CLASS IX (2019-20)
MATHEMATICS (041)
SAMPLE PAPER-4

Time : 3 Hours **Maximum Marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) The questions paper consists of 40 questions divided into 4 sections A, B, C and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

Section A

Q.1-Q.10 are multiple choice questions. Select the most appropriate answer from the given options.

1. Rational number between $\sqrt{2}$ and $\sqrt{3}$ is [1]

(a) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (b) $\frac{\sqrt{2}\times\sqrt{3}}{2}$

(c) 1.5 (d) 1.8

Ans : (c) 1.5

Since, $\sqrt{2} = 1.414..........,$

$\sqrt{3} = 1.732..........$

Rational number between $\sqrt{2}$ and $\sqrt{3}$ is 1.5.

2. If $8x^4 - 8x^2 + 7$ is divided by $2x+1$, the remainder is [1]

(a) $\frac{11}{2}$ (b) $\frac{13}{2}$

(c) $\frac{15}{2}$ (d) $\frac{17}{2}$

Ans : (a) $\frac{11}{2}$

Let, $p(x) = 8x^4 - 8x^2 + 7$

So, the remainder when $p(x)$ is divided by $2x+1$ is

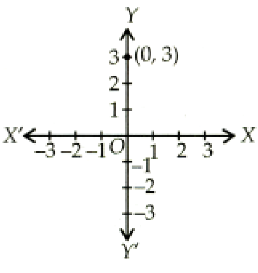
$$p\left(-\frac{1}{2}\right) = 8\left(\frac{-1}{2}\right)^4 - 8\left(\frac{-1}{2}\right)^2 + 7 = \frac{11}{2}$$

3. Point $(0,3)$ lies [1]

(a) on x-axis (b) on y-axis

(c) in I quadrant (d) at origin

Ans : (b) on y-axis



$(0,3)$ lies on y-axis.

4. The value of k, if $x=2, y=-1$ is a solution of the equation $2x+3y=k$ is [1]

(a) 6 (b) 7

(c) 5 (d) 1

Ans : (d) 1

$x=2,\ y=-1$ is solution of equation

$$\begin{aligned} 2x+3y &= k \\ 2\times 2 + 3\times(-1) &= k \\ k &= 4-3 \\ k &= 1 \end{aligned}$$

5. Which of the following needs a proof? [1]

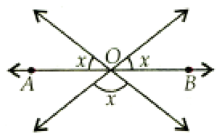
(a) Postulates (b) Definition

(c) Proposition (d) Axiom

Ans : (c) Proposition

Postulates are the universal truths specific to geometry. Axiom are also universal truths. These truths need not to be proved. Definitions also does not require proof. Only propositions or theorems can be proved using axioms, postulates and definitions.

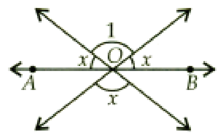
6. The value of x if AOB is a straight line, is [1]



(a) $36°$ (b) $60°$

(c) $30°$ (d) $35°$

Ans : (b) $60°$



$\angle 1 = x$ [Vertically opposite angles]

Since, AOB is a straight line

$$\begin{aligned} x+x+x &= 180° \\ 3x &= 180° \\ x &= 60° \end{aligned}$$

7. Which of the following is a correct statement? [1]

(a) Two triangles having same shape are congruent.

(b) If two sides of a triangle are equal to the corresponding sides of another triangle, then the two triangles are congruent.

(c) If the hypotenuse and one side of one right triangle are equal to the hypotenuse and one side of the other triangle, then the triangles are not congruent.
(d) None of these

Ans : (d) None of these

8. Which of the following statements is true? [1]
(a) In a parallelogram, the diagonals are equal
(b) In a parallelogram, the diagonals bisect each other.
(c) In a parallelogram, the diagonals intersect each other at right angles.
(d) In any quadrilateral, if a pair of opposite sides are equal, it is parallelogram.

Ans : (b) In a parallelogram, the diagonals bisect each other.

9. The area of a rhombus if the lengths of whose diagonals are 16 cm and 24 cm, is [1]
(a) 180 cm^2 (b) 184 cm^2
(c) 198 cm^2 (d) 192 cm^2

Ans : (d) 192 cm^2

$$\text{Area of rhombus } = \frac{1}{2}\times d_1\times d_2$$
$$= \frac{1}{2}\times 16\times 24\,\text{cm}^2 = 192\,\text{cm}^2$$

10. In a cyclic quadrilateral, the difference between two opposite angles is 58°, the measures of opposite angles are [1]
(a) $158^\circ, 22^\circ$ (b) $129^\circ, 51^\circ$
(c) $109^\circ, 71^\circ$ (d) $119^\circ, 61^\circ$

Ans : (d) $119^\circ, 61^\circ$

If $\angle A - \angle C = 58^\circ$ ...(1)
$\angle A + \angle C = 180^\circ$ ...(2)

Adding (1) & (2), we get $2\angle A = 238^\circ$
$\angle A = 119^\circ$
Subtracting (1) from (2), we get $2\angle C = 122^\circ$
$\angle C = 61^\circ$

$\angle A = 119^\circ$ and $\angle C = 61^\circ$

(Q.11-Q.15) Fill in the blanks :

11. The construction of a triangle ABC, given that $BC = 6$ cm, $\angle B = 45^\circ$ is not possible when difference of AB and AC is equal to .... cm. [1]

Ans : 6.9 cm

It is not possible to construct triangle whose difference of two side is more than the third side.

12. The percentage increase in the area of a triangle, if its each side is quadrupled, is equal to .... percentage. [1]

Ans : 1500%

$$s = \frac{1}{2}(a+b+c)$$
$$s' = \frac{1}{2}(4a+4b+4c)$$
$$= 2(a+b+c) = 4s$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} \text{ and}$$
$$\Delta' = \sqrt{s'(s'-4s)(s'-4a)(s'-4b)(s'-4c)}$$
$$\Delta' = \sqrt{4s(4s-4a)(4s-4b)(4s-4c)}$$
$$= 16\sqrt{s(s-a)(s-b)(s-c)} = 16\,\Delta$$

Increase in the area of the triangle
$$= \Delta' - \Delta = 16\,\Delta - \Delta = 15\,\Delta$$
$$\text{Percentage increase } = \frac{15\,\Delta}{\Delta}\times 100 = 1500\%$$

or

If length of hypotenuse of an isosceles right angled triangle is $10\sqrt{2}$ cm then its perimeter will be ....

Ans : $10\sqrt{2}(\sqrt{2}+1)$ cm.

13. The curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm is .... [1]

Ans : 220 cm^2

14. .... can also be drawn independently without drawing a histogram. [1]

Ans : Frequency polygon

15. A .... is an action which results in one of several outcomes. [1]

Ans : Trial

(Q.16-Q.20) Answer the following :

16. If the volume of a cuboid is $2x^2 - 16$, then find its possible dimensions. [1]

SOLUTION :

Let $p(x) = 2x^2 - 16 = 2(x^2 - 8) = 2\{x^2 - (2\sqrt{2})^2\}$
$= 2(x + 2\sqrt{2})(x - 2\sqrt{2})$

Hence, the possible dimensions of a cuboid are 2, $(x + 2\sqrt{2})$ and $(x - 2\sqrt{2})$.

17. On which axes do the points $(3, 0)$ and $(0, 4)$ lie? [1]

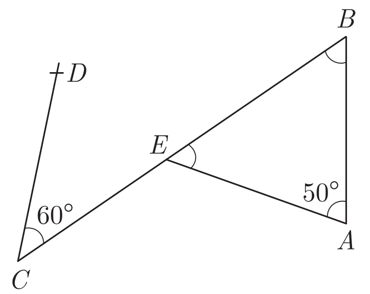
SOLUTION :

In point $(3, 0)$, y-coordinate is zero, so it lies on x-axis.
In point $(0, 4)$ x-coordinate is zero, so it lies on y-axis.

18. In the given figure, $AB \parallel CD, \angle EAB = 50^\circ$. If $\angle ECD = 60^\circ$ [1]

SOLUTION :

Given, $AB \parallel CD$ and BC is a transversal.
$\angle DCE = \angle EBA$ [Alternate interior angles]
$\angle EBA = 60^\circ$ [$\angle DCE = 60^\circ$]
In $\Delta\, ABE$, we have



$$\angle EBA + \angle EAB + \angle AEB = 180^\circ$$
$$60 + 50^\circ + \angle AEB = 180^\circ$$
$$\angle AEB = 180^\circ - 110^\circ = 70^\circ$$

19. The area of the base of a right circular cylinder is $154\,\text{cm}^2$ and its height is $15\,\text{cm}$. Find the volume of the cylinder. [1]

SOLUTION :

We know that,

Volume of a cylinder = Area of the base $\times$ height.

Here,area of the base $= 154\,\text{cm}^2$

and height $= 15\,\text{cm}$

Volume of a cylinder $= 154\,\text{cm}^2 \times 15\,\text{cm} = 2310\,\text{cm}^3$

20. The class marks of a frequency distribution are 15, 20, 25, ... Find the class corresponding to the class mark 20. [1]

SOLUTION :

Since, the difference between mid values is 5. So, the corresponding class to the class mark 20 must have difference 5.

$$\frac{17.5 + 22.5}{2} = \frac{40}{2} = 20$$

Hence, the required class is $17.5 - 22.5$.

or

If mean of $3, 5, 7, 9, x$, is 5 then find the value of x.

SOLUTION :

$$\text{Mean} = \frac{3 + 5 + 7 + 9 + x}{5} = 5$$
$$\frac{24 + x}{5} = 5$$
$$24 + x = 25$$
$$x = 25 - 24$$
$$x = 1$$

Section B

21. If $\frac{\sqrt{3} - 1}{\sqrt{3} + 1} = a + b\sqrt{3}$, then find the values of a and b . [2]

SOLUTION :

We have, $$\frac{\sqrt{3} - 1}{\sqrt{3} + 1} = a + b\sqrt{3}$$

$$\Rightarrow \quad \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \times \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = a + b\sqrt{3}$$

[By rationalising the denominator]

$$\frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - (1)^2} = a + b\sqrt{3}$$
$$\frac{3 + 1 - 2\sqrt{3}}{2} = a + b\sqrt{3}$$
$$\frac{4 - 2\sqrt{3}}{2} = a + b\sqrt{3}$$
$$\frac{2(2 - \sqrt{3})}{2} = a + b\sqrt{3}$$
$$\Rightarrow \quad 2 - \sqrt{3} = a + b\sqrt{3}$$

On comparing both sides, we get $a = 2$ and $b = -1$.

or

Simplify : $\frac{7 + \sqrt{3}}{7 - \sqrt{3}} + \frac{7 - \sqrt{3}}{7 + \sqrt{3}}$

SOLUTION :

We have,

$$\frac{7 + \sqrt{3}}{7 - \sqrt{3}} + \frac{7 - \sqrt{3}}{7 + \sqrt{3}}$$
$$= \frac{(7 + \sqrt{3})^2 + (7 - \sqrt{3})^2}{49 - 3}$$
$$= \frac{49 + 3 + 14\sqrt{3} + 49 + 3 - 14\sqrt{3}}{46}$$
$$= \frac{104}{46} = \frac{52}{23}$$

22. Write the coordinates of a point on x-axis at a distance of 4 units from the origin in the positive direction of x-axis and then justify your answer. [2]

SOLUTION :

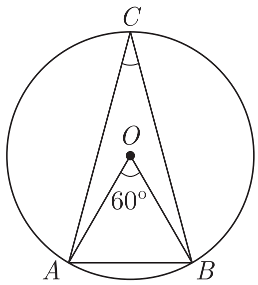
As, any point on x-axis has coordinates $(x, 0)$ where x is the distance from origin, so required coordinates are (4, 0).

23. A chord of a circle is equal to its radius. Find the angle subtended by this chord at a point in major segment. [2]

SOLUTION :

Given : A circle with centre O and AB is a chord such that $AB = OA = OB$

To find : $\angle ACB$



As ΔAOB is an equilateral triangle.

Also, $$\angle AOB = 2\angle ACB$$
$$= 2 \times 30^\circ = 60^\circ$$
$$\Rightarrow \quad \angle AOB = 60^\circ$$

[Angle subtended at the centre of circle is twice the angle subtended at the circumference]

$$\Rightarrow \quad \angle ACB = \frac{1}{2}\angle AOB$$
$$= \frac{1}{2} \times 60^\circ = 30^\circ$$

24. The sides of a triangle are 11 cm, 60 cm and 61 cm. Find the altitude of the smallest side. [2]

SOLUTION :

Let the sides of triangle are

$$a = 11\,\text{cm},\; b = 60\,\text{cm} \text{ and } c = 61\,\text{cm}$$

Then, semi-perimeter of triangle,

$$s = \frac{a+b+c}{2}$$

$$= \frac{11+60+61}{2} = \frac{132}{2} = 66 \text{ cm}$$

Now, area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{66(66-11)(66-60)(66-61)}$$
$$= \sqrt{66 \times 55 \times 6 \times 5}$$
$$= \sqrt{11 \times 6 \times 11 \times 5 \times 6 \times 5}$$
$$= 11 \times 6 \times 5$$
$$= 330 \text{ cm}^2$$

Here, we have to find the altitude of the smallest side, so we consider the base as smallest side.

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\Rightarrow \quad 330 = \frac{1}{2} \times 11 \times h$$

$$\Rightarrow \quad h = \frac{330 \times 2}{11} = 60 \text{ cm}$$

Hence, the altitude of the smallest side is 60 cm.

or

The length of the sides of a triangle are $5x$, $5x$ and $8x$. Find the area of triangle.

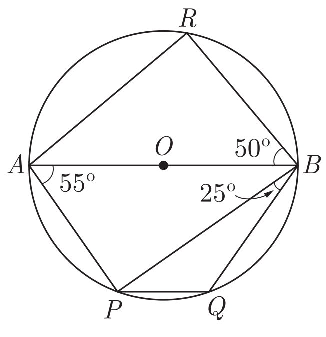
SOLUTION :

$$s = \frac{5x+5x+8x}{2} = \frac{18x}{2} = 9x$$

$\therefore$ Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{9x(9x-5x)(9x-5x)(9x-8x)}$$
$$= \sqrt{9x \times 4x \times 4x \times x}$$
$$= \sqrt{144x^4} = 12x^2 \text{ sq. units}$$

25. In the adjoining figure, AB is a diameter of a circle with centre O. If $\angle PAB = 55^\circ$, $\angle PBQ = 25^\circ$ and $\angle ABR = 50^\circ$, then find $\angle PBA$ and $\angle BAR$. [2]



SOLUTION :

Given, $\angle PAB = 55^\circ$,

$\angle PBQ = 25^\circ$

and $\angle ABR = 50^\circ$

In ΔABR,

$\angle ARB = 90^\circ$ [Angle in semi-circle]

$\angle RBA = 50^\circ$ [Given]

$\therefore \quad \angle BAR = 180^\circ - (90^\circ + 50^\circ)$

$= 40^\circ$

In ΔABP,

$\angle APB = 90^\circ$ [Angle in semi-circle]

$\angle PAB = 55^\circ$ [Given]

$\therefore \quad \angle PBA = 180^\circ - (90^\circ + 55^\circ)$

$= 35^\circ$

26. Find a point on x-axis from where graph of linear equation $2x = 1 - 5y$ will pass. [2]

SOLUTION :

Let the point be $(m, 0)$

Since, the equation passes through this point, so put $x = m$ and $y = 0$ in $2x = 1 - 5y$

$$2m = 1 - 5 \times 0$$

$$\Rightarrow \quad 2m = 1$$

$$m = \frac{1}{2}$$

Hence, $\left(\frac{1}{2}, 0\right)$ is the required point.

or

If the points (1, 0) and (2, 1) lie on the graph of $\frac{x}{a} + \frac{y}{b} = 1$, then find the values of a and b.

SOLUTION :

Since, the points (1, 0) and (2, 1) lie on the graph of the equation $\frac{x}{a} + \frac{y}{b} = 1$

$$\text{Therefore,} \quad \frac{1}{a} + \frac{0}{b} = 1$$

$$\Rightarrow \quad \frac{1}{a} = 1$$

$$a = 1$$

$$\text{and} \quad \frac{2}{a} + \frac{1}{b} = 1$$

$$\Rightarrow \quad \frac{1}{b} = 1 - 2 = -1$$

$$b = -1$$

Section C

27. Draw the graph of linear equation $x + 2y = 8$. From the graph, check whether $(-1, -2)$ is a solution of this equation. [3]

SOLUTION :

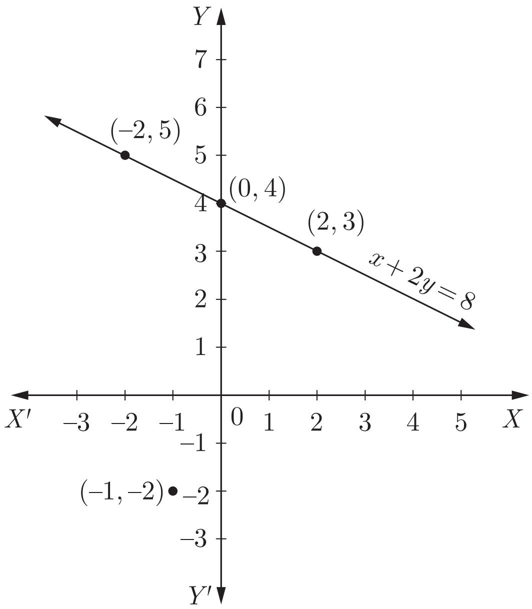
Given equation is

$$x + 2y = 8$$

$$\Rightarrow \quad y = \frac{1}{2}(8 - x)$$

Let us make table of values of x and y.

x	0	2	–2
y	4	3	5
(x, y)	(0, 4)	(2, 3)	(–2, 5)



From graph it is clear that (–1, –2) does not lie on the line, therefore, it is not a solution of given equation.

or

Solve : $\frac{5}{x} + 6y = 13$, $\frac{3}{x} + 4y = 7$.

SOLUTION :

The given equations are

$$\frac{5}{x} + 6y = 13 \quad ...(1)$$

$$\frac{3}{x} + 4y = 7 \quad ...(2)$$

Multiplying (1) by 3 and (2) by 5, we get

$$\frac{15}{x} + 18y = 39 \quad ...(3)$$

$$\frac{15}{x} + 20y = 35 \quad ...(4)$$

Subtracting (4) from (3), we get

$$-2y = 4$$

$$\Rightarrow \quad y = -2$$

Substituting $y = -2$ in (1), we get

$$\frac{5}{x} + 6 \times (-2) = 13$$

$$\Rightarrow \quad \frac{5}{x} - 12 = 13$$

$$\frac{5}{x} = \frac{25}{1}$$

$$25x = 5$$

$$x = \frac{5}{25} = \frac{1}{5}$$

28. A teak wood log is cut first in the form of a cuboid of length 2.3 m, width 0.75 m and of a certain thickness. Its volume is 1.104 m^3. How many rectangular planks of size $2.3\text{ m} \times 0.75\text{ m} \times 0.04\text{ m}$ can be cut from the cuboid ? [3]

SOLUTION :

Let the thickness of the log be h metre
Then, we have

$$\text{Volume} = 1.104\text{ m}^3$$

$$\Rightarrow \quad 2.3 \times 0.75 \times h = 1.104$$

$$h = \frac{1.104}{2.3 \times 0.75} = 0.64\text{ m}$$

Number of rectangular planks

$$= \frac{\text{Volume of cuboid}}{\text{Volume of a plank}}$$

$$= \frac{1.104}{2.3 \times 0.75 \times 0.04}$$

$$= \frac{1.104}{0.069} = \frac{1104}{69} = 16$$

Hence, 16 rectangular planks of given size can be cut from teak wood log.

or

A cylindrical roller 2.5 m in length, 1.5 m in radius when rolled on a road was found to cover the area of 16500 m^2. How many revolutions does it make ?

SOLUTION :

Given, radius of cylindrical roller $= 1.5\text{ m}$ and height of cylindrical roller $= 2.5\text{ m}$

$\therefore$ Area covered in one revolution

$$= \text{Curved surface area of cylinder}$$

$$= 2\pi rh = 2 \times \frac{22}{7} \times 1.5 \times 2.5\text{ m}^2$$

Let in n number of revolutions, area covered is 16500 m^2.

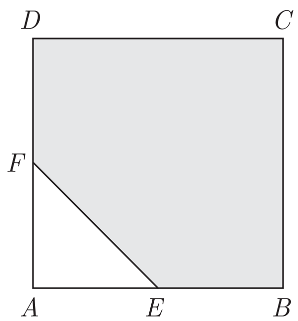
Hence, $$n \times \left(2 \times \frac{22}{7} \times 1.5 \times 2.5\right) = 16500$$

$$n = \frac{16500 \times 7}{2 \times 22 \times 1.5 \times 2.5}$$

$$= 700$$

$\therefore$ A cylindrical roller makes 700 revolutions.

29. In the given figure, $ABCD$ is a square of side 4 cm. E and F are the mid points of AB and AD respectively. Find the area of the shaded region. [3]



SOLUTION :

Given, a square of sides

$$AB = BC = CD = DA = 4\text{ cm}$$

$$\therefore \quad \text{Area of square} = (\text{side})^2$$

$$= (4\text{ cm})^2 = 16\text{ cm}^2$$

Also, given E and F are the mid points of AB and AD.

$$\therefore \quad AE = \frac{AB}{2} = \frac{4}{2} = 2\text{ cm}$$

and $$AF = \frac{AD}{2} = \frac{4}{2} = 2\text{ cm}$$

$\therefore$ Area of $\Delta AEF = \frac{1}{2} \times AE \times AF$

$= \frac{1}{2} \times 2 \times 2$

$= 2\,\text{cm}^2$

Now area of shaded region

$=$ Area of square $ABCD$ $-$ Area of ΔAEF

$= 16 - 2 = 14\,\text{cm}^2$

30. Find the median of descending order 34, 32, x, $x-1$, 19, 15, 11 where x is the mean of 10, 20, 30, 40, 50. [3]

SOLUTION :

Given x be the mean of 10, 20, 30, 40 and 50.

$$\therefore \quad x = \frac{10+20+30+40+50}{5}$$

$$= \frac{150}{5} = 30$$

and median of 34, 32, x, $x-1$, 19, 15, 11

$$= \left(\frac{7+1}{2}\right)^{\text{th}} \text{ term}$$

$$= 4^{\text{th}} \text{ term}$$

$$= x - 1$$

$$= 30 - 1 = 29$$

or

A bag contains 12 balls out of which x balls are white. If one ball is taken out from the bag, find the probability of getting a white ball. If 6 more white balls are added to the bag and the probability now for getting a white ball is double the previous one, find the value of x.

SOLUTION :

Total number of balls $= 12$

Number of white balls $= x$

$\therefore$ P(getting a white ball) $= \frac{x}{12} = P(E_1)$

Now, 6 more white balls are added in that bag

$\therefore$ Total number of balls $= 12 + 6 = 18$

$\therefore$ P(getting a white ball) $= \frac{6+x}{18} = P(E_2)$

According to the given condition,

$$P(E_2) = 2P(E_1)$$

$$\frac{6+x}{18} = 2 \times \frac{x}{12}$$

$$\Rightarrow \quad \frac{6+x}{18} = \frac{x}{6}$$

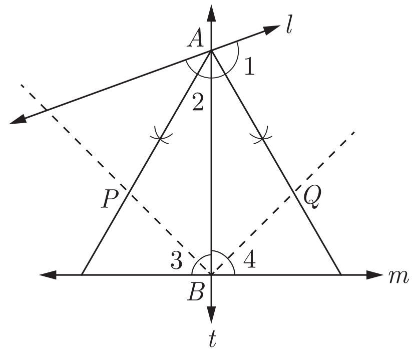
$$6 + x = 3x$$

$$2x = 6$$

$$\Rightarrow \quad x = 3$$

31. Draw line l and m intersected by a transversal t. Construct angle bisectors of the interior angle on same side of the transversal. [3]

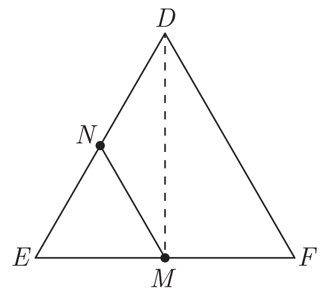
SOLUTION :



Steps of Construction :

1. Draw any two lines l and m and a transversal line t. Clearly, interior angle on the same side of transversal are $\angle 2$ and $\angle 3$ (or $\angle 1$ and $\angle 4$).
2. Draw the angle bisector of $\angle 2$ and $\angle 3$ which intersect at P (or draw the angle bisector of $\angle 1$ and $\angle 4$ which intersect at (Q).

32. In ΔDEF, M and N are mid-points of sides EF and DE respectively. If $ar(\Delta ENM) = 4\,\text{cm}^2$, find $ar(\Delta DEF)$. [3]



SOLUTION :

We have, M and N are the mid-points of EF and ED respectively.

$\therefore$ $EM = MF$ and $EN = ND$. Join DM,

Now, in ΔDEF, M is the mid-point of EF.

$\therefore$ DM is the median of ΔDEF.

$$\Rightarrow \quad ar(\Delta EDM) = ar(\Delta DMF)$$

$$= \frac{1}{2} ar(\Delta DEF) \quad ...(1)$$

[$\because$ Median divides a triangle in two triangles of equal area]

Similarly, in ΔDEM, MN is the median.

$$\therefore \quad ar(\Delta ENM) = ar(\Delta MND)$$

$$= \frac{1}{2} ar(\Delta EMD) \quad ...(2)$$

or $$ar(\Delta ENM) = \frac{1}{2}\left[\frac{1}{2} ar(\Delta DEF)\right] \text{ [Using eq.(1)]}$$

or $$ar(\Delta ENM) = \frac{1}{4} ar(\Delta DEF)$$

$$\Rightarrow \quad ar(\Delta DEF) = 4 \times ar(\Delta ENM)$$

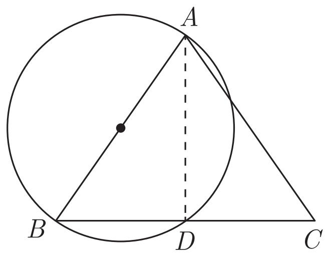
$$= 4 \times 4 = 16\,\text{cm}^2$$

33. Prove that the circle drawn on any of the equal sides of an isosceles triangle as diameter bisects the base. [3]

SOLUTION :

Given : A ΔABC, in which $AB = AC$ and a circle is drawn by taking AB as diameter which intersects the side BC of triangle at D.

To prove : $BD = DC$
Construction : Join AD
Proof : Since, angle in a semi-circle is a right angle.



$\therefore \quad \angle ADB = 90°$

But $\angle ADB + \angle ADC = 180°$ [Linear pair axiom]

$\therefore \quad 90° + \angle ADC = 180°$

$\Rightarrow \quad \angle ADC = 90°$

Now, in ΔADB and ΔADC, we have

$AB = AC$ [Given]

$\angle ADB = \angle ADC$ [Each 90°]

and $AD = AD$ [Common sides]

$\therefore \quad \Delta ADB \cong \Delta ADC$

[By RHS congruence rule]

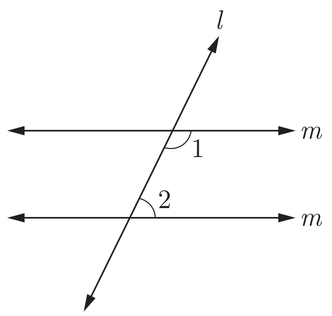
Then, $BD = DC$ [By CPCT]

Hence proved.

34. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 3 : 2, then find the greater of the two angles. [3]

SOLUTION :

Let l be a transversal intersecting two parallel lines m and n.



Let $\angle 1 = \angle 3x$ and $\angle 2 = \angle 2x$

Also, $\angle 1 + \angle 2 = 180°$

$3x + 2x = 180°$

$\Rightarrow \quad x = 36°$

$\therefore \quad \angle 1 = 3 \times 36° = 180°$

and $\angle 2 = 2 \times 36° = 72°$

So, the greater of the two angles is 108°.

Section D

35. If $x = (2+\sqrt{5})^{1/2} + (2-\sqrt{5})^{1/2}$ and $y = (2+\sqrt{5})^{1/2} - (2-\sqrt{5})^{1/2}$ evaluate $x^2 + y^2$. [4]

SOLUTION :

We know that,

$$x^2 + y^2 = (x+y)^2 - 2xy$$

$$\begin{aligned}(x+y)^2 &= \left[(2+\sqrt{5})^{1/2} + (2-\sqrt{5})^{1/2} + (2+\sqrt{5})^{1/2} - (2-\sqrt{5})^{1/2}\right]^2 \\ &= \left[2(2+\sqrt{5})^{1/2}\right]^2 \\ &= 4(2+\sqrt{5})^{1/2 \times 2} \\ &= 4(2+\sqrt{5}) = 8 + 4\sqrt{5} \qquad \text{...(1)}\end{aligned}$$

and
$$\begin{aligned}2xy &= 2\left[(2+\sqrt{5})^{1/2} + (2-\sqrt{5})^{1/2}\right] \times \left[(2+\sqrt{5})^{1/2} - (2-\sqrt{5})^{1/2}\right] \\ &= 2\left[\left((2+\sqrt{5})^{1/2}\right)^2 - \left((2-\sqrt{5})^{1/2}\right)^2\right] \\ &\qquad [\because (a+b)(a-b) = a^2 - b^2] \\ &= 2[2+\sqrt{5} - 2 + \sqrt{5}] \\ &= 2 \times 2\sqrt{5} = 4\sqrt{5} \qquad \text{...(2)}\end{aligned}$$

[From eqs.(1) and (2)]

Hence,
$$\begin{aligned}x^2 + y^2 &= (x+y)^2 - 2xy \\ &= 8 + 4\sqrt{5} - 4\sqrt{5} = 8\end{aligned}$$

or

If $a = \dfrac{1}{7-4\sqrt{3}}$ and $b = \dfrac{1}{7+4\sqrt{3}}$, find the values of the following :

(i) $a^2 + b^2$

(ii) $a^3 + b^3$

SOLUTION :

Given,
$$\begin{aligned}a &= \frac{1}{7-4\sqrt{3}} \\ &= \frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} \\ &\qquad \text{[By rationalising]} \\ &= \frac{7+4\sqrt{3}}{(7)^2 - (4\sqrt{3})^2} \\ &\qquad [\because (a-b)(a+b) = (a^2 - b^2)] \\ &= 7 + 4\sqrt{3}\end{aligned}$$

Similarly

$$\begin{aligned}b &= \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} \\ &= \frac{7-4\sqrt{3}}{49-48} = 7 - 4\sqrt{3}\end{aligned}$$

$\therefore \quad a + b = 7 + 4\sqrt{3} + 7 - 4\sqrt{3} = 14$

and
$$\begin{aligned}ab &= (7+4\sqrt{3})(7-4\sqrt{3}) \\ &= (7)^2 - (4\sqrt{3})^2 \\ &= 49 - 48 = 1 \\ &\qquad [\because (a-b)(a+b) = a^2 - b^2]\end{aligned}$$

(i) $\because \quad (a+b)^2 = a^2 + b^2 + 2ab$

$\because \quad (14)^2 = a^2 + b^2 + 2$

$a^2 + b^2 = 196 - 2 = 194$

(ii) $\because \quad (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\therefore$
$$\begin{aligned}a^3 + b^3 &= (a+b)^3 - 3ab(a+b) \\ &= (14)^3 - 3 \times 1 \times 14 \\ &= 2744 - 42 = 2702\end{aligned}$$

36. Simplify : $\left[\dfrac{(4x^2-9y^2)^3+(9y^2-16z^2)^3+(16z^2-4x^2)^3}{(2x-3y)^3+(3y-4z)^3+(4z-2x)^3}\right]$. [4]

SOLUTION :

We have,

$$(4x^2-9y^2)+(9y^2-16z^2)+(16z^2-4x^2) = 0$$

$$\therefore\ (4x^2-9y^2)^3+(9y^2-16z^2)^3+(16z^2-4x^2)^3$$
$$= 3(4x^2-9y^2)(9y^2-16z^2)(16z^2-4x^2)$$
$$= 3\{(2x)^2-(3y)^2\}\{(3y)^2-(4z)^2\}\{(4z)^2-(2x)^2\}$$
$$= 3(2x-3y)(2x+3y)(3y-4z)(3y+4z)(4z-2x)(4z+2x)$$

Similarly, we have

$$(2x-3y)+(3y-4z)+(4z-2x) = 0$$

$$\Rightarrow\ (2x-3y)^3+(3y-4z)^3+(4z-2x)^3 = 3(2x-3y)(3y-4z)(4z-2x)$$

$$\therefore\ \left[\frac{(4x^2-9y^2)^3+(9y^2-16z^2)^3+(16z^2-4x^2)^3}{(2x-3y)^3+(3y-4z)^3+(4z-2x)^3}\right]$$
$$= \frac{3(2x-3y)(2x+3y)(3y-4z)(3y+4z)(4z-2x)(4z+2x)}{3(2x-3y)(3y-4z)(4z-2x)}$$
$$= (2x+3y)(3y+4z)(4z+2x)$$

37. The linear equation that converts Fahrenheit (F) to Celsius (C) is given by the relation $C = \dfrac{5F-160}{9}$ [4]

(i) If the temperature is 86°F, what is the temperature in Celsius ?
(ii) If the temperature is 35°C, what is the temperature in Fahrenheit ?
(iii) If the temperature is 0°F, what is the temperature in Celsius ?
(iv) What is the numerical value of the temperature which is same in both the scales ?

SOLUTION :

(i) $$C = \frac{5F-160}{9}$$
$$C = \frac{5\times 86-160}{9} = \frac{430-160}{9} = 30^\circ C$$

(ii) $$C = \frac{5F-160}{9}$$
$$35 = \frac{5F-160}{9}$$
$$5F-160 = 315$$
$$\Rightarrow\ 5F = 475$$
$$F = 95^\circ F$$

(iii) $$C = \frac{5F-160}{9}$$
$$C = \frac{5\times 0-160}{9} = \frac{-160}{9} = -\left(\frac{160}{9}\right)^\circ F$$

(iv) Let the temperature on both the scales numerically be x. Then,

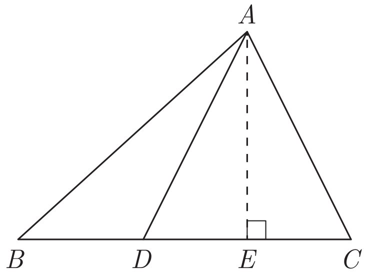
$$C = \frac{5F-160}{9}$$
$$\Rightarrow\ x = \frac{5x-160}{9}$$
$$9x = 5x-160$$
$$4x = -160$$
$$x = -40$$

Hence, numerical value of the required temperature is -40.

38. In ΔABC, if AD is the median, then prove that $AB^2+AC^2 = 2AD^2+\frac{1}{2}BC^2$. [4]

SOLUTION :

Draw $AE \perp BC$. Then, $\angle AED = 90^\circ$.



$\therefore\ \angle ADE < 90^\circ$
$\Rightarrow \angle ADB$ is an obtuse angle.
Thus, $\angle ADB$ is obtuse and $\angle ADE$ is acute.
Now, ΔABD is obtuse-angled at D and $AE \perp BD$ produced.

$$\therefore\ AB^2 = AD^2+BD^2+2BD\times DE \quad ...(1)$$

Also, ΔADC is acute-angle at D and $AE \perp DC$.

$$\therefore\ AC^2 = AD^2+DC^2-2DC\times DE$$
$$= AD^2+BD^2-2BD\times DE \quad ...(2)$$
$$[\because\ DC = BD]$$

Adding (1) and (2), we get

$$AB^2+AC^2 = 2AD^2+2BD^2$$
$$= 2AD^2+2\left(\frac{1}{2}BC\right)^2 = 2AD^2+\frac{1}{2}BC^2$$

Hence, $AB^2+AC^2 = 2AD^2+\frac{1}{2}BC^2$

39. A random survey of the number of children of various age groups playing football match in a park was found as follows [4]

Age (in years)	**Number of children**
1-2	5
2-3	4
3-5	10
5-7	12
7-10	9
10-15	10
15-17	8

Draw a histogram to represent the above data.

SOLUTION :

Here, minimum class size $= 2 - 1 = 1$

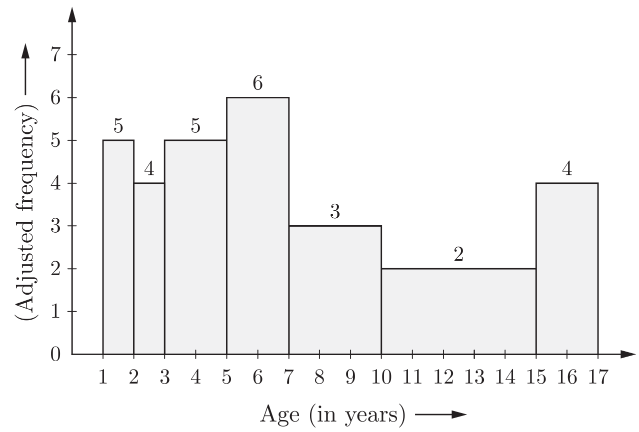
Adjusted frequency of a class

$$= \frac{\text{Minimum class size}}{\text{Class size of the class}} \times \text{Frequency of the class}$$

Frequency distribution after adjusting frequency

Age (in years)	Number of children (Frequency)	Width of the class	Adjusted frequency
1-2	5	1	$\frac{1}{1} \times 5 = 5$
2-3	4	1	$\frac{1}{1} \times 4 = 4$
3-5	10	2	$\frac{1}{2} \times 10 = 5$
5-7	12	2	$\frac{1}{2} \times 12 = 6$
7-10	9	3	$\frac{1}{3} \times 9 = 3$
10-15	10	5	$\frac{1}{5} \times 10 = 2$
15-17	8	2	$\frac{1}{2} \times 8 = 4$

The required histogram is as follows :



or

If the mean of the following frequency distribution is 28.25, find the value of p.

x_i	15	20	25	30	35	40
f_i	8	7	p	14	15	6

SOLUTION :

We prepare the table as under :

x_1	f_i	$f_i x_i$
15	8	120
20	7	140
25	p	$25p$
30	14	420
35	15	525
40	6	240
	$\Sigma f_i = (50 + p)$	$\Sigma f_i x_i = 1445 + 25p$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1445 + 25p}{50 + p}$$

But, mean $= 28.25$

$$\therefore \quad \frac{1445 + 25p}{50 + p} = 28.25$$

$$\Rightarrow \quad 1445 + 25p = 1412.5 + 28.25p$$

$$3.25p = 32.5$$

$$p = \left(\frac{32.5}{3.25}\right) = 10$$

Hence, $p = 10$

40. While selling clothes for making flags, a shopkeeper claims to sell each piece of cloth in the shape of an equilateral triangle of each side 10 cm while actually he was selling the same in the shape of an isosceles triangle with sides 10 cm, 10 cm and 8 cm. How much cloth was he saving in selling each flag ? [4]

SOLUTION :

Actually, he was selling the cloth in the shape of an isosceles triangle, whose sides are 10 cm, 10 cm and 8 cm.

Let $a = 10\text{ cm}$, $b = 10\text{ cm}$ and $c = 8\text{ cm}$

Then, semi-perimeter of isosceles triangle,

$$s = \frac{a + b + c}{2} = \frac{10 + 10 + 8}{2}$$

$$= 14\text{ cm}$$

Area of isosceles triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{14(14-10)(14-10)(14-8)}$$
$$= \sqrt{14 \times 4 \times 4 \times 6}$$
$$= \sqrt{14 \times 16 \times 6} = 8\sqrt{21}$$
$$= 8 \times 4.58$$
$$= 36.64\text{ cm}^2$$

But the shopkeeper claims to sell the cloth in the shape of an equilateral triangle, whose sides are 10 cm each.

$\therefore$ Area of equilateral triangle

$$= \frac{\sqrt{3}}{4} \times (10)^2$$
$$= 25\sqrt{3} = 25 \times 1.73$$
$$= 43.25\text{ cm}^2$$

Hence, the area of the cloth, he was saving

= Area of an equilateral triangle

− Area of an isosceles triangle

$$= 43.25 - 36.64$$
$$= 6.61\text{ cm}^2$$