into equation (1) and solve to calculate the amount of heat absorbed from the hot reservoir.

$$\frac{Q_2-Q_1}{Q_2}=1-\frac{T_1}{T_2}$$

$$\frac{2\ \text{kJ}}{Q_2}=1-\frac{(27+273)\ \text{K}}{(127+273)\ \text{K}}$$

$$=1-\frac{300}{400}$$

$$=\frac{1}{4}$$

$$\Rightarrow Q_2=4\times2\ \text{kJ}$$

$$=8\ \text{kJ}$$

Hence, the amount of heat rejected to the cold reservoir is given by

$$Q_1=Q_2-W$$

$$=8\ \text{kJ}-2\ \text{kJ}$$

$$=6\ \text{kJ}$$

Q.7. Match column I with column II and choose the correct option.

Column I	Column II
(I) Torque	(a) $\left[MLT^{-3}A^{-1}\right]$
(II) Stress	(b) $\left[ML^{-1}T^{-1}\right]$
(III) Coefficient of viscosity	(c) $\left[ML^{-1}T^{-2}\right]$
(IV) Potential gradient	(d) $\left[ML^{2}T^{-2}\right]$

A) $I\rightarrow a, II\rightarrow c, III\rightarrow b, IV\rightarrow d$

B) $I\rightarrow d, II\rightarrow b, III\rightarrow c, IV\rightarrow a$

C) $I\rightarrow d, II\rightarrow c, III\rightarrow b, IV\rightarrow a$

D) $I\rightarrow a, II\rightarrow c, III\rightarrow d, IV\rightarrow b$

Answer: $I\rightarrow d, II\rightarrow c, III\rightarrow b, IV\rightarrow a$

Solution: The dimensions of the given quantities can be calculated as follows:

$$\begin{aligned}[\tau] &= [Fr] \\ &= \left[\mathrm{MLT^{-2}}\right][\mathrm{L}] \\ &= \left[\mathrm{ML^2T^{-2}}\right] \\ [\sigma] &= \left[\frac{F}{A}\right] \\ &= \frac{\left[\mathrm{MLT^{-2}}\right]}{\left[\mathrm{L^2}\right]} \\ &= \left[\mathrm{ML^{-1}T^{-2}}\right] \\ [\eta] &= \frac{[F]}{[rv]} \\ &= \frac{\left[\mathrm{MLT^{-2}}\right]}{[\mathrm{L}]\left[\mathrm{LT^{-1}}\right]} \\ &= \left[\mathrm{ML^{-1}T^{-1}}\right] \\ [E] &= \frac{[V]}{[L]} \\ &= \frac{\left[\mathrm{ML^2T^{-3}A^{-1}}\right]}{[\mathrm{L}]} \\ &= \left[\mathrm{MLT^{-3}A^{-1}}\right]\end{aligned}$$

Q.8. Statement $-\mathrm{I}$: Electromagnets are made of soft iron

Statement $-\mathrm{II}$: Soft iron has lower permeability and high retentivity.

Choose the correct option related to statements.

A) Statement $-\mathrm{I}$ is true and Statement $-\mathrm{II}$ is also true

B) Statement $-\mathrm{I}$ is true but Statement $-\mathrm{II}$ is false

C) Statement $-\mathrm{I}$ is false but Statement $-\mathrm{II}$ is true

D) Statement $-\mathrm{I}$ is false and Statement $-\mathrm{II}$ is also false

Answer: Statement $-\mathrm{I}$ is true but Statement $-\mathrm{II}$ is false

Solution: The core of electromagnet is made of soft iron (ferromagnetic material), because the soft iron possesses the magnetic properties only when an electric current flows through the solenoid and loses the magnetic properties immediately when the current is switched off.

Hence, statement $-\mathrm{I}$ is true.

Iron becomes magnetised quickly but loses its magnetism as soon as the inducing magnet is removed. So, soft iron is said to have high susceptibility but low retentivity. Soft iron is known to have very high susceptibility and very low retentivity to magnetism, as there is a direct relation between permeability and susceptibility, so it will have very high permeability. This is the reason for which soft iron is used in making temporary electromagnets where we need strong but temporary magnets.

Hence, statement $-\mathrm{II}$ is false.

Q.9. A body of mass $5\ \mathrm{kg}$ has the linear momentum of $100\ \mathrm{kg\ m\ s^{-1}}$ and acted upon by the force of $2\ \mathrm{N}$ for $2\ \mathrm{s}$. Then change in kinetic energy in joule is

Answer: 81.6

Solution: Linear momentum is given by, $p = mu$.

Therefore,

$$mu = 100$$
$$\Rightarrow u = \frac{100}{5} = 20 \text{ m s}^{-1}$$

Now, acceleration due to force will be $a = \frac{F}{m} = \frac{2}{5} = 0.4 \text{ m s}^{-2}$.

Using equation of motion, we can write final velocity of the particle as $v = u + at = 20 + (0.4) \times 2 = 20.8$.

Therefore,

change in kinetic energy will be $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2} \times 5 \times (432.64 - 400) = 81.6 \text{ J}$

Q.10. Consider the given below statements.

Statement 1: We can get displacement from acceleration-time graph.

Statement 2: We can get acceleration from velocity-time graph.

A) Both statements are true

B) Both statements are false

C) Statement 1 is true & statement 2 is false

D) Statement 1 is false & statement 2 is true

Answer: Statement 1 is false & statement 2 is true

Solution: Acceleration of an object is defined as the rate of change of its displacement with respect to time.

In acceleration-time graph, the dependent variable is the acceleration and the independent variable is the time. On the other hand, in velocity-time graph, the dependent variable is the velocity and the independent variable is time.

The area under the acceleration-time graph indicates the velocity of the object, whereas the slope of the velocity-time graph indicates the acceleration of the object.

Hence, statement 1 is false and statement 2 is true.

Q.11. A body moving with speed 1 m s^{-1} comes to rest after moving for 20 cm over a rough surface. The coefficient of friction between the block and surface is ______.

Answer: 0.25

Solution: As the body is moving on the horizontal surface with friction, the acceleration (a) of the object can be written as

$$a = -\mu g \quad \ldots(1)$$

where, μ is the coefficient of friction and g is the acceleration due to gravity.

The equation of motion for the body is given by

$$v^2 = u^2 + 2aS \quad \ldots\left(2\right)$$

Substitute the expression for the acceleration from equation (1) into equation (2) and simplify to obtain the expression for the coefficient of viscosity.

$$0 = u^2 - 2\mu g S$$
$$\mu = \frac{u^2}{2gS} \quad \ldots\left(3\right)$$

Substitute the values of the known parameters into equation (3) to calculate the required coefficient of friction.

$$\mu = \frac{\left(1 \text{ m s}^{-1}\right)^2}{2 \times 10 \text{ m s}^{-2} \times 0.2 \text{ m}}$$
$$= 0.25$$

Chemistry

Q.12. Which of the acts as a stabiliser in the decomposition of H_2O_2 ?

A) urea

B) Alkali

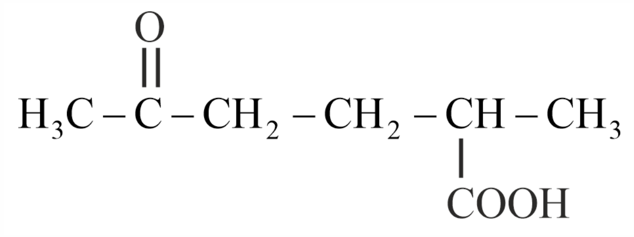
C) Glass

D) Dust

Answer: urea

Solution: H_2O_2 decomposes slowly on exposure to light. In the presence of metal surfaces or traces of alkali (present in glass containers), the said reaction is catalysed. It is, therefore, stored in wax-lined glass or plastic vessels in dark. Urea can be added as a stabiliser.

Q.13. The IUPAC name of the following is :



A) 2-pentanone

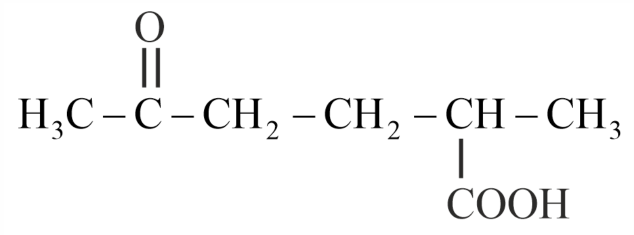
B) 5-oxo-2-methylpentanoic acid

C) 5-oxo-2-methylhexanoic acid

D) 2-methyl-5-oxohexanoic acid

Answer: 2-methyl-5-oxohexanoic acid

Solution:



In the given compound there are two functional groups, out of which carboxylic acid group is major functional group according to IUPAC rules. The main chain contains six carbon atoms and all are saturated as all carbon conncected with single bonds. One methyl group at second carbon and one ketone group at fifth carbon acts as substituents. Therefore, the IUPAC name of the compound is

2-methyl-5-oxohexanoic acid.

Q.14. What is the ratio of σ and pi bonds in pyrophosphoric acid?

A) $8:2$

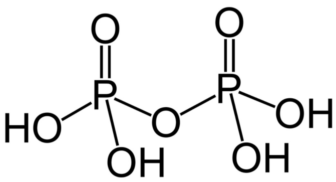
B) $10:2$

C) $12:2$

D) $8:4$

Answer: $12:2$

Solution:



In the structure that it contains four $P-OH$ bonds, two $P=O$ bonds and one $P-O-P$ bond and we know that sigma bonds are the result of head to head overlapping of orbitals thus it generally forms a single bond. Therefore we can say that all the single bonds present in pyrophosphoric acid are sigma bonds. So, a total of 10 single bonds are there between oxygen and hydrogen and phosphorus and oxygen.

The double bonds are formed by sideways overlapping of orbitals and their axes are parallel to each other resulting in double bond which generally exist in combination with a sigma bond thus one of the double bond is sigma and the other is pi. Therefore, in pyrophosphoric acid there are a total two pi bonds between phosphorus and oxygen and two more sigma bonds make it a total of 12 sigma bonds.

So the ratio is $12:2$

Q.15. Which of the following has maximum dispersion forces of attraction?

A) CH_4

B) Hexane

C) Ar

D) Water

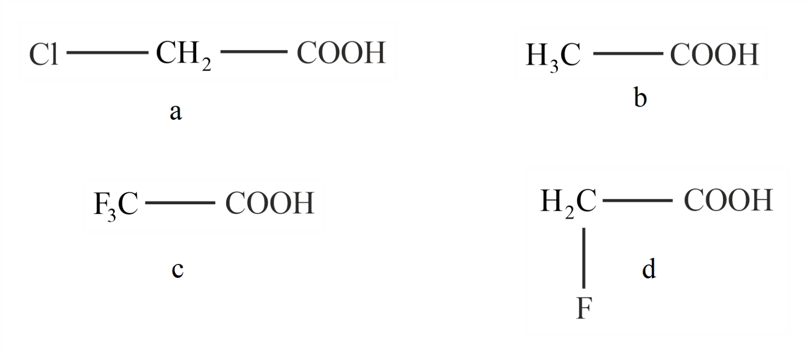
Answer: Hexane

Solution: Van der Waals forces of attraction arise due to the temporary or instantaneous dipoles in the molecules. These forces increase with increasing molecular size and surface area.

Out of the given options, hexane (C_6H_{14}) has the maximum number of atoms and the highest molecular weight, and therefore it will have the maximum Van der Waals forces of attraction. The larger surface area of the molecule allows for more contact with other molecules, increasing the potential for Van der Waals forces.

The other options, CH_4 (methane), Ar (argon), and water (H_2O), have smaller molecular weights and lesser surface areas as compared to hexane, and therefore will have weaker Van der Waals forces of attraction.

Q.16. Find the correct order of acidity of the following compounds :



A) $c > d > a > b$ B) $b > a > d > c$ C) $a > b > c > d$ D) $b > c > a > d$

Answer: $c > d > a > b$

Solution: In general, the acidic strength in an organic compound is directly related to the stability of the acid's conjugate base. In other words, an acid having a more stable conjugate base will be more acidic in comparison to an acid possessing a less stable conjugate base.

Trifluoroacetic acid is highly acidic. Since $-I$ effect of fluorine is more than that of chlorine, fluoroacetic acid is more acidic in comparison to chloroacetic acid.

Therefore, the correct anwer is A.

Q.17. Compare the acid strength of:

$1. o-$ flourobenzoic acid
$2. o-$ chlorobenzoic acid
$3. o-$ bromobenzoic acid
$4. o-$ iodobenzoic acid

A) $1 > 4 > 3 > 2$ B) $3 > 4 > 2 > 1$ C) $1 > 3 > 4 > 2$ D) $2 > 4 > 3 > 1$

Answer: $3 > 4 > 2 > 1$

Solution: Acid strength of different halogen substituted benzoic acid is $Br > I > Cl > F$.

In case of flouro benzoic acid $H-$bonding will decrease the ionization of $COOH$ group so it is least acidic. In chloro benzoic acid Cl exerts $+M$ effect. There is negligible $+M$ effect on bromo and iodo benzoic acids. So $+M$ decreases acidic nature of chloro derivative than bromo and iodo derivative.

So the order is $3 > 4 > 2 > 1$.

The pKa values of the compounds are

o-fluorobenzoic acid is 3.27, o-chlorobenzoic acid is 2.96, o-bromobenzoic acid is 2.85 and o-iodobenzoic acid is 2.86.

Q.18. Which of the following is used for the reduction of Al_2O_3?

A) Na_3AlF_6 B) CaF_2 C) Graphite D) Mg

Answer: Graphite

Solution: The Hall-Herault process is widely used in the extraction of aluminium. In Hall-Heroults process, pure Al_2O_3 is mixed with CaF_2 or Na_3AlF_6. This results in lowering the melting point of the mixture and increases its ability to conduct electricity. A steel vessel with a lining of carbon and graphite rods is used. The carbon lining acts as a cathode and graphite act as an anode.

The overall reaction is:

$2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

Q.19. Which of the following is correct?

1.Photocurrent $\propto$ Intensity of photon electrons

2.Kinetic energy is dependent on frequency

3.Kinetic energy is independent of frequency

A) 1 and 2 only B) 3 and 1only C) 2 only D) 3 only

Answer: 1 and 2 only

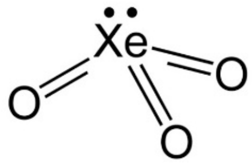
Solution: Photocurrent is directly proportional to intensity of incident light. The more the incident light then the more will be the intensity. So Statement 1 is correct. According to the photoelectric effect, when a photon of sufficient energy is absorbed by an atom or a material, it can eject an electron from the material. The ejected electron will carry kinetic energy, which depends on the energy of the absorbed photon and the energy required to overcome the binding energy of the electron in the material. When a higher frequency photon is absorbed, it imparts more energy to the electron, and hence the ejected electron has a higher kinetic energy. Thus, the kinetic energy of photoelectrons increases with an increase in the frequency of incident radiation. So statement 2 is also correct.

Q.20. Compounds of xenon having one electron pair on central atom

$XeO_3, XeOF_2, XeF_4, XeF_5^-$

Answer: 1

Solution: In the Xenon trioxide molecule, the central xenon atom forms three double bonds with three oxygen atoms. Thus, there are three $Xe = O$ double bonds in one molecule of xenon trioxide. The central xenon atom has one electron pair. The structure of XeO_3 is pyramidal, with three oxygen atoms bonded to the central xenon atom and one lone pair of electrons on the xenon atom.



The central Xe atom is having 2 lone pairs in the molecules, $XeOF_2$, XeF_4 and XeF_5^-.

Q.21. Find out the oxidation number of central metal atom of $[Fe(CO)_5]$, VO^{+2}, WO_3 ?

Calculate the sum of the oxidation states.

Answer: 10

Solution: The oxidation state of iron in $[Fe(CO)_5]$ is zero as the ligand is neutral in nature.

The oxidation state of vanadium in VO^{+2} :

$x - 2 = +2$
$x = +4$

The oxidation state of vanadium is $+4$.

The oxidation state of wolfram in $WO_3 = +6$

Therefore, the sum of the oxidation states is 10.

Q.22. How many of the following have five radical nodes ?

5s, 6s, 7s, 6p and 4p

Answer: 1

Solution: The number of radial nodes can be calculated by using the formula :

$\mathrm{n-l-1}$

Here n =Principal quantum number

l =Aziuthal quantum number

$5s = 5 - 0 - 1 = 4$
$6s = 6 - 0 - 1 = 5$
$7s = 7 - 0 - 1 = 6$
$6p = 6 - 1 - 1 = 4$
$4p = 4 - 1 - 1 = 2$

Therefore, the only $6s$ is having five radical nodes.

Mathematics

Q.23. The absolute difference of the coefficient of x^7 and x^9 in the expansion of $\left(2x + \frac{1}{2x}\right)^{11}$ is

A) 11×2^5 B) 11×2^4 C) 11×2^7 D) 11×2^3

Answer: 11×2^7

Solution: The given expansion is $\left(2x + \frac{1}{2x}\right)^{11}$.

The general term in the binomial expansion of $(x + a)^n$ is given by $T_{r+1} = {}^nC_r x^{n-r} a^r$.

$\Rightarrow T_{r+1} = {}^{11}C_r (2x)^{11-r} \left(\frac{1}{2x}\right)^r$

$= {}^{11}C_r \left(2^{11-r-r}\right) x^{11-r-r}$

Now for coefficient of $x^7 \Rightarrow 11 - 2r = 7$

$\Rightarrow r = 2$

Coefficient of x^7 is ${}^{11}C_2 \left(2^{11-2\times2}\right)$

Now for coefficient of $x^9 \Rightarrow 11 - 2r = 9$

$\Rightarrow r = 1$

Coefficient of x^9 is ${}^{11}C_1 \left(2^{11-2\times1}\right)$

We need the absolute difference, we get it by ${}^{11}C_2 \left(2^{11-2\times2}\right) - {}^{11}C_1 \left(2^{11-2\times1}\right)$.

$= \frac{11\times10}{2} \times 2^7 - 11 \times 2^9$

$= 2^7 (55 - 44)$

$= 11 \times 2^7$

Hence the absolute difference is $= 11 \times 2^7$.

Q.24. Let $f(x) = \{1, 2, 3, 4, 5, 6, 7\}$ the relation $R = \{(x, y) \in A \times A,\ x + y = 7\}$

A) Symmetric B) Reflexive C) Transitive D) Equivalence

Answer: Symmetric

Solution: We have, $f(x)=\{1,2,3,4,5,6,7\}$

Reflexive: A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, $R \text{ is reflexive} \Leftrightarrow (a,a)\in R \text{ for all } a\in A$

$\because (1,1),(2,2),(3,3),\ldots\ldots(7,7)$ does not satisfy $x+y=7$

Hence R is not reflexive.

Symmetric: A relation R is symmetric on a set A iff

$(a,b)\in R \Rightarrow (b,a)\in R \text{ for all } a,b\in A$

$\Rightarrow x+y=7$

Now on interchanging y and x we get, $y+x=7$ which is always true for given set,

Hence R is symmetric.

Transitive: A relation R on A is said to be transitive relation iff

$(a,b)\in R \text{ and } (b,c)\in R$

$\Rightarrow (a,c)\in R \text{ for all } a,b,c\in A$

Now taking $(a,b)\equiv(3,4)$ and $(b,c)\equiv(4,3)$ so $(a,c)\equiv(3,3)$ does not satisfy $x+y=7$,

Hence, R is not transitive and not equivalence.

Therefore, R is only Symmetric.

Q.25. The number of words with or without meaning can be formed from the word $\mathrm{MATHEMATICS}$ when C, S does not come together is

A) $\frac{9}{8}\times 10!$ B) $\frac{1}{8}\times 10!$ C) $\frac{5}{8}\times 10!$ D) $\frac{7}{8}\times 10!$

Answer: $\frac{9}{8}\times 10!$

Solution: The word 'MATHEMATICS' consists of 11 letters including two M's, two Ts and two T's.

Now, first we will arrange letters $\_M\_A\_T\_H\_E\_M\_A\_T\_I\_$.

Here, we have 10 gaps to fill C & S.

The number of words with or without meaning can be formed from the word $\mathrm{MATHEMATICS}$ when C, S does not come together is

$=\frac{9!}{2!\cdot 2!\cdot 2!}\times {}^{10}C_2\times 2!$

$=\frac{9!}{2!\cdot 2!}\times\frac{10\times 9}{2}$

$=\frac{9}{8}\times 10!$

Q.26. If α and β are the roots of the equation $ax^2+bx+c=0$, then $\lim\limits_{x\to\frac{1}{\alpha}}\left(\frac{1-\cos\left(cx^2+bx+a\right)}{2(1-\alpha x)^2}\right)$ is

A) $\frac{c^2(\alpha-\beta)^2}{\alpha^4\beta^2}$ B) $\frac{c(\alpha-\beta)^2}{4\alpha^4\beta^2}$ C) $\frac{c^2(\alpha-\beta)^2}{4\alpha^4\beta^2}$ D) $\frac{c^2(\alpha-\beta)^2}{2\alpha^4\beta^2}$

Answer: $\frac{c^2(\alpha-\beta)^2}{4\alpha^4\beta^2}$

Solution: Since, α and β are the roots of the equation $ax^2+bx+c=0$, therefore $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ would be the roots of $cx^2+bx+a=0$.

Let

$$L=\lim_{x\to\frac{1}{\alpha}}\left[\frac{1-\cos\left(cx^2+bx+a\right)}{2(1-\alpha x)^2}\right]\to\frac{0}{0}$$

$$\Rightarrow L=\lim_{x\to\frac{1}{\alpha}}\left[\frac{2\sin^2\left(\frac{cx^2+bx+a}{2}\right)}{2(1-\alpha x)^2}\right]$$

$$\Rightarrow L=\lim_{x\to\frac{1}{\alpha}}\left[\frac{\sin^2\left(\frac{c}{2}\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right)\right)}{\alpha^2\left(x-\frac{1}{\alpha}\right)^2}\right]$$

$$\Rightarrow L=\lim_{x\to\frac{1}{\alpha}}\left[\frac{c^2\left(x-\frac{1}{\beta}\right)^2}{4\alpha^2}\times\frac{\left\{\sin\left(\frac{c}{2}\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right)\right)\right\}^2}{\left(\frac{c}{2}\right)^2\left(x-\frac{1}{\alpha}\right)^2\left(x-\frac{1}{\beta}\right)^2}\right]$$

$$\Rightarrow L=\frac{c^2\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2}{4\alpha^2}$$

$$\Rightarrow L=\frac{c^2(\alpha-\beta)^2}{4\alpha^4\beta^2}$$

Q.27. Let $a_n=5+8+14+23+\ldots\ldots\text{upto } n \text{ terms}$. If $S_n=\sum_{k=1}^{n}a_k$ then $S_{30}-a_{40}$ is equal to

A) 78025 B) 12800 C) 11600 D) 12100

Answer: 78025

Solution: The given series is a_n, then we can write as

$a_n = 5 + 8 + 14 + 23 + \ldots\ldots\ldots\ldots + T_n$
$a_n = \quad 5 + 8 + 14 + 23 + \ldots\ldots + T_{n-1} + T_n$

Subtracting above equations, we get

$0 = 5 + 3 + 6 + 9 + \ldots. + (T_n - T_{n-1}) - T_n$

$\Rightarrow T_n = 5 + \left[\frac{(n-1)}{2}(2 \times 3 + (n - 1 - 1)3)\right]$

$\Rightarrow T_n = 5 + \frac{(n-1)}{2}(3n)$

$\Rightarrow T_n = \frac{1}{2}\left(3n^2 - 3n + 10\right)$

So,

$a_{40} = \sum T_r = \frac{1}{2}\sum\left(3r^2 - 3r + 10\right)$

$= \frac{1}{2}\left(\frac{3n(n+1)(2n+1)}{6} - \frac{3n(n+1)}{2} + 10n\right)$

$= \frac{n}{2}\left(n^2 + 9\right)$

$\Rightarrow a_{40} = \frac{40}{2}\left(40^2 + 9\right) = 32180$

$\Rightarrow S_{30} = \sum_{n=1}^{30} a_n$

$\Rightarrow S_{30} = \frac{1}{2}\sum\left(n\left(n^2 + 9\right)\right) = \frac{1}{2}\sum\left(n^3 + 9n\right)$

$= \frac{1}{2}\left(\left(\frac{n(n+1)}{2}\right)^2 + \frac{9n(n+1)}{2}\right)$

$\Rightarrow S_{30} = \frac{1}{2}\left(\left(\frac{30\times31}{2}\right)^2 + \frac{9\times30\times31}{2}\right)$

$\Rightarrow S_{30} = 110205$

$\Rightarrow S_{30} - a_{40} = 110205 - 32180 = 78025$

Therefore the required value is 78025

Q.28. The statement $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to

A) $p \wedge q$ B) $\sim p \vee \sim q$ C) $p \vee q$ D) $\sim p \wedge \sim q$

Answer: $\sim p \vee \sim q$

Solution: The given statement is $(p \wedge (\sim q)) \vee (\sim p)$

Let us take $(A \cap B^c) \cup (A^c)$.

Now let us apply distributive law to the above statement.

$\equiv (A \cup A^c) \cap (B^c \cup A^c)$

$\equiv (U) \cap (B^c \cup A^c)$

$\equiv (B^c \cup A^c)$

Now the given statement $(p \wedge (\sim q)) \vee (\sim p)$ will be equivalent to $\sim q \vee \sim p$ or $\sim p \vee \sim q$.

Hence the given statement is equivalent to $\sim p \vee \sim q$.

Q.29. If $\frac{1+2i\sin\theta}{1-i\sin\theta}$; $\theta \in (0, 2\pi)$ is a purely imaginary number, then the value of θ is

A) 0 B) $\frac{\pi}{2}$ C) π D) $\frac{\pi}{4}$

Answer: $\frac{\pi}{4}$

Solution: Let

$z = \frac{1+2i \sin\theta}{1-i \sin\theta}$

$\Rightarrow z = \frac{1+2i \sin\theta}{1-i \sin\theta} \times \frac{1+i \sin\theta}{1+i \sin\theta}$

$\Rightarrow z = \frac{1-2\sin^2\theta+3i \sin\theta}{1+\sin^2\theta}$

$\Rightarrow z = \left(\frac{1-2\sin^2\theta}{1+\sin^2\theta}\right) + i\left(\frac{3\sin\theta}{1+\sin^2\theta}\right)$

Since, z is a purely imaginary number, so real part must be zero, hence

$\frac{1-2\sin^2\theta}{1+\sin^2\theta} = 0$

$\Rightarrow 1 - 2\sin^2\theta = 0$

$\Rightarrow \cos 2\theta = 0$

$\Rightarrow \theta = \frac{\pi}{4}$

Q.30. From $O(0,0)$ two tangents OA and OB are drawn to a circle $x^2 + y^2 - 6x + 4y + 8 = 0$ then the equation of circumircle of ΔOAB is

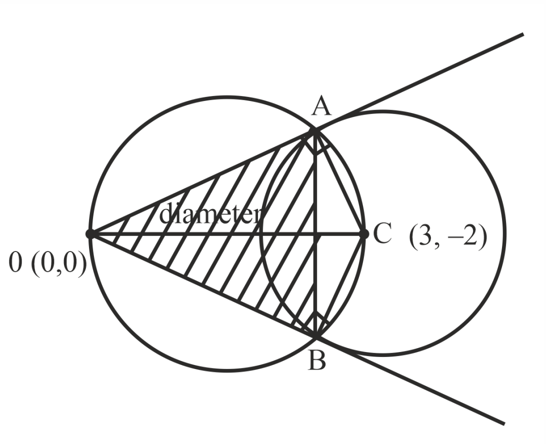
A) $x^2 + y^2 - 3x + 2y = 0$ B) $x^2 + y^2 + 3x - 2y = 0$ C) $x^2 + y^2 + 3x + 2y = 0$ D) $x^2 + y^2 - 3x - 2y = 0$

Answer: $x^2 + y^2 - 3x + 2y = 0$

Solution: The given equation of circle is $x^2 + y^2 - 6x + 4y + 8 = 0$.

The centre of the circle is $C \equiv (3, -2)$.

Now two tangents are drawn to the circle and the diagram will be



Since angle in a semicircle is a right angle.

Hence, the other circle will be passing through the centre of the first circle.

The two ends of diameter of the circle is $O(0,0)$ and $C(3,-2)$.

Hence, the required equation of circle is $(x-0)(x-3) + (y-0)(y-(-2)) = 0$

$\Rightarrow x^2 + y^2 - 3x + 2y = 0$

Therefore, the required equation is $x^2 + y^2 - 3x + 2y = 0$

Q.31. If $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}; & 0 < x < 1 \\ 3mx^2 + k^2; & x \geq 1 \end{cases}$ is differentiable at $x = 1$, then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ for $k \neq 0$ is

A) 310 B) 309 C) 311 D) 312

Answer: 309

Solution:

Since, $f(x)=\begin{cases}3x^2+k\sqrt{x+1};\ 0<x<1\\ 3mx^2+k^2;\quad x\geq 1\end{cases}$ is differentiable at $x=1$, so function must be continuous at $x=1$, hence

$\mathrm{LHL}=\mathrm{RHL}$

$3+k\sqrt{2}=3m+k^2$

$\Rightarrow k^2-k\sqrt{2}+3m-3=0\quad ....(1)$

And,

$$f'(x)=\begin{cases}6x+\frac{k}{2\sqrt{x+1}};\ 0<x<1\\ 6mx;\quad x>1\end{cases}$$

So,

$f'(1^-)=f'(1^+)$

$\Rightarrow 6+\frac{k}{2\sqrt{2}}=6m$

$\Rightarrow m=1+\frac{k}{12\sqrt{2}}\quad ...(2)$

Putting in (1), we get

$k^2-k\sqrt{2}+3+\frac{k}{4\sqrt{2}}-3=0$

$\Rightarrow k^2-\left(\sqrt{2}-\frac{1}{4\sqrt{2}}\right)k=0$

$\Rightarrow k^2-\left(\frac{7\sqrt{2}}{8}\right)k=0$

$\Rightarrow k\left(k-\frac{7\sqrt{2}}{8}\right)=0$

$\Rightarrow k=\frac{7\sqrt{2}}{8}$

So,

$m=1+\frac{k}{12\sqrt{2}}=1+\frac{7\sqrt{2}}{96\sqrt{2}}=\frac{103}{96}$

Hence,

$$f'(x)=\begin{cases}6x+\frac{7\sqrt{2}}{16\sqrt{x+1}};\ 0<x<1\\ \frac{103x}{16};\quad x>1\end{cases}$$

So,

$$\frac{f'(8)}{f'\left(\frac{1}{8}\right)}=\frac{103\times 8}{16}\times\frac{1}{\left(\frac{6}{8}+\frac{7\sqrt{2}}{16\sqrt{\frac{9}{8}}}\right)}$$

$$\Rightarrow\frac{f'(8)}{f'\left(\frac{1}{8}\right)}=\frac{103}{2}\times\frac{3}{4}$$

$$\Rightarrow\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}=309$$

Q.32. Area of quadrilateral having vertices $(1,2),\ (5,6),\ (7,6)$ and $(-1,-6)$ is ________

Answer: 24

Solution: We know that,

Area of quadrilateral having vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ is given by,

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\left|x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_4 - x_4y_3 + x_4y_1 - x_1y_4\right|$$

So, Area of quadrilateral having vertices $(1, 2),\ (5, 6),\ (7, 6)$ and $(-1, -6)$ is

$$= \frac{1}{2}\begin{vmatrix} 1 & 2 \\ 5 & 6 \\ 7 & 6 \\ -1 & -6 \\ 1 & 2 \end{vmatrix}$$

$$= \frac{1}{2}\left|6 + 30 - 42 - 2 + 6 + 6 - 42 - 10\right|$$

$$= \frac{1}{2}\left|-48\right|$$

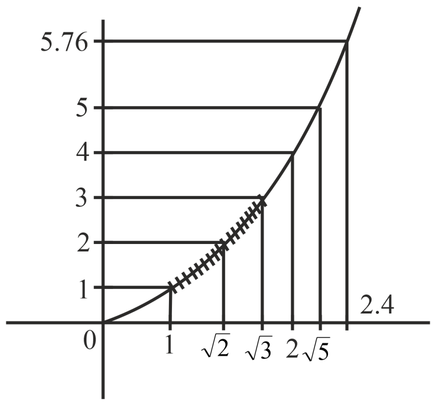
$= 24$ sq. units

Q.33. If the value of $\int_0^{2.4}\left[x^2\right]\mathrm{d}x$ is $\alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5} + \phi$, then $(\alpha + \beta + \gamma + \delta + \phi)$ will be,

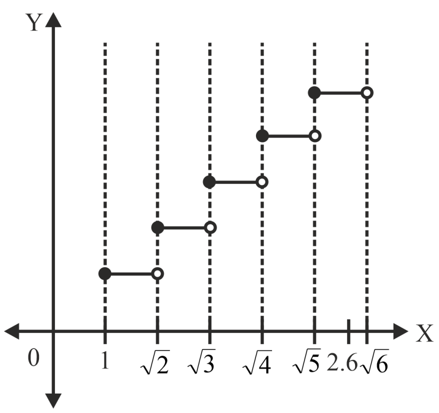
Note- $[.]$ denotes the greatest integer function

Answer: 6

Solution: Plotting the graph of x^2 for $x \in (0, 2.4)$ we get,



Now plotting the diagram for $\left[x^2\right]$ we get,



So, from above diagram we get,

$\int_0^{2.4}\left[x^2\right] \mathrm{d}x = \int_0^1\left[x^2\right] \mathrm{d}x + \int_1^{\sqrt{2}}\left[x^2\right] \mathrm{d}x + \int_{\sqrt{2}}^{\sqrt{3}}\left[x^2\right] \mathrm{d}x + \int_{\sqrt{3}}^{2}\left[x^2\right] \mathrm{d}x + \int_2^{\sqrt{5}}\left[x^2\right] \mathrm{d}x + \int_{\sqrt{5}}^{2.4}\left[x^2\right] \mathrm{d}x$

$\Rightarrow \int_0^{2.4}\left[x^2\right] \mathrm{d}x = \int_0^1 0 \, \mathrm{d}x + \int_1^{\sqrt{2}} 1 \, \mathrm{d}x + \int_{\sqrt{2}}^{\sqrt{3}} 2 \, \mathrm{d}x + \int_{\sqrt{3}}^{2} 3 \, \mathrm{d}x + \int_2^{\sqrt{5}} 4 \, \mathrm{d}x + \int_{\sqrt{5}}^{2.4} 5 \, \mathrm{d}x$

$\Rightarrow \int_0^{2.4}\left[x^2\right] \mathrm{d}x = \left(\sqrt{2}-1\right) + 2\left(\sqrt{3}-\sqrt{2}\right) + 3\left(\sqrt{4}-\sqrt{3}\right) + 4\left(\sqrt{5}-\sqrt{4}\right) + 5\left(2.4-\sqrt{5}\right)$

$\Rightarrow \int_0^{2.4}\left[x^2\right] \mathrm{d}x = 9 - \sqrt{2} - \sqrt{3} - \sqrt{5}$

Now on comparing with $\int_0^{2.4}\left[x^2\right] \mathrm{d}x = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5} + \phi$

We get, $\alpha = 9,\ \beta = -1,\ \gamma = -1,\ \delta = -1\ \&\ \phi = 0$

So, $\alpha + \beta + \gamma + \delta + \phi = 6$