CLASS IX (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-2

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.

(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.

(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. Set of natural numbers is a subset of [1]

(a) Set of even numbers

(b) Set of odd numbers

(c) Set of composite numbers

(d) Set of real numbers

Ans : (d) Set of real numbers

Since, set of real numbers contains all natural numbers, integers, rational and irrational numbers.

2. Degree of the polynomial $p(x)=(x+2)(x-2)$ is [1]

(a) 2 (b) 1

(c) 0 (d) 3

Ans : (a) 2

$$p(x) = (x+2)(x-2) = x^2-4$$

The highest power of the variable x is 2. So the degree of the polynomial, $p(x)=2$

3. A point lies on negative side of x-axis. Its distance from origin is 10 units. The coordinates of the point are [1]

(a) $(10,0)$ (b) $(-10,0)$

(c) $(0,10)$ (d) $(0,-10)$

Ans : (b) $(-10,0)$

The required points is $(-10,0)$.

4. If $(a,1)$ lies on the graph of $3x-2y+4=0$, then $a=$ [1]

(a) $\frac{-2}{3}$ (b) $\frac{2}{3}$

(c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

Ans : (a) $\frac{-2}{3}$

Since $(a,1)$ lies on

$$3x-2y+4 = 0$$

$$3\times a-2\times 1+4 = 0$$

$$3a = -4+2 = -2$$

$$a = \frac{-2}{3}$$

5. If a point C lies between two point A and B such that $AC=BC$, then [1]

(a) $AC=AB$ (b) $AC=\frac{1}{2}AB$

(c) $AB=\frac{1}{2}AC$ (d) $AC=\frac{1}{3}AB$

Ans : (b) $AC=\frac{1}{2}AB$

If $AC = BC$

Then, C is a midpoint of AB.

and $AC = \frac{1}{2}AB$

6. If $l\parallel m$, then value of x is [1]

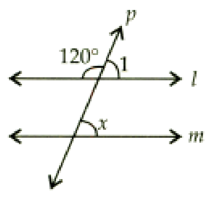
(a) 60°

(b) 120°

(c) 40°

(d) Cannot be determined

Ans : (a) 60°



$$\angle 1+120^\circ = 180^\circ \text{ [Linear pair]}$$

$$\angle 1 = 180^\circ-120^\circ = 60^\circ$$

Since $l\parallel m$

$$\angle x = \angle 1 = 60^\circ$$

[Corresponding Angles]

7. Which of the following is not a criterion for congruence of triangles? [1]

(a) SSA (b) SAS

(c) ASA (d) SSS

Ans : (a) SSA

8. The angles of a quadrilateral are x°, $(x-10)^\circ$, $(x+30)^\circ$ and $(2x)^\circ$, the smallest angle is equal to [1]

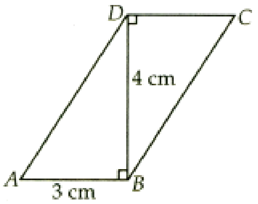
(a) 68° (b) 52°
(c) 58° (d) 47°

Ans : (c) 58°

Sum of the angles of a quadrilateral is 360°. So,

$$x^\circ + (x-10)^\circ + (x+30)^\circ + 2x^\circ = 360^\circ$$
$$5x + 20 = 360$$
$$5x = 340$$
$$x = 68$$
$$\text{smallest angles is } (x-10)^\circ = 58^\circ$$

9. In the adjoining figure, $ABCD$ is a parallelogram. Then its area is equal to [1]

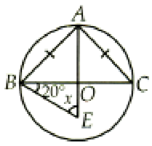


(a) 9 cm^2 (b) 12 cm^2
(c) 15 cm^2 (d) 36 cm^2

Ans : (b) 12 cm^2

$$\text{Area of parallelogram} = \text{Base} \times \text{Height}$$
$$= AB \times BD = 4 \times 3\text{ cm}^2$$
$$= 12\text{ cm}^2$$

10. In the given figure, E is any point in the interior of the circle with centre O. Chord $AB = AC$. If $\angle OBE = 20^\circ$, the value of x is [1]



(a) 40° (b) 45°
(c) 50° (d) 70°

Ans : (d) 70°

Since, $AB = AC$

Hence, $\angle AOB = \angle AOC$

[Equal chords subtend equal angles at the centre]

$AO \perp BC$ $[\angle BOA + \angle COA = 180^\circ]$

Now, in $\Delta\, OBE$

$$20^\circ + x + \angle BOE = 180^\circ$$

$$20^\circ + x + 90^\circ = 180^\circ$$
$$x = 70^\circ$$

(Q.11-Q.15) Fill in the blanks :

11. The construction of a $\Delta\, DEF$ in which $DE = 7$ cm, $\angle D = 75^\circ$ is possible when $(DE - EF)$ is equal to .... cm. [1]

Ans : 6.5 cm

We know that in a triangle, the difference of two sides is never greater than any side.
i.e., $EF - DF < DE$ i.e., 7 cm
$EF + DF$ will be 6.5 cm.

12. The sides of a triangular field are 33 m, 44 m and 55 m. the cost of levelling the field at the rate of ₹ 1.20 per m² is ₹........... [1]

Ans : ₹1480

$$s = \frac{33+44+55}{2} = 66\text{ m}$$

$$\text{Area of triangle}, A = \sqrt{66(66-33)(66-44)(66-55)}$$
$$= \sqrt{66 \times 33 \times 22 \times 11}$$
$$= 726\text{ m}^2$$

Cost of levelling = ₹ (726 × 1.20) = ₹ 871.20

or

If height of a triangle is doubled and base in tripled then its area become .... times.

Ans : six

13. The volume of a rectangular solid measuring 1 m by 50 cm by 0.5 m is .... cm³. [1]

Ans : 250, 000

14. The .... is the most frequently occurring observation. [1]

Ans : mode

15. Total number of results are called .... [1]

Ans : Outcomes

(Q.16-Q.20) Answer the following :

16. Simplify : $\sqrt[5]{243a^{10}b^5c^{10}}$ [1]

SOLUTION :

$$I \quad \sqrt[5]{243a^{10}b^5c^{10}}$$
$$= (3 \times 3 \times 3 \times 3 \times 3 \times a^{10} \times b^5 \times c^{10})^{1/5}$$
$$= (3^5 \cdot a^{10} \cdot b^5 \cdot c^{10})^{1/5}$$
$$= (3)^{5 \times \frac{1}{5}} \cdot (a)^{10 \times \frac{1}{5}} \cdot (b)^{5 \times \frac{1}{5}} (c)^{10 \times \frac{1}{5}}$$
$$= 3 \times a^2 \times b \times c^2 = 3a^2bc^2$$

17. If $p(x) = x^2 - 2\sqrt{2}\,x + 1$, then find $p(2\sqrt{2})$. [1]

SOLUTION :

$$p(x) = x^2 - 2\sqrt{2}\,x + 1$$
$$p(2\sqrt{2}) = 2(\sqrt{2})^2 - 2\sqrt{2} \times 2\sqrt{2} + 1$$
$$= 8 - 8 + 1 = 1$$

or

Find the remainder when $x^3 - px^2 + 6x - p$ is divided by $x - p$.

SOLUTION :

$$x - p = 0$$
$$x = p$$

Putting $x = p$ in $x^3 - px^2 + 6x - p$, we get

$$p^3 - p^3 + 6p - p = 5p$$

18. 'Two intersecting lines cannot be parallel to the same lines' is stated in which form. [1]

SOLUTION :

This statement is stated in the form of a postulate.

19. An isosceles right triangle has area $8\,\text{cm}^2$. Find the length of its hypotenuse. [1]

SOLUTION :

$$\text{Area of triangle} = 8\,\text{cm}^2$$

$$\frac{1}{2} \times x \times x = 8$$

$$x = 4$$

Hypotenuse of the triangle

$$= \sqrt{4^2 + 4^2}\,\text{cm}$$

$$= \sqrt{32}\,\text{cm}$$

or

The base of a right triangle is $8\,\text{cm}$ and hypotenuse is $10\,\text{cm}$. What is its area?

SOLUTION :

$$\text{Altitude of the triangle} = \sqrt{100 - 64}\,\text{cm} = 6\,\text{cm}$$

$$\text{Area of the triangle} = \frac{1}{2} \times 8 \times 6\,\text{cm}^2$$

$$= 24\,\text{cm}^2$$

20. Two coins are tossed simultaneously. List all possible outcomes. [1]

SOLUTION :

All possible outcomes are HH, HT, TH, TT.

Section B

21. If $x = \frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} - \sqrt{6}}$, then find the value of $\left(x + \frac{1}{x}\right)^2$. [2]

SOLUTION :

We have,

$$x = \frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} - \sqrt{6}}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{\frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} - \sqrt{6}}} = \frac{\sqrt{7} - \sqrt{6}}{\sqrt{7} + \sqrt{6}}$$

$$\therefore \quad x + \frac{1}{x} = \frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} - \sqrt{6}} + \frac{\sqrt{7} - \sqrt{6}}{\sqrt{7} + \sqrt{6}}$$

$$= \frac{(\sqrt{7} + \sqrt{6})^2 + (\sqrt{7} - \sqrt{6})^2}{7 - 6}$$

$$= 7 + 6 + 2\sqrt{42} + 7 + 6 - 2\sqrt{42}$$

$$= 26$$

$$\therefore \quad \left(x + \frac{1}{x}\right)^2 = 26^2 = 676$$

22. Find the value of k, for which the polynomial $x^3 - 3x^2 + 3x + k$ has 3 as its zero. [2]

SOLUTION :

Let $\quad p(x) = x^3 - 3x^2 + 3x + k$

Since, 3 is a zero of $p(x)$

$$\therefore \quad p(3) = 0$$

$$\Rightarrow \quad (3)^3 - 3(3)^2 + 3(3) + k = 0$$

$$27 - 27 + 9 + k = 0$$

$$9 + k = 0$$

$$\therefore \quad k = -9$$

or

Give the equations of two lines passing through (2, 14). How many more such lines are there, and why ?

SOLUTION :

Let $x + y = k$ be such a line, then

$$2 + 14 = k \quad \Rightarrow \quad k = 16$$

$\therefore$ $x + y = 16$ passes through (2, 14).

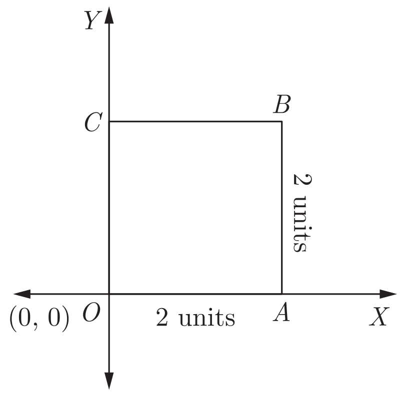
Let $2x + 3y = k'$ be another line through (2, 14).

$$2 \times 2 + 3 \times 14 = k'$$

$$\Rightarrow \quad k' = 4 + 42 = 46$$

$\Rightarrow$ $2x + 3y = 46$ passes through (2, 14).
There are infinitely many such lines, as through a point infinite number of straight lines can be drawn.

23. In the figure, O is the origin and $OABC$ is a square of side 2 units. Find the co-ordinates of A, B and C. [2]



SOLUTION :

As the point A lies on the x-axis at a distance of 2 units from the origin, its coordinates will be (2, 0), point B lies 2 units away from both the axes, its coordinate will be (2, 2) and point C lies on the y-axis at a distance of 2 units from the origin, its coordinates will be (0, 2).

24. One of the three angles of a triangle is twice the smallest and another is three times the smallest. Find the angles. [2]

SOLUTION :

Let the smallest angle be $\angle A = x$
Then, according to the question, other two angles will

be $\angle B = 2x$ and $\angle C = 3x$.

Also, $\angle A + \angle B + \angle C = 180^\circ$

[Since, sum of all three angles of a triangle is 180°]

$$\Rightarrow \quad x + 2x + 3x = 180^\circ$$
$$6x = 180^\circ$$
$$x = \frac{180^\circ}{6} = 30^\circ$$

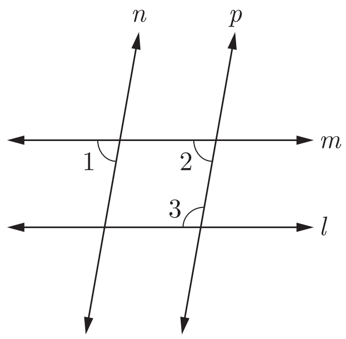
Now, $\angle A = x = 30^\circ$

$\angle B = 2x = 2 \times 30^\circ = 60^\circ$

and $\angle C = 3x = 3 \times 30^\circ = 90^\circ$

Hence, three angles of a triangle are 30°, 60° and 90°.

25. In the given figure, if $l || m$, $n || p$ and $\angle 1 = 75^\circ$, then find $\angle 3$. [2]



SOLUTION :

Given $n || p$

Since, line m is transversal of lines n and p.

$\therefore \quad \angle 1 = \angle 2 = 75^\circ$ [Corresponding angles]

As, $l || m$

Since, line p is transversal of lines m and l. [given]

$\therefore \quad \angle 2 + \angle 3 = 180^\circ$

[Since, sum of two co-interior angles is 180°]

$$\Rightarrow \quad 75^\circ + \angle 3 = 180^\circ$$
$$\angle 3 = 180^\circ - 75^\circ = 105^\circ$$

Hence, the value of $\angle 3$ is 105°.

or

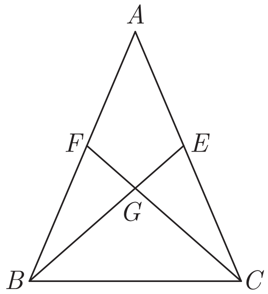
The medians BE and CF of a ΔABC intersect at G.

Prove that $ar(\Delta GBC) = ar(\text{quad } AFGE)$.

SOLUTION :

Given, BE and CF are medians of a ΔABC.

We know that median of a triangle divides it into two parts of equal area.



$$\therefore \quad ar(\Delta FBC) = ar(\Delta AFC)$$
$$\Rightarrow \quad ar(\Delta FBC) = \frac{1}{2} ar(\Delta ABC) \qquad ...(1)$$

Similarly,

$$\Rightarrow \quad ar(\Delta AEB) = \frac{1}{2} ar(\Delta ABC) \qquad ...(2)$$

From equation (1) and (2), we get

$$ar(\Delta FBC) = ar(\Delta AEB)$$

On subtracting $ar(\Delta GBF)$ from both sides, we get,

$$ar(\Delta FBC) - ar(\Delta GBF) = ar(\Delta AEB) - ar(\Delta GBF)$$
$$\Rightarrow \quad ar(\Delta GBC) = ar(\text{quadrilateral } AFGE)$$

26. A solid right circular cone of radius 4 cm and height 7 cm is melted to form a sphere. Find the radius of sphere. [2]

SOLUTION :

Let r and R be radii of cone and sphere respectively and h be the height of cone.

Given, radius of cone $(r) = 4$ cm

and height of cone $(h) = 7$ cm

$$\therefore \quad \text{Volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (4)^2 \times 7$$
$$= \frac{112}{3}\pi \text{ cm}^3$$
$$\text{Volume of sphere} = \frac{4}{3}\pi R^3$$

According to question,

$$\text{Volume of cone} = \text{Volume of sphere}$$
$$\Rightarrow \quad \frac{112}{3}\pi = \frac{4}{3}\pi R^3$$
$$R^3 = 28 \text{ cm}^3$$
$$\therefore \quad R = \sqrt[3]{28} \text{ cm}$$

Hence, the radius of the sphere is $\sqrt[3]{28}$ cm.

or

The sides of a triangle are in the ratio 3 : 5 : 7 and its perimeter is 300 m. Find its area.

SOLUTION :

Let the sides of the triangle be $3x$ m, $5x$ m and $7x$ m

Perimeter of the triangle $= 300$ m.

$$\therefore \quad 3x + 5x + 7x = 300$$
$$\Rightarrow \quad 15x = 300$$
$$x = 20$$

$\therefore$ sides of triangle are (3×20) m, (5×20) m and (7×20) m

i.e., 60 m, 100 m and 140 m

Now, suppose $a = 60$ m, $b = 100$ m and $c = 140$ m

$$\therefore \quad s = \frac{60 + 100 + 140}{2} \text{ m}$$
$$= \frac{300}{2} \text{ m} = 150 \text{ m}$$

Area of triangle

$$A = \sqrt{150(150-60) \times (150-100) \times (150-140)} \text{ m}^2$$
$$= \sqrt{150 \times 90 \times 50 \times 10} \text{ m}^2 = 1500\sqrt{3} \text{ m}^2$$

Section C

27. The points $A(a, b)$ and $B(b, 0)$ lie on the linear equation $y = 8x + 3$.

(i) Find the value of a and b
(ii) Is (2, 0) a solution of $y = 8x + 3$?
(iii) Find two solutions of $y = 8x + 3$ [3]

SOLUTION :

Given : $y = 8x + 3$ ...(1)

(i) On putting $x = a$ and $y = b$ in equation (1), we have

$$b = 8a + 3$$

On putting $x = b$ and $y = 0$ in equation (1), we have

$$0 = 8b + 3$$

$$\Rightarrow \quad b = \frac{-3}{8} \quad ...(2)$$

By putting $b = \frac{-3}{8}$ in equation (2), we have

$$\frac{-3}{8} = 8a + 3$$

$$\Rightarrow \quad \frac{-3}{8} - 3 = 8a$$

$$\frac{-27}{8} = 8a$$

$$a = \frac{-27}{64}$$

(ii) On putting $x = 2$ and $y = 0$ in (1), we have

$$0 = 8 \times 2 + 3$$

$$\Rightarrow \quad 0 = 16 + 3$$

$$0 = 19, \text{ false}$$

Hence, $(2, 0)$ is not a solution of the linear equation $y = 8x + 3$.

(iii) $y = 8x + 3$

Let $x = 0$, then

$$y = 8 \times 0 + 3$$

$$y = 3$$

Hence, (0, 3) is a solution of the linear equation $y = 8x + 3$.

Let $y = 0$, then

$$0 = 8x + 3$$

$$-3 = 8x$$

$$x = \frac{-3}{8}$$

Hence, $\left(\frac{-3}{8}, 0\right)$ is a solution of the linear equation $y = 8x + 3$.

or

Draw graphs of $3x + 2y = 0$ and $2x - 3y = 0$ and what is the point of intersection of the two lines representing the above equation.

SOLUTION :

Table of values for $3x + 2y = 0$

$$\Rightarrow \quad 3x = -2y$$

$$y = \frac{-3x}{2}$$

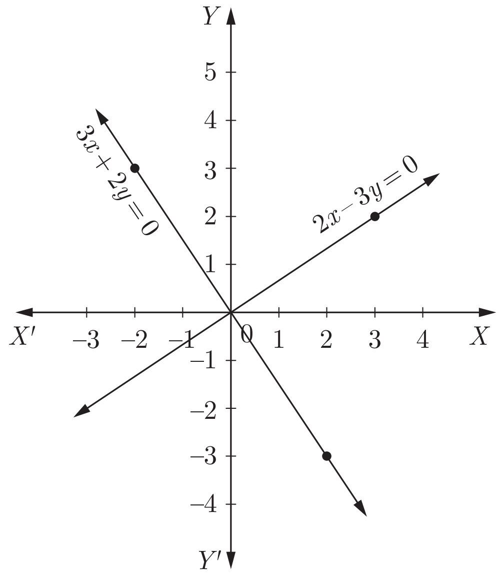
x	0	2	–2
y	0	–3	3

Table of values for $2x - 3y = 0$

$$\Rightarrow \quad 2x = 3y$$

$$y = \frac{2x}{3}$$

x	0	3	6
y	0	2	4



We see that, from graph point of intersection is (0, 0).

28. The sides of a triangular park are 8 m, 10 m and 6 m respectively. A small circular area of diameter 2 m is to be left out and the remaining area is to be used for growing roses. How much area is used for growing roses ? [Take $\pi = 3.14$] [3]

SOLUTION :

Let sides of a triangle be $a = 8\,\text{m}$, $b = 10\,\text{m}$ and $c = 6\,\text{m}$

Now, semi-perimeter of a triangle,

$$s = \frac{a + b + c}{2} = \frac{8 + 10 + 6}{2}$$

$$= \frac{24}{2} = 12\,\text{m}$$

Area of a triangle $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

[By Heron's formula]

$$= \sqrt{12(12-8)(12-10)(12-6)}$$

$$= \sqrt{12 \times 4 \times 2 \times 6} = 24\,\text{m}^2$$

and area of a circle $= \pi r^2 = 3.14 \times 1^2$

$$= 3.14\,\text{m}^2 \qquad \left[\because r = \frac{d}{2}\right]$$

$\therefore$ Area for growing roses = Area of triangle − Area of circle

$$= (24 - 3.14)\,\text{m}^2$$

$$= 20.86\,\text{m}^2$$

or

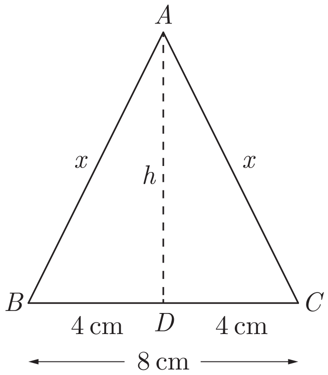
The area of an isosceles triangle is $8\sqrt{15}\text{ cm}^2$. If the base is 8 cm, find the length of each of its equal sides.

SOLUTION :

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\Rightarrow \quad 8\sqrt{15} = \frac{1}{2} \times 8 \times h$$

$$h = 2\sqrt{15}\text{ cm}$$



Using Pythagoras theorem in right angles ΔADB, we have

$$AB^2 = BD^2 + AD^2$$

$$\Rightarrow \quad x^2 = 4^2 + (2\sqrt{15})^2$$

$$x^2 = 16 + 60 = 76\text{ cm}$$

$$x = \sqrt{76} = 8.72\text{ cm}$$

$\therefore$ Length of each of its equal sides $= 8.72\text{ cm}$.

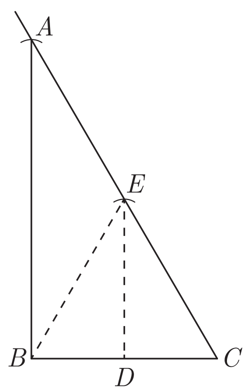
29. Draw a ΔABC, in which $BC = 4\text{ cm}$, $AB = 5\text{ cm}$ and the median $BE = 3.5\text{ cm}$. [3]

SOLUTION :

Given : ΔABC, in which $BC = 4\text{ cm}$, $AB = 5\text{ cm}$ and the median $BE = 3.5\text{ cm}$.

Steps of construction

(i) Take $BC = 4\text{ cm}$
(ii) Divide BC at D.
(iii) With B as centre and the radius equal to median (3.5 cm) draw an arc.
(iv) With D as the centre and the radius equal to the half of AB (2.5 cm), draw another arc intersecting the first arc at E.
(v) Join CE and produce to A, such that $CE = EA$.
(vi) Join A and B.



Thus, the ΔABC is the required triangle. [by mid point theorem $DE = \frac{1}{2}AB$ and $DE \,||\, AB$.]

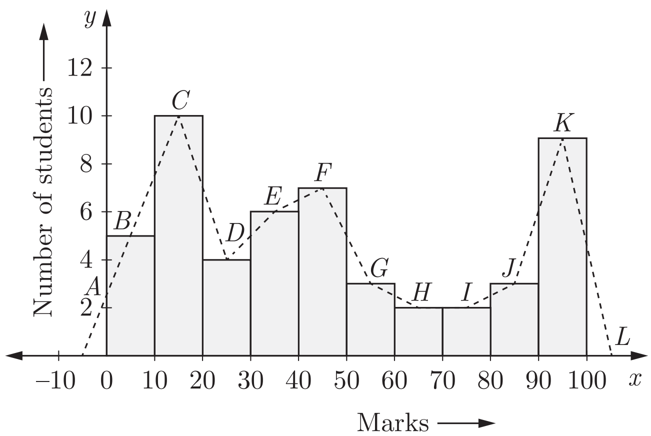
30. Consider the marks, out of 100, obtained by 51 students of a class in a test, given below. [3]

Marks	Number of students
0-10	5
10-20	10
20-30	4
30-40	6
40-50	7
50-60	3
60-70	2
70-80	2
80-90	3
90-100	9
Total	51

Draw a histogram and frequency polygon for the above data on a same scale.

SOLUTION :

The required graph is shown below :



or

For a particular year, following is the frequency distribution table of ages (in years) of primary school teachers in a district :

Age (in years)	Number of teachers
15-20	10
20-25	30
25-30	50
30-35	50
35-40	30
40-45	6
45-50	4

(i) Write the lower limit of the first class interval.
(ii) Determine the class limits of the fourth class interval.
(iii) Find the class mark of the class 45-50.

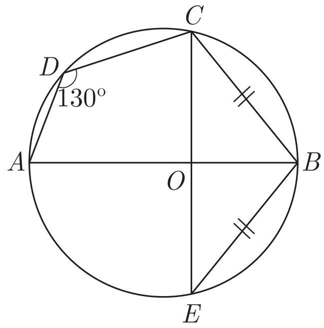
SOLUTION :

(i) 15 is the lower limit of the first class interval.

(ii) Fourth class interval is 30-35.

(iii) Class mark $= \frac{45+50}{2} = \frac{95}{2} = 47.5$

31. In the given figure, $\angle ADC = 130^\circ$ and chord BC = chord BE. Find $\angle CBE$. [3]



SOLUTION :

$ABCD$ is a cyclic quadrilateral.

$\Rightarrow \quad \angle ADC + \angle OBC = 180^\circ$

[Opposite angles of cyclic quadrilateral]

$\Rightarrow \quad 130^\circ + \angle OBC = 180^\circ$

$\Rightarrow \quad \angle OBC = 50^\circ$

In ΔOBC and ΔOBE,

$BC = BE$ [Given]

$OB = OB$ [Common]

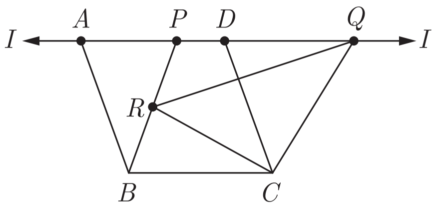
$OC = OE$ [AB act as perpendicular bisector of CE]

$\therefore \quad \Delta OBC \cong \Delta OBE$ [By SSS congruence]

$\Rightarrow \quad \angle OBE = \angle OBC = 50^\circ$

$\therefore \quad \angle CBE = \angle OBC + \angle OBE = 50^\circ + 50^\circ = 100^\circ$

32. In the given figure, parallelogram $ABCD$ and $PBCQ$ are given. If R is a point on PB, then show that $ar(\Delta QRC) = \frac{1}{2} ar(|| gm\ ABCD)$. [3]



SOLUTION :

Given : $ABCD$ and $PBCQ$ are two parallelograms, and R is a point on PB.

To prove :

$$ar(\Delta QRC) = \frac{1}{2} ar(|| gm\ ABCD)$$

Proof :

Here, parallelogram $PBCQ$ and $ABCD$ lie on the same base BC and between the same parallels BC and AQ.

So, $ar(|| gm\ PBCQ) = ar(|| gm\ ABCD)$ ...(1)

Now, ΔQRC and parallelogram $PBCQ$ lie on the same base CQ and between the same parallels CQ and BP.

So, $ar(\Delta QRC) = \frac{1}{2} ar(|| gm\ PBCQ)$ ...(2)

From equation (1) and (2), we get

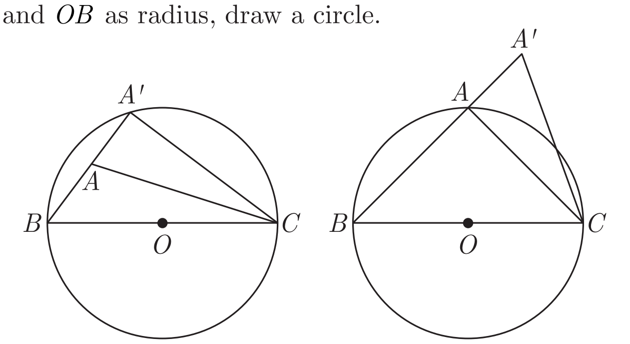
$$ar(\Delta QRC) = \frac{1}{2} ar(|| gm\ ABCD)$$

Hence proved.

33. Prove that the mid point of the hypotenuse of a right angled triangle is equidistant from its vertices. [3]

SOLUTION :

Let ΔABC be a right angled triangle such that $\angle BAC = 90^\circ$. Let O be the mid-point of the hypotenuse BC. Then, $OB = OC$. With O as centre and OB as radius, draw a circle.



Clearly, the circle passes through the points B and C . If possible, suppose this circle does not pass through A.

Let it meets BA or BA' produced at A'.

Then $\quad \angle BA'C = 90^\circ$

[Since, angle in a semi-circle is 90°]

But $\quad \angle BAC = 90^\circ$

$\therefore \quad \angle BA'C = \angle BAC$

This is not possible unless A coincide A'.

So, the circle which passes through B and C also passes through A.

Consequently, $\quad OA = OB = OC$

$= $ Radius of the circle

Hence, the mid-point O of the hypotenuse BC of right angled ΔABC is equidistant from its vertices.

34. Prove that the sum of any two sides of a triangle is greater than the third side. [3]

SOLUTION :

Given : ABC is a triangle

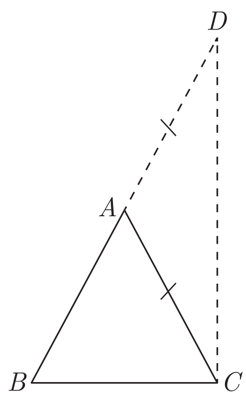
To prove :

(i) $AB + AC > BC$

(ii) $AB + BC > AC$

(iii) $BC + AC > AB$

Construction : Produce side BA to D such that $AD = AC$. Join CD.



Proof :

In ΔACD, $AC = AD$ [By construction]

$\Rightarrow$ $\angle ADC = \angle ACD$

[Since, angles opposite to equal sides of a triangle are equal]

$\because$ $\angle ACD = \angle ADC$

From figure,

$$\angle BCA + \angle ACD > \angle ACD$$

$\therefore$ $\angle BCA + \angle ACD > \angle ADC$

$\Rightarrow$ $\angle BCD > \angle ADC$

$\angle BCD > \angle BDC$ [$\because \angle ADC = \angle BDC$]

$BD > BC$

[Since, side opposite to greater angle in a triangle is greater]

$\Rightarrow$ $BA + AD > BC$

$BA + AC > BC$ [$\because AC = AD$, by construction]

Thus, $AB + AC > BC$

Similarly, $AB + BC > AC$

and $BC + AC > AB$

Section D

35. Simplify : [4]

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + ... + \frac{1}{\sqrt{8}+\sqrt{9}}$$

SOLUTION :

$$\frac{1}{1+\sqrt{2}} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1}$$
$$= \sqrt{2}-1$$

$$\frac{1}{\sqrt{2}+\sqrt{3}} = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2}$$
$$= \sqrt{3}-\sqrt{2}$$

$$\frac{1}{\sqrt{3}+\sqrt{4}} = \frac{1}{\sqrt{4}+\sqrt{3}} \times \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}}$$
$$= \frac{\sqrt{4}-\sqrt{3}}{4-3} = \sqrt{4}-\sqrt{3}$$

....

....

....

$$\frac{1}{\sqrt{8}+\sqrt{9}} = \frac{1}{\sqrt{9}+\sqrt{8}} \times \frac{\sqrt{9}-\sqrt{8}}{\sqrt{9}-\sqrt{8}}$$
$$= \frac{\sqrt{9}-\sqrt{8}}{1} = \sqrt{9}-\sqrt{8}$$

Given expression :

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + ... + \frac{1}{\sqrt{8}+\sqrt{9}}$$
$$= (\sqrt{2}-1) + (\sqrt{3}-\sqrt{2}) + (\sqrt{4}-\sqrt{3}) + $$
$$+ ... + (\sqrt{8}-\sqrt{7}) + (\sqrt{9}-\sqrt{8})$$
$$= \sqrt{9}-1 = 3-1 = 2$$

36. Find the value of $x^3 - 8y^3 - 36xy - 220$, when $x = 2y + 6$. [4]

SOLUTION :

Given,

$$x = 2y + 6$$

$\Rightarrow$ $x - 2y = 6$ ...(1)

On cubing both the sides of equation (1), we get

$$(x-2y)^3 = (6)^3$$
$$x^3 - (2y)^3 - 3x2y(x-2y) = 6^3$$

[$\because$ $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$]

$\Rightarrow$ $x^3 - 8y^3 - 6xy(x-2y) = 216$

$x^3 - 8y^3 - 6xy(6) = 216$ [From eq.(1)]

$$x^3 - 8y^3 - 36xy = 216$$
$$x^3 - 8y^3 - 36xy - 216 = 0$$
$$x^3 - 8y^3 - 36xy - 216 - 4 = -4$$
$$x^3 - 8y^3 - 36xy - 220 = -4$$

Thus, the required value of the given expression is -4.

or

Which of the following points $A\left(0, \frac{17}{3}\right)$, $B(2,6)$, $C(1,5)$ and $D(5,1)$ lie on the linear equation $2(x+1) + 3(y-2) = 13$.

SOLUTION :

$$2(x+1) + 3(y-2) = 13$$

$\Rightarrow$ $2x + 2 + 3y - 6 = 13$

$$2x + 3y = 13 + 4$$

$2x + 3y = 17$ ...(1)

On putting $x = 0$ and $y = \frac{17}{3}$ in (1), we have

$$2 \times 0 + 3 \times \frac{17}{3} = 17$$
$$0 + 17 = 17$$

$17 = 17$, true

Therefore, $\left(0, \frac{17}{3}\right)$ lies on the given linear equation

$2(x+1) + 3(y-2) = 13$.

On putting $x = 2$ and $y = 6$ in (1), we have

$$2 \times 2 + 3 \times 6 = 17$$

$\Rightarrow$ $4 + 18 = 17$

$22 = 17$, false

Therefore, (2, 6) does not lie on the given linear equation $2(x+1) + 3(y-2) = 13$.

On putting $x = 1$ and $y = 5$ in (1), we have

$$2 \times 1 + 3 \times 5 = 17$$

$\Rightarrow$ $2 + 15 = 17$

$17 = 17$, true

Therefore, (1, 5) lies on the given linear equation $2(x+1) + 3(y-2) = 13$.

On putting $x = 5$ and $y = 1$ in (1), we have

$$2 \times 5 + 3 \times 1 = 17$$

$\Rightarrow \quad 10+3 = 17$

$13 = 17$, false

Therefore, (5, 1) does not lie on the given linear equation $2(x+1)+3(y-2) = 13$.

37. Factorise : $4x^4+7x^2-2$. [4]

SOLUTION :

$$4x^4+7x^2-2 = 4(x^2)^2+7x^2-2$$

[Making quadratic polynomial]

On putting $x^2 = y$, we get

$$4x^4+7x^2-2 = 4y^2+7y-2$$

Here, $4(-2) = -8$

So, we split –8 into two parts whose sum is 7 and product is –8.

Clearly, $8+(-1)=7$ and $8(-1)=-8$

$$\therefore \quad 4y^2+7y-2 = 4y^2+8y-y-2$$
$$= 4y(y+2)-1(y+2)$$
$$= (y+2)(4y-1)$$
$$= (x^2+2)(4x^2-1) \quad [\text{Put } y = x^2]$$
$$= (x^2+2)[(2x)^2-1^2]$$
$$= (x^2+2)(2x+1)(2x-1)$$
$$[\because \ a^2-b^2 = (a-b)(a+b)]$$

Hence, $4x^4+7x^2-2 = (x^2+2)(2x-1)(2x+1)$.

38. The sum of the height and radius of the base of a solid cylinder is 37 cm. If the total surface area of the cylinder is 1628 cm^2, then find its volume. [4]

SOLUTION :

Let the radius and height of a cylinder be r cm and h cm respectively.

According to the question,

$$r+h = 37 \quad ...(1)$$

and total surface area of the cylinder $= 1628\,cm^2$

$$\therefore \quad 2\pi r(r+h) = 1628$$
$$2\pi r(37) = 1628$$
$$2\pi r = \frac{1628}{37}$$
$$2\times\frac{22}{7}\times r = 44$$
$$r = \frac{44\times7}{2\times22} = 7\text{ cm}$$

On putting the value of r in eq (1), we get

$$7+h = 37$$
$$\Rightarrow \quad h = 37-7 = 30\text{ cm}$$

$\therefore$ Volume of solid cylinder

$$= \pi r^2 h = \frac{22}{7}\times7\times7\times30$$
$$= 22\times7\times30$$
$$= 4620\text{ cm}^3$$

Hence, the volume of a solid cylinder is 4620 cm^3.

or

Three cubes of metal whose edges are in the ratio 3 : 4 : 5 are melted down into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.

SOLUTION :

Given ratio of the edges of three cubes $= 3:4:5$

Let, Edge of 1st cube $= 3x$ cm

Edge of 2nd cube $= 4x$ cm

Edge of 3rd cube $= 5x$ cm

$\therefore$ Total volume of all the three cubes

$$= (3x)^3+(4x)^3+(5x)^3$$
$$= 27x^3+64x^3+125x^3$$
$$= 216x^3\text{ cm}^3$$

Let edge of new cube formed be y cm.

$\therefore$ Length of diagonal of new cube $= \sqrt{3}\,y$

$$\sqrt{3}\,y = 12\sqrt{3}$$

[$\because$ Given diagonal of new cube $= 12\sqrt{3}$ cm]

$$\Rightarrow \quad y = 12\text{ cm}$$

$\therefore$ Volume of new cube $= (12)^3\text{ cm}^3$

According to questions,

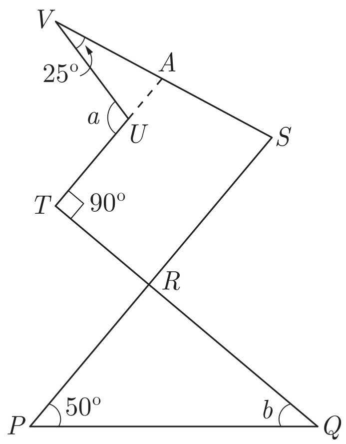
$$216x^3 = (12)^3$$
$$\Rightarrow \quad 216x^3 = 12\times12\times12$$
$$x^3 = \frac{12\times12\times12}{216}$$
$$x^3 = 2\times2\times2$$
$$x = 2\text{ cm}$$

$\therefore$ Edge of 1st cube $= 3x = 3\times2 = 6$ cm

Edge of 2nd cube $= 4x = 4\times2 = 8$ cm

Edge of 3rd cube $= 5x = 5\times2 = 10$ cm

39. In the given figure, if $TU||SR$ and $TR||SV$, then find $\angle a$ and $\angle b$. [4]



SOLUTION :

Given, $TU||RS$

$$\angle UTR = \angle SRQ$$

[By corresponding angle axiom]

$$\Rightarrow \quad \angle SRQ = 90^\circ \quad [\because \ \angle UTR = 90^\circ]$$

In ΔRPQ, we have

$$\angle SRQ = \angle RPQ + RQP$$

[$\because$ Exterior angle = Sum of interior opposite angles]

$$90^\circ = 50^\circ + b$$
$$\Rightarrow \quad b = 90^\circ - 50^\circ = 40^\circ$$

Also, given that $TR||SV$

$$\angle UTR = \angle VAU$$

[Alternate interior angles]

$$\angle VAU = 90^\circ \qquad [\because \angle UTR = 90^\circ]$$

In ΔVAU,

$$\angle VUT = \angle UVA + \angle VAU$$

[$\because$ Exterior angle = Sum of interior opposite angles]

$\because$ $a = 25^\circ + 90^\circ = 115^\circ$

Hence, $a = 115^\circ$ and $b = 40^\circ$.

40. The percentage of salary donated by twelve different households to an orphanage every month are : 2, 5, 3, 5, 6, 1, 2, 4, 3, 5, 2, 2.
Find the mean, median and mode of the data. [4]

SOLUTION :

$$\text{Mean} = \frac{\text{Sum of observation}}{\text{Number of observation}}$$

$$= \frac{2+5+3+5+6+1+2+4+3+5+2+2}{12}$$

$$= \frac{40}{12} = 3.3$$

Mean % of salary donated = 3.3 %
Arranging the data in ascending order,
We get : 1, 2, 2, 2, 2, 3, 3, 4, 5, 5, 5, 6

Median

$$= \frac{\left(\frac{n}{2}\right)^{\text{th}}\text{ observation} + \left(\frac{n}{2}+1\right)^{\text{th}}\text{ observation}}{2}$$

$$= \frac{6^{\text{th}}\text{ observation} + 7^{\text{th}}\text{ observation}}{2}$$

$$= \frac{3+3}{2} = 3$$

Median % of salary donated = 3 %
The maximum occurring observation = 2
Modal % of salary donated = 2 %