CLASS IX (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-1

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. $0.12\overline{3}$ can be expressed in rational form as [1]

(a) $\frac{900}{111}$ (b) $\frac{111}{900}$

(c) $\frac{123}{10}$ (d) $\frac{121}{900}$

Ans : (b) $\frac{111}{900}$

Let, $x = 0.12333......$ ...(1)

Multiply (1) by 10 on both sides, we get

$10x = 1.2333.......$ ...(2)

Subtracting (1) from (2), we get

$$9x = 1.11$$

$$x = 111/900$$

2. Which one of the following algebraic expressions is a polynomial in variable x? [1]

(a) $x^2+\frac{2}{x^2}$ (b) $\sqrt{x}+\frac{1}{\sqrt{x}}$

(c) $x^2+\frac{3x^{3/2}}{\sqrt{x}}$ (d) None of these

Ans : (c) $x^2+\frac{3x^{3/2}}{\sqrt{x}}$

$x^2+\frac{3x^{3/2}}{\sqrt{x}}$ can be written as x^2+3x, which is a polynomial in x.

3. If $p(a,b)$ lies in II quadrant then which of the following is true about a and b? [1]

(a) $a>0, b>0$ (b) $a>0, b<0$

(c) $a<0, b>0$ (d) $a<0, b<0$

Ans : (c) $a<0, b>0$

In second quadrant, abscissa is negative and ordinate is a positive number.

$\Rightarrow$ $a < 0, b > 0$

4. If $P(x,y)$ and $P'(y,x)$ are same points then which of the following is true? [1]

(a) $x+y=0$ (b) $xy=0$

(c) $x-y=0$ (d) $\frac{x}{y}=0$

Ans : (c) $x-y=0$

$$P(x,y) = P'(y,x)$$

$$x = y \text{ and } y = x$$

$$x-y = 0$$

5. According to Euclid's definition, the ends of a line are [1]

(a) breadth less (b) points

(c) length less (d) None of these

Ans : (b) points

By definitions given by Euclid, line ends in points.

6. An angle is 18° less than its complementary angle. The measure of this angle is [1]

(a) 36° (b) 48°

(c) 83° (d) 81°

Ans : (a) 36°

Let the angle be x.

its complement $= x+18^\circ$

Now, $x+x+18^\circ = 90^\circ$

$$2x = 90^\circ - 18^\circ$$

$$2x = 72^\circ$$

$$x = 36^\circ$$

7. Can we draw a triangle ABC with $AB = 3\text{ cm}$, $BC = 3.5\text{ cm}$ and $CA = 6.5\text{ cm}$? [1]

(a) Yes (b) No

(c) Can't be determined (d) None of these

Ans : (b) No

In $\triangle ABC$, $AB = 3\text{ cm}$,

$BC = 3.5\text{ cm}$, $CA = 6.5\text{ cm}$

Since $AB+BC > CA$

as $3\text{ cm}+3.5\text{ cm} = 6.5\text{ cm} = CA$

$\Rightarrow$ $\triangle ABC$ is not possible.

8. If in a quadrilateral, two adjacent sides are equal and the opposite sides are unequal, then it is called a [1]

(a) parallelogram (b) square

(c) rectangle (d) kite

Ans : (d) kite

In kite, adjacent sides are equal but opposite sides are not equal.

9. The area of a rhombus is 20 cm^2. If one of its diagonals is 5 cm, the other diagonal is [1]

(a) 5 cm (b) 6 cm

(c) 8 cm (d) 10 cm

Ans : (c) 8 cm

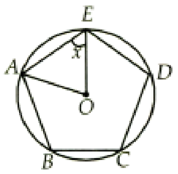
$$\text{Area of rhombus} = \frac{1}{2}\times d_1 \times d_2;$$

where d_1, d_2 are lengths of diagonals.

$$20 = \frac{1}{2}\times 5 \times d_2 \qquad [\text{Since, } d_1 = 5]$$

$$d_2 = 8\text{ cm}$$

10. In the given pentagon $ABCDE$, $AB = BC = CD = DE = AE$. The value of x is [1]



(a) 36° (b) 54°

(c) 72° (d) 108°

Ans : (b) 54°

Since, equal chords subtend equal angles at the centre.

$$\angle AOE = \frac{360^\circ}{5} = 72^\circ$$

Now, $OE = OA$

$$\angle OEA = \angle OAE = x$$

In ΔOAE, $x + x + \angle AOE = 180^\circ$

$$2x + 72^\circ = 180^\circ$$

$$x = \frac{108^\circ}{2} = 54^\circ$$

(Q.11-Q.15) Fill in the blanks :

11. The construction of a ΔLMN in which $LM = 8$ cm, $\angle L = 45^\circ$ is possible when $(MN + LN)$ is .... cm. [1]

Ans : 9 cm

We know that sum of two sides of a triangle is always greater than third side.

$MN + LN > LM$ i.e., 8 cm

$MN + LN$ will be 9 cm

12. The sides of a triangle are 25 cm, 17 cm and 12 cm. The length of the altitude on the longest side is equal to .... cm. [1]

Ans : 7.2 cm

$$s = \frac{25 + 17 + 12}{2} = 27\text{ cm}$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of triangle} = \sqrt{27(27-25)(27-17)(27-12)}$$

$$= \sqrt{27\times 2\times 10\times 15}$$

$$= 90\text{ cm}^2$$

Also, area of triangle $= \frac{1}{2}\times 25\times h = 90$

$$h = 90\times\frac{2}{25} = 7.2\text{ cm}$$

or

Perimeter of an equilateral triangle is always equal to .... times of length of sides.

Ans : three

13. .... of a solid is the amount of space enclosed by the bounding surface. [1]

Ans : Volume

14. .... is the value of the middle most observation (s). [1]

Ans : Median

15. An activity which results in a well defined end is called an .... [1]

Ans : Experiment

(Q.16-Q.20) Answer the following :

16. What is the degree of zero polynomial? [1]

SOLUTION :

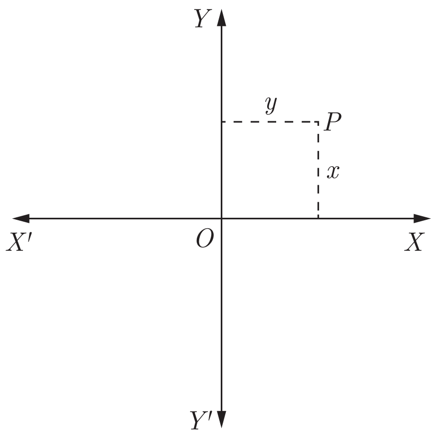
Degree of zero polynomial is not defined, as $p(x)$ can be written as

$$p(x) = 0 = 0.x = 0.x^2 = 0.x^3 = \ldots\ldots$$

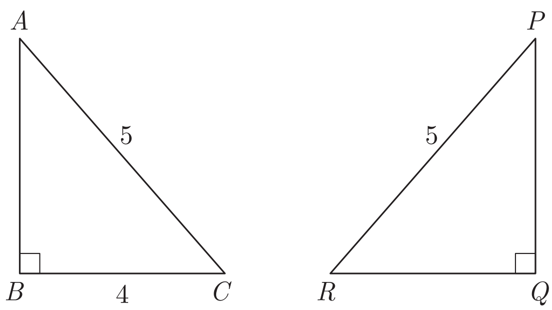
17. Write the coordinates of the point which lies at a distance of x units from X-axis and y units from Y-axis. [1]

SOLUTION :

Required point is $P(x, y)$.



18. If ΔABC is congruent to ΔPQR, find the length of QR. [1]



SOLUTION :

Since, $\Delta ABC \cong \Delta PQR$

Thus, $BC = QR$ [By CPCT]

$QR = 4\text{ cm}.$

19. The volume of a sphere is 38808 cm^3. Find its radius. [1]

SOLUTION :

Volume of a sphere $= \frac{4}{3}\pi r^3$

Where, r is the radius of a sphere.

$$\frac{4}{3}\pi r^3 = 38808$$

$$r^3 = \frac{38808}{\frac{4}{3}\pi} = \frac{38808 \times 3 \times 7}{4 \times 22}$$

$$= 9261$$

$$r = \sqrt[3]{9261} = 21$$

20. Find the range of the following data; [1]
25, 18, 10, 20, 22, 16, 6, 17, 12, 30, 29, 32, 10, 19, 13, 31.

SOLUTION :

Given, highest value $= 32$

and lowest value $= 6$

Range = Highest value − Lowest value

$= 32 - 6 = 26$

Section B

21. Simplify : $\sqrt{2a^2 + 2\sqrt{6}\,ab + 3b^2}$. [2]

SOLUTION :

$$\text{Let, } I = \sqrt{2a^2 + 2\sqrt{6}\,ab + 3b^2}$$

$$= \sqrt{2a^2 + (\sqrt{6} + \sqrt{6})ab + 3b^2}$$

[By splitting the middle term]

$$= \sqrt{2a^2 + \sqrt{6}\,ab + \sqrt{6}\,ab + 3b^2}$$

$$= \sqrt{\sqrt{2}\,a(\sqrt{2}\,a + \sqrt{3}\,b) + \sqrt{3}\,b(\sqrt{2}\,a + \sqrt{3}\,b)}$$

$$= \sqrt{(\sqrt{2}\,a + \sqrt{3}\,b)(\sqrt{2}\,a + \sqrt{3}\,b)}$$

$$= \sqrt{(\sqrt{2}\,a + \sqrt{3}\,b)^2}$$

$$= (\sqrt{2}\,a + \sqrt{3}\,b)$$

or

Simplify : $\frac{4+\sqrt{6}}{4-\sqrt{6}} + \frac{4-\sqrt{6}}{4+\sqrt{6}}$

SOLUTION :

We have,

$$I \; \frac{4+\sqrt{6}}{4-\sqrt{6}} + \frac{4+\sqrt{6}}{4-\sqrt{6}}$$

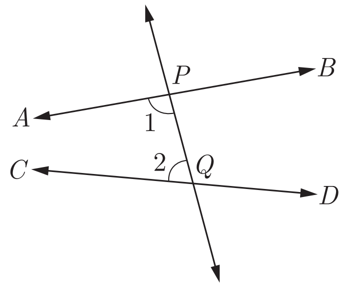
$$= \frac{(4+\sqrt{6})^2 + (4-\sqrt{6})^2}{(4-\sqrt{6})(4+\sqrt{6})}$$

$$= \frac{16 + 6 + 8\sqrt{6} + 16 + 6 - 8\sqrt{6}}{16 - 6}$$

$$= \frac{32 + 12}{10} = \frac{44}{10} = 4.4$$

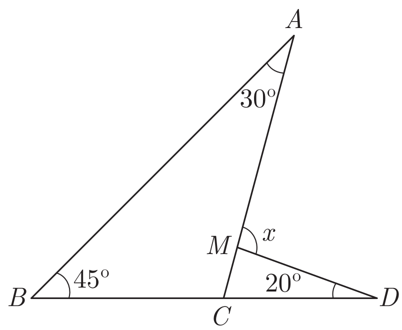
22. State Euclid's fifth postulate. [2]

SOLUTION :



If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

23. In the given figure, find the value of x. [2]



SOLUTION :

In ΔABC, we have

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

$$30 + 45\angle ACB = 180^\circ$$

$$\Rightarrow \quad \angle ACB = 105^\circ$$

Also, $\angle ACB + \angle ACD = 180^\circ$

$$105^\circ + \angle ACD = 180^\circ$$

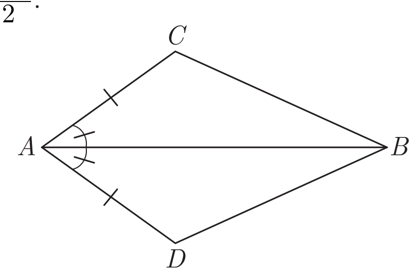
$$\Rightarrow \quad \angle ACD = 75^\circ$$

Now, $x = \angle MCD + \angle CDM$

$$x = 75^\circ + 20^\circ = 95^\circ$$

or

In the given figure, if $BC = 2.6\text{ cm}$, then find $2BD + \frac{BC}{2}$.



SOLUTION :

In ΔACB and ΔADB

$$AC = AD \quad \text{(Given)}$$
$$AB = AB \quad \text{(Common side)}$$
and $$\angle BAC = \angle BAD \quad \text{(Given)}$$
$\therefore$ $$\Delta ACB \cong \Delta ADB$$
(By SAS congruence rule)

Then, $$BC = BD \quad \text{(By CPCT)}$$
Given, $$BC = 2.6\text{ cm}$$
$$BD = 2.6\text{ cm}$$
Now, $$2BD + \frac{BC}{2} = 2 \times 2.6 + \frac{2.6}{2}$$
$$= 5.2 + 1.3 = 6.5\text{ cm}$$

24. Find the remainder when $3x^3 - 6x^2 + 3x - \frac{7}{9}$ is divided by $3x - 4$. [2]

SOLUTION :

$$(3x - 4) = 0$$
$\Rightarrow$ $$x = \frac{4}{3}$$
$\therefore$ $$p(x) = 3\left(\frac{4}{3}\right)^3 - 6\left(\frac{4}{3}\right)^2 + 3\left(\frac{4}{3}\right) - \frac{7}{9}$$
$$= 3 \times \frac{64}{27} - 6 \times \frac{16}{9} + 3 \times \frac{4}{3} - \frac{7}{9}$$
$$= \frac{64}{9} - \frac{32}{3} + 4 - \frac{7}{9} = -\frac{1}{3}$$

25. Find the coordinates of the point : [2]
(i) Which lies on x axes both.
(ii) Whose abscissa is 2 and which lies on the x-axis.

SOLUTION :

(i) The coordinates of the points which lies on the x and y-axes both are (0, 0).
(ii) Since the point lies on the x-axis therefore, its ordinate $= 0$. So, the coordinates of the given point are (2, 0).

26. The sides of a triangular field are 51 m, 37 m and 20 m. Find the number of flower beds that can be prepared, if each bed is to occupy 9 m^2 of space. [2]

SOLUTION :

Let the sides of triangular field be
$$a = 51\text{ m},\ b = 37\text{ m and } c = 20\text{ m}$$
Then, semi-perimeter of triangular field,
$$s = \frac{a+b+c}{2} = \frac{51+37+20}{2}$$
$$= \frac{108}{2} = 54\text{ m}$$
$\therefore$ Area of triangular field
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
(By Heron's formula)
$$= \sqrt{54(54-51)(54-37)(54-20)}$$
$$= \sqrt{54 \times 3 \times 17 \times 34}$$
$$= \sqrt{2 \times 3 \times 3 \times 3 \times 3 \times 17 \times 17 \times 2}$$
$$= (17 \times 2 \times 3 \times 3)\text{ m}^2$$
Now, number of flower beds,
$$n = \frac{\text{Area of triangular field}}{\text{Space occupied by each flower bed}}$$
$$= \frac{2 \times 3 \times 3 \times 17}{9} = \frac{306}{9} = 34$$
Hence, 34 flower beds can be prepared.

or

Two cylindrical vessels have their base radii as 16 cm and 8 cm respectively. If their heights are 8 cm and 16 cm respectively, then find the ratio of their volumes.

SOLUTION :

Volume of first vessel $= \pi r_1^2 h_1$
$$= \frac{22}{7} \times 16 \times 16 \times 8\text{ cm}^3$$
Volume of the second vessel $= \pi r_2^2 h_2$
$$= \frac{22}{7} \times 8 \times 8 \times 16\text{ cm}^3$$
$\therefore$ Required ratio $= \dfrac{\frac{22}{7} \times 16 \times 16 \times 8}{\frac{22}{7} \times 8 \times 8 \times 16} = 2:1$

Section C

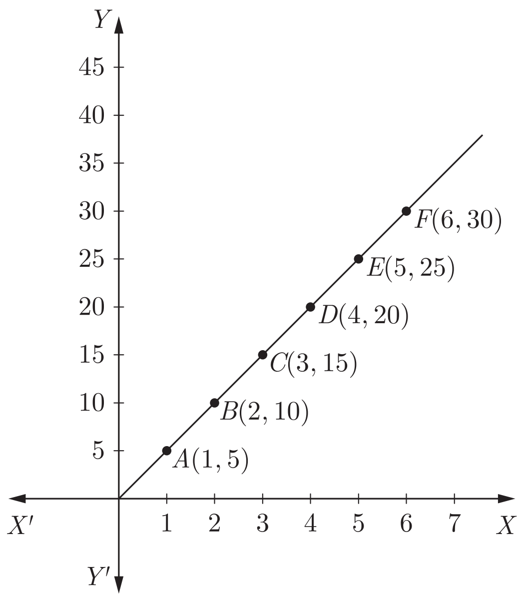
27. The following table gives the number of pairs of shoes and their corresponding price. [3]

Number of pair of shoes	1	2	3	4	5	6
Corresponding price (₹ in hundred)	5	10	15	20	25	30

Plot these as ordered pairs and join them. What type of graph do you get ?

SOLUTION :

Let us draw the coordinates axes XOX' and YOY', and choose a suitable units of distance on the axes.



The points $A(1,5)$, $B(2, 10)$, $C(3, 15)$, $D(4, 20)$, $E(5, 25)$ and $F(6, 30)$ can be plotted as shown below. Now, join all these points in order to get a straight line.

or

Draw the graph of the linear equation $x+2y=8$ and find the point on the graph where abscissa is twice the value of ordinate.

SOLUTION :

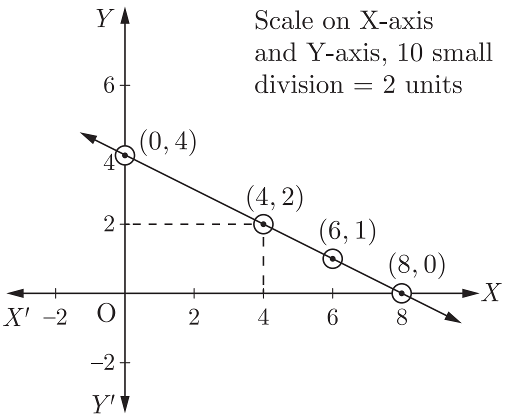
We have, $x+2y = 8$ ...(1)

x	0	8	6
y	4	0	1

Given, $x = 2y$

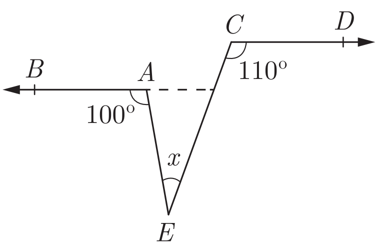
Putting $x=2y$ in eq. (1), we have

$$2y+2y = 8$$
$$4y = 8$$
$$y = 2$$
$$x = 2\times 2 = 4$$



Hence, point (4, 2) is the required point on the graph.

28. In the given figure, find $\angle x$ if $AB \,||\, CD$. [3]



SOLUTION :

Produce BA to meet CE at P. Now $BA \,||\, CD$ and EC is a transversal.

$\therefore$ $\angle APC = \angle DCP$ (Alternate angles)

$= 110°$ ...(1)

Also, $\angle APC + \angle APE = 180°$ ...(2) (Linear pair)

From (1) and (2), we get

$\angle APE = 180° - 110°$

$= 70°$

Again, $\angle BAE + \angle PAE = 180°$ (Linear pair)

$\therefore$ $\angle PAE = 180° - \angle BAE$

$= 180° - 100°$

$= 80°$ ...(3)

But, $\angle PAE + \angle APE + \angle x = 180°$ (Angles of a Δ)

$\therefore$ $\angle x = 180° - \angle PAE - \angle APE$

$= 180° - 80° - 70°$

$= 180° - 150°$

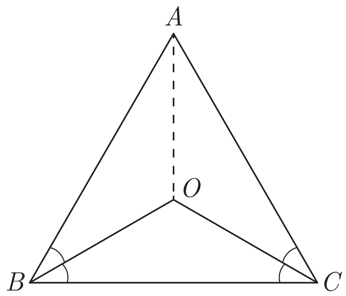
[Using (3) and (1)]

$= 30°$

29. In an isosceles triangle ABC, with $AB = AC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that :

(i) $OB = OC$

(ii) AO bisects $\angle A$ [3]



SOLUTION :

(i) $AC = AB$

$\angle ABC = \angle ACB$

[Angles opposite to equal sides are equal]

$\Rightarrow$ $\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow$ $\angle CBO = \angle BCO$

[$\because$ OB and OC are bisectors of $\angle B$ and $\angle C$ respectively]

$\Rightarrow$ $OB = OC$

[Sides opposite to equal angles are equal]

Again, $\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\angle ABO = \angle ACO$

[$\therefore$ OB and OC are bisectors of $\angle B$ and $\angle C$ respectively]

In ΔABO and ΔACO, we have

$AB = AC$ (Given)

$\angle ABO = \angle ACO$ (Proved above)

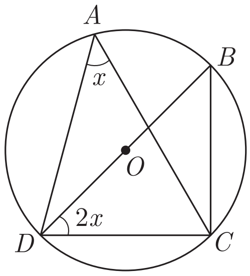
$OB = OC$ (Proved above)

$\therefore$ $\Delta ABO \cong \Delta ACO$ (SAS congruence)

$\Rightarrow$ $\angle BAO = \angle CAO$ (CPCT)

$\Rightarrow$ AO bisects $\angle A$ **Proved.**

30. In the given figure, O is the centre of the circle. Find the value of x. [3]



SOLUTION :

It is given that O is the centre of the circle.

$\therefore$ DB is the diameter of the circle.

So, $\angle DCB = 90^\circ$ (Angle in semi-circle)

Also, $\angle DBC = \angle DAC = x$

(Angles in same segment are equal)

$$\angle BDC + \angle DBC + \angle BCD = 180^\circ$$

(Angle sum property of a triangle)

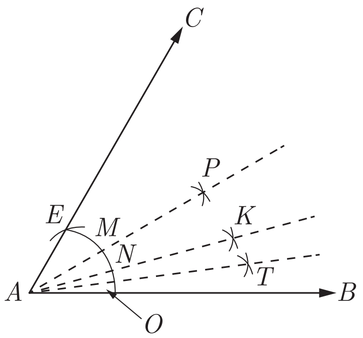
$$2x + x + 90^\circ = 180^\circ$$

$$x = \frac{180^\circ - 90^\circ}{3}$$

$$= \frac{90^\circ}{3} = 30^\circ$$

31. Construct an angle of $7\frac{1}{2}^\circ$, using compass and rules only. [3]

SOLUTION :



Here, $7\frac{1}{2}^\circ = \frac{15}{2} = \frac{15 \times 4}{2 \times 4} = \frac{60^\circ}{8}$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 60^\circ$$

so to construct an angle of $7\frac{1}{2}^\circ$, first draw an angle of 60°.

Say $\angle BAC$ and then bisect to get $\angle BAP = 30^\circ$

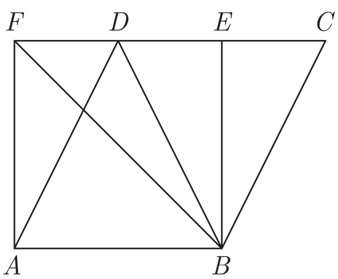
Again bisect $\angle BAP$, to get $\angle BAK = 15^\circ$. Further bisect $\angle BAK$ to get $BAT = 7\frac{1}{2}^\circ$.

32. The area of the parallelogram $ABCD$ is 90 cm^2. Find

(i) $ar(||\,gm\,ABEF)$

(ii) $ar(\Delta ABD)$

(iii) $ar(\Delta BEF)$ [3]



SOLUTION :

Given area of parallelogram $ABCD = 90\text{ cm}^2$

(i) We know that parallelogram on the same base and between the same parallel lines are equal in area

$$\therefore \quad ar(||\,gm\,ABEF) = ar(||\,gm\,ABCD)$$

$$= 90\text{ cm}^2$$

(ii) We know that if a triangle and a parallelogram are on the same base and between the same parallels, then the area of triangle is equal to half of the area of the parallelogram.

$$\therefore \quad ar(\Delta ABD) = \frac{1}{2} ar(||\,gm\,ABEF)$$

$$= \frac{1}{2} \times 90\text{ cm}^2 = 45\text{ cm}^2$$

(iii) Similarly, ΔBEF and parallelogram $ABEF$ are on the same base EF and between same parallels EF and AB.

$$\therefore \quad ar(\Delta BEF) = \frac{1}{2} ar(||\,gm\,ABEF)$$

$$= \frac{1}{2} \times 90\text{ cm}^2 = 45\text{ cm}^2$$

33. Find the ratio of the curved surface areas of two cones, if the diameters of their bases are equal and slant heights are in the ratio 3 : 4. [3]

SOLUTION :

Let the diameter of each cone be r.

$\therefore$ Radius of each cone $= \frac{r}{2}$

Also, let the slant heights of each cone be $3x$ and $4x$ respectively.

i.e., $l_1 = 3x$ and $l_2 = 4x$

Curved surface area of the first cone

$$= \pi r l_1 = \pi \times \left(\frac{r}{2}\right) \times 3x$$

and curved surface area of the second cone

$$= \pi r l_2 = \pi \times \left(\frac{r}{2}\right) \times 4x$$

$\therefore$ Required ratio of the curved surface areas

$$= \frac{\pi \times \left(\frac{r}{2}\right) \times 3x}{\pi \times \left(\frac{r}{2}\right) \times 4x} = \frac{3}{4} \text{ or } 3 : 4$$

or

The sides of a triangle are x, $x + 1$, $2x - 1$ and its area is $x\sqrt{10}$. Find the value of x.

SOLUTION :

Let the sides of triangle are $a = x$, $b = x + 1$ and $c = 2x - 1$

Then, semi-perimeter,

$$s = \frac{a + b + c}{2}$$

$$= \frac{x + x + 1 + 2x - 1}{2}$$

$$= \frac{4x}{2} = 2x$$

Now, area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

But given, area of triangle $= x\sqrt{10}$

$$\therefore \quad x\sqrt{10} = \sqrt{2x(2x-x)\{2x-(x+1)\}\{2x-(2x-1)\}}$$
$$= \sqrt{2x \times x \times (2x-x-1)(2x-2x+1)}$$
$$= \sqrt{2x \times x \times (x-1) \times 1}$$
$$= \sqrt{2x^2(x-1)}$$

On squaring both sides, we get

$$(x\sqrt{10})^2 = \left[\sqrt{2x^2(x-1)}\right]^2$$
$$\Rightarrow \quad 10x^2 = 2x^2(x-1)$$
$$10 = 2(x-1)$$

[dividing both sides by x^2]

$$x-1 = \frac{10}{2}$$
$$\Rightarrow \quad x = 5+1 = 6$$

Hence, the value of x is 6.

34. A batsman in his 12^{th} inning makes a score of 63 runs and thereby increases his average score by 2. What is his average after the 12^{th} inning ? [3]

SOLUTION :

Let the average score of 12 innings be x.
Then, the average score of 11 innings $= (x-2)$
Total score of 12 innings $= 12x$

Total score of 11 innings $= 11(x-2) = 11x-22$

Score of the 12^{th} inning= Total score of 12 innings − Total score of 11 innings

$$= [12x-(11x-22)]$$
$$= x+22$$

According to the questions,

$$x+22 = 63$$
$$x = 41$$

Hence, the average score after 12^{th} inning is 41.

or

A die is rolled 300 times and following outcomes are recorded:

Outcomes	1	2	3	4	5	6
Frequency	42	60	55	53	60	30

Find the probability of getting a number (i) more than 4 (ii) less than 3.

SOLUTION :

(i) Number of possible outcomes to get a number more than 4 $= 60+30 = 90$
Total number of times die rolled $= 300$

$$\therefore P(\text{getting a number more than 4}) = \frac{90}{300} = \frac{3}{10} = 0.3$$

(ii) Number of possible outcomes to get a number less than 3 $= 42+60 = 102$

$$\therefore P(\text{getting a number less than 3}) = \frac{102}{300} = \frac{51}{150} = 0.34$$

Section D

35. Simplify : $\dfrac{-3}{\sqrt{3}+\sqrt{2}} - \dfrac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} + \dfrac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$ [4]

SOLUTION :

We have,
$$I = \frac{-3}{\sqrt{3}+\sqrt{2}} - \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} + \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$$
$$= \frac{-3}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} \times \frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}} + \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

[By rationalising]

$$= \frac{-3(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2} - \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6})^2-(\sqrt{3})^2} + 4\sqrt{3} \times \frac{(\sqrt{6}-\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}$$

[$\because (a-b)(a+b) = a^2-b^2$]

$$= \frac{-3(\sqrt{3}-\sqrt{2})}{1} - \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{6-3} + \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{6-2}$$
$$= -3(\sqrt{3}-\sqrt{2}) - \sqrt{2}(\sqrt{6}-\sqrt{3}) + \sqrt{3}(\sqrt{6}-\sqrt{2})$$
$$= -3\sqrt{3}+3\sqrt{2}-\sqrt{12}+\sqrt{6}+\sqrt{18}-\sqrt{6}$$
$$= -3\sqrt{3}+3\sqrt{2}-2\sqrt{3}+\sqrt{6}+3\sqrt{2}-\sqrt{6}$$
$$= -5\sqrt{3}+6\sqrt{2} \text{ or } 6\sqrt{2}-5\sqrt{3}$$

36. If (x^3+ax^2+bx+6) has $(x-2)$ as a factor and leaves a remainder 3 when divided by $(x-3)$, then find the values of a and b. [4]

SOLUTION :

Let $\quad f(x) = x^3+ax^2+bx+6 \qquad ...(1)$

Since $(x-2)$ is a factor of $f(x)$, then

$$f(2) = 0$$
$$\therefore \quad 2^3+a \times 2^2+b \times 2+6 = 0$$
$$8+4a+2b+6 = 0$$
$$4a+2b+14 = 0$$
$$\Rightarrow \quad 2a+b = -7 \qquad ...(2)$$

[Divided both sides by 2]

Since $f(x)$ leaves remainder 3 when divided by $(x-3)$.

On putting $x=3$ in eq. (1), we get

$$f(3) = (3)^3+a \times (3)^2+b \times 3+6$$
$$= 27+9a+3b+6$$
$$= 9a+3b+33$$

But $f(3) = 3$

$$\therefore \quad 9a+3b+33 = 3$$
$$9a+3b = -30$$
$$\Rightarrow \quad 3a+b = -10 \qquad ...(3)$$

[Dividing both sides by 3]

On subtracting eq.(3) from eq.(2), we get

$$\begin{aligned} 2a + b &= -7 \\ 3a + b &= -10 \\ - \quad - & \quad + \\ \hline \therefore \quad -a &= 3 \quad \text{or } a = -3 \end{aligned}$$

On putting $a = -3$ in eq.(2), we get

$$-6 + b = -7$$

$$\Rightarrow \quad b = -1$$

Hence, the required values of a and b are -3 and -1 respectively.

37. Draw the graph of equation $5x + 3y = 4$ and check whether
(a) $x = 2,\ y = 5$
(b) $x = -1,\ y = 3$ are solution. [4]

SOLUTION :

Given equation is

$$5x + 3y = 4$$

$$3y = 4 - 5x$$

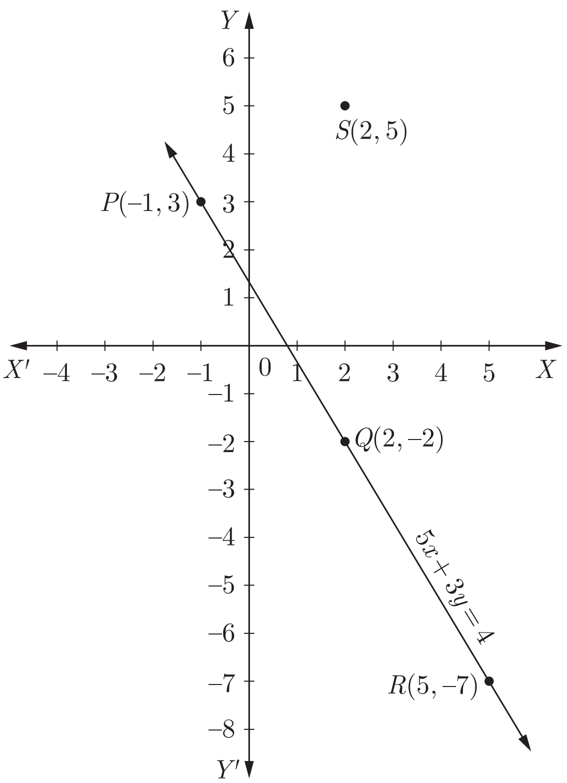
$$\Rightarrow \quad y = \frac{4 - 5x}{3}$$

Let us draw the table of given equation in which values of dependent variable y are determined corresponding to the independent variable x.

x	–1	2	5
y	3	–2	–7
(x, y)	$P(-1, 3)$	$Q(2, -2)$	$R(5, -7)$

Now, we plot the points P, Q and R. Join all these points to obtain the graph of line $5x + 3y = 4$

On plotting the points we see that $P(-1, 3)$ lies on the graph but point $S(2, 5)$ does not lie on it.



Hence, $x = -1$, $y = 3$ is a solution but $x = 2$, $y = 5$ is not a solution of $5x + 3y = 4$.

or

In a class, number of girls is x and that of boys is y. Also, the number of girls is 10 more than the number of boys. Write the given data in the form of a linear equation in two variables. Also, represent it graphically. Find graphically the number of girls, if the number of boys in 20.

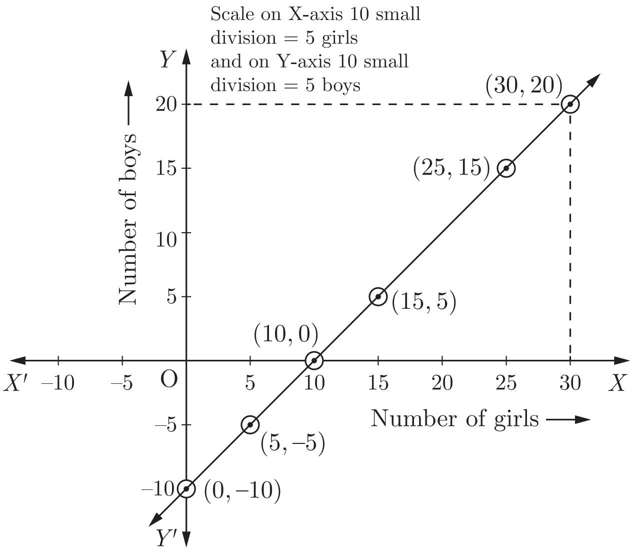
SOLUTION :

Given number of girls and boys are x and y respectively.
According to the question,

$$x - y = 10$$

x	0	10	5	15	25
y	–10	0	–5	5	15

Hence, from the graph, if the number of boys is 20,, then the number of girls is 30.



38. Prove that the quadrilateral formed by the internal angle bisectors of any quadrilateral is cyclic. [4]

SOLUTION :

Let $ABCD$ be a quadrilateral in which the angle bisectors AH, BF, CF and DH of internal angles A, B, C and D respectively form a quadrilateral $EFGH$. $EFGH$ is a cyclic quadrilateral.

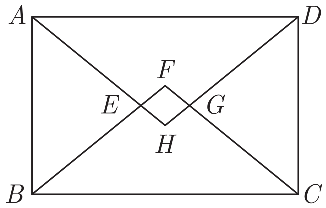
i.e., $\angle E + \angle G = 180°$

or $\angle F + \angle H = 180°$

Since

$$\begin{aligned} \angle FEH &= \angle AEB \\ &= 180° - \angle EAB - \angle EBA \\ &= 180° - \frac{1}{2}(2\angle EAB + 2\angle EBA) \\ &= 180° - \frac{1}{2}(\angle A + \angle B) \quad \text{...(1)} \end{aligned}$$

[$\because$ AH and BF are bisectors of $\angle A$ and $\angle B$ respectively]



Similarly, $\angle FGH = \angle CGD$

$$= 180^\circ - \angle GCD - \angle GDC$$

$$= 180^\circ - \frac{1}{2}(\angle C + \angle D) \quad ...(2)$$

On adding eq.(1) and (2), we get

$$\angle FEH + \angle FGH = 180^\circ - \frac{1}{2}(\angle A + \angle B) + $$

$$+180^\circ - \frac{1}{2}(\angle C + \angle D)$$

$$= 360^\circ - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D)$$

$$= 360^\circ - \frac{1}{2} \times 360^\circ = 180^\circ$$

[$\because$ Sum of angles of a quadrilateral is 360°]

39. Find the mean, median and mode for the following data. [4]
10, 15, 18, 10, 10, 20, 10, 20, 15, 21, 15, 25

SOLUTION :

As we know that,

$$\text{Mean} = \frac{\text{Sum of all the observations}}{\text{Total number of observations}}$$

$$= \frac{10 + 15 + 18 + 10 + 10 + 20 + 10 + 20 + 15 + 21 + 15 + 25}{12}$$

$$= \frac{189}{12} = 15.75$$

For Median,
Arranging the given data in ascending order, we have
10, 10, 10, 10, 15, 15, 15, 18, 20, 20, 21, 25
$\because$ Number of observations $(n) = 12$ [even]

$\therefore$ Median

$$= \frac{\left(\frac{n}{2}\right)^{\text{th}}\text{observation} + \left(\frac{n}{2}+1\right)^{\text{th}}\text{observation}}{2}$$

$$= \frac{\left(\frac{12}{2}\right)^{\text{th}}\text{observation} + \left(\frac{12}{2}+1\right)^{\text{th}}\text{observation}}{2}$$

$$= \frac{6^{\text{th}}\text{ observation} + 7^{\text{th}}\text{ observation}}{2} = \frac{15+15}{2}$$

$$= 15$$

For Mode,
Arranging the given data in ascending order, we have
10, 10, 10, 10, 15, 15, 15, 18, 20, 20, 21, 25
Here, 10 occurs most frequently (4 times)

$\therefore$ Mode $= 10$

40. 50 students of class IX planned to visit an old age home and to spend the whole day with their inmates. Each one prepared a cylindrical flower base using cardboard to gift the inmates. The radius of the cylindrical flower base is 4.2 cm and the height is 11.2 cm.
What is the amount spent for purchasing the cardboard at the rate of ₹ 20 per 100 cm^2 ?

SOLUTION :

Given, radius of the cylindrical base $(r) = 4.2$ cm
and height of the cylindrical flower base $(h) = 11.2$ cm
$\therefore$ Surface area of cylindrical base

A = curved surface area of cylindrical flower base + area of base

$$= 2\pi rh + \pi r^2$$

$$= 2\pi \times 4.2 \times 11.2 + \pi(4.2)^2$$

$$= 4.2\pi(22.4 + 4.2) = 4.2 \times \frac{22}{7} \times 26.6$$

$$= 351.12\text{ cm}^2$$

Now, surface area of 50 cylinders

$$= 50 \times 351.12\text{ cm}^2 = 17556\text{ cm}^2$$

So, Total cost $= ₹\left(17556 \times \frac{20}{100}\right) = ₹\,3511.20$

or

Water is flowing at the rate of 3 km/hour through a circular pipe of 20 cm internal diameter into a circular cistern of diameter 10 m and depth 2 m. In how much time will the cistern be filled ?

SOLUTION :

Given,

Internal diameter of circular pipe, $d = 20$ cm

Internal radius $= r$

$$= \frac{20}{2} = 10\text{ cm}$$

$$= \frac{10}{100}\text{ m}$$

$$= \frac{1}{10}\text{ m}$$

Water is flowing at the rate of 3 km/hour = 3000 m/hr
Volume of water flowing in one hour

$$= \pi r^2 h$$

$$= \pi\left(\frac{1}{10}\right)^2 \times 3000\text{ m}^3$$

Diameter of cistern $= 10$ m

Radius of cistern $= \frac{10}{2} = 5$ m

Depth of cistern $= 2$ m

Volume of water in cistern $= \pi r^2 h = \pi(5)^2 \times 2\text{ m}^3$

Let the time taken to fill the cistern $= t$ hours

$$t = \frac{\text{Volume of water in cistern}}{\text{Volume of water flowing from pipe in 1 hour}}$$

$$= \frac{\pi(5)^2 \times 2}{\pi\left(\frac{1}{100}\right) \times 3000}$$

$$= 1\frac{2}{3}\text{ hours} = 1\text{ hour 40 minutes}$$