CLASS IX (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-3

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. The rationalising factor of $\sqrt[5]{a^2b^3c^4}$ is [1]

(a) $\sqrt[5]{a^3b^2c}$ (b) $\sqrt[4]{a^3b^2c}$
(c) $\sqrt[3]{a^3b^2c}$ (d) $\sqrt{a^3b^2c}$

Ans : (a) $\sqrt[5]{a^3b^2c}$

Since, multiplication of $\sqrt[5]{a^2b^3c^4}$ by $\sqrt[5]{a^3b^2c}$ gives rational number.

R.F. of $\sqrt[5]{a^2b^3c^4} = \sqrt[5]{a^3b^2c}$

2. Factorisation of $a^{2x}-b^{2x}$ is [1]

(a) $(a^x+b^x)(a^x-b^x)$ (b) $(a^x-b^x)^2$
(c) $(a^x+b^x)(a^2-b^2)$ (d) $(a^x-b^x)(a^2+b^2)$

Ans : (a) $(a^x+b^x)(a^x-b^x)$

$$a^{2x}-b^{2x} = (a^x)^2-(b^x)^2 = (a^x+b^x)(a^x-b^x)$$

3. In which quadrant will $(-3,4)$ lie? [1]

(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

Ans : (b) II quadrant

Since, x-coordinate of $(-3,4)$ is negative and y -coordinate is positive.
Point $(-3,4)$ lies in II quadrant.

4. The number of solutions, the equation $3x+5y+15=0$ can have [1]

(a) one only (b) exactly two
(c) zero (d) infinite

Ans : (d) infinite

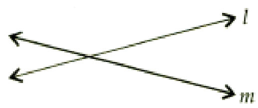
$3x+5y+15=0$ is a linear equation in two variables and every linear equation in two variables has infinite many solutions.

5. Two distinct intersecting lines l and m cannot have [1]

(a) any point in common (b) one point in common
(c) two points in common (d) None of these

Ans : (c) two points in common

Two distinct intersecting lines can have almost one point in common. If they have more than one points in common then they coincide with each other.



6. Supplement of angle is one fourth of itself. The measure of the angle is [1]

(a) 18° (b) 36°
(c) 144° (d) 72°

Ans : (c) 144°

Let the angle be x

$$\text{its supplement} = \frac{1}{4} \text{ of } x = \frac{1}{4}x$$
$$x+\frac{1}{4}x = 180^\circ$$
$$\frac{5x}{4} = 180^\circ$$
$$x = \frac{180^\circ \times 4}{5} = 144^\circ$$

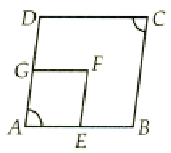
7. In $\triangle ABC$, if $\angle B < \angle A$, then [1]

(a) $BC > CA$ (b) $BC < CA$
(c) $BC > AB + CA$ (d) $AB < CA$

Ans : (a) $BC > CA$

$BC > CA$ (opposite side of larger angle is greater then the opposite side of smaller angle)

8. In the following figure, $ABCD$ and $AEFG$ are two parallelograms. If $\angle C = 55^\circ$, find $\angle F$. [1]



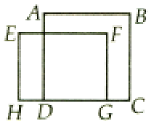
(a) 65° (b) 75°
(c) 85° (d) 55°

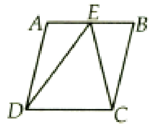
Ans : (d) 55°

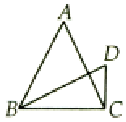
Given, $ABCD$ is a parallelogram.

$$\angle A = \angle F = 55^\circ$$

9. Which of the following figures lie on the same base and between the same parallels? [1]

(a) 

(b) 

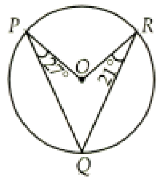
(c) 

(d) All of these

Ans : (b)

Common base $= DC$ and two parallels are AB and DC

10. In the given figure, O is the centre of circle. $\angle OPQ = 27^\circ$ and $\angle ORQ = 21^\circ$. The values of $\angle POR$ and $\angle PQR$ respectively are [1]

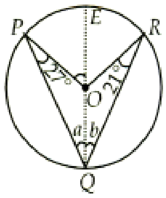


(a) $84^\circ, 42^\circ$ (b) $96^\circ, 48^\circ$
(c) $54^\circ, 42^\circ$ (d) $108^\circ, 54^\circ$

Ans : (b) $96^\circ, 48^\circ$

Draw a line passing through Q and O.

$$a = 27^\circ \qquad [OP = OQ]$$
$$b = 21^\circ \qquad [OR = OQ]$$



$$\angle PQR = a + b = 27^\circ + 21^\circ = 48^\circ$$
$$\angle POR = 2 \times \angle PQR = 2 \times 48^\circ$$
$$= 96^\circ$$

(Q.11-Q.15) Fill in the blanks :

11. If the lengths of two sides of an isosceles triangle are 4 cm and 10 cm, then the length of the third side is .... cm. [1]

Ans : 10 cm

As triangle is isosceles, thus two of its sides must be equal. If the length of third side is taken to be 4 cm, then sum of two sides that is $(4 + 4 = 8)$ will be less than third side which is not possible. Thus, third side must be 10 cm.

12. The perimeter of a right angled triangle is 450 m. If its sides are in the ratio 5 : 12 : 13, then area of the triangle is .... m^2. [1]

Ans : $6750\,\text{m}^2$

Let the three sides be $5x$, $12x$ and $13x$

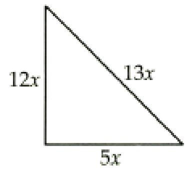
$$5x + 12x + 13x = 450$$
$$30x = 450$$
$$x = 15\,\text{m}$$

Sides are $5 \times 15 = 75$ cm,

$$12 \times 15 = 195\,\text{cm}$$
$$s = \frac{75 + 180 + 195}{2} = \frac{450}{2}$$
$$= 225\,\text{cm}$$

Area of triangle

$$= \sqrt{225\,(225 - 75)\,(225 - 180)\,(225 - 195)}$$
$$= \sqrt{225 \times 150 \times 45 \times 30}$$
$$= 15 \times 30 \times 5 \times 3 = 6750\,\text{m}^2$$



or

If each side of a scalene triangle is halved then its area will reduced by .... percentage.

Ans : 75%

13. The sum of the areas of the plane and curved surfaces (faces) of a solid is called its .... surface area. [1]

Ans : total

14. .... is found by adding all the values of the observations and dividing this by the total number of observations. [1]

Ans : Mean

15. Probability of an event can be any .... from 0 to 1. [1]

Ans : Fraction

(Q.16-Q.20) Answer the following :

16. If $125^x = \frac{25}{5^x}$, find the value of x. [1]

SOLUTION :

Given, $125^x = \frac{25}{5^x}$

$$(5^3)^x = \frac{5^2}{5^x}$$
$$5^{3x} = 5^{2-x}$$

On equating power from both sides,

We get $3x = 2 - x$

$$4x = 2$$
$$x = \frac{2}{4} = \frac{1}{2}$$

or

What is the best way to evaluate $(996)^2$?

SOLUTION :

The best way to evaluate $(996)^2$ is $(1000 - 4)^2$, which is easy to simplify.

17. In which quadrants, abscissa of a point is negative? [1]

SOLUTION :

Abscissa of a point is negative in II and III quadrant.

18. If two angles of a triangle are complementary, then what type of triangle will be formed? [1]

SOLUTION :

If two angles of a triangle are complementary i.e., their sum is 90°, then third angle will be 90°.
Hence, triangle is right angled triangle.

19. What is the lateral surface area of a cuboid with dimensions l, b and h? [1]

SOLUTION :

Lateral surface area of a cuboid

$$= 2(lb + bh + hl) - 2lb$$
$$= 2(bh + hl) = 2(l + b)h.$$

20. If each observation of the data is decreased by 5, then what is the effect on the mean? [1]

SOLUTION :

Mean is also decreased by 5.

Section B

21. Without actually calculating the cubes, find the value of $48^3 - 30^3 - 18^3$. [2]

SOLUTION :

We know that

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

Also, we know that, if

$$x + y + z = 0$$

Then, $x^3 + y^3 + z^3 = 3xyz$

Given expression is $(48)^3 + (-30)^3 + (-18)^3$

Here, $48 - 30 - 18 = 0$

$\therefore$ $48^3 - 30^3 - 18^3 = 3 \times 48 \times (-30) \times (-18)$
$= 77760$

or

Find the value of x, if $5^{x-3} \times 3^{2x-8} = 225$.

SOLUTION :

$$5^{x-3} \times 3^{2x-8} = 225$$
$$\Rightarrow \quad 5^x \cdot 5^{-3} \times 3^{2x} \times 3^{-8} = 5^2 \times 3^2$$
$$5^x \cdot 3^{2x} = \frac{5^2 \times 3^2}{5^{-3} \times 3^{-8}}$$
$$= 5^2 \times 3^2 \times 5^3 \times 3^8$$
$$= 5^{2+3} \times 3^{2+8}$$
$$= 5^5 \times 3^{10} = 5^5 \times 3^{2 \times 5}$$

On comparing the exponents both sides we get $x = 5$.

22. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$, leaves the remainder 19. Find the value of a. Also, find the remainder when $p(x)$ is divided by $x + 2$. [2]

SOLUTION :

We $p(x)$ is divided by $x + 1$ the remainder is 19.

$$\therefore \quad p(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7 = 19$$
$$\Rightarrow \quad 1 + 2 + 3 + a + 3a - 7 = 19$$
$$4a - 1 = 19$$
$$4a = 20$$
$$a = 5$$

The remainder when $p(x)$ is divided by $(x + 2)$ is equal to $p(-2)$

$$p(-2) = (-2)^4 - 2 \times (-2)^3 + 3(-2)^2 - a(-2) + 3a - 7$$
$$= 46 + 16 + 12 + 2a + 3a - 7$$
$$= 44 + 5a - 7$$
$$= 44 + 25 - 7 = 62$$

or

Factorise : $2x^3 - 5x^2 - 19x + 42$

SOLUTION :

Let $p(x) = 2x^3 - 5x^2 - 19x + 42$

Now, factors of 42 are 1, 2, 3, 6, 7, 14, 21, 42

$$\therefore \quad p(1) = 2(1)^3 - 5(1)^2 - 19 + 42$$
$$= 2 - 5 - 19 + 42 = 20 \neq 0$$

$\therefore$ $(x - 1)$ is not a factor of $p(x)$.

Now, $p(2) = 2(2)^3 - 5(2)^2 - 19(2) + 42$
$= 16 - 20 - 38 + 42 = 0$

$\therefore$ $(x - 2)$ is a factor of $p(x)$.
Dividing $p(x)$ by $x - 2$, we get the other factors.

$$p(x) = (x - 2)(2x^2 - x - 21)$$
$$= (x - 2)(2x^2 - 7x + 6x - 21)$$
$$= (x - 2)[x(2x - 7) + 3(2x - 7)]$$
$$= (x - 2)(2x - 7)(x + 3)$$

23. Find the coordinates of the point : [2]
(i) Which lies on x and y axes both.
(ii) Whose abscissa is 2 and which lies on the x-axis.

SOLUTION :

(i) The coordinates of the points which lies on the x and y-axes both are (0, 0).
(ii) Since the point lies on the x-axis therefore, its ordinate $= 0$. So, the coordinates of the given point are (2, 0).

24. If the complement of an angle is one-third of its supplement, find the angle ? [2]

SOLUTION :

Let the angle be x

$\therefore$ Complement of an angle $= 90^\circ - x$
Supplement of an angle $= 180^\circ - x$
By the given condition,

$$90^\circ - x = \frac{1}{3}(180^\circ - x)$$

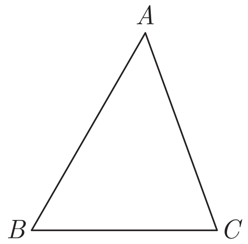
$x = 45^\circ$

$\therefore$ Required angle $= 45^\circ$

or

In ΔABC, if $\angle A = 50^\circ$ and $\angle B = 60^\circ$, determine the shortest and the longest side of the triangle.

SOLUTION :



In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$ [Angle sum property]

$\Rightarrow$ $50^\circ + 60^\circ + \angle C = 180^\circ$

$\angle C = 70^\circ$

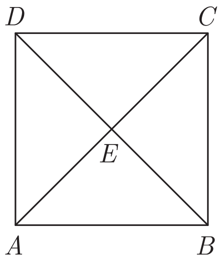
$\angle C = 70^\circ \Rightarrow$ AB is the longest side

$\angle A = 50^\circ \Rightarrow$ BC is the shortest side

25. $ABCD$ is a rhombus. If $AC = 8$ cm, $DB = 6$ cm, find the length of BC. [2]

SOLUTION :

We know that, the diagonals of a rhombus bisect each other at 90°.



$\therefore$ In right angled $\Delta ABCE$

$$BE = \frac{1}{2}BD = \frac{1}{2} \times 6 = 3 \text{ cm}$$

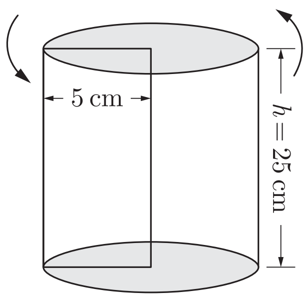
$$CE = \frac{1}{2}AC = \frac{1}{2} \times 8 = 4 \text{ cm}$$

Now, $BC = \sqrt{BE^2 + CE^2} = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$ cm

$\therefore$ $BC = 5$ cm

26. A rectangle strip 5 cm $\times$ 25 cm is rotated completely about the 25 cm side. Find the total surface area of the solid thus generated. [2]



SOLUTION :

Clearly, the solid thus generated will be a right circular cylinder having its radius as 5 cm and height 25 cm.

$\therefore$ Radius, $r = 5$ cm

and height, $h = 25$ cm

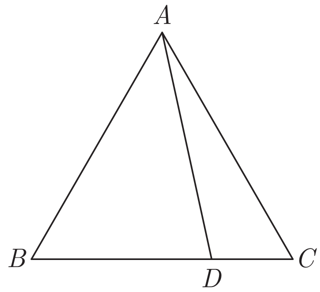
Total surface area of cylinder generated

$$= 2\pi r(r+h) = 2 \times \frac{22}{7} \times 5(5+25)$$

$$= \frac{220 \times 30}{7} \text{ cm}^2 = 942.86 \text{ cm}^2$$

Section C

27. In the given figure, $AB > AC$ and D is any point on side BC of ΔABC. Prove that $AB > AD$. [3]



SOLUTION :

Given, $AB > AC$

$\therefore$ $\angle C > \angle B$ [Angle opposite to longer side is larger] ...(1)

Now, $\angle ADB$ is the exterior angle of ΔADC

$\Rightarrow$ $\angle ADB = \angle DAC + \angle C$

$\angle ADB > \angle C$ ...(2)

Therefore, from equation (1), we get

$\Rightarrow$ $\angle ADB > \angle B$

Now in ΔABD

$\angle ADB > \angle B$

$\Rightarrow$ $AB > AD$

[Side opposite to greater angle is longer]

Hence proved.

28. The remainder of the polynomial $5 + bx - 2x^2 + ax^3$, when divided by $(x-2)$ is twice the remainder when it is divided by $(x+1)$. Show that $10a + 4b = 9$. [3]

SOLUTION :

Let $f(x) = 5 + bx - 2x^2 + ax^3$

When $f(x)$ is divided by $(x-2)$, then the remainder is $f(2)$.

When $f(x)$ is divided by $(x+1)$, then the remainder is $f(-1)$.

Now, $f(2) = 5 + b(2) - 2(2)^2 + a(2)^3$

$= 8a + 2b - 3$

and $f(-1) = 5 + b(-1) - 2(-1)^2 + a(-1)^3$

$= -a - b + 3$

According to the question,

$f(2) = 2f(-1)$

$\therefore$ $8a + 2b - 3 = 2(-a-b+3) = -2a - 2b + 6$

$$10a + 4b = 9$$

Hence proved.

29. The mean of first 8 observations is 18 and last 8 observation is 20. If the mean of all 15 observations is 19, find the 8^{th} observation. [3]

SOLUTION :

Mean of first 8 observations $= 18$
Sum of first 8 observation $= 8 \times 18 = 144$
Similarly, sum of last 8 observations $= 8 \times 20 = 160$
Sum of all 15 observations $= 15 \times 19 = 285$

$$8^{th} \text{ observation } = (144 + 160) - 285 = 19$$

or

Two coins are tossed simultaneously 200 times and the following outcomes are recorded :

HH	HT/TH	TT
56	110	34

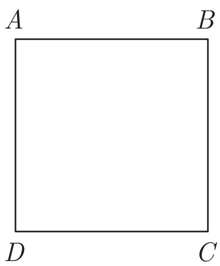
What is the empirical probability of occurrence of at least one head in the above case ?

SOLUTION :

Total number of possible outcomes with at least one head $= 56 + 110 = 166$
Total number of outcomes $= 200$

$\therefore$ P (getting at least one head) $= \frac{166}{200} = 0.83$

30. In the given figure, $AB || DC$ and $AD || BC$. Prove that, $\angle DAB = \angle DCB$. [3]



SOLUTION :

Given, $AB || DC$ and $AD || BC$

To prove : $\angle DAB = \angle DCB$

Proof :

Since, $AD || BC$ and AB is the transversal.

Then, $\angle DAB + \angle ABC = 180°$ ...(1)

[Since, sum of two co-interior angles is 180^o]

Similarly, since, $AB || CD$ and BC is the transversal.

Then, $\angle ABC + \angle DCB = 180°$ ...(2)

[Since, sum of two co-interior angles is 180^o]

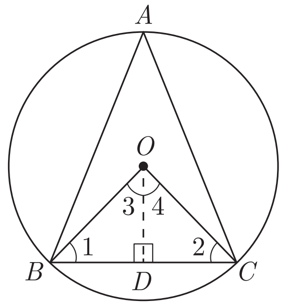
From Eqs.(1) and (2),

$$\angle DAB + \angle ABC = \angle ABC + \angle DCB$$

$\therefore$ $\angle DAB = \angle DCB$

Hence proved.

31. The circumcentre of the triangle ABC is O. Prove that $\angle OBC + \angle BAC = 90°$. [3]



SOLUTION :

Given : O is the circumcentre of ΔABC.

To prove : $\angle OBC + \angle BAC = 90°$

Construction : Draw $OD \perp BC$

Proof : In ΔOBD and ΔOCD, we have

$OB = OC$ [Radii of the same circle]

$\angle ODB = \angle ODC = 90°$

[By construction]

$OD = OD$ [Common side]

$\therefore$ $\Delta OBD \cong \Delta OCD$ [By RHS congruence]

$\angle 3 = \angle 4$ [C.P.C.T.]

$\angle 1 = \angle 2$ [C.P.C.T.]

$\angle BOC = 2\angle BAC$

$\Rightarrow$ $2\angle 3 = 2\angle BAC$

$[\because \angle BOC = \angle 3 + \angle 4 = \angle 3 + \angle 3 = 2\angle 3]$

$\Rightarrow$ $\angle 3 = \angle BAC$

$\angle 4 = \angle BAC$

$\angle 3 + \angle 2 = \angle BAC + \angle 2$

[Adding $\angle 2$ both side]

$90° = \angle BAC + \angle 1$

$[\because \angle 1 = \angle 2$ and $\angle 3 = \angle 4$ Also, $\angle 3 + \angle 4 = 90°]$

$\Rightarrow$ $\angle OBC + \angle A = 90°$

Hence proved.

32. A spherical canon ball, 28 cm, in diameter is melted into a right circular conical mould, the base of which is 35 cm in diameter. Find the height of the cone, correct to one place of decimal. [3]

SOLUTION :

Let h be the height of the cone.
We have, the diameter of spherical canon ball $= 28$ cm

$\therefore$ Radius of the base of the cone $= \frac{35}{2}$ cm.

Now according to question,
Volume of the cone $=$ Volume of spherical canon ball

$$\Rightarrow \quad \frac{1}{3}\pi\left(\frac{35}{2}\right)^2 h = \frac{4}{3}\pi(14)^3$$

$$\Rightarrow \quad \left(\frac{35}{2}\right)^2 h = 4 \times (14)^3$$

$$\therefore \quad h = \left(4 \times 14 \times 14 \times 14 \times \frac{2}{35} \times \frac{2}{35}\right)$$

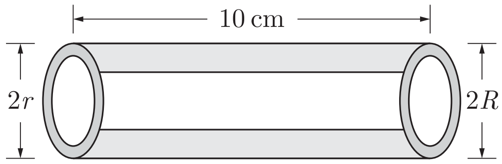
$$= \left(4 \times 2 \times 2 \times 14 \times \frac{2}{5} \times \frac{2}{5}\right)$$

$$= \frac{896}{25} = 35.84 \text{ cm}$$

Hence, the height of the cone is 35.84 cm.

or

The total surface area of a hollow metal cylinder open at both ends of external radius 8 cm and height 10 cm is 338π cm^2. Taking r to be inner radius, find the thickness of the metal in the cylinder.



SOLUTION :

Given that r is a inner radius of a hollow metal cylinder.

Then external radius $= R = 8$ cm

Total surface area of hollow metal cylinder = (Outer + Inner curved surface area of cylinder) + Area of base rings

$$\Rightarrow \quad 338\pi = [2\pi Rh + 2\pi rh] + 2\pi(R^2 - r^2)$$
$$= 2\pi h[R + r] + 2\pi(R^2 - r^2)$$
$$= 2 \times \pi \times 10 \times [8 + r] + 2\pi(8^2 - r^2)$$
$$= \pi[20(8 + r) + 2(64 - r^2)]$$
$$338 = 160 + 20r + 2(64 - r^2)$$
$$\Rightarrow \quad 2r^2 - 20r + 338 - 160 - 128 = 0$$
$$2r^2 - 20r + 50 = 0$$
$$2r^2 - 10r - 10r + 50 = 0$$
$$2r(r - 5) - 10(r - 5) = 0$$
$$(r - 5)(2r - 10) = 0$$
$$\Rightarrow \quad r = 5 \text{ cm}$$

$\therefore$ Thickness of the metal in the cylinder

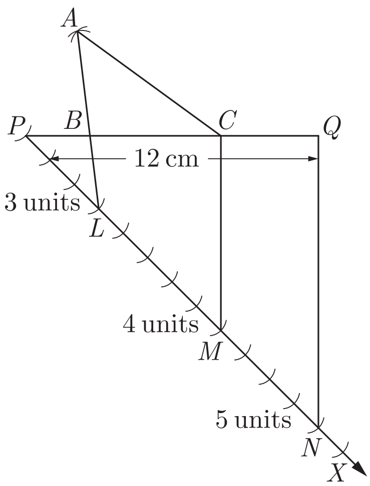
$$= R - r = 8 - 5 = 3 \text{ cm}$$

33. Construct a ΔABC whose perimeter is 12 cm and sides are in the ratio 3 : 4 : 5. [3]

SOLUTION :

Steps of Construction :

1. Draw a line segment $PQ = 3 + 4 + 5 = 12$ cm
2. Draw a ray PX in the downward direction making an acute angle with PQ.
3. From P, cut $(3 + 4 + 5) = 12$, equal distance mark on PX.
4. Denote the points L, M and N on PX such that $PL = 3$ units, $LM = 4$ units and $MN = 5$ units.
5. Join NQ.



6. Through L and M, draw $LB || NQ$ and $MC || NQ$, cutting PQ at B and C, respectively.
7. With B as centre and radius BP, draw an arc and with C as centre and radius CQ, draw another arc cutting the previous arc at A on the upward side of PQ.
8. Join AB and AC. Thus, ABC is the required triangle.

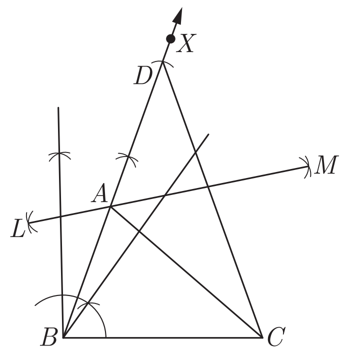
or

Construct a triangle ABC in which $BC = 7$ cm, $\angle B = 75°$ and $AB + AC = 13$ cm.

SOLUTION :

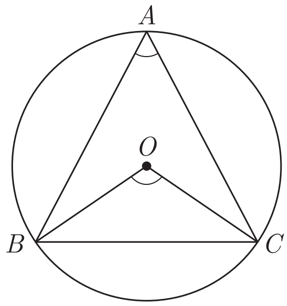
Steps of Construction :

1. Draw a line segment $BC = 7$ cm.
2. At B, draw $\angle CBX = 75°$.
3. Cut a line segment $BD = 13$ cm from BX



4. Join DC.
5. Draw the perpendicular bisector LM of CD, which intersects BD at A.
6. Join AC. Then ABC is the required triangle.

34. 3 STD booths situated at A, B and C in the figure are operated by handicapped persons. These three booths are equidistant from each other as shown in the figure. [3]



(i) Find $\angle BAC$.
(ii) Find $\angle BOC$

SOLUTION :

Here, A, B and C represented 3 STD booths.

(i) Given, A, B and C are equidistant from each other.

$$\therefore \quad AB = BC = CA$$

Then, ΔABC is an equilateral triangle.

Hence, $\quad \angle BAC = \angle ABC = \angle BCA = 60°$

(ii) We know that, the angle subtended by an arc at the centre of the circle is twice the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad \angle BOC = 2\angle BAC$

From part (i), $\angle BAC = 60^\circ$

Hence, $\angle BOC = 2 \times 60^\circ = 120^\circ$

Section D

35. If $x = (5 + 2\sqrt{6})$, then show that $\sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{3}$. [4]

SOLUTION :

We have, $x = 5 + 2\sqrt{6}$

Now, $\frac{1}{x} = \frac{1}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}}$ [by rationalising]

$$= \frac{5-2\sqrt{6}}{(5)^2 - (2\sqrt{6})^2}$$

$[\because (a-b)(a+b) = a^2 - b^2]$

$$= \frac{5-2\sqrt{6}}{25-24} = 5 - 2\sqrt{6}$$

$\therefore \quad x + \frac{1}{x} = 5 + 2\sqrt{6} + 5 - 2\sqrt{6} = 10$

$\Rightarrow \quad x + \frac{1}{x} + 2 = 12$ [adding 2 on both sides]

$\Rightarrow \quad \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = (2\sqrt{3})^2$

$\left[\because \left(\sqrt{a} + \frac{1}{\sqrt{a}}\right)^2 = a + \frac{1}{a} + 2\right]$

On taking positive square root both sides, we get

$$\sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{3}$$

Hence proved.

36. Factorise : $a^7 - ab^6$. [4]

SOLUTION :

$$a^7 - ab^6 = a(a^6 - b^6)$$
$$= a[(a^3)^2 - (b^3)^2]$$
$$= a(a^3 - b^3)(a^3 + b^3)$$

$[\because a^2 - b^2 = (a-b)(a+b)]$

$$= a[(a-b)(a^2+ab+b^2)][(a+b)(a^2-ab+b^2)]$$

$\left[\because (a^3 - b^3) = (a-b)(a^2+ab+b^2) \text{ and } (a^3+b^3) = (a+b)(a^2-ab+b^2)\right]$

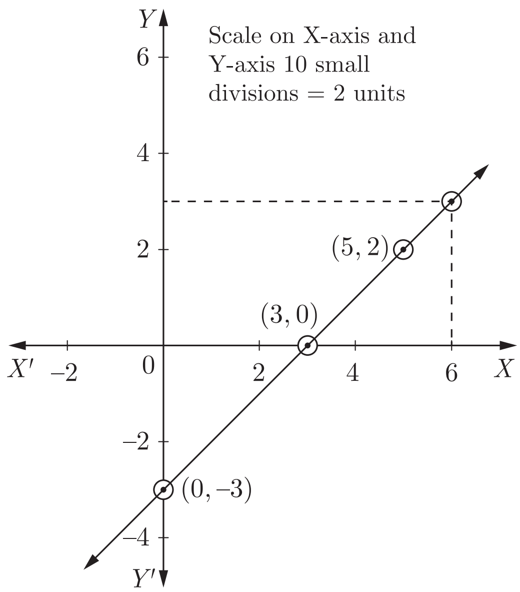
$$= a(a-b)(a+b)(a^2+ab+b^2)(a^2-ab+b^2)$$

37. Draw the graph of the equation $x - y = 3$. If $y = 3$, then find the value of x from the graph. [4]

SOLUTION :

Given equation is $x - y = 3$.

x	0	3	5
y	–3	0	2



From the graph, we can see that the value of x is 6 for $y = 3$.

or

A and B are friends A is elder to B by 5 years. B's sister C is half the age of B while A's father D is 8 years older than twice the age of B. If the present age of D is 48 years, find the present ages of A, B and C.

SOLUTION :

Present age of $D = 48$ years

According to question :

$$A = B + 5 \quad ...(1)$$
$$C = \frac{1}{2}B \quad ...(2)$$
$$D = 2B + 8 \quad ...(3)$$

From eq. (3),

$\Rightarrow \quad D = 2B + 8$

$48 = 2B + 8$

$2B = 48 - 8 = 40$

$B = 20$ years

From eq. (1),

$A = B + 5$

$= 20 + 5 = 25$ years

From eq. (2),

$C = \frac{1}{2}B$

$= \frac{1}{2} \times 20 = 10$ years

Hence, present ages of A, B and C are 25 years, 20 years and 10 years respectively.

38. Draw a frequency polygon representing the following frequency distribution. [4]

Class intervals	30-34	35-39	40-44	45-49	50-54	55-59
Frequency	12	16	20	8	10	4

SOLUTION :

Given, class intervals are discontinuous, so we make continuous by subtracting and adding 0.5 in lower and upper limits of each class interval.

We take the imagined class 24.5 - 29.5 at the beginning and 59.5 - 64.5 at the end, both with frequency zero.

Class intervals	Class marks	Frequency
24.5 - 29.5	27	0
29.5 - 34.5	32	12
34.5 - 39.5	37	16
39.5 - 44.5	42	20
44.5 - 49.5	47	8
49.5 - 54.5	52	10
54.5 - 59.5	57	4
59.5 - 64.5	62	0

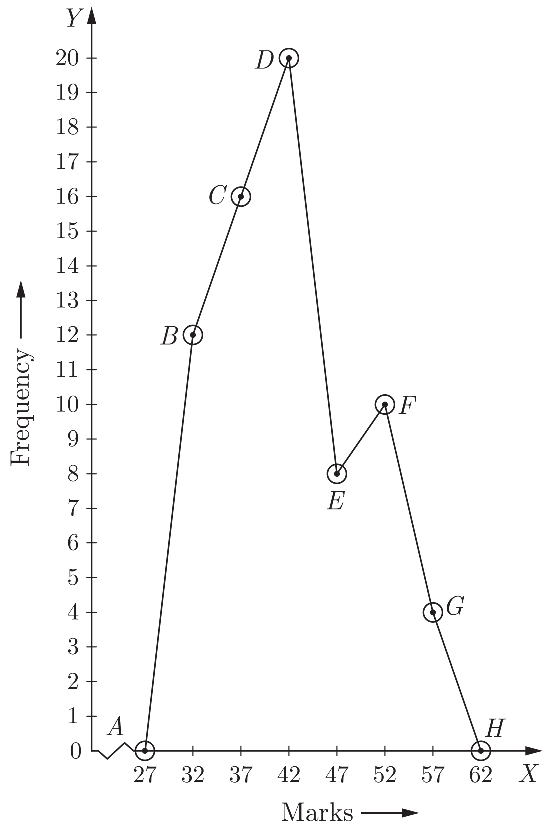
Along the X-axis, we take class marks 27, 32, 37, 42, 47, 52, 57 and 62 with a suitable scale.

Along the Y-axis, we take corresponding frequencies 0, 12, 16, 20, 8, 10, 4 and 0 with a suitable scale.

Now, plot the points

$A(27, 0)$, $B(32, 12)$, $C(37, 16)$, $D(42, 20)$, $E(47, 8)$, $F(52, 10)$, $G(57, 4)$ and $H(62, 0)$.

We join line segments AB, BC, CD, DE, EF, FG and GH to obtain the required frequency polygon as shown.



or

The mean of 1, 7, 5, 3, 4 and 4 is m. The observations 3, 2, 4, 2, 3, 3 and p have mean $(m-1)$ and median q. Find p and q.

SOLUTION :

We know that,

$$\text{Mean } \overline{x}_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\Rightarrow \quad m = \frac{1}{6}(1+7+5+3+4+4)$$

$$= \frac{1}{6} \times 24 = 4$$

Again, mean of another data $= m-1 = 4-1 = 3$

$$\therefore \quad \overline{x}_2 = 3$$

$$\Rightarrow \quad 3 = \frac{1}{7}(3+2+4+2+3+3+p)$$

$$21 = 17 + p$$

$$p = 21 - 17 = 4$$

$\therefore$ Observations are 3, 2, 4, 2, 3, 3 and 4.

Total number of observations $= 7$

Arrange data in ascending order, we get

2, 2, 3, 3, 3, 4, 4

$$\therefore \quad \text{Median, } q = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term}$$

$$= \frac{1}{2}[7+1]^{\text{th}} \text{ term}$$

$$= 4^{\text{th}} \text{ term} = 3$$

Hence, $p = 4$ and $q = 3$.

39. The length of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. Find [4]

(i) the area of the triangle

(ii) the height corresponding to the longest side

SOLUTION :

Given, perimeter $= 144\text{ cm}$

and ratio of the sides $= 3:4:5$

sum of the terms $= 3+4+5 = 12$

$$\therefore \quad 1^{\text{st}} \text{ side} = a = 144 \times \frac{3}{12} = 36\text{ cm}$$

$$2^{\text{nd}} \text{ side} = b = 144 \times \frac{4}{12} = 48\text{ cm}$$

$$3^{\text{rd}} \text{ side} = c = 144 \times \frac{5}{12} = 60\text{ cm}$$

Now, semi-perimeter of the triangle

$$s = \frac{a+b+c}{2}$$

$$= \frac{36+48+60}{2}$$

$$= \frac{144}{2} = 72\text{ cm}$$

(i) Area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{72(72-36)(72-48)(72-60)}$$

$$= \sqrt{72 \times 36 \times 24 \times 12}$$

$$= \sqrt{(36)^2 \times (24)^2} = 36 \times 24 = 864\text{ cm}^2$$

Hence, the area of the given triangle is 864 cm^2.

(ii) Let height of the triangle be h cm.

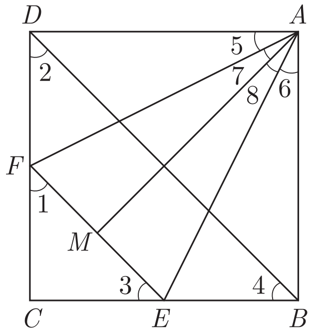
$$\text{Then, area of triangle} = \frac{1}{2} \times \text{Base} \times \text{height}$$

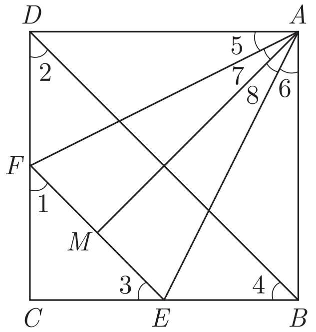
$$864 = \frac{1}{2} \times 60 \times h = 30h$$

$$h = 28.8\text{ cm}$$

Hence, the height corresponding to the longest side is 28.8 cm.

40. In the given figure, $ABCD$ is a square, EF is parallel to diagonal BD and $EM = FM$. [4]





Prove that
(i) $DF = BE$
(ii) AM bisects $\angle BAD$.

SOLUTION :

$ABCD$ is a square and $EF || BD$, $EM = FM$
To prove :
(i) $DF = BE$
(ii) AM bisects $\angle BAD$

Proof :

(i) $EF \parallel BD$ [Given]

$\therefore$ $\angle 1 = \angle 2$ [Corresponding angles] ...(1)

and $\angle 3 = \angle 4$ [Corresponding angles] ...(2)

Now, $\angle 2 = \angle 4$
[$\because$ In a square, diagonals bisect the opposite angles and all angles are equal]

$\Rightarrow$ $\angle 1 = \angle 3$ [Using eqs.(1) and (2)]

$\Rightarrow$ $CE = CF$ ...(3)
[$\because$ Side opposite to equal angles of a triangle are equal]

Now, $CD = BC$ [Sides of a square]
On subtracting CF from both sides, we get

$CD - CF = BC - CF$

$\Rightarrow$ $FD = BC - CE$ [Using eqs.(3)]

$\Rightarrow$ $DF = BE$

(ii) In ΔADF and ΔABE,

$AD = AB$ [Sides of a square]

$FD = BE$ [From part (i)]

and $\angle D = \angle B = 90°$

$\therefore$ $\Delta ADF \cong \Delta ABE$ [By SAS congruence rule]

Then, $AF = AE$

and $\angle 5 = \angle 6$ [CPCT]

Also, $\Delta AMF \cong \Delta AME$ [by SAS congruence rule]

[$\because$ $AF = AE$, $\angle AFE = \angle AEF$ (angles opposite to equal sides) and $AM = AM$]

$\therefore$ $\angle 7 = \angle 8$ [By CPCT]

$\Rightarrow \angle 7 + \angle 5 = \angle 8 + \angle 6$ [$\because$ $\angle 5 = \angle 6$]

$\Rightarrow$ $\angle MAD = \angle MAB$
Hence, AM bisect $\angle BAD$